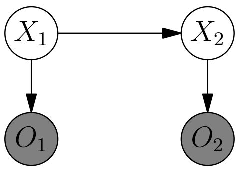
CS188 Spring 2014 Section 6: Hidden Markov Models

1 Basic computations

Consider the following Hidden Markov Model.



X_1	$\Pr(X_1)$
0	0.3
1	0.7

X_t	X_{t+1}	$\Pr(X_{t+1}\|X_t)$
0	0	0.4
0	1	0.6
1	0	0.8
1	1	0.2

X_t	O_t	$\Pr(O_t\|X_t)$
0	A	0.9
0	B	0.1
1	A	0.5
1	B	0.5

Suppose that $O_1 = A$ and $O_2 = B$ is observed.

Use the Forward algorithm to compute the probability distribution $\Pr(X_2, O_1 = A, O_2 = B)$. Show your work. You do not need to evaluate arithmetic expressions involving only numbers.

X_1	$\Pr(X_1, O_1 = A)$
0	$0.3 \cdot 0.9$
1	$0.7 \cdot 0.5$

X_2	$\Pr(X_2, O_1 = A, O_2 = B)$
0	$0.1 \cdot [0.4 \cdot (0.3 \cdot 0.9) + 0.8 \cdot (0.7 \cdot 0.5)] = 0.0388$
1	$0.5 \cdot [0.6 \cdot (0.3 \cdot 0.9) + 0.2 \cdot (0.7 \cdot 0.5)] = 0.1160$

Use the Viterbi algorithm to compute the maximum probability sequence X_1, X_2. Show your work.

X_1	$\Pr(X_1, O_1 = A)$
0	$0.3 \cdot 0.9$
1	$0.7 \cdot 0.5$

X_2	$\max_{x_1} \Pr(X_1 = x_1, X_2, O_1 = A, O_2 = B)$	arg max
0	$0.1 \cdot \max(0.4 \cdot (0.3 \cdot 0.9), 0.8 \cdot (0.7 \cdot 0.5)) = 0.1 \cdot \max(0.108, 0.28) = 0.028$	$X_1 = 1$
1	$0.5 \cdot \max(0.6 \cdot (0.3 \cdot 0.9), 0.2 \cdot (0.7 \cdot 0.5)) = 0.5 \cdot \max(0.162, 0.07) = 0.081$	$X_1 = 0$

Thus, in the maximum probability sequence, $X_2 = 1$ and $X_1 = 0$.

True or false: Variable elimination is generally more accurate than the Forward algorithm. Explain your answer.

They both perform exact inference.

2 Tracking a Jabberwock

You have been put in charge of a Jabberwock for your friend Lewis. The Jabberwock is kept in a large tugley wood which is conveniently divided into an $N \times N$ grid. It wanders freely around the N^2 possible cells. At each time step $t = 1, 2, 3, \ldots$, the Jabberwock is in some cell $X_t \in \{1, \ldots, N\}^2$, and it moves to cell X_{t+1} randomly as follows: with probability $1 - \epsilon$, it chooses one of the (up to 4) valid neighboring cells uniformly at random; with probability ϵ, it uses its magical powers to teleport to a random cell uniformly at random among the N^2 possibilities (it might teleport to the same cell). Suppose $\epsilon = \frac{1}{2}$, $N = 10$ and that the Jabberwock always starts in $X_1 = (1, 1)$.

1. Compute the probability that the Jabberwock will be in $X_2 = (2, 1)$ at time step 2. What about $\Pr(X_2 = (4, 4))$?

   $\Pr(X_2 = (2, 1)) = 1/2 \cdot 1/2 + 1/2 \cdot 1/100 = 0.255$

   $\Pr(X_2 = (4, 4)) = 1/2 \cdot 1/100 = 0.005$

   At each time step t, you dont see X_t but see E_t, which is the row that the Jabberwock is in; that is, if $X_t = (r, c)$, then $E_t = r$. You still know that $X_1 = (1, 1)$.

2. Suppose we see that $E_1 = 1$, $E_2 = 2$, $E_3 = 10$. Fill in the following table with the distribution over X_t after each time step, taking into consideration the evidence. Your answer should be concise. *Hint*: you should not need to do any heavy calculations.

t	$\Pr(X_t, e_{1:t-1})$	$\Pr(X_t, e_{1:t})$
1	(1,1) : 1.0, (others) : 0.0	(1,1): 1.0, (others): 0.0
2	(1,2), (2,1): 51/200, (others): 1/200	(2,1): 51/200, (2,2+): 1/200