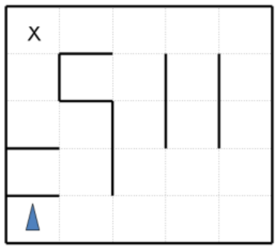
CS188 Spring 2014 Section 1: Search

1 Search and Heuristics

Imagine a car-like agent wishes to exit a maze like the one shown below:



The agent is directional and at all times faces some direction $d \in (N, S, E, W)$. With a single action, the agent can *either* move forward at an adjustable velocity v *or* turn. The turning actions are *left* and *right*, which change the agent's direction by 90 degrees. Turning is only permitted when the velocity is zero (and leaves it at zero). The moving actions are *fast* and *slow*. *Fast* increments the velocity by 1 and *slow* decrements the velocity by 1; in both cases the agent then moves a number of squares equal to its NEW adjusted velocity. Any action that would result in a collision with a wall crashes the agent and is illegal. Any action that would reduce v below 0 or above a maximum speed $V_{\max}$ is also illegal. The agent's goal is to find a plan which parks it (stationary) on the exit square using as few actions (time steps) as possible.

As an example: if the agent shown were initially stationary, it might first turn to the east using (*right*), then move one square east using *fast*, then two more squares east using *fast* again. The agent will of course have to *slow* to turn.

1. If the grid is M by N, what is the size of the state space? Justify your answer. You should assume that all configurations are reachable from the start state.

   The size of the state space is $4MN(V_{max} + 1)$. The state representation is (direction facing, x, y, speed). Note that the speed can take any value in $\{0, ..., V_{max}\}$.

2. What is the maximum branching factor of this problem? You may assume that illegal actions are simply not returned by the successor function. Briefly justify your answer.

   The maximum branching factor is 3, and this happens when the agent is stationary. While stationary it can take the following 3 actions - $fast$, $left$, $right$.

3. Is the Manhattan distance from the agent's location to the exit's location admissible? Why or why not?

   No, Manhattan distance is not an admissible heuristic. The agent can move at an average speed of greater than 1 (by first speeding up to V_{max} and then slowing down to 0 as it reaches the goal), and so can reach the goal in less time steps than there are squares between it and the goal. A specific example: the target is 6 squares away, and the agent's velocity is already 4. By taking only 4 *slow* actions, it reaches the goal with a velocity of 0.

4. State and justify a non-trivial admissible heuristic for this problem which is not the Manhattan distance to the exit.

   There are many answers to this question. Here are a few, in order of weakest to strongest:

   (a) The number of turns required for the agent to face the goal.

   (b) Consider a relaxation of the problem where there are no walls, the agent can turn and change speed arbitrarily. In this relaxed problem, the agent would move with V_{max}, and then suddenly stop at the goal, thus taking $d_{manhattan}/V_{max}$ time.

   (c) We can improve the above relaxation by accounting for the deceleration dynamics. In this case the agent will have to slow down to 0 when it is about to reach the goal. Note that this heuristic will always return a greater value than the previous one, but is still not an overestimate of the true cost to reach the goal. We can say that this heuristic *dominates* the previous one.

5. If we used an inadmissible heuristic in A* tree search, could it change the completeness of the search?

   No! If the heuristic function is bounded, then A* tree search would visit all the nodes eventually, and would find a path to the goal state if there exists one.

6. If we used an inadmissible heuristic in A* tree search, could it change the optimality of the search?

   Yes! It can make the good optimal goal look as though it is very far off, and take you to a suboptimal goal.

7. Give a general advantage that an inadmissible heuristic might have over an admissible one.

   The time to solve an A* tree search problem is a function of two factors: the number of nodes expanded, and the time spent per node.

   An inadmissible heuristic may be faster to compute, leading to a solution that is obtained faster due to less time spent per node. It can also be a closer estimate to the actual cost function (even though at times it will overestimate!), thus expanding less nodes.

   We lose the guarantee of optimality by using an inadmissible heuristic. But sometimes we may be okay with finding a suboptimal solution to a search problem.

2 Expanded Nodes

Consider tree search (i.e. no closed set) on an arbitrary search problem with max branching factor b. Each search node n has a backward (cumulative) cost of $g(n)$, an admissible heuristic of $h(n)$, and a depth of $d(n)$. Let c be a minimum-cost goal node, and let s be a shallowest goal node.

For each of the following, you will give an expression that characterizes the set of nodes that are expanded before the search terminates. For instance, if we asked for the set of nodes with positive heuristic value, you could say $h(n) \geq 0$. Don't worry about ties (so you won't need to worry about $>$ versus $\geq$). If there are no nodes for which the expression is true, you must write "none."

1. Give an expression (i.e. an inequality in terms of the above quantities) for which nodes n will be expanded in a breadth-first tree search.

   $d(n) \leq d(s)$: BFS expands all nodes which are shallower than the shallowest goal. Recall that our search performs the $goal - test$ after popping nodes from the fringe, so we typically expand some nodes at depth s, before we expand the optimal goal node.

2. Give an expression for which nodes n will be expanded in a uniform cost search.

   $g(n) \leq g(c)$: Uniform cost search expands all nodes that are closer than the closest goal node. Recall that our search performs the $goal - test$ after popping nodes from the fringe (this ensures optimality!), so we might expand some nodes of cost $g(c)$, before we expand the optimal goal node.

3. Give an expression for which nodes n will be expanded in an A* tree search with heuristic $h(n)$.

   $g(n) + h(n) \leq g(c)$: All nodes with a total cost of less than g(c), get expanded before the goal node is expanded. This can be proved by induction on the cost $g(n) + h(n)$. Consider a node n_1 which satisfies this property. Note that its parent n_0, will also satisfy this inequality, and by the induction hypothesis, n_0 will be expanded before the goal is expanded, which means that it will put n_1 on the fringe, which will get expanded before the goal node is expanded.

4. Let h_1 and h_2 be two admissible heuristics such that $\forall n, h_1(n) \geq h_2(n)$. Give an expression for the nodes which will be expanded in an A* tree search using h_1 but not when using h_2.

   Let S, be the set of all the nodes. Using the above part, set of nodes expanded by h_1 is $N_1 = \{n : g(n) + h_1(n) \leq g(c)\}$, and, set of nodes expanded by h_2 is $N_2 = \{n : g(n) + h_2(n) \leq g(c)\}$. The set of nodes expanded using h_1 but not using h_2, is $N_1 \cap (S - N_2)$. Since, $h_1 \geq h_2$, $N_1 \subseteq N_2$, hence $N_1 \cap (S - N_2) = \phi$.

5. Give an expression for the nodes which will be expanded in an A* tree search using h_2 but not when using h_1.

   As above, set of nodes expanded using h_2 but not using h_1, is $N_2 \cap (S - N_1) = \{n : g(n) + h_2(n) \leq g(c)$ and $g(c) \leq g(n) + h_1(n)\}$.