GRADE 11
PHYSICAL SCIENCES

SIYAVULA
TECHNOLOGY-POWERED LEARNING

TEACHERS' GUIDE

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

METROPOLITAN

EVERYTHING SCIENCE

GRADE 11 PHYSICAL SCIENCES

TEACHERS' GUIDE

VERSION 1 CAPS

WRITTEN BY VOLUNTEERS

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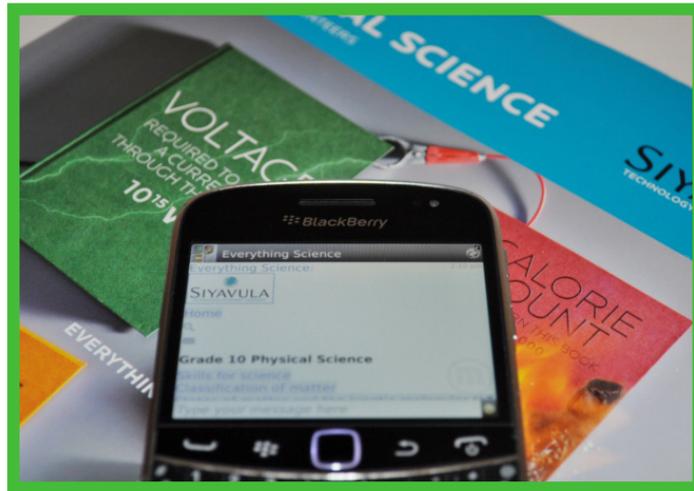
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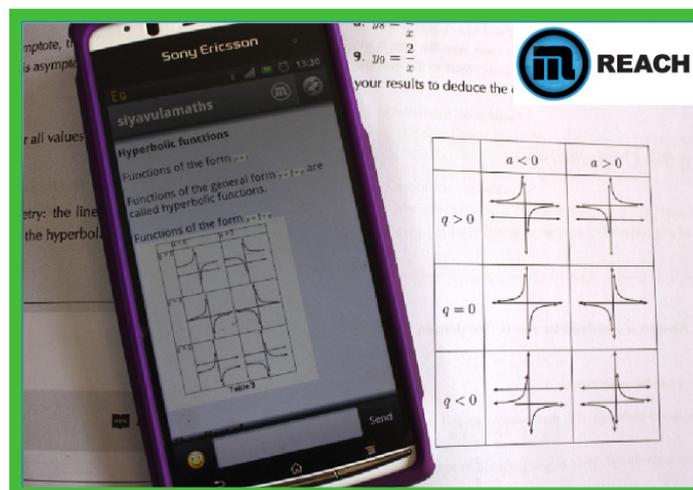
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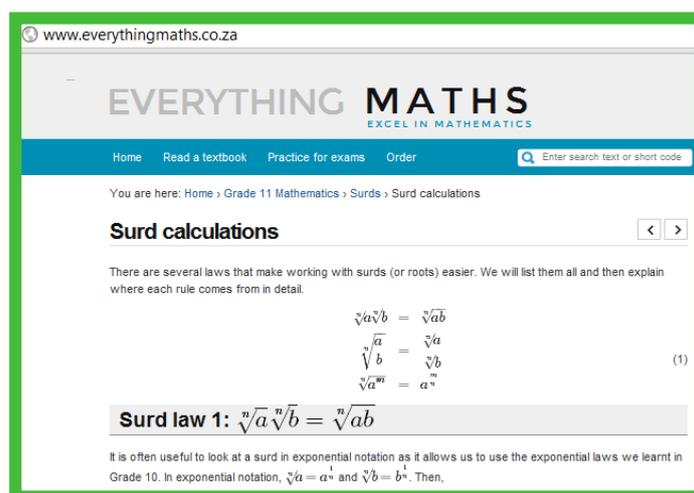
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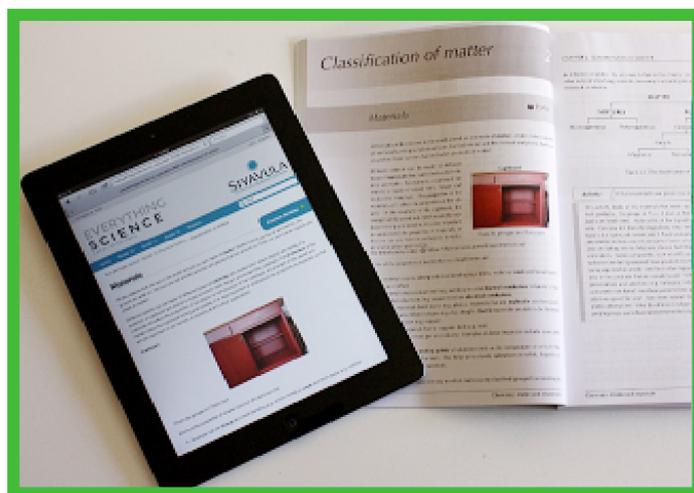
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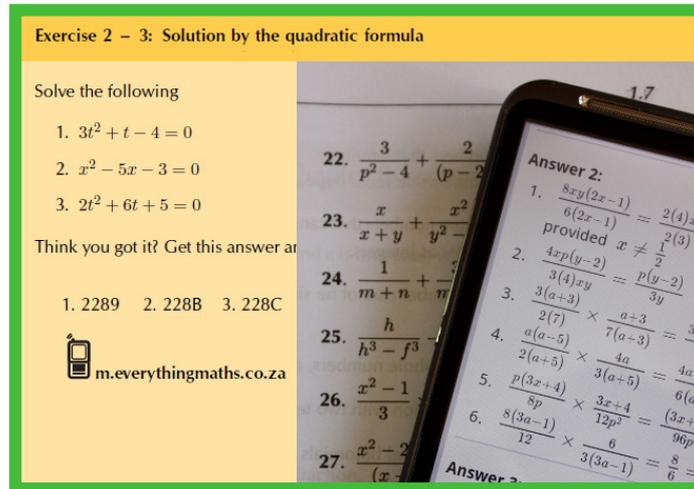
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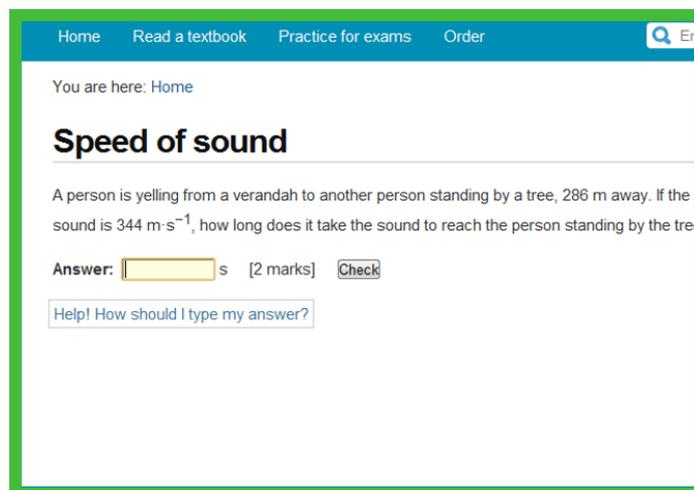
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EVERYTHING SCIENCE

When we look outside at everything in nature, look around us at everything manufactured or look up at everything in space we cannot but be struck by the incredible diversity and complexity of life; so many things, that look so different, operating in such unique ways. The physical universe really contains incredible complexity.

Yet, what is even more remarkable than this seeming complexity is the fact that things in the physical universe are knowable. We can investigate them, analyse them and understand them. It is this ability to understand the physical universe that allows us to transform elements and make technological progress possible.

If we look back at some of the things that developed over the last century ñ space travel, advances in medicine, wireless communication (from television to mobile phones) and materials a thousand times stronger than steel we see they are not the consequence of magic or some inexplicable phenomena. They were all developed through the study and systematic application of the physical sciences. So as we look forward at the 21st century and some of the problems of poverty, disease and pollution that face us, it is partly to the physical sciences we need to turn.

For however great these challenges seem, we know that the physical universe is knowable and that the dedicated study thereof can lead to the most remarkable advances. There can hardly be a more exciting challenge than laying bare the seeming complexity of the physical universe and working with the incredible diversity therein to develop products and services that add real quality to peopleís lives.

Physical sciences is far more wonderful, exciting and beautiful than magic! It is everywhere.

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CHAPTER 1

Physical Sciences - Teachers guide

1.1 On the Web, Everyone can be a Scientist

Did you know that you can fold protein molecules, hunt for new planets around distant suns or simulate how malaria spreads in Africa, all from an ordinary PC or laptop connected to the Internet? And you don't need to be a certified scientist to do this. In fact some of the most talented contributors are teenagers. The reason this is possible is that scientists are learning how to turn simple scientific tasks into competitive online games.

This is the story of how a simple idea of sharing scientific challenges on the Web turned into a global trend, called citizen cyberscience. And how you can be a scientist on the Web, too.

Looking for Little Green Men

A long time ago, in 1999, when the World Wide Web was barely ten years old and no one had heard of Google, Facebook or Twitter, a researcher at the University of California at Berkeley, David Anderson, launched an online project called SETI@home. SETI stands for Search for Extraterrestrial Intelligence. Looking for life in outer space.

Although this sounds like science fiction, it is a real and quite reasonable scientific project. The idea is simple enough. If there are aliens out there on other planets, and they are as smart or even smarter than us, then they almost certainly have invented the radio already. So if we listen very carefully for radio signals from outer space, we may pick up the faint signals of intelligent life.

Exactly what radio broadcasts aliens would produce is a matter of some debate. But the idea is that if they do, it would sound quite different from the normal hiss of background radio noise produced by stars and galaxies. So if you search long enough and hard enough, maybe you'll find a sign of life.

It was clear to David and his colleagues that the search was going to require a lot of computers. More than scientists could afford. So he wrote a simple computer program which broke the problem down into smaller parts, sending bits of radio data collected by a giant radio-telescope to volunteers around the world. The volunteers agreed to download a programme onto their home computers that would sift through the bit of data they received, looking for signals of life, and send back a short summary of the result to a central server in California.

The biggest surprise of this project was not that they discovered a message from outer space. In fact, after over a decade of searching, no sign of extraterrestrial life has been found, although there are still vast regions of space that have not been looked at. The biggest surprise was the number of people willing to help such an endeavour. Over a million people have downloaded the software, making the total computing power of SETI@home rival that of even the biggest supercomputers in the world.

David was deeply impressed by the enthusiasm of people to help this project. And he realized that searching for aliens was probably not the only task that people would

be willing to help with by using the spare time on their computers. So he set about building a software platform that would allow many other scientists to set up similar projects. You can read more about this platform, called BOINC, and the many different kinds of volunteer computing projects it supports today, at `http://boinc.berkeley.edu/`.

There's something for everyone, from searching for new prime numbers (PrimeGrid) to simulating the future of the Earth's climate (ClimatePrediction.net). One of the projects, MalariaControl.net, involved researchers from the University of Cape Town as well as from universities in Mali and Senegal.

The other neat feature of BOINC is that it lets people who share a common interest in a scientific topic share their passion, and learn from each other. BOINC even supports teams – groups of people who put their computer power together, in a virtual way on the Web, to get a higher score than their rivals. So BOINC is a bit like Facebook and World of Warcraft combined – part social network, part online multiplayer game.

Here's a thought: spend some time searching around BOINC for a project you'd like to participate in, or tell your class about.

You are a Computer, too

Before computers were machines, they were people. Vast rooms full of hundreds of government employees used to calculate the sort of mathematical tables that a laptop can produce nowadays in a fraction of a second. They used to do those calculations laboriously, by hand. And because it was easy to make mistakes, a lot of the effort was involved in double-checking the work done by others.

Well, that was a long time ago. Since electronic computers emerged over 50 years ago, there has been no need to assemble large groups of humans to do boring, repetitive mathematical tasks. Silicon chips can solve those problems today far faster and more accurately. But there are still some mathematical problems where the human brain excels.

Volunteer computing is a good name for what BOINC does: it enables volunteers to contribute computing power of their PCs and laptops. But in recent years, a new trend has emerged in citizen cyberscience that is best described as volunteer thinking. Here the computers are replaced by brains, connected via the Web through an interface called eyes. Because for some complex problems – especially those that involve recognizing complex patterns or three-dimensional objects – the human brain is still a lot quicker and more accurate than a computer.

Volunteer thinking projects come in many shapes and sizes. For example, you can help to classify millions of images of distant galaxies (GalaxyZoo), or digitize handwritten information associated with museum archive data of various plant species (Herbaria@home). This is laborious work, which if left to experts would take years or decades to complete. But thanks to the Web, it's possible to distribute images so that hundreds of thousands of people can contribute to the search.

Not only is there strength in numbers, there is accuracy, too. Because by using a technique called validation – which does the same sort of double-checking that used to be done by humans making mathematical tables – it is possible to practically eliminate the effects of human error. This is true even though each volunteer may make quite a few mistakes. So projects like Planet Hunters have already helped astronomers

pinpoint new planets circling distant stars. The game FoldIt invites people to compete in folding protein molecules via a simple mouse-driven interface. By finding the most likely way a protein will fold, volunteers can help understand illnesses like Alzheimer's disease, that depend on how proteins fold.

Volunteer thinking is exciting. But perhaps even more ambitious is the emerging idea of volunteer sensing: using your laptop or even your mobile phone to collect data – sounds, images, text you type in – from any point on the planet, helping scientists to create global networks of sensors that can pick up the first signs of an outbreak of a new disease (EpiCollect), or the initial tremors associated with an earthquake (QuakeCatcher.net), or the noise levels around a new airport (NoiseTube).

There are about a billion PCs and laptops on the planet, but already 5 billion mobile phones. The rapid advance of computing technology, where the power of a ten-year old PC can easily be packed into a smart phone today, means that citizen cyberscience has a bright future in mobile phones. And this means that more and more of the world's population can be part of citizen cyberscience projects. Today there are probably a few million participants in a few hundred citizen cyberscience initiatives. But there are more than seven billion brains on the planet. That is a lot of potential citizen cyberscientists.

You can explore much more about citizen cyberscience on the Web. There's a great list of all sorts of projects, with brief summaries of their objectives, at `http://distributedcomputing.info/`. BBC Radio 4 produced a short series on citizen science `http://www.bbc.co.uk/radio4/science/citizensciences.html` and you can subscribe to a newsletter about the latest trends in this field at `http://scienceforcitizens.net/` . The Citizen Cyberscience Centre, `www.citizencyberscience.net` which is sponsored by the South African Shuttleworth Foundation, is promoting citizen cyberscience in Africa and other developing regions.

1.2   Blog posts and other interesting online content

General blog posts

- Educator's Monthly - Education News and Resources (`http://www.teachersmonthly.com`)
    - "We eat, breathe and live education! "
    - "Perhaps the most remarkable yet overlooked aspect of the South African teaching community is its enthusiastic, passionate spirit. Every day, thousands of talented, hard-working educators gain new insight from their work and come up with brilliant, inventive and exciting ideas. Educator's Monthly aims to bring educators closer and help them share knowledge and resources.
    - Our aim is twofold:
        * To keep South African educators updated and informed.
        * To give educators the opportunity to express their views and cultivate their interests."
- Head Thoughts – Personal Reflections of a School Headmaster (`http://headthoughts.co.za/`)
    - blog by Arthur Preston

- "Arthur is currently the headmaster of a growing independent school in Worcester, in the Western Cape province of South Africa. His approach to primary education is progressive and is leading the school through an era of new development and change."

- Reflections of a Science Teacher - Scientist, Educator, Life-Long Learner (`http://sanmccarron.blogspot.com/`)

  - blog by Sandra McCarron
  - "After 18 years as an Environmental Consultant, I began teaching high school science and love it. My writings here reflect some of my thoughts about teaching, as they occur. I look forward to conversations with other thoughtful teachers."

- René Toerien – Resources for science teachers (`http://renetoerien.net/`)

  - blog by René Toerien
  - "I am the coordinator of the UCT Chemical Engineering Schools Project. We develop resource materials for the South African Physical Sciences curriculum."

- The Naked Scientists - Science Radio and Naked Science Podcasts (`http://www.thenakedscientists.com/`)

  - "The Naked Scientists" are a media-savvy group of physicians and researchers from Cambridge University who use radio, live lectures, and the Internet to strip science down to its bare essentials, and promote it to the general public. Their award winning BBC weekly radio programme, The Naked Scientists, reaches a potential audience of 6 million listeners across the east of England, and also has an international following on the web."

- Wired Science - News for your neurons (`http://www.wired.com/wiredscience/`)

  - Blog posts and interesting pictures about science as it happens.

Chemistry blog posts

- Chemical Heritage Foundation – We Tell the Story of Chemistry (`http://www.chemheritage.org/`)

  - "The Chemical Heritage Foundation (CHF) fosters an understanding of chemistry's impact on society. An independent, nonprofit organization, CHF maintains major collections of instruments, fine art, photographs, papers, and books. We host conferences and lectures, support research, offer fellowships, and produce educational materials. Our museum and public programs explore subjects ranging from alchemy to nanotechnology."

- ChemBark – A Blog About Chemistry and Chemical Research (`http://blog.chembark.com/`)

  - blog maintained by Paul Bracher
  - "The scope of this blog is the world of chemistry and chemical research. Common subjects of discussion include ideas, experiments, data, publications, writing, education, current events, lab safety, scientific policy, academic politics, history, and trivia."

- Chemistry World Blog (`http://www.rscweb.org/blogs/cw/`)

  - "This blog provides a forum for news, opinions and discussion about the chemical sciences. Chemistry World is the monthly magazine of the UK's Royal Society of Chemistry."

- Chemistry Blog (`http://www.chemistryblog.net/`)

  - "A brand new site for chemists and the home of the international chemistry societies' electronic network. The site provides interesting features and useful services for the chemistry community. The information you find has been made available by various national chemistry societies for dissemination on a single site. Currently around 30 such societies are providing varying levels of information."

- About.com Chemistry (`http://chemistry.about.com/`)

  - This website is full of great chemistry information, including Chem 101, science projects, elements, plus many interesting articles, including a daily "This Day in Science History"

Physics blog posts

- dotphysics (`http://scienceblogs.com/dotphysics/`)

  - blog by Rhett Allain
  - "This blog is about physics. Not crazy hard physics, but nice physics. You know, like physics you would take home to your mom. I try to aim most of the posts at the physics level an advanced high school student could understand."

- Think Thnk Thunk – Dealing with the Fear of Being a Boring Teacher (`http://101studiostreet.com/wordpress/`)

  - blog by Shawn Cornally
  - "I am Mr. Cornally. I desperately want to be a good teacher. I teach Physics, Calculus, Programming, Geology, and Bioethics. Warning: I have problem with using colons. I proof read, albeit poorly."

Interesting online content

- XKCD What if?

  Answering your hypothetical questions with physics, every Tuesday.

  http://what-if.xkcd.com/

- MinutePhysics

  Cool videos and other sweet science - all in a minute!

  MinutePhysics

- Science Alert

  We connect, engage and inspire science enthusiasts worldwide by sharing reputable science news, information and entertainment. (On facebook.)

  ScienceAlert

1.3 Overview

Dear educator, welcome to the force of educators that make a difference by unlocking the marvels of the Physical Sciences to learners. What a privilege you have to guide the learners in becoming critical thinkers!

To improve curriculum implementation and to meet the vision for our nation, the National Curriculum Statement Grades R - 12 (NCS) was revised, changed and is replaced by a national policy document developed for each subject. All Physical Sciences educators in the country have to use the National Curriculum and Assessment Policy Statement for Physical Sciences. This policy document replaces all old Subject Statements, Learning Programme Guidelines and Subject Assessment Guidelines in Grades R - 12. These changed curriculum and assessment requirements come into effect in January 2012. As a Physical Sciences educator for Grade 11, you need to have a sound understanding of the National Curriculum and Assessment Policy Statement for Physical Sciences.

This teachers' guide is divided into two main parts:

- Part 1 deals with the policy document; and
- Part 2 with the learners' textbook.

Part 1

The National Curriculum and Assessment Policy Statement for Physical Sciences has four sections:

Section 1: Curriculum overview

Section 2: Physical Sciences

Section 3: Physical Sciences Content (Grades 10 - 12)

Section 4: Assessment

This part will assist you in getting to grips with the objectives and requirements laid down for the Physical Sciences at national level, and how to implement the prescribed policy document.

Part 2

Each chapter in the textbook addresses prescribed content, concepts and skills. The range of activities includes practical activities, experiments, and informal and formal assessment tasks.

1.4 Curriculum overview

From the beginning of January 2012, all learning and teaching in public and independent schools in South Africa is laid down in the National Curriculum and Assessment Policy Statements (January 2012) (CAPS) document. National Curriculum and As-

sessment Policy Statements were developed for each subject and replace all previous policy statements including:

1. National Senior Certificate: a qualification at Level 4 on the National Qualifications Framework (NQF);

2. An addendum to the policy document, the National Senior Certificate: a qualification at Level 4 on the National Qualifications Framework (NQF), regarding learners with special needs, published in the Government Gazette, No. 29466 of 11 December 2006;

3. The Subject Statements, Learning Programme Guidelines and Subject Assessment Guidelines for Grades R - 9 and Grades 10 - 12.

The following sections in this document set out the expected norms and standards and minimum outcomes, as well as processes and procedures for the assessment of learner achievement in public and independent schools.

The national agenda and how the curriculum can serve this agenda:

1. The knowledge, skills and values worth learning for learners in South Africa are clearly set out in the National Curriculum and Assessment Policy Statement for Physical Sciences. The content links to the environment of the learners and is presented within local context, with awareness of global trends.

2. The National Curriculum Statement Grades R - 12 undertakes to:

   - equip all learners, irrespective of their socio-economic background, race, gender, physical ability or intellectual ability, with the knowledge, skills and values necessary for self-fulfilment to participate meaningfully in society as citizens of a free country;

   - provide access to higher education;

   - facilitate the transition of learners from education institutions to the workplace; and

   - provide employers with a sufficient profile of a learner's competencies.

3. The key principles (fuller described in the document) of the National Curriculum Statement for Grades R - 12 are:

   - social transformation: making sure that the educational differences of the past are put right, by providing equal educational opportunities to all;

   - active and critical learning: encouraging an active and critical approach to learning, not only rote learning of given facts;

   - high knowledge and high skills: specified minimum standards of knowledge and skills are set to be achieved at each grade;

   - progression: content and context of each grade shows progression from simple to complex;

   - human rights, inclusivity, environmental and social justice: being sensitive to issues such as poverty, inequality, race, gender, language, age, disability and other factors;

   - valuing indigenous knowledge systems: acknowledging the rich history and heritage of this country; and

- credibility, quality and efficiency: providing an education that is comparable in quality, breadth and depth to those of other countries.

4. The aims as listed in the National Curriculum Statement Grades R - 12 interpret the kind of citizen the education systems tries to develop. It aims to produce learners that are able to:

  - identify and solve problems and make decisions using critical and creative thinking;
  - work effectively as individuals and with others as members of a team;
  - organise and manage themselves and their activities responsibly and effectively;
  - collect, analyse, organise and critically evaluate information;
  - communicate effectively using visual, symbolic and/or language skills in various modes;
  - use science and technology effectively and critically showing responsibility towards the environment and the health of others; and
  - demonstrate an understanding of the world as a set of related systems by recognising that problem solving contexts do not exist in isolation.

5. Inclusivity is one of the key principles of the National Curriculum Statement Grades R - 12 and should become a central part of the organisation, planning and teaching at each school. Educators need to:

  - have a sound understanding of how to recognise and address barriers to learning;
  - know how to plan for diversity;
  - address barriers in the classroom;
  - use various curriculum differentiation strategies; (Consult the Department of Basic Education's Guidelines for Inclusive Teaching and Learning (2010))
  - address barriers to learning using the support structures within the community; District-Based Support Teams, Institutional-Level Support Teams, parents and Special Schools as Resource Centres.

Physical Sciences

As economic growth is stimulated by innovation and research which is embedded in the Physical Sciences, this subject plays an increasingly important role to meet the country's needs. The nature of the Physical Sciences and the needs of the country are reflected in the curriculum. The specific aims direct the classroom activities that intend to develop higher order cognitive skills of learners, needed for higher education.

The nature of the Physical Sciences is to:

- investigate physical and chemical phenomena through scientific inquiry, application of scientific models, theories and laws in order to explain and predict events in the physical environment;

- deal with society's need to understand how the physical environment works in order to benefit from it and responsibly care for it;

- use all scientific and technological knowledge, including Indigenous Knowledge Systems (IKS) to address challenges facing society.

The specific aims of Physical Sciences

The specific aims provide guidelines on how to prepare learners to meet the challenges of society and the future during teaching, learning and assessment. The Specific Aims of the Physical Sciences (CAPS document, stated below) are aligned to the three Learning Outcomes (NCS document) with which you are familiar. Developing language skills as such is not a specific aim for the Physical Sciences, but we know that cognitive skills are rooted in language; therefore language support is crucial for success in this subject.

The specific aims for the Physical Sciences are:

- to promote knowledge and skills in scientific inquiry and problem solving; the construction and application of scientific and technological knowledge; an understanding of the nature of science and its relationships to technology, society and the environment.

- to equip learners with investigating skills relating to physical and chemical phenomena. These skills are: classifying, communicating, measuring, designing an investigation, drawing and evaluating conclusions, formulating models, hypothesising, identifying and controlling variables, inferring, observing and comparing, interpreting, predicting, problem solving and reflective skills.

- to prepare learners for future learning (including academic courses in Higher Education), specialist learning, employment, citizenship, holistic development, socio-economic development, and environmental management. Learners choosing Physical Sciences as a subject in Grades 10 - 12, including those with barriers to learning, can have improved access to professional career paths related to applied science courses and vocational career paths.

Within each of these aims, specific skills or competences have been identified. It is not advisable to try to assess each of the skills separately, nor is it possible to report on individual skills separately. However, well designed assessments must show evidence that, by the end of the year, all of the skills have been assessed at a grade-appropriate level. Study the next section that deals with assessment.

Developing language skills: reading and writing

As a Physical Sciences educator you need to engage in the teaching of language. This is particularly important for learners for whom the Language of Learning and Teaching (LoLT) is not their home language. It is important to provide learners with opportunities to develop and improve their language skills in the context of learning Physical Sciences. It will therefore be critical to afford learners opportunities to read scientific texts, to write reports, paragraphs and short essays as part of the assessment, especially (but not only) in the informal assessments for learning.

Six main knowledge areas inform the Physical Sciences. These are:

- Matter and Materials
- Chemical Systems
- Chemical Change

- Mechanics

- Waves, Sound and Light

- Electricity and Magnetism

Time Allocation of the Physical Sciences in the Curriculum

The teaching time for Physical Sciences is 4 hours per week, with 40 weeks in total per grade. The time allocated for the teaching of content, concepts and skills includes the practical work. These are an integral part of the teaching and learning process.

Grade	No. of weeks allocated	Content, concepts and skills (Weeks)	Formal assessment
10	40	30	10
11	40	30	10
12	40	28	12

Topics and Content to be Dealt with in Grade 11

(Consult the National Curriculum and Assessment Policy Statement for Physical Sciences for an overview of Grades 10 - 12)

Topic	Content
Mechanics	**Vectors in two dimensions** (resultant of perpendicular vectors, resolution of a vector into its parallel and perpendicular components), **Newton's Laws and Application of Newton's Laws** (Newton's first, second and third laws and Newton's law of universal gravitation, different kinds of forces: weight, normal force, frictional force, applied (push, pull), tension (strings or cables), force diagrams, free body diagrams and application of Newton's laws(equilibrium and non-equilibrium)) **27 hours**
Waves, sound and light	**Geometrical Optics** (Refraction, Snell's Law, Critical angles and total internal reflection), **2D and 3D Wave fronts** (Diffraction) **13 hours**
Electricity and magnetism	**Electrostatics** (Coulomb's Law, Electric field), **Electromagnetism** (Magnetic field associated with current-carrying wires, Faraday's Law), **Electric circuits** (Energy, Power) **20 hours**
Matter and materials	**Molecular structure** (a chemical bond; molecular shape; electronegativity and bond polarity; bond energy and bond length) **Intermolecular forces** (chemical bonds revised; types of intermolecular forces; states of matter; density; kinetic energy; temperature; three phases of water (macroscopic properties related to sub-microscopic structure)), **Ideal gases** (motion and kinetic theory of gases; gas laws; relationship between T and P) **23 hours**
Chemical systems	**Lithosphere** (mining; energy resources) **8 hours**
Chemical change	**Stoichiometry** (molar volume of gases; concentration; limiting reagents; volume relationships in gaseous reactions) **Energy and chemical change** (energy changes related to bond energy; exothermic and endothermic reactions; activation energy), **Types of reactions**(acid-base; redox reactions; oxidation numbers) **28 hours**

An overview of practical work

Educators now have clarity regarding the role and assessment of practical work. This document specifies that practical work must be integrated with theory to strengthen the concepts being taught. Practical work can be: simple practical demonstrations; an experiment or practical investigation. In Section 3 practical activities are outlined alongside the content, concepts and skills column. The table below lists prescribed practical activities for formal assessment as well as recommended practical activities for informal assessment in Grade 11.

Term	Prescribed activities for Formal Assessment	Prescribed activities for Informal Assessment
Term 1	**Experiment (Physics):** Investigate the relationship between force and acceleration (Newton 2)	**Practical demonstration (Physics):** Investigate the relationship between normal force and static friction. Investigate the effect of different surfaces on maximum static friction by keeping the object the same. **or** **Experiment (Chemistry):** Investigate the physical properties of water (density, BP, MP, effective as solvent)
Term 2	**Experiment (Chemistry):** The effects of intermolecular forces: boiling points, melting points; surface tension, solubility, capilarity	**Experiment (Physics):** Determine the critical angle of a rectangular glass (clear) block **or** **Experiment (Chemistry):** Boyle's law OR prepartion of PbO_2 from $Pb(NO_3)_2$
Term 3	**Project (Chemistry):** Exothermic and endothermic reactions (examples and applications) **or** **Project (Physics):** Snell's law	**Experiment (Physics):** Obtain current and voltage data for a resistor and a light bulb and determine which one obeys Ohm's law **or** **Experiment (Chemistry):** Investigate natural indicators for acids and bases
Term 4		**Experiment (Chemistry):** Redox reactions: one synthesis, one decomposition and one displacement reaction

The assessment rubric below could be used for the above project (poster), as adapted from the UCT Chemical Engineering Mining and Mineral Processing Resource Pack.

Project - poster

Research (individual) (30 marks)	Not achieved (0% - 29%)		Moderate (30% - 49%)	Adequate (50% - 69%)		Outstanding (70% - 100%)	
Use of available resources	0	1	2	3	4	5	6
Accuracy and interpretation of information	0 1	2 3	4 5	6 7 8		9 10	11 12
Coverage of the topic	0	1	2	3	4	5	6
Quality of writing	0	1	2	3	4	5	6

Research (group) (25 marks)	Not achieved (0% - 29%)		Moderate (30% - 49%)	Adequate (50% - 69%)	Outstanding (70% - 100%)	
The research question was thoroughly addressed and expressed on the poster	0	1	2	3	4	5
Content is coherent and reads and is presented well	0	2	4	6	8	10
Group members worked together to produce the poster	0	1	2	3	4	5
Creative approach to topic, write-up and/or overall presentation of poster. Project stands out from the rest	0	1	2	3	4	5

The poster (10 marks)	Not achieved	Acceptable	Good
The format of the poster is correct according to the instructions given	0	1	2
Poster is well presented, on suitably sized piece of paper	0	1	2
Poster has appropriate illustrations	0	1	2
Poster flows in a logical fashion and presents information in an eye catching manner	0	1	2
Reference list is included and shows a variety of sources	0	1	2
		Total	65

The assessment rubric below could be used for the above project (presentation), as taken from the UCT Chemical Engineering Mining and Mineral Processing Resource Pack.

Class presentation (group mark)	Not achieved (0% - 29%)	Moderate (30% - 49%)	Adequate (50% - 69%)	Outstanding (70% - 100%)
The content was presented in a logical and well-organised way.	0	1	2	3
The most important and relevant content was presented.	0	1	2	3
The visual aids were effective, appropriate and supported the presentation.	0	1	2	3
The socio-political issues were addressed by the group and own opinions were presented.	0	1	2	3
Appropriate conclusions were drawn.	0	1	2	3
Class presentation (individual mark)				
A highly enthusiastic learner; the presentation is interesting, capturing the attention.	0	1	2	3
Accurate information was presented. It is clear that the learner has mastered the content.	0	1	2	3
The learner is able to answer questions knowledgeably.	0	1	2	3
The learner is able to converse in a scientific language.	0	1	2	3
			Total	30

Weighting of topics [40 week programme]:

	Grade 11	Time
Mechanics	16,87%	27 hours
Waves, Sound and Light	8,13%	13 hours
Electricity and Magnetism	12,50%	20 hours
Matter and Materials	15,00%	24 hours
Chemical Change	17,50%	28 hours
Chemical Systems	5,00%	8 hours
Teaching time (theory and practical work)	75%	120 hours
Time for examinations and control tests	25%	40 hours

Total time $=$ 40 hours per term $\times$ 4 terms $=$ 160 hours per year

1.5   Physical Sciences content (Grade 11)

This section of the CAPS document provides a complete plan for: time, topics, content, concepts and skills, practical activities, resource material and guidelines for educators. You need to consult this section of the document regularly to check whether your classroom activities fall within the requirements and objectives of the prescribed curriculum. Use the condensed work schedule below which is aligned with Section 3 and the learner's book as a pacesetter to check your progress.

Term 1: 43 hours or 11 weeks			
Physics (Mechanics)			
Weeks	Topics	Practical activities	Assessment
Week 1 (4 h)	*Vectors in 2 dimensions* Resultant of perpendicular vectors Resolution of a vector into its parallel and perpendicular components	**Recommended experiment for informal assessment** Determine the resultant of three non-linear force vectors	**Recommended Formal Assessment:** 1. Control Test 2. Prescribed experiment in Physics on Newton's second law
Week 2 and 3 (8 h)	*Newton's laws* Different kinds of forces: weight, normal forces, frictional force, applies (push, pull), tension (strings or cables). (5 h) Force diagrams, free body diagrams (3 h)	**Recommended investigation for informal assessment** 1. Investigate the relationship between normal force and maximum static friction. Investigate the effect of different surfaces on maximum static friction by keeping the object the same. and/or 2. Investigate the relationship between normal force and force of dynamic friction.	**Recommended Informal Assessment:** 1. At least two problem-solving exercises as homework and/or class work (every day, if possible cover all cognitive levels) 2. **One** practical activity per term. 3. At least **one** informal test per term.

Weeks 4, 5 and 6 (12 h)	Newton's first, second and third laws. (11 h)	**Recommended experiment for formal assessment** Investigate the relationship between force and acceleration (Verification of Newton's second law).	
Week 7 (4 h)	Newton's Law of Universal Gravitation		
Chemistry (Matter and Materials)			
Week 8 (4 h)	*Atomic combinations* A chemical bond (is seen as the net electrostatic force two atoms sharing electrons exert on each other (2 h) Molecular shape as predicted using the Valence Shell Electron Pair Repulsion (VSEPR) theory. (2 h)		
Week 9 (2 h)	Electronegativity of atoms to explain the polarity of bonds. (1 h) Bond energy and length (1 h)		
Week 9 contd. and Week 10 (6 h)	*Intermolecular forces* Intermolecular and interatomic forces (chemical bonds). Physical state and density explained in terms of these forces. Particle kinetic energy and temperature.	**Recommended experiment for formal assessment** Investigate and explain intermolecular forces and the effects of intermolecular forces on evaporation, surface tension, solubility, boiling points and capilarity.	
Week 11 (4 h)	The chemistry of water. (Macroscopic properties of the three phases of water related to their submicroscopic structure.)	**Recommended experiment for informal assessment** Investigate the physical properties of water (density, BP, MP, effectivity as a solvent, etc.)	

Newton's first, second and third laws are given 11 hours in CAPS. However to avoid starting a new topic at the end of a week the extra hour can be used for further teaching, experiments or assessment.

Term 2: 33 hours or 9 weeks			
Physics (Waves, Sound and Light)			
Weeks	**Topics**	**Practical activities**	**Assessment**
Weeks 1, 2 and 3 (10 h)	*Geometrical Optics* Refraction (3 h) Snell's Law (4 h) Critical angles and total internal reflection (3 h)	**Recommended project for formal assessment** Verifying Snell's Law and determine the refractive index of an unknown solid transparent material using Snell's Law. **Recommended experiment for informal assessment** Determine the critical angle of a rectangular glass (clear) block.	**Recommended Formal Assessment:** 1. Control Test 2. Prescribed experiment in Chemistry on Intermolecular forces **Recommended Informal Assessment:** 1. At least two problem-solving exercises as homework and/or class work (every day, if possible cover all cognitive levels) 2. **One** practical activity per term. 3. At least **one** informal test per term.
Week 3 contd. (2 h)	*2D and 3D wavefronts* Diffraction (3 h)		
Chemistry (Matter and Materials)			
Weeks 4 and 5 (8 h)	*Ideal gases* Motion of particles; kinetic theory of gases. (1 h) Ideal gas law (6 h) Temperature and heating, pressure (1 h)	**Recommended experiment for informal assessment** Verify Boyle's law.	
Chemistry (Chemical change)			
Weeks 6, 7 and 8 (12 h)	*Quantitative aspects of chemical change* Molar volume of gases, concentration of solutions (3 h) More complex stoichiometric calculations (6 h) Volume relationships in gaseous reactions. (3 h)	**Recommended experiment for informal assessment** Determine the mass of PbO prepared from a certain mass of $Pb(NO_3)_2$. (In the learners book this has been changed).	
Week 9, 10 and 11 (12 h)	Mid year examination.		

The topic of diffraction has 3 hours assigned in CAPS. However, this topic is given only 2 hours in this table. You can use the first part of week 4 to finish diffraction, before starting on the chemistry content. This will use some of week 9 before the mid-year examination begins.

Term 3: 36 hours or 9 weeks			
Physics (Electricity and magnetism)			
Weeks	**Topics**	**Practical activities**	**Assessment**
Weeks 1 and 2 (6 h)	*Electrostatics* Coulomb's Law (3 h) Electric field (3 h)		**Recommended Formal Assessment:** 1. Control Test 2. Prescribed experiment in Chemistry on exothermic and endothermic reactions (examples of each type of reaction). **Recommended Informal Assessment:** 1. At least two problem-solving exercises as homework and/or class work (every day, if possible cover all cognitive levels) 2. **One** practical activity per term. 3. At least **one** informal test per term.
Week 2 contd. and week 3 (6 h)	*Electromagnetism* Magnetic field associated with current carrying wires (3 h) Faraday's Law (3 h)		
Weeks 4 and 5 (8 h)	*Electric circuits* Ohm's Law (4 h) Power, Energy (4 h)	**Recommended experiment for informal assessment** Obtain current and voltage data for a resistor and light bulb and determine which one obeys Ohm's Law.	
Chemistry (Chemical change)			
Weeks 6 (4 h)	*Energy and chemical change* Energy changes in reactions related to bond energy changes. (2 h) Exothermic and endothermic reactions (1 h) Activation energy (1 h)	**Recommended project for formal assessment** 1. Investigate endothermic reactions as for example ammonium nitrate and water, potassium nitrate and water and magnesium sulfate and water. AND 2. Investigate exothermic reactions as for example calcium chloride and water, dry copper(II) sulfate and water and lithium and water. (Identify and explain the applications of exothermic and endothermic reactions in everyday life and industry.)	
Weeks 7, 8 and 9 (12 h)	*Types of reactions* Acids and bases. (6 h) Acid and base reactions. (6 h)	**Recommended experiment for informal assessment** Discover your own effective natural acid-base indicator by using coloured plants. Do experiments using natural indicators (don't use only red cabbage, investigate with different coloured plants to find new indicators that might be useful and compare their usefulness as acid-base indicators.)	

Term 4: 14 hours or 4 weeks			
Chemistry (Chemical change)			
Weeks	**Topics**	**Practical activities**	**Assessment**
Weeks 1, 2 and 3 (6 h)	*Types of reactions* Oxidation number of atoms in molecules to explain their relative richness in electrons. (1 h) Redox reactions (5 h)	**Recommended experiment for informal assessment** Do redox reactions that include synthesis reactions, decomposition reactions and displacement reactions (for informal assessment do at least ONE synthesis, ONE decomposition and ONE displacement reactions.)	**Formal Assessment:** Final examinations. **Recommended Informal Assessment:** 1. At least two problem-solving exercises as homework and/or class work (every day, if possible cover all cognitive levels) 2. **One** practical activity per term. 3. At least **one** informal test per term.
Chemistry (Chemical systems)			
Weeks 4 and 5 (8 h)	*The lithosphere* Mining and mineral processing. The choices are the following: Gold, iron, phosphate, coal, diamond, copper, platinum, zinc, chrome, asbestos and manganese mining industries.		
Week 6 up to end of term	Revision and formal assessment.		

1.6   Assessment

Dear Educator, as the Programme of Assessment (PoA) is the driving force of teaching and learning in the classroom, you need to familiarise yourself with the requirements specified in the National Curriculum and Assessment Policy Statement for Physical Sciences (CAPS) document. It is important that you take notice of the significant role of practical work in the Physical Sciences.

Assessment is a continuous planned process of identifying, gathering and interpreting information about the performance of learners, using various forms of assessment

It involves four steps:

- generating and collecting evidence of achievement;

- evaluating this evidence;

- recording the findings; and

- using this information to understand and assist the learner's development to improve the process of learning and teaching.

Assessment should be both informal (Assessment for Learning) and formal (Assessment of Learning). To enhance the learning experience, learners need regular feedback from both informal and formal assessment.

Assessment is a process that measures individual learners' attainment of knowledge (content, concepts and skills) in a subject by collecting, analysing and interpreting the data and information obtained from this process to:

- enable the educator to make reliable judgements about a learner's progress;

- inform learners about their strengths, weaknesses and progress;

- assist educators, parents and other stakeholders in making decisions about the learning process and the progress of the learners.

Assessment should be mapped against the content, concepts and skills and the aims specified for Physical Sciences and in both informal and formal assessments. It is important to ensure that in the course of a school year:

- all of the subject content is covered;

- the full range of skills is included;

- a variety of different forms of assessment are used.

Informal or daily assessment

Assessment for learning has the purpose of continuously collecting information on a learner's achievement, that can be used to improve their learning. Informal assessment is a daily monitoring of learners' progress. This is done through observations, discussions, practical demonstrations, learner-educator conferences, informal classroom interactions, etc.

Informal assessment may be as simple as stopping during the lesson to observe learners, or to discuss with them how learning is progressing. Informal assessment should be used to provide feedback to the learners and to inform planning for teaching, but need not be recorded. It should not be seen as separate from learning activities taking place in the classroom.

Informal assessment tasks can consist of:

- homework, class work, practical investigations, experiments and informal tests.

Informal assessment tasks will assess:

- structured problem solving involving calculations, include problem-solving exercises that do not involve calculations, practical investigations, experiments, projects, scientific arguments, ability to predict, observe and explain.

Learners or educators can mark these assessment tasks.

Self-assessment and peer assessment actively involves learners in assessment. This is important as it allows learners to learn from and reflect on their own performance. The results of the informal daily assessment tasks are not formally recorded unless the educator wishes to do so. The results of daily assessment tasks are not taken into account for promotion and certification purposes. Informal, on-going assessments should be used to structure the gaining of knowledge and skills, and should precede formal tasks in the Programme of Assessment.

Formal assessment

Formal assessment tasks form part of a year-long formal Programme of Assessment in each grade and subject. Examples of formal assessments include tests, examinations, practical tasks, projects, oral presentations, demonstrations, performances, etc. Formal assessment tasks are marked and formally recorded by the educator for progression and certification purposes. All Formal Assessment tasks are subject to moderation for the purpose of quality assurance and to ensure that appropriate standards are maintained. Formal assessment provides educators with a systematic way of evaluating how well learners are progressing in a grade and in a particular subject.

Control Tests and Examinations

Control tests and examinations are written under controlled conditions within a specified period of time. Questions in tests and examinations should assess performance at different cognitive levels with an emphasis on process skills, critical thinking, scientific reasoning and strategies to investigate and solve problems in a variety of scientific, technological, environmental and everyday contexts. The table below shows recommended weighting of cognitive levels for formal assessment.

Cognitive level	Description	Paper 1 (Physics)	Paper 2 (Chemistry)
1	Recall	15%	15%
2	Comprehension	35%	40%
3	Analysis, appplication	40%	35%
4	Evaluation, synthesis	10%	10%

The recommended weighting of cognitive levels for examinations and control tests in the Physical Sciences in Grades 10 - 12.

See Appendix 1 of the CAPS document for a detailed description of the cognitive levels.

Practical Investigations and Experiments

Practical investigations and experiments should focus on the practical aspects and the process skills required for scientific inquiry and problem solving. Assessment activities should be designed so that learners are assessed on their use of scientific inquiry

skills, like planning, observing and gathering information, comprehending, synthesising, generalising, hypothesising and communicating results and conclusions.

Practical investigations should assess performance at different cognitive levels and focus on process skills, critical thinking, scientific reasoning and strategies to investigate and solve problems in a variety of scientific, technological, environmental and everyday contexts. The CAPS document distinguishes between a practical investigation and an experiment: an experiment is conducted to verify or test a known theory; an investigation is an experiment that is conducted to test a hypothesis i.e. the result or outcome is not known beforehand.

Requirements for grade 11 practical work

Two prescribed experiments for formal assessment (**one** Chemistry and **one** Physics experiment) and **one** project on either Physics or Chemistry. This gives a total of **three** formal assessments in practical work in Physical Sciences. It is recommended that Grade 11 learners also do **four** experiments for informal assessment (**two** Chemistry and **two** Physics experiments).

A summary to use as a checklist for practical work in Grade 11:

Practical work	Chemistry	Term	Mark (x)	Physics	Term	Mark (x)
Prescribed experiments (Formal assessment)	1	2		1	1	
Experiments (Informal assessment)	2	1, 2, 3 or 4		2	1, 2 or 3	
Project (Formal assessment)	One either Physics or Chemistry	3				
Total	7 practical activities					

Projects

As taken from CAPS, "a project is an integrated assessment task that focuses on process skills, critical thinking and scientific reasoning as well as strategies to investigate and solve problems in a variety of scientific, technological, environmental and everyday contexts. This requires a learner to follow the scientific method to produce either a poster, a device, a model or to conduct a practical investigation."

A project will entail only one of the following:

- Making of a scientific poster

- Construction of a device e.g. electric motor

- Building a physical model in order to solve a challenge you have identified using concepts in the FET Physical Sciences curriculum

- Practical investigation

Note: The assessment tools used, specifying the assessment criteria for each task, will be dictated by the nature of the task and the focus of assessment.

Assessment Tools

You use an assessment tool to record information during an assessment. Assessment tools can be:

- Check lists

- Assessment grids/sheets

- Rubrics

- Observation books or notebooks

- Completed tasks, assignments of worksheets

- Conferencing or interviews

- Self or Peer Assessment Sheets

- Recordings, photographs, written descriptions

- Portfolios

Before you use the tool the learners must know:

- When he/she is to be assessed

- When he/she is to be assessed

- How she/he will be assessed

- The consequences of the assessment

- The expected mode for response (written, spoken, practical)

After using the tool, the educator needs to answer the following question:

- Were the criteria used adequate to assess the outcome, and were the levels appropriate?

- Is appropriate feedback given to learners?

- Are learning difficulties identified and action planned?

- What happens to the product?

- What feedback follow-up action is needed?

- Has the integrating function been addressed?

- What learner appeal process exists?

- How will assessment inform further teaching/learning?

Rubrics

A rubric is an assessment tool which defines different levels of performance. It can be used for assessing concepts and process skills during informal and formal assessment, and for practical work. Rubrics aim to make assessment more objective and consistent. Some of the advantages of using rubrics are:

- Learners become aware of the expectations of educators
- Educators become aware of learners' progress and potential
- Enhance greater learner involvement
- Learners are more focused and self-directed

Examples of rubrics:

Assessment of Practical Work

Planning and organising experimental investigations to test hypotheses

Criteria	High (3)	Medium (2)	Low (1)
Plan should reflect process of identification of variables, control of variables, range of conditions, ways in which the experiment could be improved, awareness of inaccuracies, offering of a conclusion.	Able to plan independently an experiment in which all variables are identified and controlled as necessary. Able to suggest ways in which the experiment could be improved.	Able to plan independently an experiment to test a hypothesis in which most of the variables are identified and controlled as necessary.	Able to plan a one-step experiment to test the hypothesis.

Following instructions and manipulations

Criteria	High (3)	Medium (2)	Low (1)
Accurately following a sequence of written/verbal instructions.	Following a sequence of instructions including branched instructions.	Can complete an experiment by following a sequence of instructions.	Able to follow a single written, diagrammatic or verbal instruction.
Selecting/using the appropriate apparatus.	Select in advance all the apparatus needed to execute a particular experiment and be able to use it.	Able to select/use most of the apparatus necessary; some more specialized equipment may still be needed.	Able to select/use only the most basic apparatus.
Manipulative skills include correct and safe handling of apparatus and material.	Able to use all apparatus and material correctly and safely.	Use most of the apparatus and material safely.	Able to use only the most basic equipment.

Making accurate observations and measurements, being aware of possible sources of error.

Criteria	High (3)	Medium (2)	Low (1)
Accuracy, completeness and relevance of observations.	Able to make a complete sequence of observations in a given situation and is aware of a number of sources of error.	Able to make a range of observations in a given situation and is able to suggest one possible source of error.	Able to make a single observation and more if prompted, e.g.: What did you observe regarding colour and smell or temperature in test tube?
Selection of measurement instrument, performance of measuring operation and reading scales.	Able to read a variety of scales as accurately as the scale permits.	Able to read a scale to the nearest division.	Able to read scales within ± one numbered scale division.

Criteria for check lists could be more differentiated when directed towards specific experiments in Chemistry or Physics.

Regarding the scope of observations properties, similarities and differences taking place in colour, hardness, mass, relative speed, size, smell, sound, state, temperature, texture, volume, voltages could be listed.

The performance of measuring which might be used for assessment could be listed as:

1. Is the instrument capable of measuring the correct amount?

2. Was the correct range of the instrument selected?

3. Were the necessary precautions taken to ensure that the measurements will be valid?

4. Are measurements repeated or checked?

5. Are readings made with due regard for parallax?

6. Is the scale reading translated to the correct magnitude and are the correct units assigned?

Recording accurately and clearly the results of experiments

Criteria	High (3)	Medium (2)	Low (1)
All the observations are described accurately and completely. Appropriate methods (written, tables, diagrams) used to record observations and measurements.	Able to draw fully labelled diagrams to record observations. Able to record results in neat tables with appropriate headings and units, with all measurements recorded as well as derived quantities.	Data recorded as an ordered set of statements, or in a table some data and units omitted.	Information recorded as a prose account, as a sequence of statements. Able to record data in a pre-prepared table.

Presentation of data in graphic form

Criteria	High (3)	Medium (2)	Low (1)
Acceptable scale. X-axis correctly labelled. Units on X-axis. Y-axis correctly labelled. Units on Y-axis. Points correctly marked. Points correctly linked. Contradictory results "normalised". Appropriate subtitle.	All the criteria could be met without assistance.	All the criteria could be met with help.	Graphs could only be drawn with pre-prepared axes, with a lot of assistance.

Drawing conclusions and making generalisations from experiments

Criteria	High (3)	Medium (2)	Low (1)
Valid deductions from results.	Able to identify patterns or relationships and explain fundamental principles, verbal or in mathematical terms.	Able to identify a pattern, simple or trend in the relationship between two variables. Perform simple calculations of a derived quantity.	Classifications of observations and recognition of similarities and differences. Able to explain a simple observation.

End–of–Year Examinations: Grade 11 (Internal)

The end-of-year examination papers in Grade 11 will be set, marked and moderated internally, unless otherwise instructed by provincial departments of education. The examination will consist of two papers. The table below shows the weighting of questions across cognitive levels and the specification and suggested weighting of the content.

Grade 11								
Paper	Content	Marks	Total marks per paper	Duration (hours)	Weighting of questions across cognitive levels			
					Level 1	Level 2	Level 3	Level 4
Paper 1: Physics focus	Mechanics	68	150	3	15%	35%	40%	10%
	Waves, Sound and Light	32						
	Electricity and Magnetism	50						
Paper 2: Chemistry focus	Matter and Materials	70	150	3	15%	40%	35%	10%
	Chemical systems	20						
	Chemical change	60						

Weighting of questions across cognitive levels, the specification and suggested weighting of the content for the Grade 11 end-of-year examination:

Assessment plan and weighting of tasks in the programme of assessment for Grade 11:

Term 1
Prescribed experiment: (20% of year mark) or 20 marks.
Control test: (10% of year mark) or 10 marks.
Total: 30 marks

Term 2
Prescribed experiment: (20% of year mark) or 20 marks.
Mid Year Examination: (20% of year mark) or 20 marks.
Total: 40 marks
Assessment Tasks: (25% of final mark).
Total: 100 marks

Term 3
Project: Any one of poster/construction of device/building a model/practical investigation 20 marks.
Control test: 10 marks.
Total: 30 marks
Final mark: (100%) 400 marks

Term 4
Final examination:
Paper 1 × 150 marks.
Paper 2 × 150 marks.
Total: 300 marks
End-of-Year Assessment: (75% of final mark) 300 marks

Recording and Reporting

Recording is a process in which the educator documents the level of a learner's performance in a specific assessment task. It shows learner progress towards the achievement of the knowledge and skills as prescribed in the Curriculum and Assessment Policy Statements. Records of learner performance should provide evidence of the learner's conceptual progression within a grade and her / his readiness to progress or be promoted to the next grade. Records of learner performance should also be used to verify the progress made by educators and learners in the teaching and learning process.

Reporting is a process of communicating learner performance to learners, parents, schools, and other stakeholders. Learner performance can be reported through report cards, parents' meetings, school visitation days, parent-educator conferences, phone calls, letters, class or school newsletters, etc. Educators in all grades report in percentages against the subject. The various achievement levels and their corresponding percentage bands are as shown in the table below.

Note: The seven point scale should have clear descriptions that give detailed information for each level. Educators will record actual marks against the task by using a record sheet, and report percentages against the subject on the learners' report card.

Codes and Percentages for Reporting in Grades R - 12:

Rating Code	Description of Competence	Percentage
7	Outstanding Achievement	80% - 100%
6	Meritorius Achievement	70% - 79%
5	Substantial Achievement	60% - 69%
4	Adequate Achievement	50% - 59%
3	Moderate Achievement	40% - 49%
2	Elementary Achievement	30% - 39%
1	Not Achieved	0% - 29%

Schools are required to provide quarterly feedback to parents on the Programme of Assessment using a formal reporting tool such as a report card. The schedule and the report card should indicate the overall level of performance of a learner.

Skills for physical sciences learners

This section requires you to reference Appendix 2 of the CAPS document, as it recommends that the skills it covers be incorporated in lessons in Grade 11, in order to sharpen the skills necessary for successful teaching and learning. Skills covered are scientific notation, conversion of units, changing the subject of the formula, rate and its applications in physics and chemistry, direct and inverse proportions, fractions and ratios, the use and meaning of constants in equations, skills needed for practical investigations (observation, precautions, data collection, data handling, tables, general types of graphs, analysis, writing conclusions, writing a hypothesis, identifying variables - for example independent, dependent and control variable), models in science, safety data, and basic trigonometry skills, etc.

CHAPTER 1

Vectors in two dimensions

1 Vectors in two dimensions

1.1 Introduction

In this chapter learners will explore vectors in two dimensions. In grade 10 learners were introduced to the concept of vectors and scalars and learnt techniques for calculating the resultant of several vectors in a straight line (or one dimension). This chapter will use force vectors as examples and serves as a sound introduction to the next chapter on forces, as well as much of physics.

The following topics are covered in this chapter.

- **How to sketch vectors and how to sketch vectors on the Cartesian plane.**

  In this topic learners will refresh how to sketch vectors from grade 10 and will then apply these skills to sketching the vectors on the Cartesian plane. They will learn about vectors that are perpendicular to an axis and parallel to an axis. In addition the different ways of specifying direction are given with an emphasis on the angle the vector makes with the positive x-axis.

- **How to determine the resultant of several vectors in two dimensions using graphical and algebraic techniques.**

  In grade 10 learners learnt about the resultant of vectors in one-dimension and different techniques of determining this. This work is now extended to finding the resultant of vectors in two dimensions. The same techniques from grade 10 are used and expanded on for determining the resultant.

- **Using trigonometry to determine the direction of the resultant**

  In grade 10 the direction was simply given as positive or negative. Now that learners are moving into two dimensions, new ways of specifying the direction are needed. We use the angle that the vector makes with the positive x-axis as the main way to specify direction. Learners will need to be familiar with sine, cosine and tangent from trigonometry and how to calculate these ratios for right-angled triangles.

- **How to break vectors up into components and carry out simple calculations using components**

  Vectors can be broken up into x and y components. This skill will be very useful for the next chapter on Newton's laws. In this section learners learn how to resolve vectors into components and then to calculate the resultant using components.

An experiment for informal assessment on using force boards to determine the resultant of vectors is included. In this experiment learners will use a force board to determine the resultant of three non-linear vectors. You will need blank paper, force board, 4 spring balances, an assortment of weights, gut or string and four pulleys per group. This experiment provides learners with the opportunity for them to see an abstract mathematical idea in action. Learners can use graphical techniques such as the tail-to-head method to find the resultant of three of the four measured forces. The questions at the end of the experiment guide learners in thinking about the results they have obtained.

Vectors on the Cartesian plane

Exercise 1 – 1:

1. Draw the following forces as vectors on the Cartesian plane originating at the origin:

   - $\vec{F_1} = 1{,}5$ N in the positive x-direction
   - $\vec{F_2} = 2$ N in the positive y-direction

   Solution:

   We choose a scale of 1 N:1 cm. Now we sketch the vectors on the Cartesian plane (we can place the vectors anywhere on the Cartesian plane, we will place them starting at the origin):
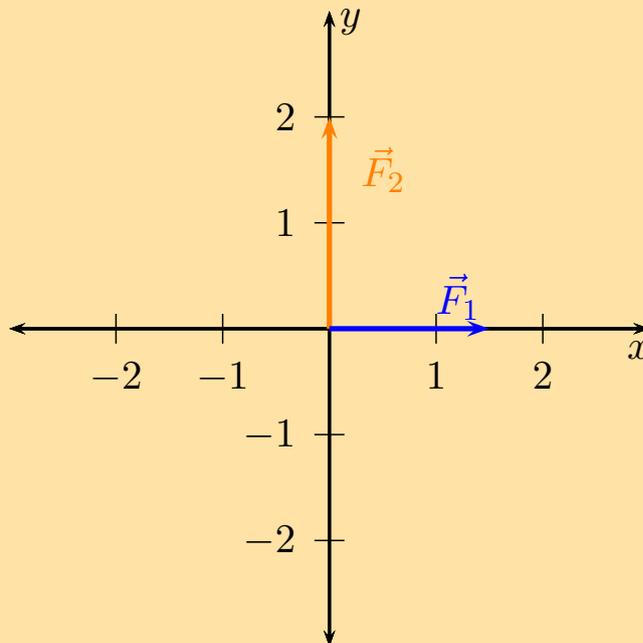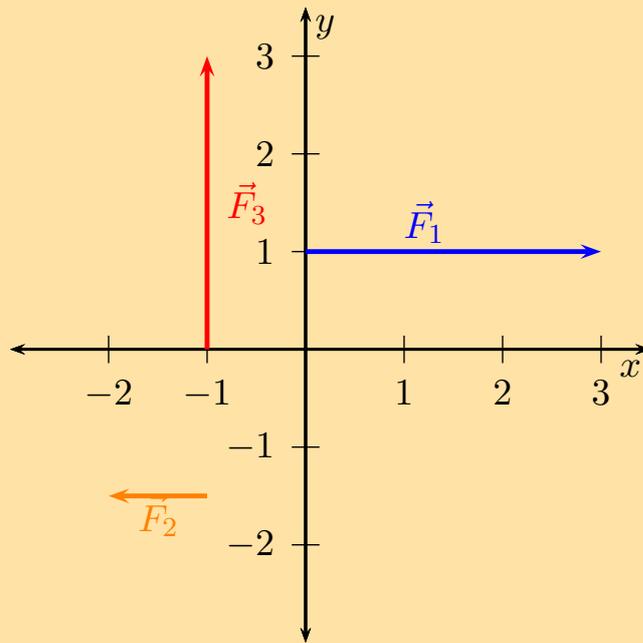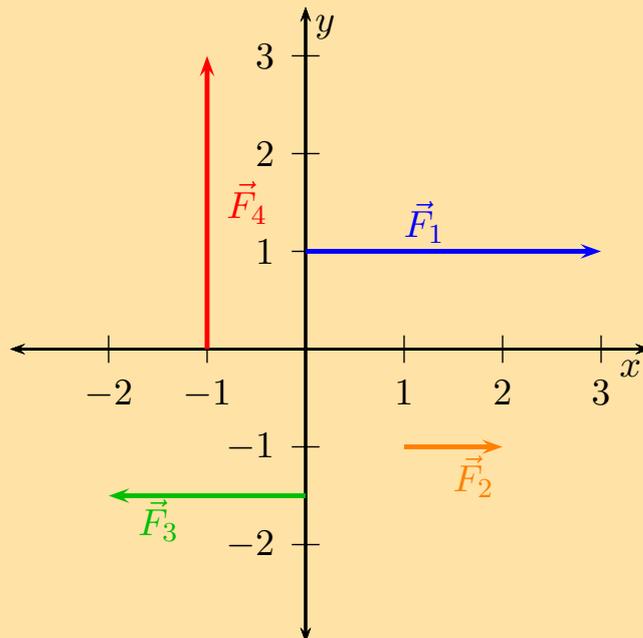

2. Draw the following forces as vectors on the Cartesian plane:

   - $\vec{F_1} = 3$ N in the positive x-direction
   - $\vec{F_2} = 1$ N in the negative x-direction
   - $\vec{F_3} = 3$ N in the positive y-direction

   Solution:

   We choose a scale of 1 N:1 cm. Now we sketch the vectors on the Cartesian plane (we can place the vectors anywhere on the Cartesian plane):

3. Draw the following forces as vectors on the Cartesian plane:

- $\vec{F_1} = 3$ N in the positive x-direction
- $\vec{F_2} = 1$ N in the positive x-direction
- $\vec{F_3} = 2$ N in the negative x-direction
- $\vec{F_4} = 3$ N in the positive y-direction

Solution:

We choose a scale of 1 N:1 cm. Now we sketch the vectors on the Cartesian plane (we can place the vectors anywhere on the Cartesian plane):

4. Draw the following forces as vectors on the Cartesian plane:

- $\vec{F_1} = 2$ N in the positive y-direction

- $\vec{F_2} = 1{,}5$ N in the negative y-direction
- $\vec{F_3} = 2{,}5$ N in the negative x-direction
- $\vec{F_4} = 3$ N in the positive y-direction

Solution:

We choose a scale of 1 N:1 cm. Now we sketch the vectors on the Cartesian plane (we can place the vectors anywhere on the Cartesian plane):
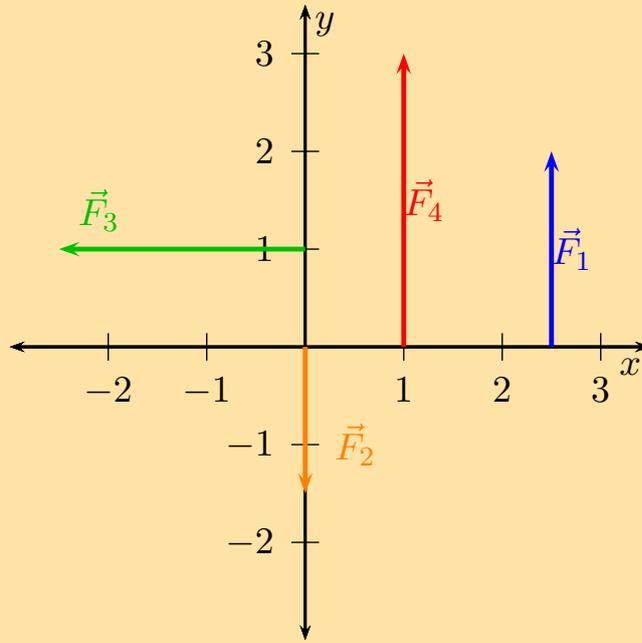
The resultant vector

Exercise 1 – 2:

1. Find the resultant in the x-direction, R_x, and y-direction, R_y for the following forces:

   - $\vec{F_1} = 1{,}5$ N in the positive x-direction
   - $\vec{F_2} = 1{,}5$ N in the positive x-direction
   - $\vec{F_3} = 2$ N in the negative x-direction

   Solution:

   We choose a scale of 1 N: 1 cm and for our diagram we will define the positive direction as *to the right*.

   We will start with drawing the vector $\vec{F_1} = 1{,}5$ N pointing in the positive direction. Using our scale of 1 N : 1 cm, the length of the arrow must be 1,5 cm pointing to the right.
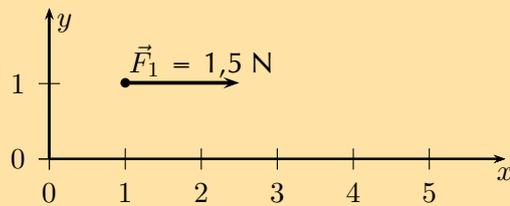
The next vector is $\vec{F_2} = 1,5$ N in the same direction as $\vec{F_1}$. Using the scale, the arrow should be 1,5 cm long and pointing to the right.
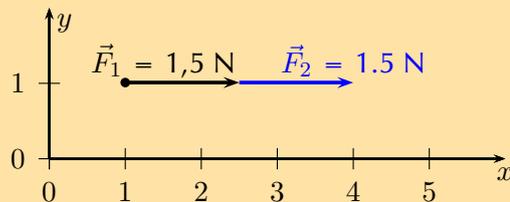
The next vector is $\vec{F_3} = 2$ N in the *opposite* direction. Using the scale, this arrow should be 2 cm long and point to the *left*.

Note: We are working in one dimension so this arrow would be drawn on top of the first vectors to the left. This will get confusing so we'll draw it next to the actual line as well to show you what it looks like.
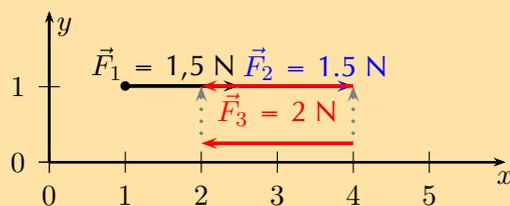
We have now drawn all the force vectors that are given. The resultant vector is the arrow which starts at the tail of the first vector and ends at the head of the last drawn vector.
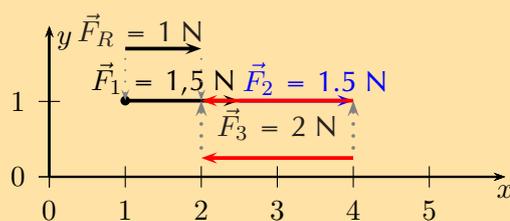

The resultant vector measures 1 cm which, using our scale is equivalent to 1 N and points to the right (*or* the positive direction). This is $\vec{R_x}$. For this set of vectors we have no vectors pointing in the y-direction and so we do not need to find $\vec{R_y}$.

2. Find the resultant in the x-direction, $\vec{R_x}$, and y-direction, $\vec{R_y}$ for the following forces:

- $\vec{F_1} = 2,3$ N in the positive x-direction
- $\vec{F_2} = 1$ N in the negative x-direction
- $\vec{F_3} = 2$ N in the positive y-direction
- $\vec{F_4} = 3$ N in the negative y-direction

Solution:

We choose a scale of 1 N: 1 cm and for our diagram we will define the positive direction as *to the right*.

Before we draw the vectors we note the lengths of the vectors using our scale:

$$\vec{F_1} = 2,3 \text{ cm}$$
$$\vec{F_2} = 1 \text{ cm}$$
$$\vec{F_3} = 2 \text{ cm}$$
$$\vec{F_4} = 3 \text{ cm}$$

We also note the direction the vectors are in:

$$\vec{F_1} = \text{positive } x\text{-direction}$$
$$\vec{F_2} = \text{negative } x\text{-direction}$$
$$\vec{F_3} = \text{positive } y\text{-direction}$$
$$\vec{F_4} = \text{negative } y\text{-direction}$$

We now look at the two vectors in the x-direction to find $\vec{R_x}$:
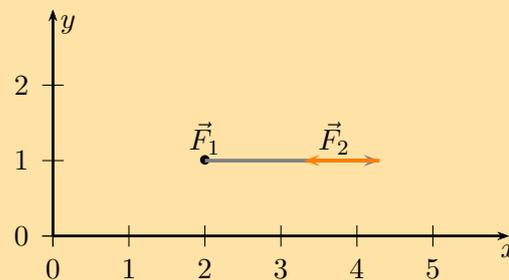

We have now drawn all the force vectors that act in the x-direction. To find $\vec{R_x}$ we note that the resultant vector is the arrow which starts at the tail of the first vector and ends at the head of the last drawn vector in that direction.
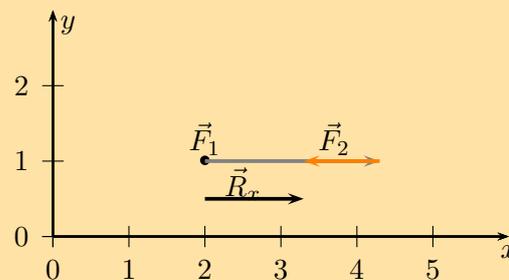
We note that $\vec{R_x}$ is 1,3 cm or 1,3 N in the positive x-direction.

We now look at the two vectors in the y-direction to find $\vec{R_y}$:
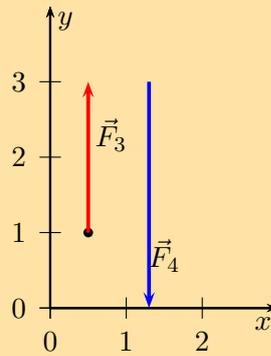
We have now drawn all the force vectors that act in the y-direction. To find $\vec{R}_y$ we note that the resultant vector is the arrow which starts at the tail of the first vector and ends at the head of the last drawn vector in that direction.
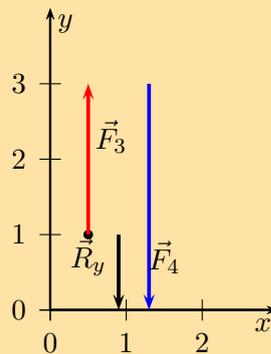
We note that $\vec{R}_y$ is 1 cm or 1 N in the negative y-direction.

$\vec{R}_x$ = 1,3 N and points in the positive x-direction. $\vec{R}_y$ = 1 N and points in the negative y-direction.

3. Find the resultant in the x-direction, $\vec{R}_x$, and y-direction, $\vec{R}_y$ for the following forces:

- $\vec{F}_1 = 3$ N in the positive x-direction
- $\vec{F}_2 = 1$ N in the positive x-direction
- $\vec{F}_3 = 2$ N in the negative x-direction
- $\vec{F}_4 = 3$ N in the positive y-direction

Solution:

We choose a scale of 1 N: 1 cm and for our diagram we will define the positive direction as *to the right*.

Before we draw the vectors we note the lengths of the vectors using our scale:

$$\vec{F}_1 = 3 \text{ cm}$$
$$\vec{F}_2 = 1 \text{ cm}$$
$$\vec{F}_3 = 2 \text{ cm}$$
$$\vec{F}_4 = 3 \text{ cm}$$

We also note the direction the vectors are in:

$$\vec{F}_1 = \text{positive } x\text{-direction}$$
$$\vec{F}_2 = \text{positive } x\text{-direction}$$
$$\vec{F}_3 = \text{negative } x\text{-direction}$$
$$\vec{F}_4 = \text{positive } y\text{-direction}$$

We now look at the three vectors in the x-direction to find $\vec{R}_x$:
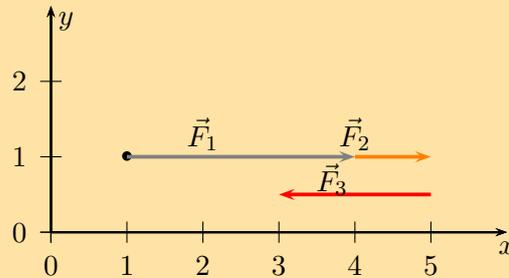
We have now drawn all the force vectors that act in the x-direction. To find $\vec{R}_x$ we note that the resultant vector is the arrow which starts at the tail of the first vector and ends at the head of the last drawn vector in that direction.
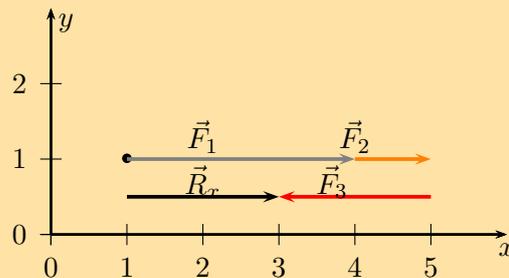
We note that $\vec{R}_x$ is 2 cm or 2 N in the positive x-direction.

We now look at the vectors in the y-direction to find $\vec{R}_y$. We notice that we only have one vector in this direction and so that vector is the resultant.
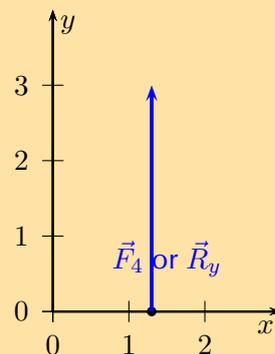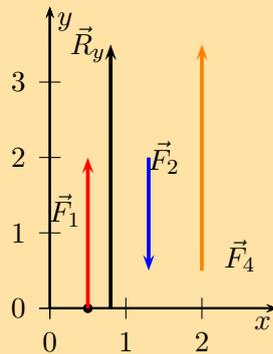

We note that $\vec{R}_y$ is 3 cm or 3 N in the positive y-direction.

$\vec{R}_x$ = 2 N and points in the positive x-direction. $\vec{R}_y$ = 3 N and points in the positive y-direction.

4. Find the resultant in the x-direction, $\vec{R}_x$, and y-direction, $\vec{R}_y$ for the following forces:

   • $\vec{F}_1$ = 2 N in the positive y-direction

- $\vec{F_2} = 1{,}5$ N in the negative y-direction
- $\vec{F_3} = 2{,}5$ N in the negative x-direction
- $\vec{F_4} = 3$ N in the positive y-direction

Solution:

We choose a scale of 1 cm: 1 N and for our diagram we will define the positive direction as *to the right*.

Before we draw the vectors we note the lengths of the vectors using our scale:

$$\vec{F_1} = 2 \text{ cm}$$
$$\vec{F_2} = 1{,}5 \text{ cm}$$
$$\vec{F_3} = 2{,}5 \text{ cm}$$
$$\vec{F_4} = 3 \text{ cm}$$

We also note the direction the vectors are in:

$$\vec{F_1} = \text{positive } y\text{-direction}$$
$$\vec{F_2} = \text{negative } y\text{-direction}$$
$$\vec{F_3} = \text{negative } x\text{-direction}$$
$$\vec{F_4} = \text{positive } y\text{-direction}$$

We look at the vectors in the x-direction to find $\vec{R_x}$. We notice that we only have one vector in this direction and so that vector is the resultant.
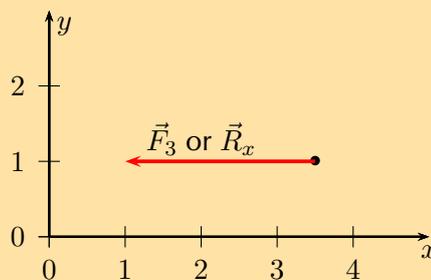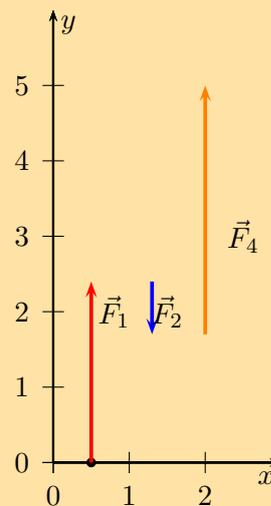

We note that $\vec{R_x}$ is 2,5 cm or 2,5 N in the negative x-direction.

We now look at the three vectors in the y-direction to find $\vec{R_y}$:
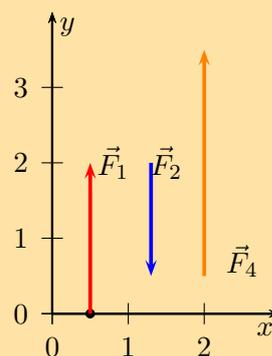
We have now drawn all the force vectors that act in the y-direction. To find $\vec{R}_y$ we note that the resultant vector is the arrow which starts at the tail of the first vector and ends at the head of the last drawn vector in that direction.

We note that $\vec{R}_y$ is 3,5 cm or 3,5 N in the positive y-direction.

$\vec{R}_x$ = 2,5 N and points in the negative x-direction. $\vec{R}_y$ = 3,5 N and points in the positive y-direction.

5. Find a force in the x-direction, F_x, and y-direction, F_y, that you can add to the following forces to make the resultant in the x-direction, R_x, and y-direction, R_y zero:

   - $\vec{F}_1 = 2,4$ N in the positive y-direction
   - $\vec{F}_2 = 0,7$ N in the negative y-direction
   - $\vec{F}_3 = 2,8$ N in the negative x-direction
   - $\vec{F}_4 = 3,3$ N in the positive y-direction

Solution:

To solve this problem we will draw the vectors on the Cartesian plane and then look at what the resultant vector is. Then we determine what force vector to add so that the resultant vector is 0.

We choose a scale of 1 cm: 1 N and for our diagram we will define the positive direction as *to the right*.

Before we draw the vectors we note the lengths of the vectors using our scale:

$$\vec{F}_1 = 2,4 \text{ cm}$$
$$\vec{F}_2 = 0,7 \text{ cm}$$
$$\vec{F}_3 = 2,8 \text{ cm}$$
$$\vec{F}_4 = 3,3 \text{ cm}$$

We also note the direction the vectors are in:

$$\vec{F}_1 = \text{positive } y\text{-direction}$$
$$\vec{F}_2 = \text{negative } y\text{-direction}$$
$$\vec{F}_3 = \text{negative } x\text{-direction}$$
$$\vec{F}_4 = \text{positive } y\text{-direction}$$

We look at the vectors in the x-direction to find $\vec{R}_x$. We notice that we only have one vector in this direction and so that vector is the resultant.
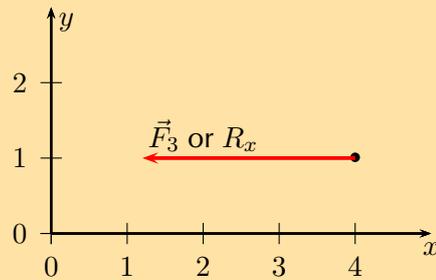
We note that $\vec{R}_x$ is 2,8 cm or 2,8 N in the negative x-direction.
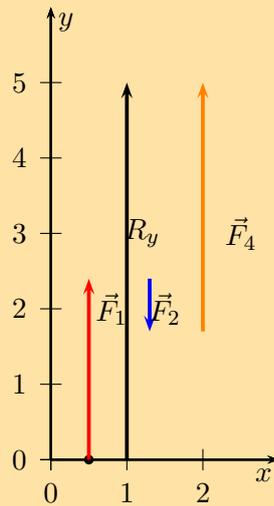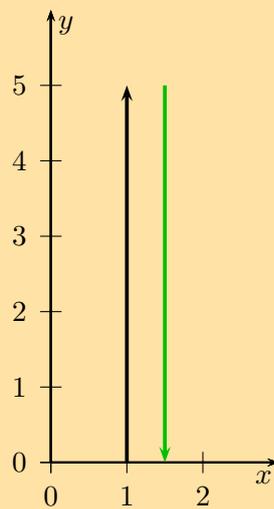
So if we add a force of 2,8 N in the positive x-direction the resultant will be 0:
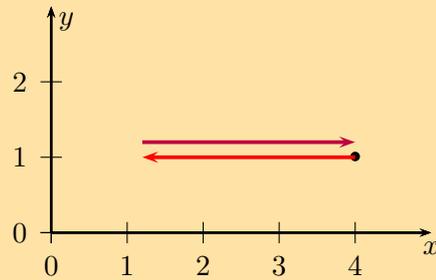
We now look at the three vectors in the y-direction to find $\vec{R}_y$:

We have now drawn all the force vectors that act in the y-direction. To find $\vec{R}_y$ we note that the resultant vector is the arrow which starts at the tail of the first vector and ends at the head of the last drawn vector in that direction.

We note that $\vec{R}_y$ is 5 cm or 5 N in the positive y-direction.

So if we add a force of 5 N in the negative y-direction the resultant will be 0:

We must add a force of 2,8 N in the positive x-direction and a force of 5 N in the negative y-direction.

Magnitude of the resultant of vectors at right angles

Exercise 1 – 3:

1. Sketch the resultant of the following force vectors using the tail-to-head method:

   - $\vec{F}_1 = 2,1$ N in the positive y-direction
   - $\vec{F}_2 = 1,5$ N in the negative x-direction

   Solution:
   We first draw a Cartesian plane with the first vector originating at the origin:
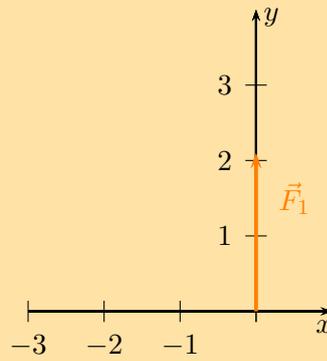
The next step is to take the second vector and draw it from the head of the first vector:
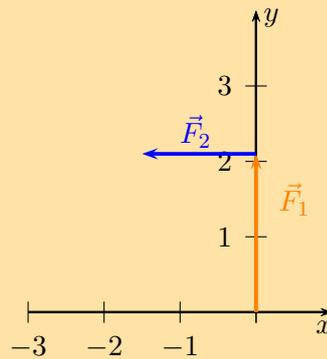
The resultant, $\vec{R}$, is the vector connecting the tail of the first vector drawn to the head of the last vector drawn:
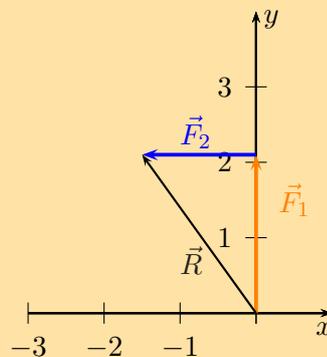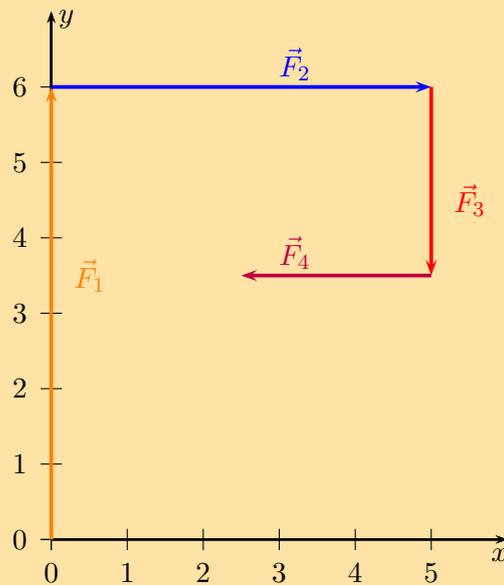

2. Sketch the resultant of the following force vectors using the tail-to-head method:
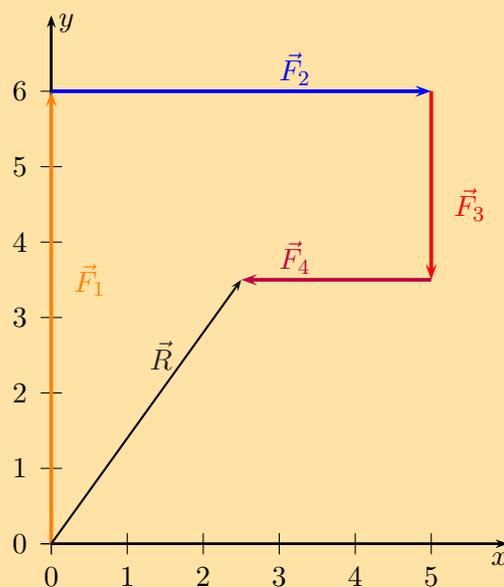
- $\vec{F_1} = 12$ N in the positive y-direction
- $\vec{F_2} = 10$ N in the positive x-direction
- $\vec{F_3} = 5$ N in the negative y-direction
- $\vec{F_4} = 5$ N in the negative x-direction

Solution:

We first sketch the vectors on the Cartesian plane. We choose a scale of 1 cm = 2 N. Remember to sketch F_2 starting at the head of F_1, F_3 starting at the head of F_2 and F_4 starting at the head of F_3.

The resultant, $\vec{R}$, is the vector connecting the tail of the first vector drawn to the head of the last vector drawn:
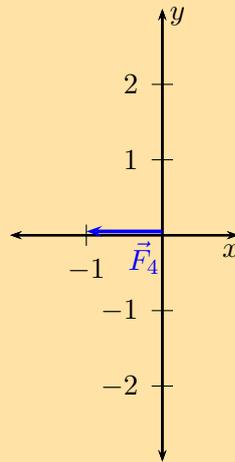

3. Sketch the resultant of the following force vectors using the tail-to-head method by first determining the resultant in the x- and y-directions:

- $\vec{F_1} = 2$ N in the positive y-direction
- $\vec{F_2} = 1,5$ N in the negative y-direction
- $\vec{F_3} = 1,3$ N in the negative y-direction
- $\vec{F_4} = 1$ N in the negative x-direction

Solution:

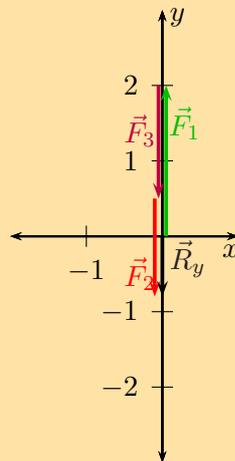We first determine $\vec{R}_x$

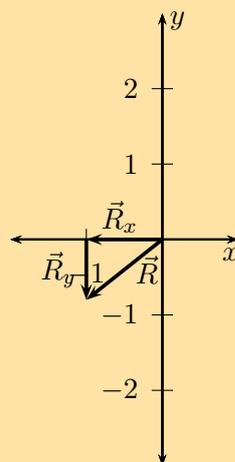Draw the Cartesian plane with the vectors in the x-direction:

This is $\vec{R}_x$ since it is the only vector in the x-direction.

Secondly determine $\vec{R}_y$

Next we draw the Cartesian plane with the vectors in the y-direction:

Now we draw the resultant vectors, $\vec{R}_y$ and $\vec{R}_x$ head-to-tail:

You can check this answer by using the tail-to-head method without first determining the resultant in the x-direction and the y-direction.

4. Sketch the resultant of the following force vectors using the tail-to-head method by first determining the resultant in the x- and y-directions:
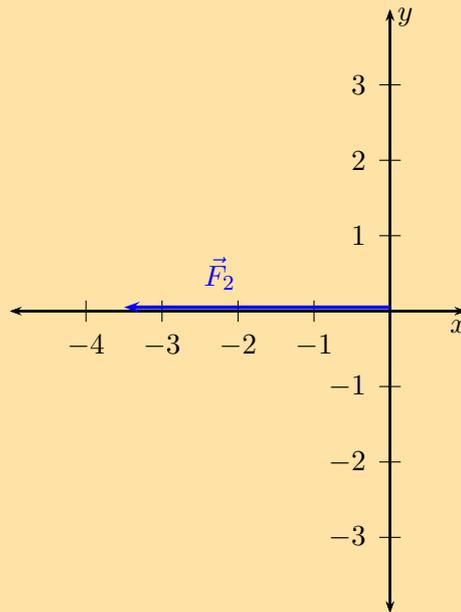
- $\vec{F_1} = 6$ N in the positive y-direction
- $\vec{F_2} = 3{,}5$ N in the negative x-direction
- $\vec{F_3} = 8{,}7$ N in the negative y-direction
- $\vec{F_4} = 3$ N in the negative y-direction

Solution:

We choose a scale of 1 cm : 2 N.

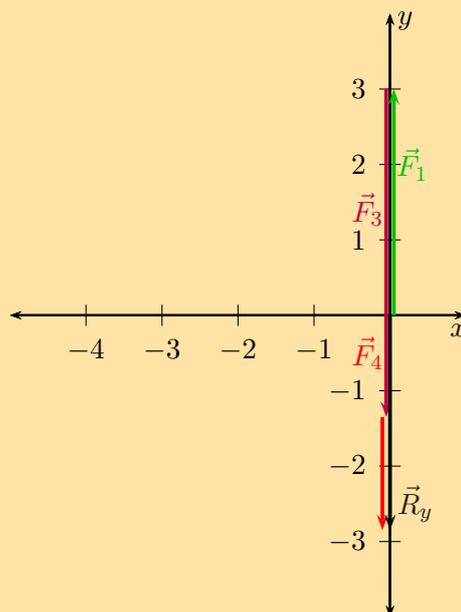We first determine $\vec{R_x}$

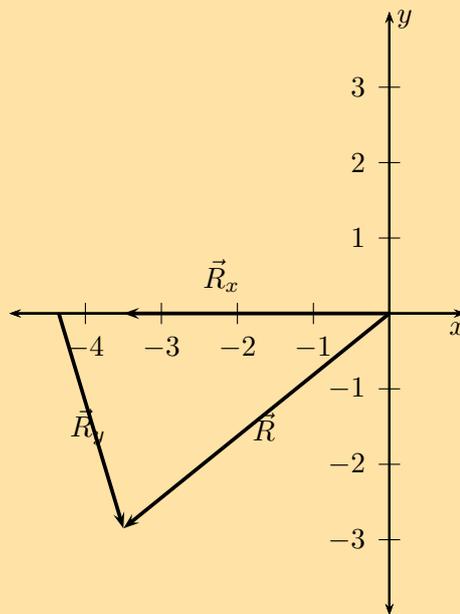Draw the Cartesian plane with the vectors in the x-direction:

This is $\vec{R_x}$ since it is the only vector in the x-direction.

Secondly determine $\vec{R_y}$

Next we draw the Cartesian plane with the vectors in the y-direction:

Now we draw the resultant vectors, $\vec{R_y}$ and $\vec{R_x}$ head-to-tail:

You can check this answer by using the tail-to-head method without first determining the resultant in the x-direction and the y-direction.
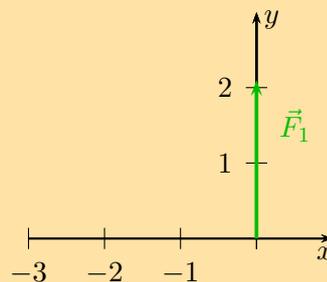
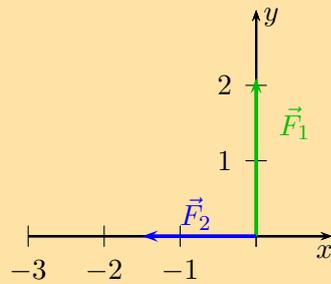1. Sketch the resultant of the following force vectors using the tail-to-tail method:

   - $\vec{F_1} = 2{,}1$ N in the positive y-direction
   - $\vec{F_2} = 1{,}5$ N in the negative x-direction
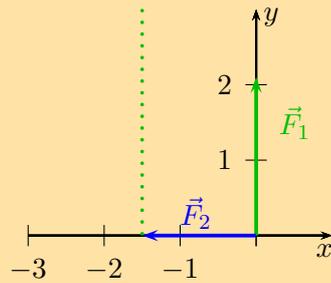
   Solution:

   We first draw a Cartesian plane with the first vector originating at the origin:
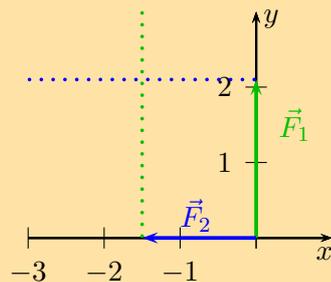

   Then we add the second vector but also originating from the origin so that the vectors are drawn tail-to-tail:

Now we draw a line parallel to $\vec{F}_1$ from the head of $\vec{F}_2$:

Next we draw a line parallel to $\vec{F}_2$ from the head of $\vec{F}_1$:

Where the two lines intersect is the head of the resultant vector which will originate at the origin so:

2. Sketch the resultant of the following force vectors using the tail-to-tail method by first determining the resultant in the x- and y-directions:

- $\vec{F}_1 = 2$ N in the positive y-direction
- $\vec{F}_2 = 1{,}5$ N in the negative y-direction
- $\vec{F}_3 = 1{,}3$ N in the negative y-direction
- $\vec{F}_4 = 1$ N in the negative x-direction

Solution:

We need to determine $\vec{R}_x$ and $\vec{R}_y$ and then use these to find the resultant.

Determine $\vec{R}_x$.

Draw the Cartesian plane with the vectors in the x-direction:

This is $\vec{R}_x$ since it is the only vector in the x-direction.

Secondly determine $\vec{R}_y$

Next we draw the Cartesian plane with the vectors in the y-direction:

Now we draw the resultant vectors, $\vec{R}_y$ and $\vec{R}_x$ tail-to-tail:

Now we can draw the lines to show us where the head of the resultant must be:

And finally we find the resultant:

3. Sketch the resultant of the following force vectors using the tail-to-tail method by first determining the resultant in the x- and y-directions:
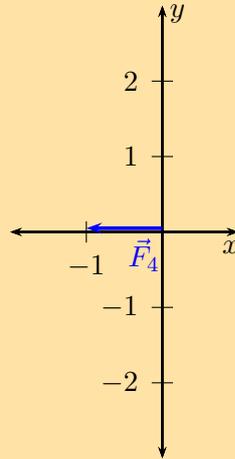
- $\vec{F_1} = 6$ N in the positive y-direction
- $\vec{F_2} = 3{,}5$ N in the negative x-direction
- $\vec{F_3} = 8{,}7$ N in the negative y-direction
- $\vec{F_4} = 3$ N in the negative y-direction

Solution:

We choose a scale of 1 cm = 2 N.

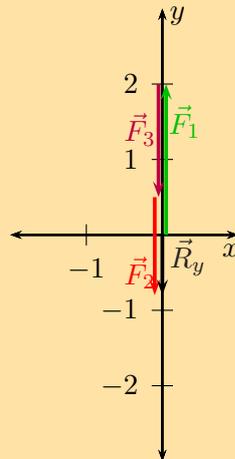We first determine $\vec{R_x}$

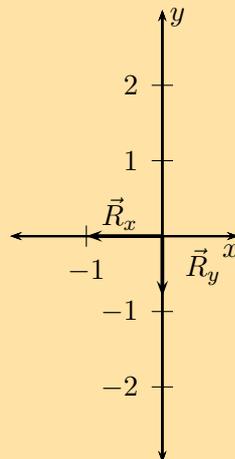Draw the Cartesian plane with the vectors in the x-direction:

This is $\vec{R}_x$ since it is the only vector in the x-direction.
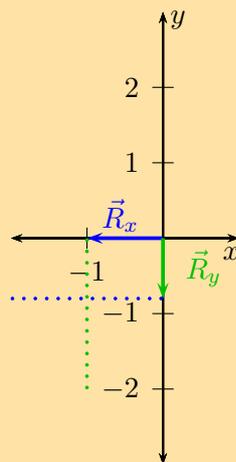
Secondly determine $\vec{R}_y$

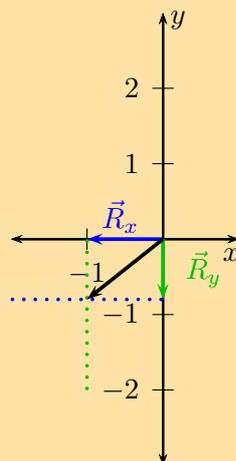Next we draw the Cartesian plane with the vectors in the y-direction:

Now we draw the resultant vectors, $\vec{R}_y$ and $\vec{R}_x$ tail-to-tail:

Now we can draw the lines to show us where the head of the resultant must be:

And finally we find the resultant:

Algebraic methods

Exercise 1 – 5: Algebraic addition of vectors

1. A force of 17 N in the positive x-direction acts simultaneously (at the same time) to a force of 23 N in the positive y-direction. Calculate the resultant force.

Solution:

We draw a rough sketch:
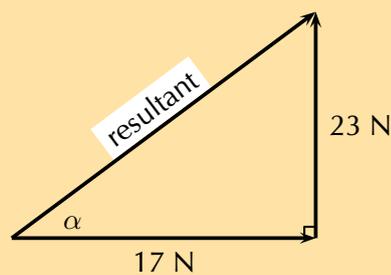
Now we determine the length of the resultant.s

We note that the triangle formed by the two force vectors and the resultant vector is a right-angle triangle. We can thus use the Theorem of Pythagoras to determine the length of the resultant. Let R represent the length of the resultant vector. Then:

$$F_x^2 + F_y^2 = R^2 \text{ Pythagoras' theorem}$$
$$(17)^2 + (23)^2 = R^2$$
$$R = 28{,}6 \text{ N}$$

To determine the direction of the resultant force, we calculate the angle α between the resultant force vector and the positive x-axis, by using simple trigonometry:

$$\tan \alpha = \frac{\text{opposite side}}{\text{adjacent side}}$$
$$\tan \alpha = \frac{23}{17}$$
$$\alpha = \tan^{-1}(1{,}353)$$
$$\alpha = 53{,}53°$$

The resultant force is then 28,6 N at $53{,}53°$ to the positive x-axis.

2. A force of 23,7 N in the negative x-direction acts simultaneously to a force of 9 N in the positive y-direction. Calculate the resultant force.

   Solution:

   We draw a rough sketch:
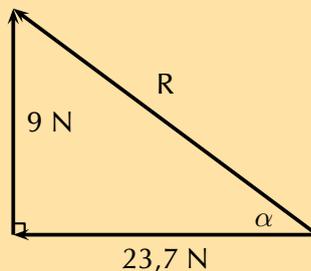
   Now we determine the length of the resultant.

   We note that the triangle formed by the two force vectors and the resultant vector is a right-angle triangle. We can thus use the Theorem of Pythagoras to determine the length of the resultant. Let R represent the length of the resultant vector. Then:

$$F_x^2 + F_y^2 = R^2 \text{ Pythagoras' theorem}$$
$$(23{,}7)^2 + (9)^2 = R^2$$
$$R = 25{,}4 \text{ N}$$

   To determine the direction of the resultant force, we calculate the angle α between the resultant force vector and the positive x-axis, by using simple trigonometry:

$$\tan \alpha = \frac{\text{opposite side}}{\text{adjacent side}}$$
$$\tan \alpha = \frac{9}{23{,}7}$$
$$\alpha = \tan^{-1}(0{,}3797)$$
$$\alpha = 20{,}79°$$

   The resultant force is then 25,4 N at $20{,}79°$ to the x-axis. However we actually give this as $159{,}21°$ (i.e. $180°$ minus $20{,}79°$) to keep with the convention we have defined of giving vector directions.

3. Four forces act simultaneously at a point, find the resultant if the forces are:

- $\vec{F_1}$ = 2,3 N in the positive x-direction
- $\vec{F_2}$ = 4 N in the positive y-direction
- $\vec{F_3}$ = 3,3 N in the negative y-direction
- $\vec{F_4}$ = 2,1 N in the negative y-direction

Solution:

We note that we have more than two vectors so we must first find the resultant in the x-direction and the resultant in the y-direction.

In the x-direction we only have one vector and so this is the resultant.

In the y-direction we have three vectors. We can add these algebraically to find $\vec{R_y}$:

$$\vec{F_1} = +4 \text{ N}$$
$$\vec{F_2} = -3{,}3 \text{ N}$$
$$\vec{F_3} = -2{,}1 \text{ N}$$

Thus, the resultant force is:

$$\vec{F_1} + \vec{F_2} + \vec{F_3} = (4) + (-3{,}3) + (-2{,}1)$$
$$= -1{,}4$$

Now we use $\vec{R_x}$ and $\vec{R_y}$ to find the resultant.

We note that the triangle formed by $\vec{R_x}$, $\vec{R_y}$ and the resultant vector is a right-angle triangle. We can thus use the Theorem of Pythagoras to determine the length of the resultant. Let R represent the length of the resultant vector. Then:

$$F_x^2 + F_y^2 = R^2 \text{ Pythagoras' theorem}$$
$$(2{,}3)^2 + (-1{,}4)^2 = R^2$$
$$R = 2{,}69 \text{ N}$$

To determine the direction of the resultant force, we calculate the angle α between the resultant force vector and the positive x-axis, by using simple trigonometry:

$$\tan \alpha = \frac{\text{opposite side}}{\text{adjacent side}}$$
$$\tan \alpha = \frac{1{,}4}{2{,}3}$$
$$\alpha = \tan^{-1}(0{,}6087)$$
$$\alpha = 31{,}33°$$

The resultant force is then 2,69 N at 31,33° to the x-axis. However we actually give this as 328,67° to keep with the convention we have defined of giving vector directions.

4. The following forces act simultaneously on a pole, if the pole suddenly snaps in which direction will it be pushed:

- $\vec{F_1}$ = 2,3 N in the negative x-direction
- $\vec{F_2}$ = 11,7 N in the negative y-direction
- $\vec{F_3}$ = 6,9 N in the negative y-direction
- $\vec{F_4}$ = 1,9 N in the negative y-direction

Solution:

To determine the answer we need to find the magnitude and direction of the resultant. This is then the direction in which the pole will be pushed.

We note that we have more than two vectors so we must first find the resultant in the x-direction and the resultant in the y-direction.

In the x-direction we only have one vector and so this is the resultant.

In the y-direction we have three vectors. We can add these algebraically to find $\vec{R_y}$:

$$\vec{F_1} = -11{,}7 \text{ N}$$
$$\vec{F_2} = -6{,}9 \text{ N}$$
$$\vec{F_3} = -1{,}9 \text{ N}$$

Thus, the resultant force is:

$$\vec{F_1} + \vec{F_2} + \vec{F_3} = (-11{,}7) + (-6{,}9) + (-1{,}9)$$
$$= -20{,}5$$

Now we use $\vec{R_x}$ and $\vec{R_y}$ to find the resultant.

We note that the triangle formed by $\vec{R_x}$, $\vec{R_y}$ and the resultant vector is a right-angle triangle. We can thus use the Theorem of Pythagoras to determine the length of the resultant. Let R represent the length of the resultant vector. Then:

$$F_x^2 + F_y^2 = R^2 \text{ Pythagoras' theorem}$$
$$(-2{,}3)^2 + (-20{,}5)^2 = R^2$$
$$R = 20{,}63 \text{ N}$$

A rough sketch will help to determine the direction.

To determine the direction of the resultant force, we calculate the angle α between the resultant force vector and the positive x-axis, by using simple trigonometry:

$$\tan \alpha = \frac{\text{opposite side}}{\text{adjacent side}}$$
$$\tan \alpha = \frac{2{,}3}{20{,}5}$$
$$\alpha = \tan^{-1}(0{,}112)$$
$$\alpha = 6{,}40°$$

The resultant force acts in a direction of $6{,}40°$ to the x-axis. However we actually give this as $186{,}4°$ to keep with the convention we have defined of giving vector directions.

1.3 Components of vectors

Exercise 1 – 6:

1. Resolve each of the following vectors into components:

   - $\vec{F_1} = 5$ N at $45°$ to the positive x-axis.
   - $\vec{F_2} = 15$ N at $63°$ to the positive x-axis.
   - $\vec{F_3} = 11{,}3$ N at $127°$ to the positive x-axis.
   - $\vec{F_4} = 125$ N at $245°$ to the positive x-axis.

   Solution:

   - Draw a rough sketch of the original vector (we use a scale of 1 N : 1 cm)
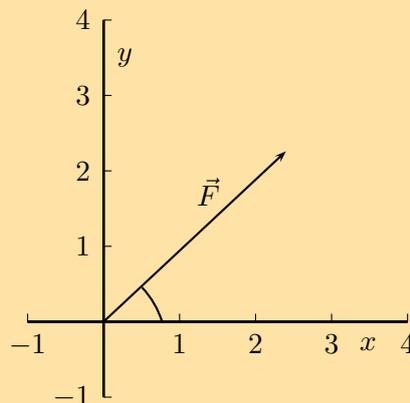
   Next we resolve the force into components parallel to the axes. Since these directions are perpendicular to one another, the components form a right-angled triangle with the original force as its hypotenuse.

   Now we can use trigonometry to calculate the magnitudes of the components of the original force:

$$F_y = 5\sin(45°)$$
$$= 3{,}54 \text{ N}$$

and

$$F_x = 5\cos(45°)$$
$$= 3{,}54 \text{ N}$$

- Draw a rough sketch of the original vector (we use a scale of 5 N : 1 cm)

Next we resolve the force into components parallel to the axes. Since these directions are perpendicular to one another, the components form a right-angled triangle with the original force as its hypotenuse.

Now we can use trigonometry to calculate the magnitudes of the components of the original force:

$$F_y = 15\sin(63°)$$
$$= 13{,}37 \text{ N}$$

and

$$F_x = 15\cos(63°)$$
$$= 6{,}81 \text{ N}$$

- Draw a rough sketch of the original vector (we use a scale of 4 N : 1 cm)

Next we resolve the force into components parallel to the axes. Since these directions are perpendicular to one another, the components form a right-angled triangle with the original force as its hypotenuse.

Now we can use trigonometry to calculate the magnitudes of the components of the original force:

$$F_y = 11{,}3 \sin(127°)$$
$$= 9{,}02 \text{ N}$$

and

$$F_x = 11{,}3 \cos(127°)$$
$$= -6{,}80 \text{ N}$$

- Draw a rough sketch of the original vector (we use a scale of 1 cm = 50 N)

Next we resolve the force into components parallel to the axes. Since these directions are perpendicular to one another, the components form a right-angled triangle with the original force as its hypotenuse.

Now we can use trigonometry to calculate the magnitudes of the components of the original force:

$$F_y = 125 \sin(245°)$$
$$= -113{,}29 \text{ N}$$

and

$$F_x = 125 \cos(245°)$$
$$= -52{,}83 \text{ N}$$

2. Resolve each of the following vectors into components:

- $\vec{F_1} = 11 \times 10^4$ N at 33° to the positive x-axis.
- $\vec{F_2} = 15$ GN at 28° to the positive x-axis.
- $\vec{F_3} = 11{,}3$ kN at 193° to the positive x-axis.
- $\vec{F_4} = 125 \times 10^5$ N at 317° to the positive x-axis.

Solution:

- Draw a rough sketch of the original vector (we use a scale of 1×10^4 N : 1 cm)
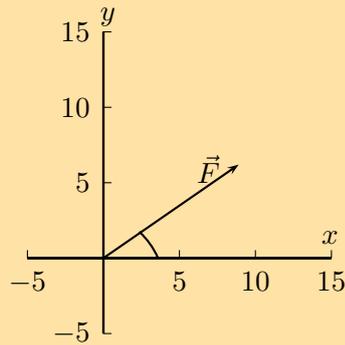
Next we resolve the force into components parallel to the axes. Since these directions are perpendicular to one another, the components form a right-angled triangle with the original force as its hypotenuse. We do not need to convert to N but we must remember to keep the same units throughout.

Now we can use trigonometry to calculate the magnitudes of the components of the original force:

$$F_y = 11 \times 10^4 \sin(33°)$$
$$= 5{,}99 \times 10^4 \text{ N}$$

and

$$F_x = 11 \times 10^4 \cos(33°)$$
$$= 9{,}22 \times 10^4 \text{ N}$$

- Draw a rough sketch of the original vector (we use a scale of 1 GN : 1 cm)

Next we resolve the force into components parallel to the axes. Since these directions are perpendicular to one another, the components form a right-angled triangle with the original force as its hypotenuse. We do not need to convert to N but we must remember to keep the same units throughout.

Now we can use trigonometry to calculate the magnitudes of the components of the original force:

$$F_y = 15 \sin(28°)$$
$$= 7{,}04 \text{ GN}$$

and

$$F_x = 15 \cos(28°)$$
$$= 13{,}2 \text{ GN}$$

- Draw a rough sketch of the original vector (we use a scale of 1 kN : 1 cm)
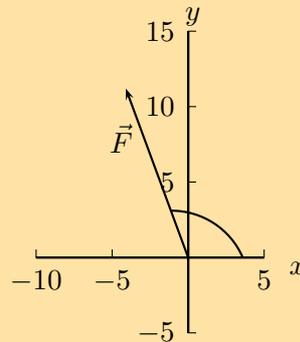
Next we resolve the force into components parallel to the axes. Since these directions are perpendicular to one another, the components form a right-angled triangle with the original force as its hypotenuse.

Now we can use trigonometry to calculate the magnitudes of the components of the original force:
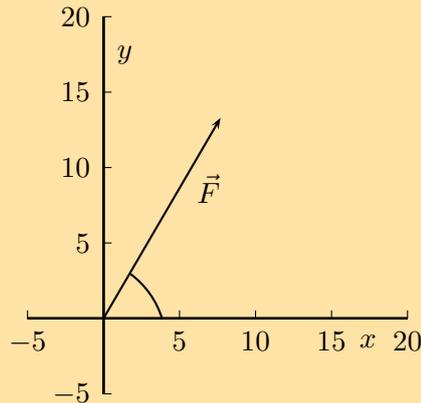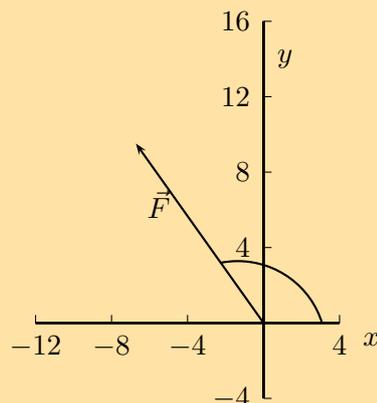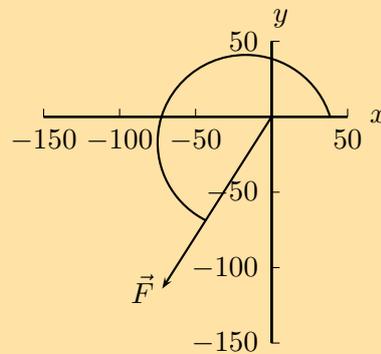
$$F_y = 11{,}3 \sin(103°)$$
$$= 11{,}01 \text{ kN}$$

and

$$F_x = 11{,}3 \cos(103°)$$
$$= -2{,}54 \text{ kN}$$

- Draw a rough sketch of the original vector (we use a scale of 50×10^5 N : 1 cm)
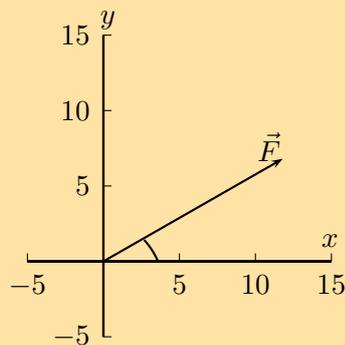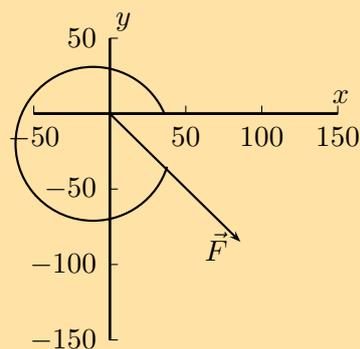

Next we resolve the force into components parallel to the axes. Since these directions are perpendicular to one another, the components form a right-angled triangle with the original force as its hypotenuse.

Now we can use trigonometry to calculate the magnitudes of the components of the original force:

$$F_y = 125 \times 10^5 \sin(317°)$$
$$= -8{,}52 \times 10^6 \text{ N}$$

and

$$F_x = 125 \times 10^5 \cos(317°)$$
$$= 9{,}14 \times 10^6 \text{ N}$$

1.4 Chapter summary

Exercise 1 – 7:

1. Draw the following forces as vectors on the Cartesian plane originating at the origin:

   - $\vec{F_1} = 3{,}7$ N in the positive x-direction
   - $\vec{F_2} = 4{,}9$ N in the positive y-direction

   Solution:

   We choose a scale of 1 N : 1 cm.
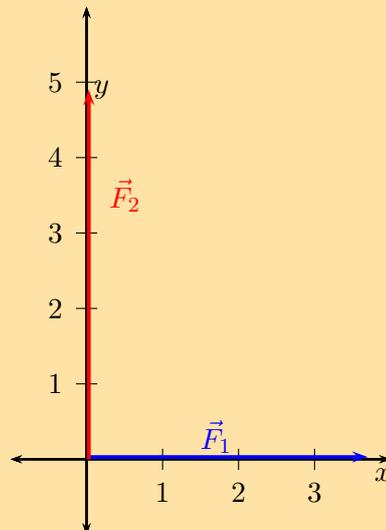
2. Draw the following forces as vectors on the Cartesian plane:

   - $\vec{F_1} = 4{,}3$ N in the positive x-direction
   - $\vec{F_2} = 1{,}7$ N in the negative x-direction
   - $\vec{F_3} = 8{,}3$ N in the positive y-direction
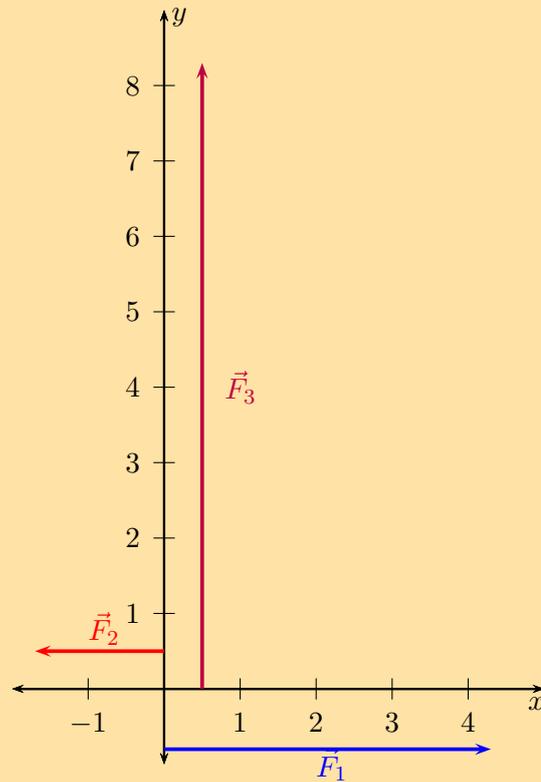
   Solution:

   We choose a scale of 1 N : 1 cm.

3. Find the resultant in the x-direction, R_x, and y-direction, R_y for the following forces:

- $\vec{F_1} = 1{,}5$ N in the positive x-direction
- $\vec{F_2} = 1{,}5$ N in the positive x-direction
- $\vec{F_3} = 2$ N in the negative x-direction

Solution:

We choose a scale of 1 N: 1 cm and for our diagram we will define the positive direction as *to the right*.

We note that there are no vectors in the y-direction and so $R_y = 0$.

We now find $\vec{R_x}$:
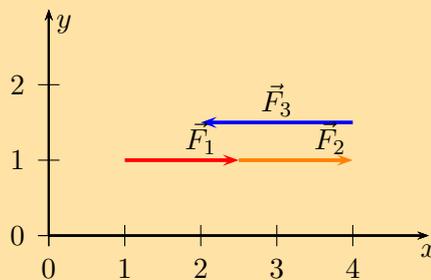
We have now drawn all the force vectors that act in the x-direction. To find $\vec{R_x}$ we note that the resultant vector is the arrow which starts at the tail of the first vector and ends at the head of the last drawn vector in that direction.

We note that $\vec{R}_x$ is 1 cm or 1 N in the positive x-direction.

$\vec{R}_x$ = 1 N and points in the positive x-direction. $\vec{R}_y$ = 0 N.

4. Find the resultant in the x-direction, R_x, and y-direction, R_y for the following forces:

- $\vec{F}_1 = 4{,}8$ N in the positive x-direction
- $\vec{F}_2 = 3{,}2$ N in the negative x-direction
- $\vec{F}_3 = 1{,}9$ N in the positive y-direction
- $\vec{F}_4 = 2{,}1$ N in the negative y-direction

Solution:

We choose a scale of 1 N: 1 cm and for our diagram we will define the positive direction as *to the right*.
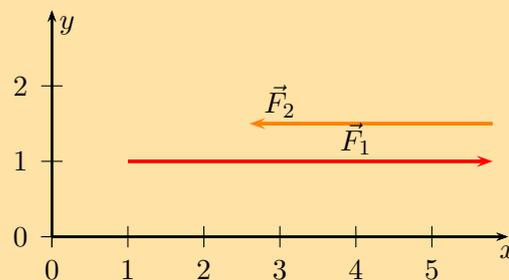
We find $\vec{R}_x$:

We have now drawn all the force vectors that act in the x-direction. To find $\vec{R}_x$ we note that the resultant vector is the arrow which starts at the tail of the first vector and ends at the head of the last drawn vector in that direction.
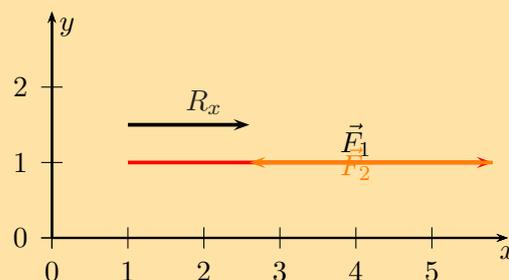
We note that $\vec{R}_x$ is 1,6 cm or 1,6 N in the positive x-direction.

We now find $\vec{R}_y$:
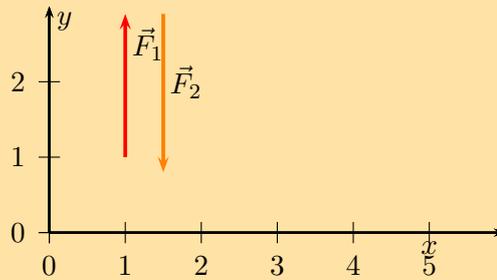
We have now drawn all the force vectors that act in the x-direction. To find $\vec{R}_x$ we note that the resultant vector is the arrow which starts at the tail of the first vector and ends at the head of the last drawn vector in that direction.
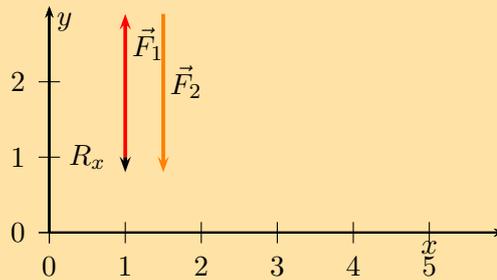
We note that $\vec{R}_y$ is 0,2 cm or 0,2 N in the negative y-direction.

$\vec{R}_x$ = 1,6 N and points in the positive x-direction. $\vec{R}_y$ = 0,2 N and points in the negative y-direction.

5. Find the resultant in the x-direction, R_x, and y-direction, R_y for the following forces:

- $\vec{F}_1 = 2,7$ N in the positive x-direction
- $\vec{F}_2 = 1,4$ N in the positive x-direction
- $\vec{F}_3 = 2,7$ N in the negative x-direction
- $\vec{F}_4 = 1,7$ N in the negative y-direction

Solution:

We choose a scale of 1 N : 1 cm and for our diagram we will define the positive direction as *to the right*.
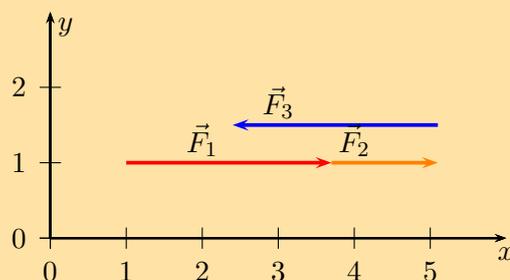
We find $\vec{R}_x$:

We have now drawn all the force vectors that act in the x-direction. To find $\vec{R}_x$ we note that the resultant vector is the arrow which starts at the tail of the first vector and ends at the head of the last drawn vector in that direction.
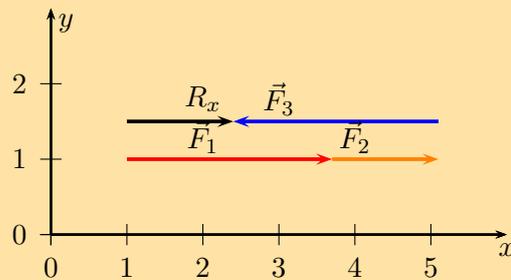
We note that $\vec{R}_x$ is 1,4 cm or 1,4 N in the positive x-direction.

There is only one vector in the y-direction and so this is $\vec{R}_y$.

$\vec{R}_x$ = 1,4 N and points in the positive x-direction. $\vec{R}_y$ = 1,7 N and points in the negative y-direction.

6. Sketch the resultant of the following force vectors using the tail-to-head method:

   - $\vec{F}_1 = 4{,}8$ N in the positive y-direction
   - $\vec{F}_2 = 3{,}3$ N in the negative x-direction

Solution:

We first sketch the vectors on the Cartesian plane. We choose a scale of 1 N : 1 cm. Remember to sketch F_2 starting at the head of F_1.
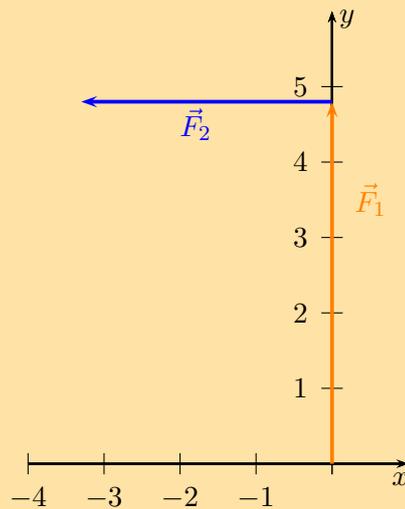
The resultant, $\vec{R}$, is the vector connecting the tail of the first vector drawn to the head of the last vector drawn:
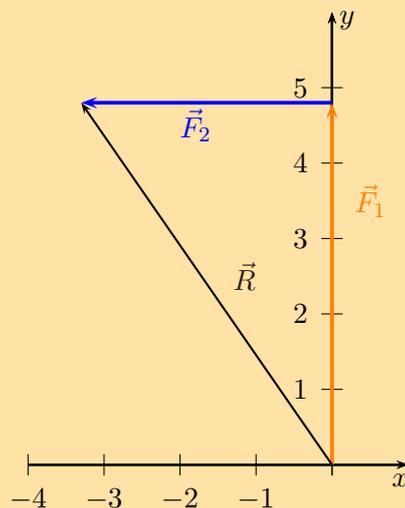
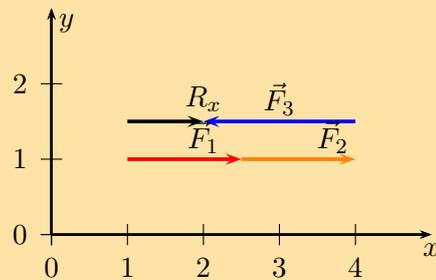7. Sketch the resultant of the following force vectors using the tail-to-head method:

- $\vec{F_1} = 0{,}7$ N in the positive y-direction
- $\vec{F_2} = 6$ N in the positive x-direction
- $\vec{F_3} = 3{,}8$ N in the negative y-direction
- $\vec{F_4} = 11{,}9$ N in the negative x-direction

Solution:

We first sketch the vectors on the Cartesian plane. We choose a scale of 1 N : 1 cm. Remember to sketch F_2 starting at the head of F_1.
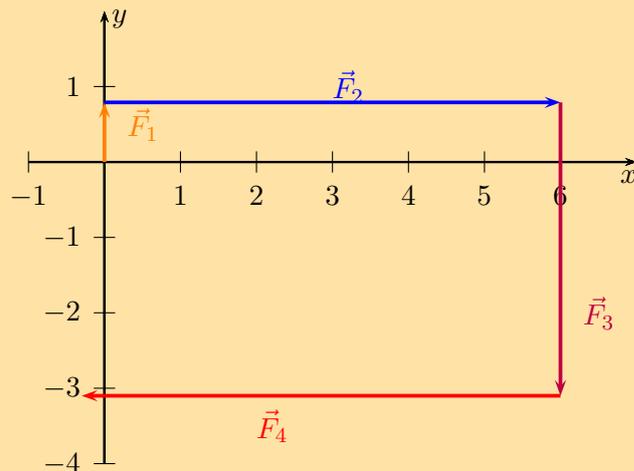
The resultant, $\vec{R}$, is the vector connecting the tail of the first vector drawn to the head of the last vector drawn:
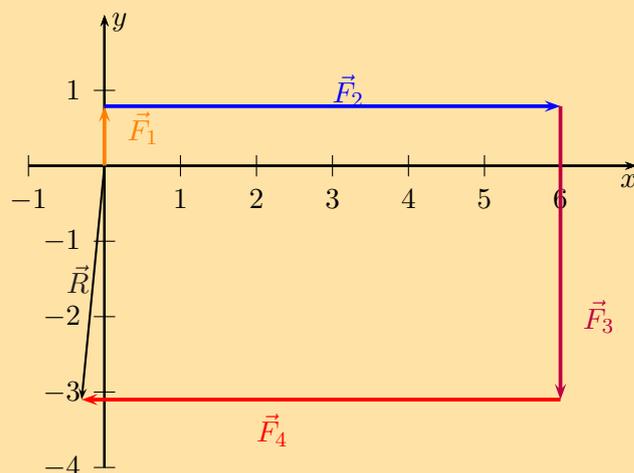
8. Sketch the resultant of the following force vectors using the tail-to-head method by first determining the resultant in the x- and y-directions:

- $\vec{F_1} = 5{,}2$ N in the positive y-direction
- $\vec{F_2} = 7{,}5$ N in the negative y-direction
- $\vec{F_3} = 4{,}8$ N in the positive y-direction
- $\vec{F_4} = 6{,}3$ N in the negative x-direction

Solution:

We choose a scale of 2 N : 1 cm.

We first determine $\vec{R}_x$
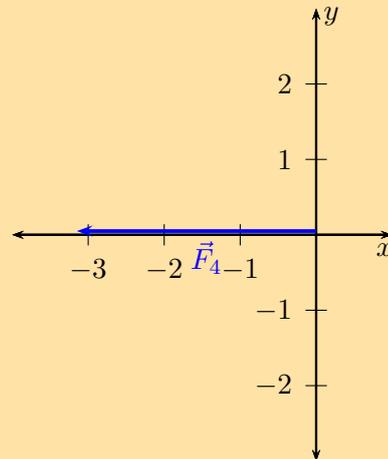
Draw the Cartesian plane with the vectors in the x-direction:

This is $\vec{R}_x$ since it is the only vector in the x-direction.

Secondly determine $\vec{R}_y$

Next we draw the Cartesian plane with the vectors in the y-direction:
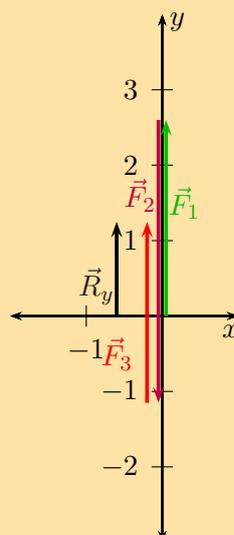
Now we draw the resultant vectors, $\vec{R}_y$ and $\vec{R}_x$ head-to-tail:
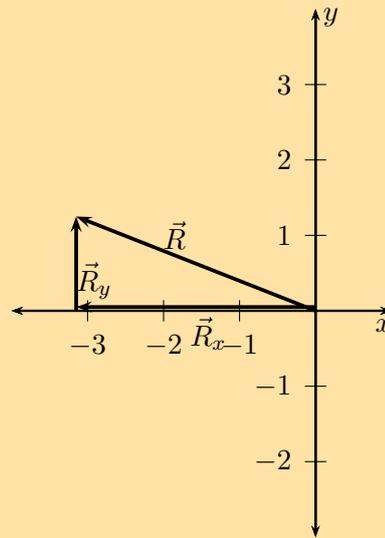
9. Sketch the resultant of the following force vectors using the tail-to-head method by first determining the resultant in the x- and y-directions:

- $\vec{F_1} = 6{,}7$ N in the positive y-direction
- $\vec{F_2} = 4{,}2$ N in the negative x-direction
- $\vec{F_3} = 9{,}9$ N in the negative y-direction
- $\vec{F_4} = 3{,}4$ N in the negative y-direction

Solution:

We choose a scale of 1 N : 1 cm.

We first determine $\vec{R_x}$
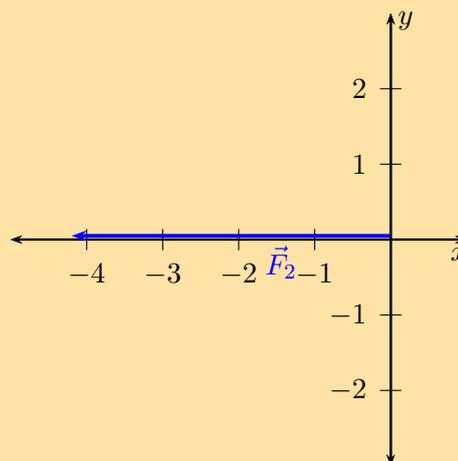
Draw the Cartesian plane with the vectors in the x-direction:

This is $\vec{R_x}$ since it is the only vector in the x-direction.

Secondly determine $\vec{R_y}$

Next we draw the Cartesian plane with the vectors in the y-direction:
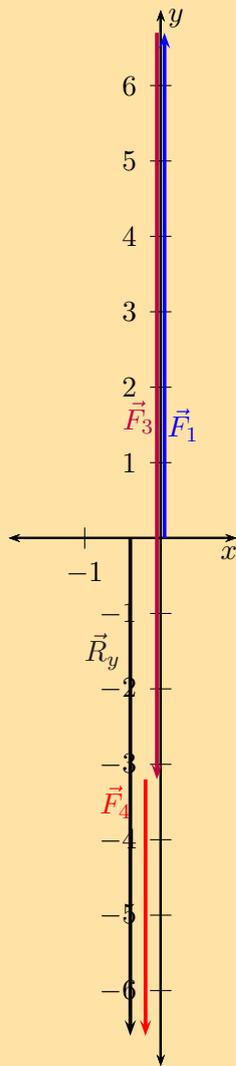
Now we draw the resultant vector, $\vec{R}_y$ and $\vec{R}_x$ head-to-tail:
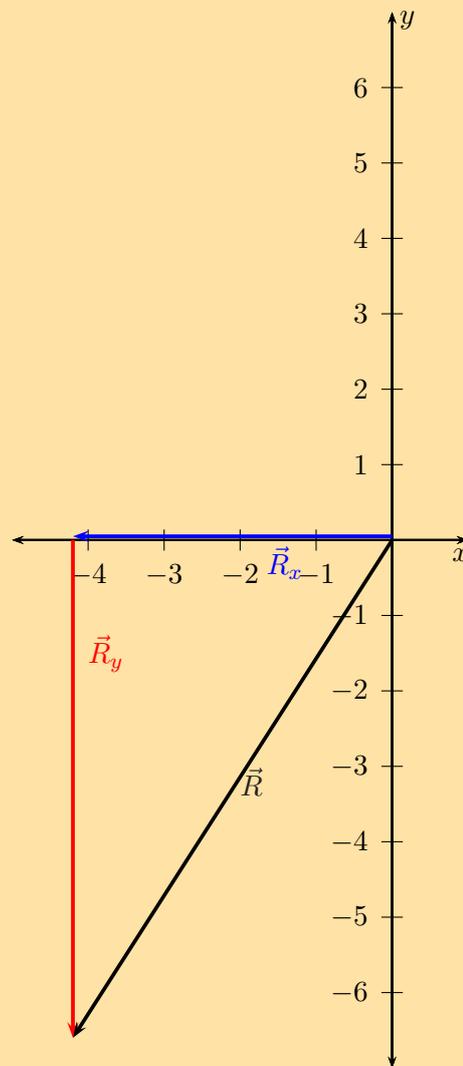
10. Sketch the resultant of the following force vectors using the tail-to-tail method:

- $\vec{F_1} = 6{,}1$ N in the positive y-direction

- $\vec{F_2} = 4{,}5$ N in the negative x-direction

Solution:

We first draw a Cartesian plane with the two vectors: (we use a scale of 1 N : 1 cm)
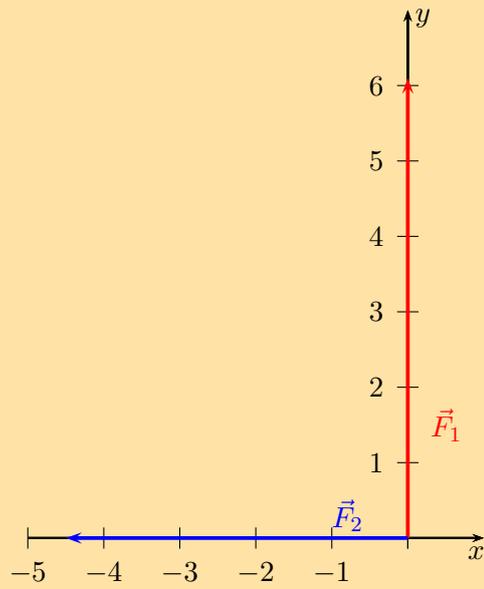
Now we draw a line parallel to $\vec{F_1}$ from the head of $\vec{F_2}$ and we draw a line parallel to $\vec{F_2}$ from the head of $\vec{F_1}$:
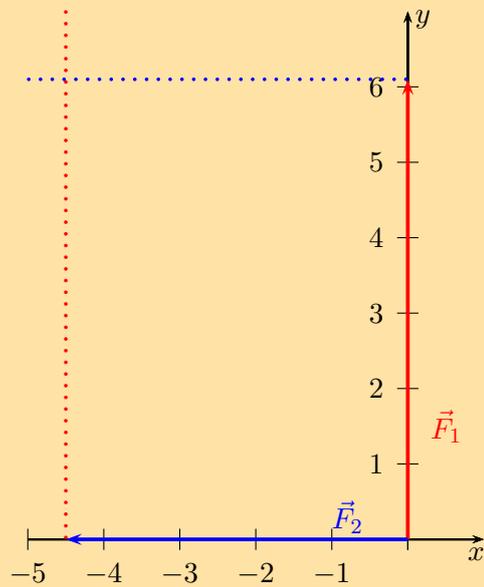
Where the two lines intersect is the head of the resultant vector which will originate at the origin so:
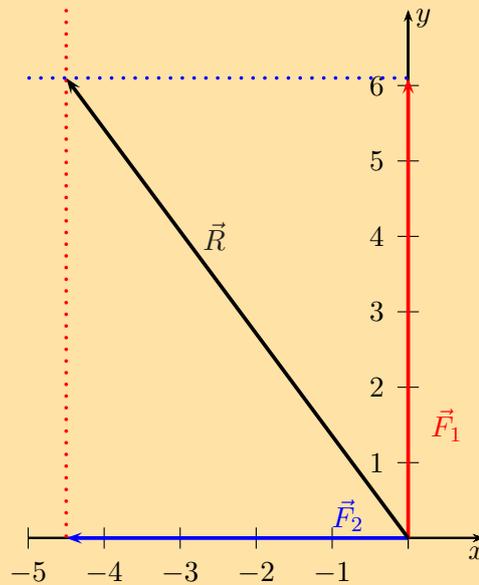
11. Sketch the resultant of the following force vectors using the tail-to-tail method by first determining the resultant in the x- and y-directions:

- $\vec{F_1} = 2{,}3$ N in the positive y-direction
- $\vec{F_2} = 11{,}8$ N in the negative y-direction
- $\vec{F_3} = 7{,}9$ N in the negative y-direction
- $\vec{F_4} = 3{,}2$ N in the negative x-direction

Solution:

We need to determine $\vec{R_x}$ and $\vec{R_y}$ and then use these to find the resultant.

Determine $\vec{R_x}$. We choose a scale of 2 N : 1 cm.

Draw the Cartesian plane with the vectors in the x-direction:
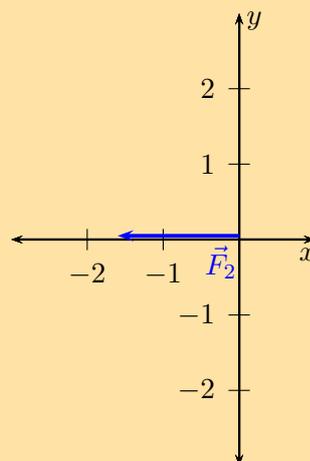
This is $\vec{R_x}$ since it is the only vector in the x-direction.

Secondly determine $\vec{R_y}$

Next we draw the Cartesian plane with the vectors in the y-direction:
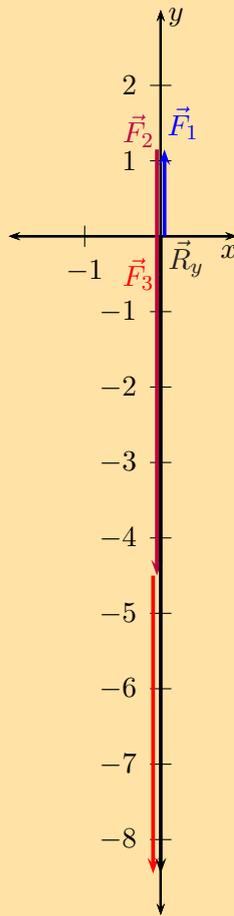
Now we draw the resultant vectors, $\vec{R}_y$ and $\vec{R}_x$ tail-to-tail:
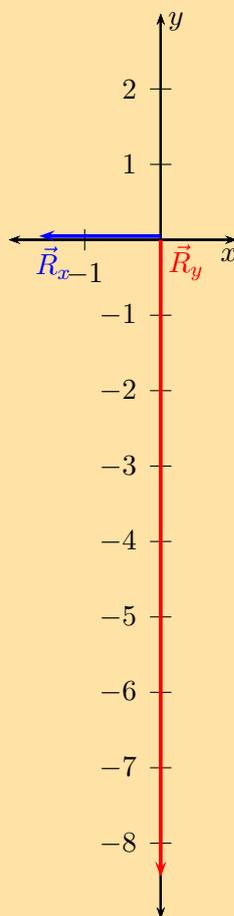
Now we can draw the lines to show us where the head of the resultant must be:
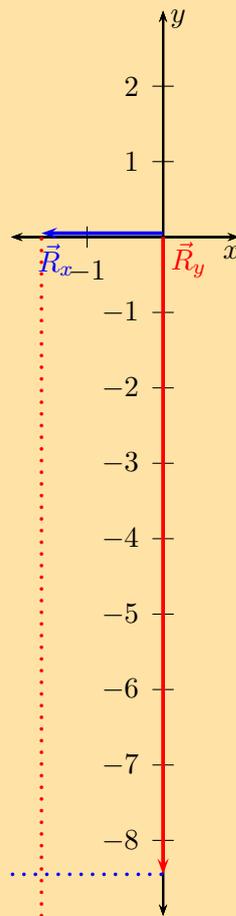
And finally we find the resultant:
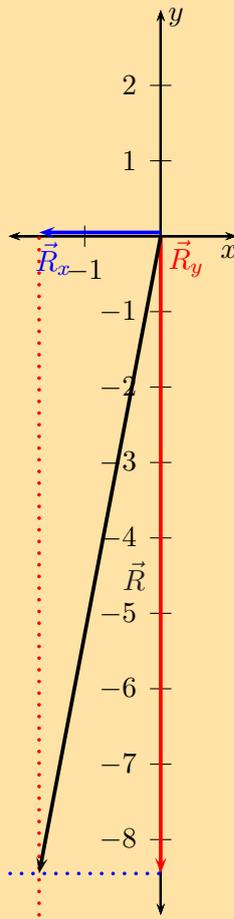
12. Four forces act simultaneously at a point, find the resultant if the forces are:

- $\vec{F_1}$ = 2,3 N in the positive x-direction
- $\vec{F_2}$ = 4,9 N in the positive y-direction
- $\vec{F_3}$ = 4,3 N in the negative y-direction
- $\vec{F_4}$ = 3,1 N in the negative y-direction

Solution:

We note that we have more than two vectors so we must first find the resultant in the x-direction and the resultant in the y-direction.

In the x-direction we only have one vector and so this is the resultant.

In the y-direction we have three vectors. We can add these algebraically to find $\vec{R_y}$:

$$\vec{F_2} = +4,9 \text{ N}$$
$$\vec{F_3} = -4,3 \text{ N}$$
$$\vec{F_4} = -3,1 \text{ N}$$

Thus, the resultant force is:

$$\vec{F_2} + \vec{F_3} + \vec{F_4} = (4,9) + (-4,3) + (-3,1)$$
$$= -2,3$$

Now we use $\vec{R}_x$ and $\vec{R}_y$ to find the resultant.

We note that the triangle formed by $\vec{R}_x$, $\vec{R}_y$ and the resultant vector is a right-angle triangle. We can thus use the Theorem of Pythagoras to determine the length of the resultant. Let R represent the length of the resultant vector. Then:

$$F_x^2 + F_y^2 = R^2 \text{ Pythagoras' theorem}$$
$$(2{,}3)^2 + (-2{,}3)^2 = R^2$$
$$R = 3{,}25 \text{ N}$$

To determine the direction of the resultant force, we calculate the angle α between the resultant force vector and the positive x-axis, by using simple trigonometry:

$$\tan \alpha = \frac{\text{opposite side}}{\text{adjacent side}}$$
$$\tan \alpha = \frac{2{,}3}{2{,}3}$$
$$\alpha = \tan^{-1}(1)$$
$$\alpha = 45°$$

The resultant force is then $3{,}25$ N at $45°$ to the x-axis.

13. Resolve each of the following vectors into components:

   a) $\vec{F}_1 = 105$ N at $23{,}5°$ to the positive x-axis.

   b) $\vec{F}_2 = 27$ N at $58{,}9°$ to the positive x-axis.

   c) $\vec{F}_3 = 11{,}3$ N at $323°$ to the positive x-axis.

   d) $\vec{F}_4 = 149$ N at $245°$ to the positive x-axis.

   e) $\vec{F}_5 = 15$ N at $375°$ to the positive x-axis.

   f) $\vec{F}_6 = 14{,}9$ N at $75{,}6°$ to the positive x-axis.

   g) $\vec{F}_7 = 11{,}3$ N at $123{,}4°$ to the positive x-axis.

   h) $\vec{F}_8 = 169$ N at $144°$ to the positive x-axis.

Solution:

   a) Draw a rough sketch of the original vector (we use a scale of 50 N : 1 cm)

Next we resolve the force into components parallel to the axes. Since these directions are perpendicular to one another, the components form a right-angled triangle with the original force as its hypotenuse.

Now we can use trigonometry to calculate the magnitudes of the components of the original force:

$$F_y = 105 \sin(23{,}5°)$$
$$= 41{,}87 \text{ N}$$

and

$$F_x = 105 \cos(23{,}5°)$$
$$= 96{,}29 \text{ N}$$

F_x = 96,29 N and F_y = 41,87 N

b) Draw a rough sketch of the original vector (we use a scale of 10 N : 1 cm)
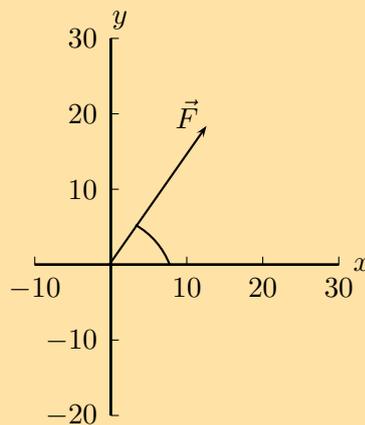
Next we resolve the force into components parallel to the axes. Since these directions are perpendicular to one another, the components form a right-angled triangle with the original force as its hypotenuse.

Now we can use trigonometry to calculate the magnitudes of the components of the original force:

$$F_y = 27 \sin(58{,}9°)$$
$$= 23{,}12 \text{ N}$$

and

$$F_x = 27 \cos(58{,}9°)$$
$$= 13{,}95 \text{ N}$$

F_x = 13,95 N and F_y = 23,12 N

c) Draw a rough sketch of the original vector (we use a scale of 5 N : 1 cm)
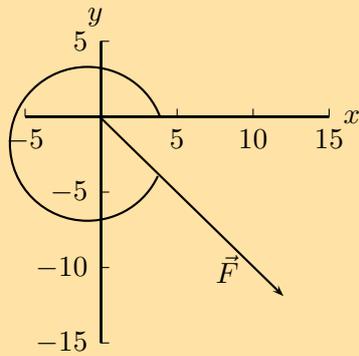
Next we resolve the force into components parallel to the axes. Since these directions are perpendicular to one another, the components form a right-angled triangle with the original force as its hypotenuse.

Now we can use trigonometry to calculate the magnitudes of the components of the original force:

$$F_y = 11{,}3 \sin(323°)$$
$$= -6{,}8 \text{ N}$$

and

$$F_x = 11{,}3 \cos(323°)$$
$$= 9{,}02 \text{ N}$$

$F_x = 9{,}02$ N and $F_y = -6{,}8$ N

d) Draw a rough sketch of the original vector (we use a scale of 5 N : 1 cm)
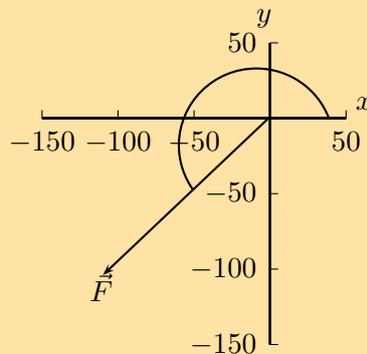
Next we resolve the force into components parallel to the axes. Since these directions are perpendicular to one another, the components form a right-angled triangle with the original force as its hypotenuse.

Now we can use trigonometry to calculate the magnitudes of the components of the original force:

$$F_y = 149 \sin(245°)$$
$$= -135{,}04 \text{ N}$$

and

$$F_x = 149 \cos(245°)$$
$$= -62{,}97 \text{ N}$$

$F_x = -62{,}97$ N and $F_y = -135{,}04$ N

e) Draw a rough sketch of the original vector (we use a scale of 5 N : 1 cm)
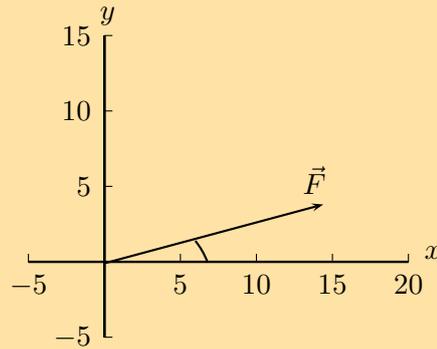
Next we resolve the force into components parallel to the axes. Since these directions are perpendicular to one another, the components form a right-angled triangle with the original force as its hypotenuse.

Now we can use trigonometry to calculate the magnitudes of the components of the original force:

$$F_y = 15\sin(375°)$$
$$= 3{,}89 \text{ N}$$

and

$$F_x = 15\cos(375°)$$
$$= 14{,}49 \text{ N}$$

Note that we would get the same result if we used $15°$ as $375° - 360° = 15°$.
$F_x = 3{,}89$ N and $F_y = 14{,}49$ N

f) Draw a rough sketch of the original vector (we use a scale of 2 N : 1 cm)
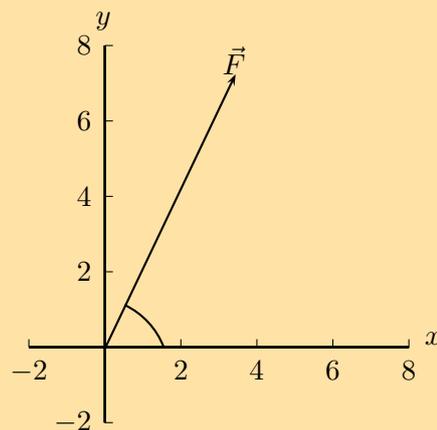
Next we resolve the force into components parallel to the axes. Since these directions are perpendicular to one another, the components form a right-angled triangle with the original force as its hypotenuse.

Now we can use trigonometry to calculate the magnitudes of the components of the original force:

$$F_y = 14{,}9\sin(75{,}6°)$$
$$= 14{,}43 \text{ N}$$

and

$$F_x = 14{,}9\cos(75{,}6°)$$
$$= 3{,}71 \text{ N}$$

F_x = 14,43 N and F_y = 3,71 N

g) Draw a rough sketch of the original vector (we use a scale of 5 N : 1 cm)
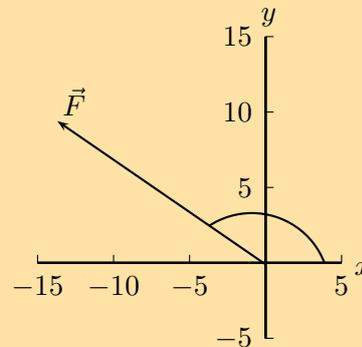
Next we resolve the force into components parallel to the axes. Since these directions are perpendicular to one another, the components form a right-angled triangle with the original force as its hypotenuse.

Now we can use trigonometry to calculate the magnitudes of the components of the original force:

$$F_y = 11{,}3\sin(123{,}4°)$$
$$= 9{,}43 \text{ N}$$

and

$$F_x = 11{,}3\cos(123{,}4°)$$
$$= -6{,}22 \text{ N}$$

F_x = −6,22 N and F_y = 9,43 N

h) Draw a rough sketch of the original vector (we use a scale of 50 N : 1 cm)
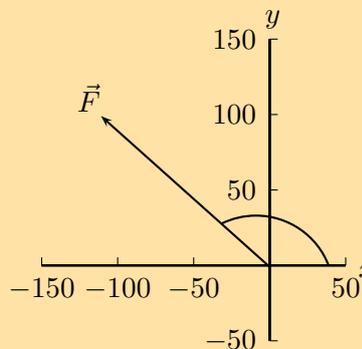
Next we resolve the force into components parallel to the axes. Since these directions are perpendicular to one another, the components form a right-angled triangle with the original force as its hypotenuse.

Now we can use trigonometry to calculate the magnitudes of the components of the original force:

$$F_y = 169\sin(144°)$$
$$= 99{,}34 \text{ N}$$

and

$$F_x = 169\cos(144°)$$
$$= -136{,}72 \text{ N}$$

$F_x = -136{,}72$ N and $F_y = 99{,}34$ N

14. A point is acted on by two forces and the resultant is zero. The forces

    a) have equal magnitudes and directions.

    b) have equal magnitudes but opposite directions.

    c) act perpendicular to each other.

    d) act in the same direction.

Solution:

have equal magnitudes but opposite directions.

15. A point in equilibrium is acted on by three forces. Force F_1 has components 15 N due south and 13 N due west. What are the components of force F_2?

    a) 13 N due north and 20 N due west

    b) 13 N due north and 13 N due west

    c) 15 N due north and 7 N due west

    d) 15 N due north and 13 N due east
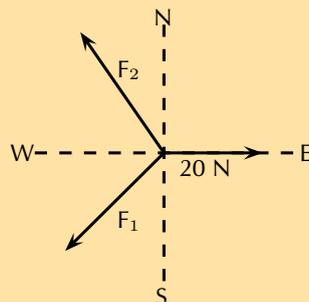
Solution:

15 N due north and 7 N due west

16. Two vectors act on the same point. What should the angle between them be so that a maximum resultant is obtained?

    a) 0°

    b) 90°

    c) 180°

    d) cannot tell

Solution:

$0°$

17. Two forces, 4 N and 11 N, act on a point. Which one of the following cannot be the magnitude of a resultant?

   a) 4 N

   b) 7 N

   c) 11 N

   d) 15 N

Solution:

4 N

18. An object of weight W is supported by two cables attached to the ceiling and wall as shown. The tensions in the two cables are T_1 and T_2 respectively. Tension $T_1 = 1200$ N. Determine the tension T_2 by accurate construction and measurement or by calculation.
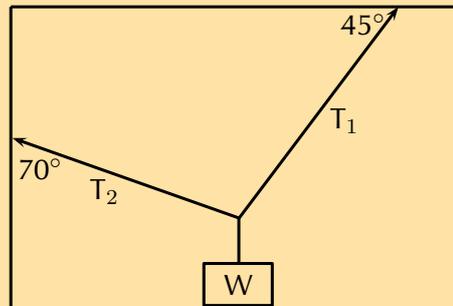
Solution:

We will use the x-component of T_2 the fact that $T_{1x} = T_{2x}$ to find T_2:

$$T_{2x} = T_{1x}$$
$$T_2 \cos(20°) = T_1 \cos(45°)$$
$$T_2 \cos(20°) = (1200)(\cos(45°))$$
$$T_2 \cos(20°) = 848{,}53$$
$$T_2 = 902{,}98 \text{ N}$$

19. An object X is supported by two strings, A and B, attached to the ceiling as shown in the sketch. Each of these strings can withstand a maximum force of 700 N. The weight of X is increased gradually.
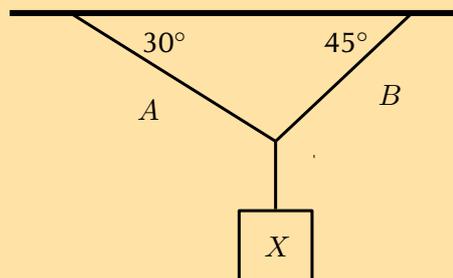
a) Draw a rough sketch of the triangle of forces, and use it to explain which string will break first.

b) Determine the maximum weight of X which can be supported.

Solution:

a) We draw a rough sketch:
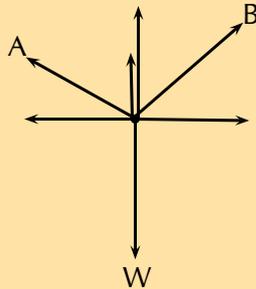
We note that $T_A \cos(30°) = T_B \cos(45°)$. Therefore:

$$\frac{T_A}{T_B} = \frac{\cos(45°)}{\cos(30°)}$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}$$

$$\therefore \frac{T_A}{T_B} < 1$$

$$\therefore T_A < T_B$$

The tension in B is greater and hence it will break first.

b) Since B will break first, the maximum weight that can be supported is when B is at a tension of 700 N.

$$\frac{T_A}{T_B} = \frac{\cos(45°)}{\cos(30°)}$$

$$\therefore T_A = 700 \times \frac{\cos(45°)}{\cos(30°)}$$

$$T_A = 572 \text{ N}$$

$$W = T_A \sin(30°) + T_B \sin(45°)$$

$$= 781 \text{ N}$$

CHAPTER 2

Newton's laws

2   Newton's laws

2.1   Introduction

In this chapter learners will explore Newtons three laws of motion and Newtons law of universal gravitation. Learners will also learn more about forces and the different kinds of forces. The following provides a summary of the topics covered in this chapter.

- **Different kinds of forces.**

  The types of forces covered are: normal force, frictional force, applied force and tension.

- **Force diagrams and free body diagrams.**

  In this section learners will see how to take a problem and draw diagrams to show all the forces acting on a body. They will learn what a force diagram is and what a free body diagram is.

- **Newton's three laws.**

  Newton's three laws of motion are discussed in this section. Each law is covered in detail and practical applications such as rockets, lifts and seat belts are covered.

- **Newton's law of universal gravitation.**

  This topic explores gravity and Newton's law of universal gravitation. Learners are introduced to the ideas of weight and mass as well as the acceleration due to gravity.

A recommended practical for informal assessment on normal forces and friction is included. In this experiment learners investigate the relationship between the normal force and the maximum static friction. Learners should use different surfaces to see the effect of static friction. If time allows learners can also investigate dynamic friction. You will need spring balances; several blocks, of the same material, with hooks attached to one end; several rough and smooth surfaces; and bricks or blocks to incline the surfaces.

A recommended experiment for formal assessment on Newton's second law of motion is also included in this chapter. In this experiment learners will investigate the relationship between force and acceleration (Newton's second law). You will need trolleys, different masses, inclined plane, rubber bands, meter ruler, ticker tape apparatus, ticker timer, graph paper.

2.2 Force

Different types of forces in physics

Exercise 2 – 1:

1. A box is placed on a rough surface. It has a normal force of magnitude 120 N. A force of 20 N applied to the right cannot move the box. Calculate the magnitude and direction of the friction forces.

   Solution:

   The normal force is given (120 N) and we know that the block does not move when an applied force of 20 N is used.

   We are asked to find the magnitude and direction of the friction forces.

   Since the box is stationary and the applied force is insufficient to move the box, we know that the kinetic friction force is 0 N.

   The frictional force is defined as the the force that opposes the motion of an object in contact with a surface and it acts parallel to the surface the object is in contact with. Therefore the static frictional force is 20 N to the left to give a resultant of zero.

2. A block rests on a horizontal surface. The normal force is 20 N. The coefficient of static friction between the block and the surface is 0,40 and the coefficient of dynamic friction is 0,20.

   a) What is the magnitude of the frictional force exerted on the block while the block is at rest?

   b) What will the magnitude of the frictional force be if a horizontal force of magnitude 5 N is exerted on the block?

   c) What is the minimum force required to start the block moving?

   d) What is the minimum force required to keep the block in motion once it has been started?

   e) If the horizontal force is 10 N, determine the frictional force.

   Solution:

   a) The block is not moving and so there is no kinetic frictional force acting on the block. There is also no force applied to the block and so the static frictional force is also zero.

   b) The block is not moving and so there is no kinetic frictional force acting on the block.

   The frictional force is defined as the the force that opposes the motion of an object in contact with a surface and it acts parallel to the surface the object is in contact with. Therefore the static frictional force is 5 N in the opposite direction to the applied force.

   c) To find the minimum force required to start the block moving we need to calculate the maximum static friction force:

$$F_s^{\max} = \mu_s N$$
$$= (0{,}40)(20)$$
$$= 8 \text{ N}$$

So we must apply a force of magnitude greater than 8 N to start the box moving.

d) To find the minimum force required to keep the block in motion we need to calculate the magnitude of the force of kinetic friction:

$$F_k = \mu_k N$$
$$= (0{,}20)(20)$$
$$= 4 \text{ N}$$

So we must apply a force of 4 N to keep the box in motion.

e) This force is greater than the force required to start the block moving. So the static frictional force is 0 N.

For kinetic friction the value of the frictional force remains the same regardless of the magnitude of the applied force. So the kinetic frictional force is 4 N.

3. A lady injured her back when she slipped and fell in a supermarket. She holds the owner of the supermarket accountable for her medical expenses. The owner claims that the floor covering was not wet and meets the accepted standards. He therefore cannot accept responsibility. The matter eventually ends up in court. Before passing judgement, the judge approaches you, a science student, to determine whether the coefficient of static friction of the floor is a minimum of $0{,}5$ as required. He provides you with a tile from the floor, as well as one of the shoes the lady was wearing on the day of the incident.

   a) Write down an expression for the coefficient of static friction.

   b) Plan an investigation that you will perform to assist the judge in his judgement. Follow the steps outlined below to ensure that your plan meets the requirements.

      i. Formulate an investigation question.
      ii. Apparatus: List *all* the other apparatus, except the tile and the shoe, that you will need.
      iii. A stepwise method: How will you perform the investigation? Include a relevant, labelled diagram.
      iv. Results: What will you record?
      v. Conclusion: How will you interpret the results to draw a conclusion?

   Solution:

   a)
   $$F_s \leq \mu_s N$$

   b) The following provides a guideline to what learners should do. This experiment is very similar to the experiment on normal forces and friction given above.

   Investigation question: What is the co-efficient of static friction for the tiles in the supermarket for different shoes and weights of people?

Apparatus: Spring balance, several blocks, of the same material, with hooks attached to one end, several rough and smooth surfaces, bricks or blocks to incline the surfaces (and the tile and shoe)

Method:

   i. Measure the frictional force for a minimum mass

   ii. Measure the frictional force for a maximum mass

   iii. Measure the frictional force for an average mass

   iv. Repeat each of the above measurements and take the average

   v. Determine the average co-efficient of friction

Results: You should record the results for each step described above

Conclusion: If the co-efficient of static friction is less than the minimum required than the supermarket owner is incorrect. If the co-efficient of static friction is greater than the minimum required than the lady is incorrect. If the co-efficient of static friction is the same as the minimum then other factors must be considered.

Resolving forces into components

Exercise 2 – 2:

1. A block on an inclined plane experiences a force due to gravity, $\vec{F}_g$ of 456 N straight down. If the slope is inclined at 67,8° to the horizontal, what is the component of the force due to gravity perpendicular and parallel to the slope?

   Solution:

   We know that for a block on a slope we can resolve the force due to gravity, $\vec{F}_g$ into components parallel and perpendicular to the slope.

   $$F_{gx} = F_g \sin(\theta)$$
   $$F_{gy} = F_g \cos(\theta)$$

   The slope is inclined at an angle of 67,8°. This is the same angle we need to use to calculate the components, therefore:

   $$\begin{aligned} F_{gx} &= F_g \sin(\theta) \\ &= (456)\sin(67{,}8°) \\ &= 422{,}20 \text{ N} \end{aligned}$$

   $$\begin{aligned} F_{gy} &= F_g \cos(\theta) \\ &= (456)\cos(67{,}8°) \\ &= 172{,}30 \text{ N} \end{aligned}$$

   The component of $\vec{F}_g$ that is perpendicular to the slope is $\vec{F}_{gy} = 172{,}30$ N in the negative y-direction.

   The component of $\vec{F}_g$ that is perpendicular to the slope is $\vec{F}_{gx} = 422{,}20$ N in the negative x-direction.

2. A block on an inclined plane is subjected to a force due to gravity, $\vec{F}_g$ of 456 N straight down. If the component of the gravitational force parallel to the slope is $\vec{F}_{gx} = 308{,}7$ N in the negative x-direction (down the slope), what is the incline of the slope?

Solution:

We know that for a block on a slope we can resolve the force due to gravity, $\vec{F}_g$ into components parallel and perpendicular to the slope.

$$F_{gx} = F_g \sin(\theta)$$
$$F_{gy} = F_g \cos(\theta)$$

We are given the horizontal component of gravity and the force due to gravity. We can use these to find the incline of the slope:

$$F_{gx} = F_g \sin(\theta)$$
$$308{,}7 = (456)\sin\theta$$
$$\sin\theta = 0{,}6769\ldots$$
$$\theta = 42{,}61°$$

The slope is at an incline of $42{,}61°$.

Finding the resultant force

Exercise 2 – 3: Forces and motion

1. A boy pushes a shopping trolley (weight due to gravity of 150 N) with a constant force of 75 N. A constant frictional force of 20 N is present.

   a) Draw a labelled force diagram to identify all the forces acting on the shopping trolley.
   b) Draw a free body diagram of all the forces acting on the trolley.
   c) Determine the resultant force on the trolley.

   Solution:
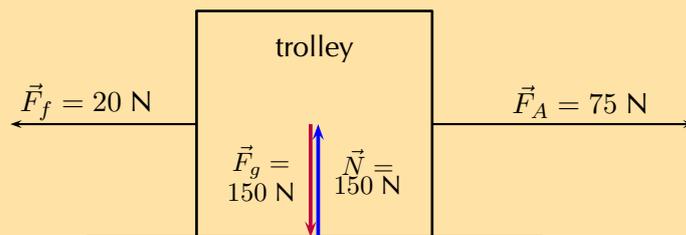
   a)
   Where:

$\vec{F}_f$ frictional force

$\vec{F}_A$ applied force

$\vec{F}_g$ gravitational force

$\vec{N}$ normal force

$$\vec{N} = 150 \text{ N}$$

$\vec{F}_f = 20$ N $\qquad \vec{F}_A = 75$ N

$\vec{F}_g = 150$ N

b)
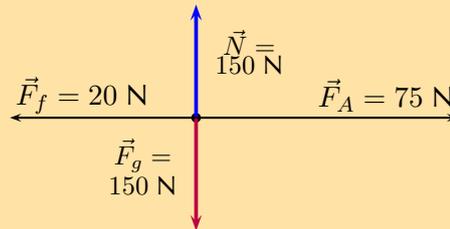
Where:

$\vec{F}_f$ frictional force

$\vec{F}_A$ applied force

$\vec{F}_g$ gravitational force

$\vec{N}$ normal force

c) The normal force and the gravitational force are balanced and produced no net force on the trolley.

We need to determine the resultant force using the force of friction and the applied force. We choose to the right as positive and assume that the applied force is applied to the right, while the frictional force acts to the left.

$$\vec{F}_R = \vec{F}_f + \vec{F}_A$$
$$= -20 + 75$$
$$= 55 \text{ N to the right}$$

2. A donkey (experiencing a gravitational force of 2500 N) is trying to pull a cart (force due to gravity of 800 N) with a force of 400 N. The rope between the donkey and the cart makes an angle of 30° with the cart. The cart does not move.

   a) Draw a free body diagram of all the forces acting on the donkey.

   b) Draw a force diagram of all the forces acting on the cart.

   c) Find the magnitude and direction of the frictional force preventing the cart from moving.

   Solution:

N: Upward force of ground on donkey (2500 N)

60°

F_1: Force of cart on donkey (60° downwards from horizontal)

F_g: Downward gravitational force of the Earth on the donkey (2500 N)

a)

b)
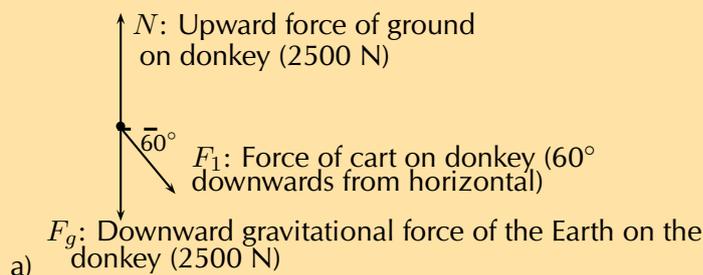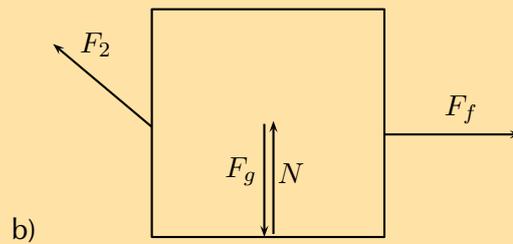
Where:

$\vec{F}_f$ frictional force

$\vec{F}_2$ force of cart on donkey

$\vec{F}_g$ gravitational force

$\vec{N}$ normal force

c) The cart is not moving so this is static friction only. We note that the frictional force acts in the x-direction only and will be equal to the x component of the force applied by the donkey. The direction will be in the opposite direction that the donkey is pulling. The frictional force is:

$$\vec{F}_f = F_A \cos\theta$$
$$= 400\cos(30)$$
$$= 346{,}41 \text{ N in the opposite direction to the applied force}$$

2.3   Newton's laws

Newton's first law

Exercise 2 − 4:

1. If a passenger is sitting in a car and the car turns round a bend to the right, what happens to the passenger? What happens if the car turns to the left?

   Solution:

   Before the car starts turning both the passenger and the car are travelling at the same velocity.

   As the car turns to the right a force acts on the car but not the passengers, hence (by Newton's first law) the passenger continues moving with the same original velocity. (In other words the car turns, but the passenger does not).

   The net result of this is that the passenger is pulled to the left as the car turns right.

   If the car instead turned to the left the passenger would be pulled to the right.

2. Helium is less dense than the air we breathe. Discuss why a helium balloon in a car driving around a corner appears to violate Newton's first law and moves towards the inside of the turn not the outside like a passenger.

Solution:

As the car goes around the corner all the air keeps moving forward (it acts in the same way that a passenger acts). This causes the air pressure on the one side of the car to increase (this will be on the opposite side to the direction the car is turning). This slight increase in the air pressure pushes the helium balloon to the other side of the car.

Because of this it appears that the helium balloon does not obey Newton's first law.

Newton's second law of motion

Exercise 2 – 5:

1. A tug is capable of pulling a ship with a force of 100 kN. If two such tugs are pulling on one ship, they can produce any force ranging from a minimum of 0 kN to a maximum of 200 kN. Give a detailed explanation of how this is possible. Use diagrams to support your result.

Solution:

We start off with the two tug boats pulling in opposite directions:
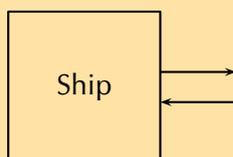
The resultant force is 0 kN since the tug boats are pulling with equal forces in opposite directions.

If the two tugboats pull in the same direction then we get:
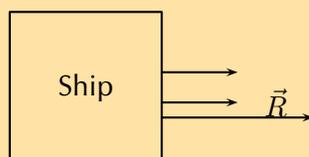
The resultant force is 200 kN since the tug boats are pulling with equal forces in the same direction.

To get any force between these two extremes, one tugboat will have to be pulling on the ship at a different angle to the second tugboat, for example:
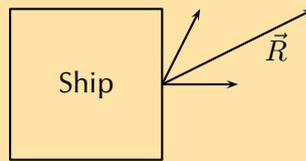
Note that the resultant force in this situation is less than 200 kN (You can check this by using any of the vector addition techniques).

2. A car of mass 850 kg accelerates at 2 m·s^{-2}. Calculate the magnitude of the resultant force that is causing the acceleration.

   Solution:

   $$F = ma$$
   $$= (850)(2)$$
   $$= 1700 \text{ N}$$

3. Find the force needed to accelerate a 3 kg object at 4 m·s^{-2}.

   Solution:

   $$F = ma$$
   $$= (3)(4)$$
   $$= 12 \text{ N}$$

4. Calculate the acceleration of an object of mass 1000 kg accelerated by a force of magnitude 100 N.

   Solution:

   $$F = ma$$
   $$a = \frac{F}{m}$$
   $$= \frac{100}{1000}$$
   $$= 0{,}1 \text{ m·s}^{-2}$$

5. An object of mass 7 kg is accelerating at 2,5 m·s^{-2}. What resultant force acts on it?

   Solution:

   $$F = ma$$
   $$= (7)(2{,}5)$$
   $$= 17{,}5 \text{ N}$$

6. Find the mass of an object if a force of 40 N gives it an acceleration of 2 m·s^{-2}.

Solution:

$$F = ma$$
$$m = \frac{F}{a}$$
$$= \frac{40}{2}$$
$$= 20 \text{ kg}$$

7. Find the acceleration of a body of mass 1000 kg that has a force with a magnitude of 150 N acting on it.

   Solution:

$$F = ma$$
$$a = \frac{F}{m}$$
$$= \frac{150}{1000}$$
$$= 0{,}15 \text{ m·s}^{-2}$$

8. Find the mass of an object which is accelerated at 3 m·s^{-2} by a force of magnitude 25 N.

   Solution:

$$F = ma$$
$$m = \frac{F}{a}$$
$$= \frac{25}{3}$$
$$= 8{,}33 \text{ kg}$$

9. Determine the acceleration of a mass of 24 kg when a force of magnitude 6 N acts on it. What is the acceleration if the force were doubled and the mass was halved?

   Solution:

   We first find the acceleration:

$$F = ma$$
$$a = \frac{F}{m}$$
$$= \frac{6}{24}$$
$$= 0{,}25 \text{ m·s}^{-2}$$

   If the force is doubled and the mass is halved then the acceleration will be four times this amount: 1 m·s^{-2}.

   You can check this by doubling the force and halving the mass.

10. A mass of 8 kg is accelerating at 5 m·s^{-2}.

   a) Determine the resultant force that is causing the acceleration.

   b) What acceleration would be produced if we doubled the force and reduced the mass by half?

   Solution:

   a)

$$F = ma$$
$$= (8)(5)$$
$$= 40 \text{ N}$$

   b) If we doubled the force and halved the mass we would get four times the acceleration or 20 m·s^{-2}. You can check this by carrying out the calculation using double the force and half the mass.

11. A motorcycle of mass 100 kg is accelerated by a resultant force of 500 N. If the motorcycle starts from rest:

   a) What is its acceleration?

   b) How fast will it be travelling after 20 s?

   c) How long will it take to reach a speed of 35 m·s^{-1}?

   d) How far will it travel from its starting point in 15 s?

   Solution:

   a)

$$F = ma$$
$$a = \frac{F}{m}$$
$$= \frac{500}{100}$$
$$= 5 \text{ m·s}^{-2}$$

   b) We can use the equations of motion (recall from grade 10: motion in one dimension) to determine how fast it will be travelling:

$$v_f = v_i + at$$
$$= 0 + (5)(20)$$
$$= 100 \text{ m·s}^{-1}$$

   c) Again, we can use the equations of motion (recall from grade 10: motion in one dimension) to determine how long it takes to reach the given speed:

$$v_f = v_i + at$$
$$35 = 0 + (5)t$$
$$t = \frac{35}{5}$$
$$= 7 \text{ s}$$

d) Again, we can use the equations of motion (recall from grade 10: motion in one dimension) to determine how far it travels:

$$\Delta x = v_i t + \frac{1}{2}at^2$$
$$35 = (0)(15) + \frac{1}{2}(5)(15)^2$$
$$= 562,5 \text{ m}$$

12. A force of 200 N, acting at 60° to the horizontal, accelerates a block of mass 50 kg along a horizontal plane as shown.
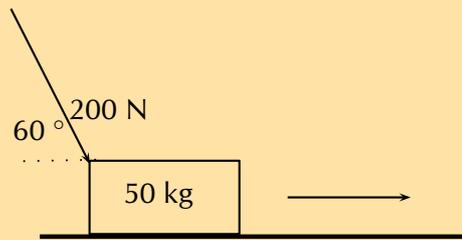
a) Calculate the component of the 200 N force that accelerates the block horizontally.

b) If the acceleration of the block is 1,5 m·s⁻², calculate the magnitude of the frictional force on the block.

c) Calculate the vertical force exerted by the block on the plane.

Solution:

a)

$$F_x = F \cos \theta$$
$$= (200) \cos(60)$$
$$= 100 \text{ N}$$

b)

$$F_R = ma$$
$$F_x + F_f = ma$$
$$100 + F_f = (50)(1,5)$$
$$F_f = 75 - 100$$
$$= -25 \text{ N}$$

c)

$$F_y = F \sin \theta$$
$$= (200) \sin(60)$$
$$= 173,2 \text{ N}$$

13. A toy rocket experiences a force due to gravity of magnitude 4,5 N is supported vertically by placing it in a bottle. The rocket is then ignited. Calculate the force that is required to accelerate the rocket vertically upwards at 8 m·s⁻².

Solution:

Taking the upwards direction as positive:

$$F_R = ma$$
$$F_1 + F_g = ma$$
$$F_1 + -4{,}5 = (0{,}5)(8)$$
$$F_1 = 4 + 4{,}5$$
$$= 8{,}5 \text{ N}$$

14. A constant force of magnitude 70 N is applied vertically to a block as shown. The block experiences a force due to gravity of 49 N. Calculate the acceleration of the block.
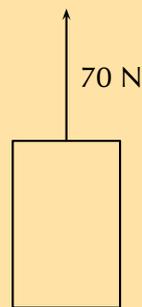
Solution:

Taking the upwards direction as positive:

$$F_R = ma$$
$$F_1 + F_g = ma$$
$$70 - 49 = 5a$$
$$21 = 5a$$
$$a = 4{,}2 \text{ m·s}^{-2}$$

15. A student experiences a gravitational force of magnitude 686 N investigates the motion of a lift. While he stands in the lift on a bathroom scale (calibrated in newton), he notes three stages of his journey.

   a) For 2 s immediately after the lift starts, the scale reads 574 N.

   b) For a further 6 s it reads 686 N.

   c) For the final 2 s it reads 854 N.

Answer the following questions:

   a) Is the motion of the lift upward or downward? Give a reason for your answer.

   b) Write down the magnitude and the direction of the resultant force acting on the student for each of the stages 1, 2 and 3.

Solution:

a) Stage 1: Downwards. The scale reads less than the gravitational force that he experiences.

Stage 2: Stationary. The scale reads the same as the gravitational force that he experiences.

Stage 3: Upwards. The scale reads more than the gravitational force that he experiences.

b) Stage 1: 574 N downwards

Stage 2: The resultant force on the student is zero.

Stage 3: 854 N upwards

16. A car of mass 800 kg accelerates along a level road at 4 m·s^{-2}. A frictional force of 700 N opposes its motion. What force is produced by the car's engine?

Solution:

$$F_R = ma$$
$$F_f + F_E = ma$$
$$700 + F_E = (800)(4)$$
$$F_E = 3200 - 700$$
$$= 2500 \text{ N}$$

17. Two objects, with masses of 1 kg and 2 kg respectively, are placed on a smooth surface and connected with a piece of string. A horizontal force of 6 N is applied with the help of a spring balance to the 1 kg object. Ignoring friction, what will the force acting on the 2 kg mass, as measured by a second spring balance, be?
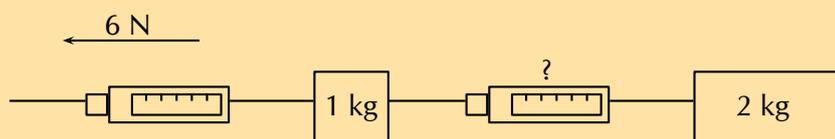
Solution:

The force acting on the 2 kg block is 6 N. Since the surface is assumed to be frictionless, the applied force on the 1 kg block is equal to the force experienced by the 2 kg block.

18. A rocket of mass 200 kg has a resultant force of 4000 N upwards on it.

a) What is its acceleration on the Earth, where it experiences a gravitational force of 1960 N?

b) What driving force does the rocket engine need to exert on the back of the rocket on the Earth?

Solution:

a) The force on the rocket is in the upwards direction, while the force due to gravity is in the downwards direction. Taking upwards as positive:

$$F_R = ma$$
$$F_g + F_{\text{rocket}} = ma$$
$$-1960 + 4000 = ma$$
$$a = \frac{2040}{200}$$
$$= 10{,}2 \text{ m·s}^{-2}$$

b) On Earth the rocket engines need to overcome the gravitational force and so need to exert a force of 1960 N or greater.

19. A car going at 20 m·s^{-1} accelerates uniformly and comes to a stop in a distance of 20 m.

    a) What is its acceleration?

    b) If the car is 1000 kg how much force do the brakes exert?

    Solution:

    a) We can use the equations of motion to find the acceleration:

    $$v_f^2 = v_i^2 + 2a\Delta x$$
    $$0 = (20)^2 + 2(20)a$$
    $$40a = -400$$
    $$a = -10 \text{ m·s}^{-2}$$

    b)

    $$F = ma$$
    $$= (1000)(10)$$
    $$= 10\ 000 \text{ N}$$

20. A block on an inclined plane experiences a force due to gravity, $\vec{F}_g$ of 300 N straight down. If the slope is inclined at $67{,}8°$ to the horizontal, what is the component of the force due to gravity perpendicular and parallel to the slope? At what angle would the perpendicular and parallel components of the force due to gravity be equal?

    Solution:

    The component parallel to the slope is:

    $$F_{gx} = F \sin\theta$$
    $$= (300)\sin(67{,}8)$$
    $$= 277{,}76 \text{ N}$$

    The component perpendicular to the slope is:

    $$F_{gy} = F \cos\theta$$
    $$= (300)\cos(67{,}8)$$
    $$= 113{,}35 \text{ N}$$

    For the two components to be equal the angle must be $45°$. ($\sin(45) = \cos(45)$).

21. A block on an inclined plane is subjected to a force due to gravity, $\vec{F}_g$ of 287 N straight down. If the component of the gravitational force parallel to the slope is $\vec{F}_{gx} = 123{,}7$ N in the negative x-direction (down the slope), what is the incline of the slope?

Solution:

$$F_{gx} = F \sin \theta$$
$$123{,}7 = (287) \sin \theta$$
$$\sin \theta = 0{,}431 \dots$$
$$\theta = 25{,}53°$$

22. A block on an inclined plane experiences a force due to gravity, $\vec{F}_g$ of 98 N straight down. If the slope is inclined at an unknown angle to the horizontal but we are told that the ratio of the components of the force due to gravity perpendicular and parallel to the slope is 7:4. What is the angle the incline makes with the horizontal?

Solution:

We first write down the equations for the parallel and perpendicular components:

$$\begin{aligned} F_{gx} &= F \sin \theta \\ &= (98) \sin \theta \end{aligned}$$

$$\begin{aligned} F_{gy} &= F \cos \theta \\ &= (98) \cos \theta \end{aligned}$$

Now we note the following:

$$7F_{gx} = 4F_{gy}$$
$$\therefore 7(98) \sin \theta = 4(98) \cos \theta$$

Now we need to solve for theta:

$$\begin{aligned} \sin \theta &= \frac{392}{686} \cos \theta \\ &= 0{,}5714 \cos \theta \end{aligned}$$
$$\frac{\sin \theta}{\cos \theta} = 0{,}5714$$
$$\tan \theta = 0{,}5714$$
$$\theta = 25{,}53°$$

Recall from trigonometry that $\frac{\sin \theta}{\cos \theta} = \tan \theta$.

23. Two crates, 30 kg and 50 kg respectively, are connected with a thick rope according to the diagram. A force, to the right, of 1500 N is applied. The boxes move with an acceleration of 7 m·s^{-2} to the right. The ratio of the frictional forces on the two crates is the same as the ratio of their masses. Calculate:
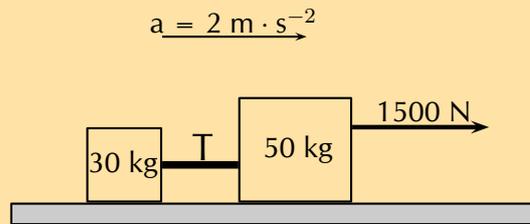
$$a = 2 \; \text{m} \cdot \text{s}^{-2}$$

a) the magnitude and direction of the total frictional force present.

b) the magnitude of the tension in the rope at T.

Solution:

a) Let the 50 kg crate be crate 1 and the 30 kg crate be crate 2.

To find the frictional force we will apply Newton's second law. We are given the mass (30 + 50 kg) and the acceleration (2 m·s^{-2}). Choose the direction of motion to be the positive direction (to the right is positive).

$$F_R = ma$$
$$F_{\text{applied}} + F_f = ma$$
$$1500 + F_f = (30 + 50)(2)$$
$$F_f = 160 - 1500$$
$$F_f = -1340 \; \text{N}$$

b) We note that $m_1 = \frac{5}{3}m_2$, therefore $F_{f1} = \frac{5}{3}F_{f2}$.

To find the tension in the rope we need to look at one of the two crates on their own. Let's choose the 30 kg crate.

The frictional force on the 30 kg block is noted above. We can calculate F_{f2}:

$$F_f = F_{f1} + F_{f2}$$
$$1340 = F_{f2} + \frac{5}{3}F_{f2}$$
$$1340 = \frac{8}{3}F_{f2}$$
$$F_{f2} = 502{,}5 \; \text{N}$$

If we apply Newton's second law:

$$F_R = ma$$
$$T + F_f = (30)(2)$$
$$T + -502{,}5 = 60$$
$$T = 562{,}5 \; \text{N}$$

24. Two crates, 30 kg and 50 kg respectively, are connected with a thick rope according to the diagram. If they are dragged up an incline such that the ratio of the parallel and perpendicular components of the gravitational force on each block are 3 : 5. The boxes move with an acceleration of 7 m·s^{-2} up the slope. The ratio of the frictional forces on the two crates is the same as the ratio of their masses. The magnitude of the force due to gravity on the 30 kg crate is 294 N and on the 50 kg crate is 490 N. Calculate:
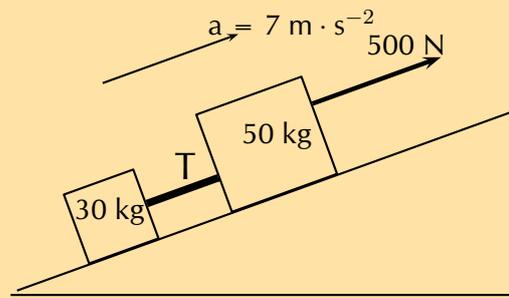
a) the magnitude and direction of the total frictional force present.

b) the magnitude of the tension in the rope at T.

Solution:

a) Let crate 1 be the 50 kg crate and crate 2 be the 30 kg crate. We will choose up the slope as positive.

We need to find all the forces that act parallel to the slope on each box in turn and use this to solve simultaneously for the frictional force.

We draw free body diagrams for each crate:

First we find the gravitational force parallel to the slope for each box:

$$F_{gx1} = \frac{3}{5}F_{gy1}$$
$$F_{g1}^2 = F_{gx1}^2 + F_{gy1}^2$$
$$= F_{gx1}^2 + \frac{9}{25}F_{gx1}^2$$
$$= \frac{34}{25}F_{gx1}^2$$
$$(490)^2 = \frac{34}{25}F_{gx1}^2$$
$$F_{gx1}^2 = 176\ 533{,}1176$$
$$F_{gx1} = 420{,}17\ \text{N}$$

$$F_{gx2} = \frac{3}{5}F_{gy2}$$
$$F_{g2}^2 = F_{gx2}^2 + F_{gy2}^2$$
$$= F_{gx2}^2 + \frac{9}{25}F_{gx2}^2$$
$$= \frac{34}{25}F_{gx2}^2$$
$$(294)^2 = \frac{34}{25}F_{gx2}^2$$
$$F_{gx2}^2 = 63\ 555{,}88$$
$$F_{gx1} = 252{,}10\ \text{N}$$

Now we can write an expression for the resultant force on each crate:

$$\vec{F_R} = ma$$
$$F_A - F_{gx1} - F_f - T = ma$$
$$500 - 420{,}17 - F_{f1} - T = (50)(7)$$
$$-T = 420{,}17 + F_f$$

$$\vec{F}_R = ma$$
$$T - F_{gx2} - F_f = ma$$
$$T - 252{,}10 - F_f = (30)(7)$$
$$T = 462{,}1 + F_f$$

And solve simultaneously for the frictional force:

$$-T + T = 420{,}17 + F_f + 462{,}1 + F_f$$
$$0 = 882{,}27 + 2F_f$$
$$F_f = -441{,}14 \text{ N}$$

b) We can use either of the two expressions above to find the tension. We will use the expression for crate 1:

$$-T = 420{,}17 + F_f$$
$$= 420{,}17 + -441{,}14$$
$$T = 20{,}97 \text{ N}$$

Newton's third law of motion

Exercise 2 – 6:

1. A fly hits the front windscreen of a moving car. Compared to the magnitude of the force the fly exerts on the windscreen, the magnitude of the force the windscreen exerts on the fly during the collision, is:

    a) zero.

    b) smaller, but not zero.

    c) bigger.

    d) the same.

    Solution:

    the same

2. Which of the following pairs of forces correctly illustrates Newton's third law?

A
A man standing still
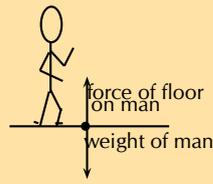
B
A book pushed against a wall

Force of book on wall

Force of wall on book

force of floor on man

weight of man
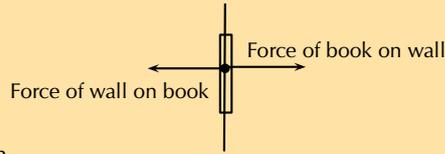
C
A bird flying at a constant height and velocity

Weight of the bird

The weight of the bird = force of Earth on bird
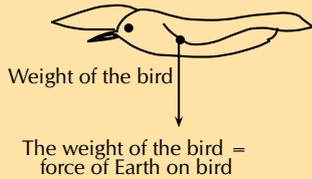
D
A crate moving at constant speed

Force used to push the crate

frictional force exerted by the floor
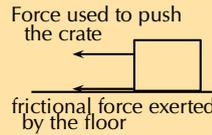
Solution:

A or B

2.4   Forces between masses

Newton's law of universal gravitation

Exercise 2 − 7:

1. When the planet Jupiter is closest to Earth it is $6,28 \times 10^8$ km away. If Jupiter has a mass of $1,9 \times 10^{27}$ kg, what is the magnitude of the gravitational force between Jupiter and the Earth?

   Solution:

   The mass of the Earth is: $5,98 \times 10^{24}$ kg. The magnitude of the gravitational force is:

   $$F = G\frac{m_1 m_2}{d^2}$$
   $$= (6,67 \times 10^{-11}) \left(\frac{(5,98 \times 10^{24})(1,9 \times 10^{27})}{(6,28 \times 10^{11})^2} \right)$$
   $$= 1,92 \times 10^{18} \text{ N}$$

2. When the planet Jupiter is furthest from the Earth it is $9,28 \times 10^8$ km away. If Jupiter has a mass of $1,9 \times 10^{27}$ kg, what is the magnitude of the gravitational force between Jupiter and the Earth?

   Solution:

The mass of the Earth is: $5{,}98 \times 10^{24}$ kg. The magnitude of the gravitational force is:

$$F = G\frac{m_1 m_2}{d^2}$$
$$= \left(6{,}67 \times 10^{-11}\right) \left(\frac{(5{,}98 \times 10^{24})(1{,}9 \times 10^{27})}{(9{,}28 \times 10^{11})^2}\right)$$
$$= 8{,}80 \times 10^{17} \text{ N}$$

3. What distance must a satellite with a mass of 80 kg be away from the Earth to feel a force of 1000 N? How far from Jupiter to feel the same force?

   Solution:

   The mass of the Earth is: $5{,}98 \times 10^{24}$ kg and the mass of Jupiter is $1{,}9 \times 10^{27}$ kg. We first find how far the satellite must be from Earth:

$$F = G\frac{m_1 m_2}{d^2}$$
$$1000 = \left(6{,}67 \times 10^{-11}\right) \left(\frac{(5{,}98 \times 10^{24})(80)}{(d)^2}\right)$$
$$1000 = \frac{3{,}19 \times 10^{16}}{(d)^2}$$
$$d^2 = 3{,}19 \times 10^{13}$$
$$d = 5{,}65 \times 10^6 \text{ m}$$

   Next we find how far the satellite must be from Jupiter:

$$F = G\frac{m_1 m_2}{d^2}$$
$$1000 = \left(6{,}67 \times 10^{-11}\right) \left(\frac{(1{,}9 \times 10^{27})(80)}{(d)^2}\right)$$
$$1000 = \frac{1{,}01 \times 10^{19}}{(d)^2}$$
$$d^2 = 1{,}01 \times 10^{16}$$
$$d = 1{,}00 \times 10^8 \text{ m}$$

4. The radius of Jupiter is $71{,}5 \times 10^3$ km and the radius of the moon is $1{,}7 \times 10^3$ km, if the moon has a mass of $7{,}35 \times 10^{22}$ kg work out the gravitational acceleration on Jupiter and on the moon.

   Solution:

   We work out the gravitational acceleration on Jupiter (remember to convert the distance to m):

$$a_o = G\frac{M_{\text{Jupiter}}}{d^2_{\text{Jupiter}}}$$
$$= \left(6{,}67 \times 10^{-11}\right) \left(\frac{1{,}9 \times 10^{27}}{(71{,}5 \times 10^6)^2}\right)$$
$$= 24{,}8 \text{ m·s}^{-2}$$

We work out the gravitational acceleration on the moon (remember to convert the distance to m):

$$a_o = G\frac{M_{\text{Moon}}}{d_{\text{Moon}}^2}$$

$$= \left(6{,}67 \times 10^{-11}\right)\left(\frac{7{,}35 \times 10^{22}}{(1{,}7 \times 10^6)^2}\right)$$

$$= 1{,}7 \text{ m·s}^{-2}$$

5. Astrology, NOT astronomy, makes much of the position of the planets at the moment of one's birth. The only known force a planet exerts on Earth is gravitational. Calculate:

   a) the gravitational force exerted on a 4,20 kg baby by a 100 kg father 0,200 m away at birth

   b) the force on the baby due to Jupiter if it is at its closest distance to Earth, some $6{,}29 \times 10^{11}$ m away.

   c) How does the force of Jupiter on the baby compare to the force of the father on the baby?

   Solution:

   a)

   $$F = G\frac{m_1 m_2}{d^2}$$

   $$= \left(6{,}67 \times 10^{-11}\right)\left(\frac{(4{,}20)(100)}{(0{,}200)^2}\right)$$

   $$= 7 \times 10^{-7} \text{ N}$$

   b)

   $$F = G\frac{m_1 m_2}{d^2}$$

   $$= \left(6{,}67 \times 10^{-11}\right)\left(\frac{(1{,}9 \times 10^{27})(4{,}2)}{(6{,}29 \times 10^{11})^2}\right)$$

   $$= 1{,}35 \times 10^{-6} \text{ N}$$

   c) The force of the father on the baby is 0,52 times the force of Jupiter on the baby. (We calculate this using: $\frac{F_{\text{father}}}{F_{\text{Jupiter}}}$)

6. The existence of the dwarf planet Pluto was proposed based on irregularities in Neptune's orbit. Pluto was subsequently discovered near its predicted position. But it now appears that the discovery was fortuitous, because Pluto is small and the irregularities in Neptune's orbit were not well known. To illustrate that Pluto has a minor effect on the orbit of Neptune compared with the closest planet to Neptune:

   a) Calculate the acceleration due to gravity at Neptune due to Pluto when they are $4{,}50 \times 10^{12}$ m apart, as they are at present. The mass of Pluto is $1{,}4 \times 10^{22}$ kg and the mass of Neptune is: $1{,}02 \times 10^{26}$ kg.

   b) Calculate the acceleration due to gravity at Neptune due to Uranus, presently about $2{,}50 \times 10^{12}$ m apart, and compare it with that due to Pluto. The mass of Uranus is $8{,}62 \times 10^{25}$ kg.

Solution:

a)

$$F = G\frac{m_1 m_2}{d^2}$$
$$= \left(6{,}67 \times 10^{-11}\right) \left(\frac{(1{,}4 \times 10^{22})(1{,}02 \times 10^{26})}{(4{,}50 \times 10^{12})^2}\right)$$
$$= 4{,}7 \times 10^{12} \text{ N}$$

b)

$$F = G\frac{m_1 m_2}{d^2}$$
$$= \left(6{,}67 \times 10^{-11}\right) \left(\frac{(8{,}62 \times 10^{25})(1{,}02 \times 10^{26})}{(2{,}50 \times 10^{12})^2}\right)$$
$$= 9{,}4 \times 10^{16} \text{ N}$$

Weight and mass

Exercise 2 − 8:

1. Jojo has a mass of $87{,}5$ kg, what is his weight on the following planets:

   a) Mercury (radius of $2{,}440 \times 10^3$ km and mass of $3{,}3 \times 10^{23}$ kg)

   b) Mars (radius of $3{,}39 \times 10^3$ km and mass of $6{,}42 \times 10^{23}$ kg)

   c) Neptune (radius of $24{,}76 \times 10^3$ km and mass of $1{,}03 \times 10^{26}$ kg)?

Solution:

   a) We first find the gravitational acceleration:

$$a_o = G\frac{M_{\text{Mercury}}}{d^2_{\text{Mercury}}}$$
$$= \left(6{,}67 \times 10^{-11}\right) \left(\frac{3{,}3 \times 10^{23}}{(2{,}440 \times 10^6)^2}\right)$$
$$= 3{,}70 \text{ m·s}^{-2}$$

   Now we can calculate Jojo's weight:

$$\vec{F}_g = m\vec{a}_o$$
$$= (87{,}5)(3{,}70)$$
$$= 323{,}75 \text{ N}$$

   b) We first find the gravitational acceleration:

$$a_o = G\frac{M_{\text{Mars}}}{d^2_{\text{Mars}}}$$

$$= \left(6{,}67 \times 10^{\text{-}11}\right)\left(\frac{6{,}42 \times 10^{23}}{(3{,}39 \times 10^6)^2}\right)$$

$$= 3{,}73 \text{ m·s}^{-2}$$

Now we can calculate Jojo's weight:

$$\vec{F}_g = m\vec{a}_o$$

$$= (87{,}5)(3{,}73)$$

$$= 326{,}04 \text{ N}$$

c) We first find the gravitational acceleration:

$$a_o = G\frac{M_{\text{Neptune}}}{d^2_{\text{Neptune}}}$$

$$= \left(6{,}67 \times 10^{\text{-}11}\right)\left(\frac{1{,}03 \times 10^{26}}{(24{,}76 \times 10^6)^2}\right)$$

$$= 11{,}21 \text{ m·s}^{-2}$$

Now we can calculate Jojo's weight:

$$\vec{F}_g = m\vec{a}_o$$

$$= (87{,}5)(11{,}21)$$

$$= 980{,}88 \text{ N}$$

2. If object 1 has a weight of $1{,}78 \times 10^3$ N on Neptune and object 2 has a weight of $3{,}63 \times 10^5$ N on Mars, which has the greater mass?

Solution:

We begin by calculating the gravitational acceleration for each planet:

$$a_o = G\frac{M_{\text{Neptune}}}{d^2_{\text{Neptune}}}$$

$$= \left(6{,}67 \times 10^{\text{-}11}\right)\left(\frac{1{,}03 \times 10^{26}}{(24{,}76 \times 10^6)^2}\right)$$

$$= 11{,}21 \text{ m·s}^{-2}$$

$$a_o = G\frac{M_{\text{Mars}}}{d^2_{\text{Mars}}}$$

$$= \left(6{,}67 \times 10^{\text{-}11}\right)\left(\frac{6{,}42 \times 10^{23}}{(3{,}39 \times 10^6)^2}\right)$$

$$= 3{,}73 \text{ m·s}^{-2}$$

Now we can calculate each objects mass:

$$\vec{F_g} = m_1 \vec{a_o}$$
$$1{,}78 \times 10^3 = 11{,}21 m_1$$
$$m_1 = 158{,}79 \text{ kg}$$

$$\vec{F_g} = m_1 \vec{a_o}$$
$$3{,}63 \times 10^5 = 3{,}73 m_2$$
$$m_2 = 9{,}73 \times 10^4 \text{ kg}$$

Object 2 has the greater mass.

Comparative problems

Exercise 2 – 9:

1. Two objects of mass $2X$ and $3X$ respectively, where X is an unknown quantity, exert a force F on each other when they are a certain distance apart. What will be the force between two objects situated the same distance apart but having a mass of $5X$ and $6X$ respectively?

    a) 0,2 F
    b) 1,2 F
    c) 2,2 F
    d) 5 F

Solution:

5 F.

We can write an expression for the force in each case in terms of the distance:

$$F_1 = G\frac{m_1 m_2}{d^2}$$
$$d^2 = \frac{G(2X)(3X)}{F_1}$$
$$F_2 = G\frac{m_1 m_2}{d^2}$$
$$d^2 = \frac{G(6X)(5X)}{F_2}$$

Since the distance is equal, the square of the distance is also equal and so we can equate these two and find the force.

$$\frac{G(2X)(3X)}{F_1} = \frac{G(6X)(5X)}{F_2}$$
$$\frac{6X^2}{F_1} = \frac{30X^2}{F_2}$$
$$F_2 = 5F_1$$

2. As the distance of an object above the surface of the Earth is greatly increased, the weight of the object would

    a) increase

    b) decrease

    c) increase and then suddenly decrease

    d) remain the same

    Solution:

    decrease

    The distance is inversely proportional to the weight (or force of gravity) and so as the distance increases the weight decreases.

3. A satellite circles around the Earth at a height where the gravitational force is a factor 4 less than at the surface of the Earth. If the Earth's radius is R, then the height of the satellite above the surface is:

    a) R

    b) 2 R

    c) 4 R

    d) 16 R

    Solution:

    The gravitational force on the satellite on the Earth's surface is:

    $$F_{surface} = G\frac{M_{Earth}m}{R^2}$$

    The gravitational force on the satellite in orbit is:

    $$F_{orbit} = G\frac{M_{Earth}m}{r^2}$$

    We know that the force from the Earth when the satellite is in orbit is a factor 4 less, therefore:

    $$F_{orbit} = \frac{1}{4}F_{surface}$$
    $$G\frac{M_{Earth}m}{r^2} = \frac{1}{4}G\frac{M_{Earth}m}{R^2}$$
    $$r = 2R$$

    The question asks for the distance above the Earth's surface so the answer is $r - R = 2R - R = R$.

4. A satellite experiences a force F when at the surface of the Earth. What will be the force on the satellite if it orbits at a height equal to the diameter of the Earth:

    a) $\frac{1}{F}$

    b) $\frac{1}{2}$ F

c) $\frac{1}{3}$ F

d) $\frac{1}{9}$ F

Solution:

The diameter of the Earth is twice the radius. This means that the distance to the satellite would be the radius of the Earth plus twice the radius of the Earth, a factor 3 increase so $\frac{1}{9}$.

5. The weight of a rock lying on surface of the Moon is W. The radius of the Moon is R. On planet Alpha, the same rock has weight 8W. If the radius of planet Alpha is half that of the Moon, and the mass of the Moon is M, then the mass, in kg, of planet Alpha is:

   a) $\frac{M}{2}$

   b) $\frac{M}{4}$

   c) 2 M

   d) 4 M

Solution:

2 M

6. Consider the symbols of the two physical quantities g and G used in Physics.

   a) Name the physical quantities represented by g and G.

   b) Derive a formula for calculating g near the Earth's surface using Newton's law of universal gravitation. M and R represent the mass and radius of the Earth respectively.

Solution:

   a) g is the acceleration due to gravity on the Earth and G is the universal gravitational constant.

   b) We note the following two formulae:

$$F_1 = G\frac{m_1 m_2}{d^2}$$

$$F = G\frac{m_o M}{R^2}$$

$$a_o = \frac{F}{m_o}$$

   Rearranging the second equation to get m_o gives:

$$m_o = \frac{F}{a_o}$$

   Now we substitute this into the first equation and solve for a_o:

$$F = G\frac{\frac{F}{a_o}M}{R^2}$$

$$FR^2 = G\frac{F}{a_o}M$$

$$a_o FR^2 = FGM$$

$$a_o = \frac{GM}{R^2}$$

   Which has been mentioned earlier in this chapter.

7. Two spheres of mass 800 g and 500 g respectively are situated so that their centres are 200 cm apart. Calculate the gravitational force between them.

Solution:

$$F = G\frac{m_1 m_2}{d^2}$$
$$= \left(6{,}67 \times 10^{-11}\right)\left(\frac{(800 \times 10^{-3})(500 \times 10^{-3})}{(200 \times 10^{-2})^2}\right)$$
$$= 6{,}67 \times 10^{-12} \text{ N}$$

8. Two spheres of mass 2 kg and 3 kg respectively are situated so that the gravitational force between them is $2{,}5 \times 10^{-8}$ N. Calculate the distance between them.

Solution:

$$F = G\frac{m_1 m_2}{d^2}$$
$$2{,}5 \times 10^{-8} = \left(6{,}67 \times 10^{-11}\right)\left(\frac{(2)(3)}{d^2}\right)$$
$$d^2 = \frac{4{,}002 \times 10^{-10}}{2{,}5 \times 10^{-8}}$$
$$= 1{,}6 \times 10^{-2}$$
$$d = 0{,}13 \text{ m}$$

9. Two identical spheres are placed 10 cm apart. A force of $1{,}6675 \times 10^{-9}$ N exists between them. Find the masses of the spheres.

Solution:

$$F = G\frac{m_1 m_2}{d^2}$$
$$1{,}6675 \times 10^{-9} = \left(6{,}67 \times 10^{-11}\right)\left(\frac{m^2}{(10 \times 10^{-2})^2}\right)$$
$$1{,}6675 \times 10^{-9} = 6 \times 10^{-9} m^2$$
$$m^2 = 0{,}25$$
$$m = 0{,}5 \text{ kg}$$

10. Halley's comet, of approximate mass 1×10^{15} kg was $1{,}3 \times 10^8$ km from the Earth, at its point of closest approach during its last sighting in 1986.

    a) Name the force through which the Earth and the comet interact.

    b) Is the magnitude of the force experienced by the comet the same, greater than or less than the force experienced by the Earth? Explain.

    c) Does the acceleration of the comet increase, decrease or remain the same as it moves closer to the Earth? Explain.

    d) If the mass of the Earth is 6×10^{24} kg, calculate the magnitude of the force exerted by the Earth on Halley's comet at its point of closest approach.

    Solution:

a) Newton's law of universal gravitation

b) It is the same, Newton's third law.

c) The force the comet experiences increases because it is inversely proportional to the square of the distance. The mass of the comet remains constant and we know that $F = ma; a = \dfrac{F}{m}$ so the acceleration increases.

d)

$$F = G\frac{m_1 m_2}{d^2}$$
$$= \left(6,67 \times 10^{-11}\right) \left(\frac{(6 \times 10^{24})(1 \times 10^{15})}{(1,3 \times 10^{11})^2}\right)$$
$$= 2,37 \times 10^7 \text{ N}$$

2.5 Chapter summary

Exercise 2 – 10: Forces and Newton's Laws

1. A force acts on an object. Name three effects that the force can have on the object.

   Solution:

   A force can change the shape of the object. A force can change the direction in which the object is moving. A force can accelerate or stop a body.

2. Identify each of the following forces as contact or non-contact forces.

   a) The force between the north pole of a magnet and a paper clip.

   b) The force required to open the door of a taxi.

   c) The force required to stop a soccer ball.

   d) The force causing a ball, dropped from a height of 2 m, to fall to the floor.

   Solution:

   a) non-contact

   b) contact

   c) contact

   d) non-contact

3. A book of mass 2 kg is lying on a table. Draw a labelled force diagram indicating all the forces on the book.

   Solution:
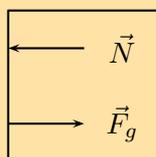
Where $\vec{F}_g$ is the force due to gravity and $\vec{N}$ is the normal force.

4. A constant, resultant force acts on a body which can move freely in a straight line. Which physical quantity will remain constant?

   a) acceleration

   b) velocity

   c) momentum

   d) kinetic energy

[SC 2003/11]

Solution:

acceleration

5. Two forces, 10 N and 15 N, act at an angle at the same point.
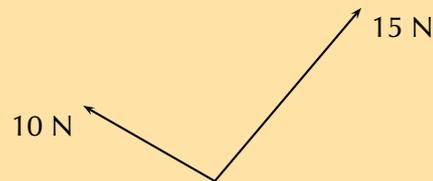
Which of the following **cannot** be the resultant of these two forces?

   a) 2 N

   b) 5 N

   c) 8 N

   d) 20 N

[SC 2005/11 SG1]

Solution:

2 N

6. A concrete block weighing 250 N is at rest on an inclined surface at an angle of 20°. The magnitude of the normal force, in newtons, is

   a) 250

   b) $250 \cos 20°$

   c) $250 \sin 20°$

   d) $2500 \cos 20°$

Solution:

250

7. A 30 kg box sits on a flat frictionless surface. Two forces of 200 N each are applied to the box as shown in the diagram. Which statement best describes the motion of the box?

   a) The box is lifted off the surface.

   b) The box moves to the right.

c) The box does not move.

d) The box moves to the left.
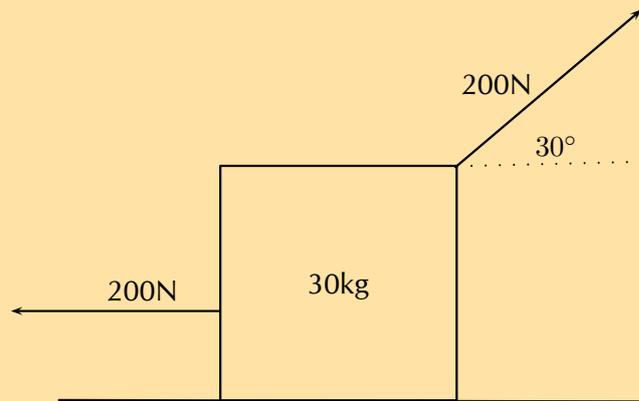
Solution:

The box moves to the left.

8. A concrete block weighing 200 N is at rest on an inclined surface at an angle of 20°. The normal force, in newtons, is

a) 200

b) $200 \cos 20°$

c) $200 \sin 20°$

d) $2000 \cos 20°$

Solution:

200

9. A box, mass m, is at rest on a rough horizontal surface. A force of constant magnitude F is then applied on the box at an angle of 60° to the horizontal, as shown.
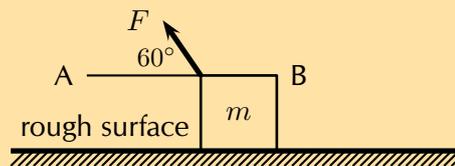
If the box has a uniform horizontal acceleration of magnitude, a, the frictional force acting on the box is...

a) $F\cos 60° - ma$ in the direction of A

b) $F\cos 60° - ma$ in the direction of B

c) $F\sin 60° - ma$ in the direction of A

d) $F\sin 60° - ma$ in the direction of B

[SC 2003/11]

Solution:

$F\cos 60° - ma$ in the direction of A

10. Thabo stands in a train carriage which is moving eastwards. The train suddenly brakes. Thabo continues to move eastwards due to the effect of:

    a) his inertia.

    b) the inertia of the train.

    c) the braking force on him.

    d) a resultant force acting on him.

[SC 2002/11 SG]

Solution:

his inertia

11. A 100 kg crate is placed on a slope that makes an angle of $45°$ with the horizontal. The gravitational force on the box is 98 N. The box does not slide down the slope. Calculate the magnitude and direction of the frictional force and the normal force present in this situation.

    Solution:

    Since the box does not move the frictional force acting on the box is equal to the component of the gravitational force parallel to the plane.

$$F_f = F_{gx}$$
$$= F \cos \theta$$
$$= (98) \cos(45)$$
$$= 69,3 \text{ N}$$

    This force acts at an an angle of $45°$ with the horizontal.

    The normal force acts in the opposite direction to the gravitational force and has a magnitude of 98 N at an angle of $45°$ with the horizontal.

12. A body moving at a *CONSTANT VELOCITY* on a horizontal plane, has a number of unequal forces acting on it. Which one of the following statements is TRUE?

    a) At least two of the forces must be acting in the same direction.

    b) The resultant of the forces is zero.

    c) Friction between the body and the plane causes a resultant force.

    d) The vector sum of the forces causes a resultant force which acts in the direction of motion.

[SC 2002/11 HG1]

Solution:

The resultant of the forces is zero.

13. Two masses of m and $2m$ respectively are connected by an elastic band on a frictionless surface. The masses are pulled in opposite directions by two forces each of magnitude F, stretching the elastic band and holding the masses stationary.
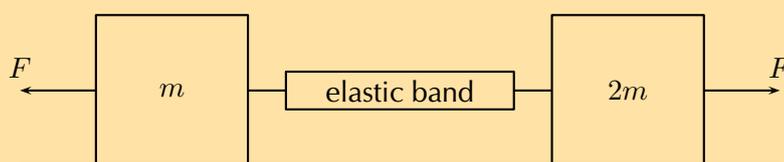
Which of the following gives the magnitude of the tension in the elastic band?

a) zero

b) $\frac{1}{2}F$

c) F

d) $2F$

[IEB 2005/11 HG]

Solution:

F

14. A rocket takes off from its launching pad, accelerating up into the air.
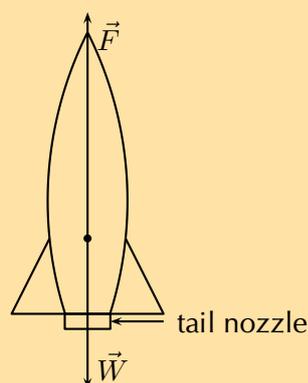
The rocket accelerates because the magnitude of the upward force, F is greater than the magnitude of the rocket's weight, W. Which of the following statements **best** describes how force F arises?

a) F is the force of the air acting on the base of the rocket.

b) F is the force of the rocket's gas jet *pushing down* on the air.

c) F is the force of the rocket's gas jet *pushing down* on the ground.

d) F is the reaction to the force that the rocket exerts on the gases which escape out through the tail nozzle.

[IEB 2005/11 HG]

Solution:

F is the reaction to the force that the rocket exerts on the gases which escape out through the tail nozzle.

15. A box of mass 20 kg rests on a smooth horizontal surface. What will happen to the box if two forces each of magnitude 200 N are applied simultaneously to the box as shown in the diagram.
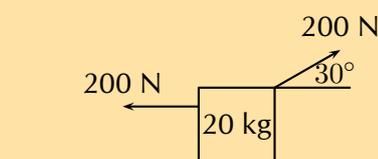
The box will:

a) be lifted off the surface.

b) move to the left.

c) move to the right.

d) remain at rest.

[SC 2001/11 HG1]

Solution:

move to the left

16. A 2 kg mass is suspended from spring balance X, while a 3 kg mass is suspended from spring balance Y. Balance X is in turn suspended from the 3 kg mass. Ignore the weights of the two spring balances.
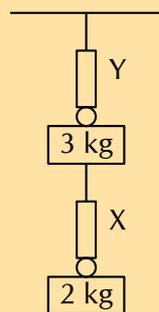
The readings (in N) on balances X and Y are as follows:

	X	Y
a)	19,6	29,4
b)	19,6	49
c)	24,5	24,5
d)	49	49

[SC 2001/11 HG1]

Solution:

b) 19,6, 49

17. P and Q are two forces of equal magnitude applied simultaneously to a body at X.
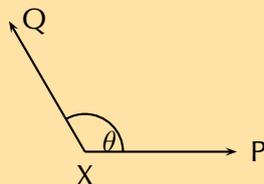
As the angle θ between the forces is **decreased** from 180° to 0°, the magnitude of the resultant of the two forces will

a) initially increase and then decrease.

b) initially decrease and then increase.

c) increase only.

d) decrease only.

Solution:

increase only

18. The graph below shows the velocity-time graph for a moving object:
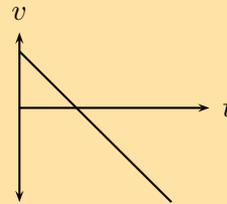
Which of the following graphs could best represent the relationship between the resultant force applied to the object and time?

| (a) | (b) | (c) | (d) |
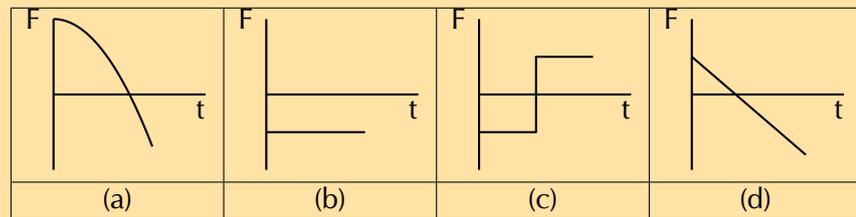
Solution:

graph (b)

19. Two blocks each of mass 8 kg are in contact with each other and are accelerated along a frictionless surface by a force of 80 N as shown in the diagram. The force which block Q will exert on block P is equal to ...
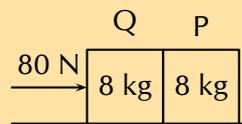
a) 0 N

b) 40 N

c) 60 N

d) 80 N

Solution:

40 N

20. A 12 kg box is placed on a rough surface. A force of 20 N applied at an angle of 30° to the horizontal cannot move the box. Calculate the magnitude and direction of the normal and friction forces.

Solution:

The normal force is:

$$N = mg$$
$$= (12)(9{,}8)$$
$$= 117{,}6 \text{ N}$$

This force points straight up from the surface.

The friction force is:

$$F_f = F_{\text{app}} \cos 30°$$
$$= (20) \cos(30°)$$
$$= 17{,}3 \text{ N}$$

The frictional force points in the opposite direction to the applied force.

21. Three 1 kg mass pieces are placed on top of a 2 kg trolley. When a force of magnitude F is applied to the trolley, it experiences an acceleration a.
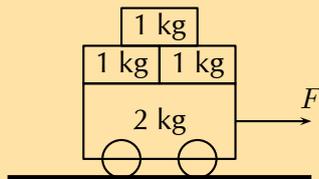
If one of the 1 kg mass pieces falls off while F is still being applied, the trolley will accelerate at ...

    a) $\frac{1}{5}a$

    b) $\frac{4}{5}a$

    c) $\frac{5}{4}a$

    d) $5a$

[SC 2002/03 HG1]

Solution:

$\frac{5}{4}a$

22. A car moves along a horizontal road at constant velocity. Which of the following statements is true?

    a) The car is not in equilibrium.

    b) There are no forces acting on the car.

    c) There is zero resultant force.

    d) There is no frictional force.

[IEB 2004/11 HG1]

Solution:

There is zero resultant force.

23. A crane lifts a load vertically upwards at constant speed. The upward force exerted on the load is F. Which of the following statements is correct?

    a) The acceleration of the load is 9.8 m·s^{-1} downwards.

    b) The resultant force on the load is F.

    c) The load has a weight equal in magnitude to F.

    d) The forces of the crane on the load, and the weight of the load, are an example of a Newton's third law 'action-reaction' pair.

    [IEB 2004/11 HG1]

    Solution:

    The load has a weight equal in magnitude to F.

24. A body of mass M is at rest on a smooth horizontal surface with two forces applied to it as in the diagram below. Force F_1 is equal to Mg. The force F_1 is applied to the right at an angle θ to the horizontal, and a force of F_2 is applied horizontally to the left.
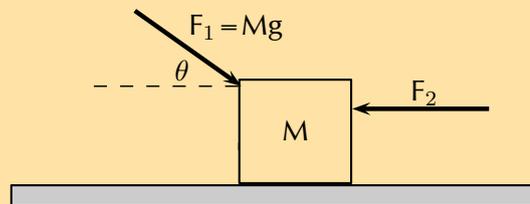
    How is the body affected when the angle θ is increased?

    a) It remains at rest.

    b) It lifts up off the surface, and accelerates towards the right.

    c) It lifts up off the surface, and accelerates towards the left.

    d) It accelerates to the left, moving along the smooth horizontal surface.

    [IEB 2004/11 HG1]

    Solution:

    It accelerates to the left, moving along the smooth horizontal surface.

25. Which of the following statements correctly explains why a passenger in a car, who is not restrained by the seat belt, continues to move forward when the brakes are applied suddenly?

    a) The braking force applied to the car exerts an equal and opposite force on the passenger.

    b) A forward force (called inertia) acts on the passenger.

    c) A resultant forward force acts on the passenger.

    d) A zero resultant force acts on the passenger.

    [IEB 2003/11 HG1]

    Solution:

    A zero resultant force acts on the passenger.

26. A rocket (mass 20 000 kg) accelerates from rest to 40 m·s^{-1} in the first 1,6 seconds of its journey upwards into space.

The rocket's propulsion system consists of exhaust gases, which are pushed out of an outlet at its base.

   a) Explain, with reference to the appropriate law of Newton, how the escaping exhaust gases exert an upwards force (thrust) on the rocket.

   b) What is the magnitude of the total thrust exerted on the rocket during the first 1,6 s?

   c) An astronaut of mass 80 kg is carried in the space capsule. Determine the resultant force acting on him during the first 1,6 s.

   d) Explain why the astronaut, seated in his chair, feels "heavier" while the rocket is launched.

[IEB 2004/11 HG1]

Solution:

   a) Newton's third law states that if body A exerts a force on body B, then body B exerts a force of equal magnitude on body A, but in the opposite direction.

   The exhaust gases exert a force on the rocket. This means that the rocket also exerts a force on the exhaust gases. Since the direction of the force from the exhaust is downwards the direction of the force from the rocket must be upwards. This is what propels the rocket upwards.

   b) We first need to find the acceleration. We can do this by using the equations of motion:

$$v_f = v_i + at$$
$$40 = 0 + (1,6)a$$
$$a = 25 \text{ m·s}^{-2}$$

   Now we can find the magnitude of the total thrust, T:

$$T = W + ma$$
$$= (20\ 000)(9,8) + (20\ 000)(25)$$
$$= 696\ 000 \text{ N}$$

   c)

$$T = W + ma$$
$$= (80)(9,8) + (80)(25)$$
$$= 2784 \text{ N}$$

   d) Weight is measured through normal forces. When the rocket accelerates upwards he feels a greater normal force acting on him as the force required to accelerate him upwards in addition to balancing out the gravitational force.

27.   a) State Newton's second law of Motion.

b) A sports car (mass 1000 kg) is able to accelerate uniformly from rest to 30 m·s^{-1} in a minimum time of 6 s.

    i. Calculate the magnitude of the acceleration of the car.

    ii. What is the magnitude of the resultant force acting on the car during these 6 s?

c) The magnitude of the force that the wheels of the vehicle exert on the road surface as it accelerates is 7500 N. What is the magnitude of the retarding forces acting on this car?

d) By reference to a suitable Law of Motion, explain why a headrest is important in a car with such a rapid acceleration.

[IEB 2003/11 HG1 - Sports Car]

Solution:

a) If a resultant force acts on a body, it will cause the body to accelerate in the direction of the resultant force. The acceleration of the body will be directly proportional to the resultant force and inversely proportional to the mass of the body. The mathematical representation is:

$$\vec{F}_{net} = m\vec{a}$$

b)   i.

$$a = \frac{\Delta v}{\Delta t}$$
$$= \frac{30}{6}$$
$$= 5 \text{ m·s}^{-2}$$

    ii.

$$F = ma$$
$$= (1000)(5)$$
$$= 5000 \text{ N}$$

c) The retarding force is the force that the cars wheels exert minus the resultant force:

$$F = F_{\text{app}} - F_{\text{res}}$$
$$= 7500 - 5000$$
$$= 2500 \text{ N}$$

d) Newton's first law states that an object continues in a state of rest or uniform motion (motion with a constant velocity) unless it is acted on by an unbalanced (net or resultant) force.

There is a resultant force on the person's head and as the car accelerates their head is pulled back. The headrest stops the person's head from being pulled to far back and prevents whiplash.

28. A child (mass 18 kg) is strapped in his car seat as the car moves to the right at constant velocity along a straight level road. A tool box rests on the seat beside him.

tool box
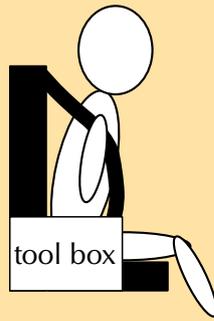
The driver brakes suddenly, bringing the car rapidly to a halt. There is negligible friction between the car seat and the box.

   a) Draw a labelled free-body diagram of **the forces acting on the child** during the time that the car is being braked.

   b) Draw a labelled free-body diagram of **the forces acting on the box** during the time that the car is being braked.

   c) *Modern cars are designed with safety features (besides seat belts) to protect drivers and passengers during collisions e.g. the crumple zones on the car's body. Rather than remaining rigid during a collision, the crumple zones allow the car's body to collapse steadily.*

   State Newton's second law of motion.

[IEB 2005/11 HG1]

Solution:
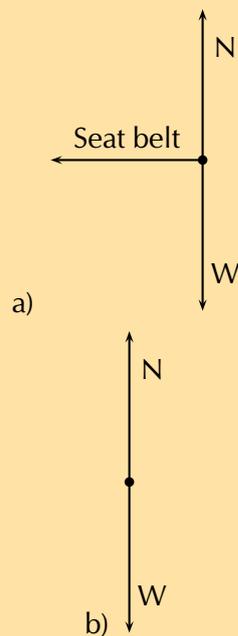
   c) If a resultant force acts on a body, it will cause the body to accelerate in the direction of the resultant force. The acceleration of the body will be directly proportional to the resultant force and inversely proportional to the mass of the body. The mathematical representation is:

$$\vec{F}_{net} = m\vec{a}$$

29. The total mass of a lift together with its load is 1200 kg. It is moving downwards at a constant velocity of 9 m·s^{-1}.

1 200 kg

$9 \text{ m} \cdot \text{s}^{-1}$
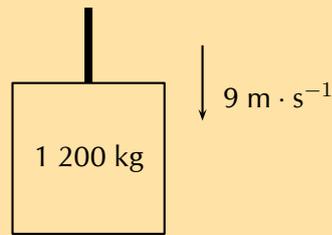
a) What will be the magnitude of the force exerted by the cable on the lift while it is moving downwards at constant velocity? Give an explanation for your answer.

b) *The lift is now uniformly brought to rest over a distance of 18 m.*
Calculate the magnitude of the acceleration of the lift.

c) Calculate the magnitude of the force exerted by the cable while the lift is being brought to rest.

[SC 2003/11 HG1]

Solution:

a) We take upwards as the positive direction.
The acceleration is 0 since the lift is moving at constant velocity.

$$\Sigma F = T - W$$
$$(0)(m) = T - W$$
$$\therefore T = W$$
$$= (9{,}8)(1200)$$
$$= 11\ 760 \text{ N}$$

b)

$$v_f^2 = v_i^2 + 2a\Delta x$$
$$0 = (9)^2 + 2a(18)$$
$$36a = -81$$
$$a = \frac{-81}{36}$$
$$= -2{,}25 \text{ m·s}^{-2}$$

c)

$$T - W = ma$$
$$T = ma + mg$$
$$= (1200)(-2{,}25) + (9{,}8)(1200)$$
$$= 9060 \text{ N}$$

30. A driving force of 800 N acts on a car of mass 600 kg.

a) Calculate the car's acceleration.

b) Calculate the car's speed after 20 s.

c) Calculate the new acceleration if a frictional force of 50 N starts to act on the car after 20 s.

d) Calculate the speed of the car after another 20 s (i.e. a total of 40 s after the start).

Solution:

a)

$$F = ma$$
$$a = \frac{F}{m}$$
$$= \frac{800}{600}$$
$$= 1{,}33 \text{ m·s}^{-2}$$

b)

$$v_f = v_i + at$$
$$v_f = 0 + (20)(1{,}33)$$
$$= 26{,}7 \text{ m·s}^{-1}$$

c)

$$a_2 = a_1 - \frac{F_f}{m}$$
$$= 1{,}33 - \frac{50}{600}$$
$$= 1{,}25 \text{ m·s}^{-2}$$

d)

$$v_{f2} = a_1 t + a_2 t$$
$$v_{f2} = (20)(1{,}33) + (20)(1{,}25)$$
$$= 51{,}6 \text{ m·s}^{-1}$$

31. A stationary block of mass 3 kg is on top of a plane inclined at $35°$ to the horizontal.
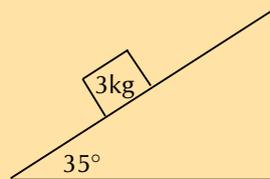
a) Draw a force diagram (not to scale). Include the weight of the block as well as the components of the weight that are perpendicular and parallel to the inclined plane.

b) Determine the values of the weight's perpendicular and parallel components.

Solution:

a)
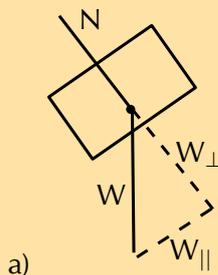
b) The perpendicular component is:

$$W_\perp = F \sin(\theta)$$
$$= mg \sin(\theta)$$
$$= (3)(9.8) \sin(35)$$
$$= 16{,}86 \text{ N}$$

The parallel component is:

$$W_{||} = F \cos(\theta)$$
$$= mg \cos(\theta)$$
$$= (3)(9.8) \cos(35)$$
$$= 24{,}01 \text{ N}$$

32. **A crate on an inclined plane**

Elephants are being moved from the Kruger National Park to the Eastern Cape. They are loaded into crates that are pulled up a ramp (an inclined plane) on frictionless rollers.

The diagram shows a crate being held stationary on the ramp by means of a rope parallel to the ramp. The tension in the rope is 5000 N.
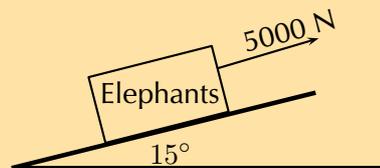
a) Explain how one can deduce the following: "The forces acting on the crate are in equilibrium".

b) Draw a labelled free-body diagram of the forces acting on the elephant. (Regard the crate and elephant as one object, and represent them as a dot. Also show the relevant angles between the forces.)

c) The crate has a mass of 800 kg. Determine the mass of the elephant.

d) The crate is now pulled up the ramp at a constant speed. How does the crate being pulled up the ramp at a constant speed affect the forces acting on the crate and elephant? Justify your answer, mentioning any law or principle that applies to this situation.

[IEB 2002/11 HG1]

Solution:

a) The sum of the forces is equal to the mass times the acceleration. Since the acceleration is 0, the sum of the forces is 0. This means that the forces acting on the crate are in equilibrium.

b)

c)

$$W_{E\parallel} = W \sin \theta$$
$$5000 = W \sin(15)$$
$$W = \frac{5000}{\sin(15)}$$
$$= 19\ 318{,}52 \text{ N}$$
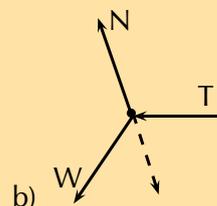
$$m_E = \frac{W}{g} - m_{\text{crate}}$$
$$= \frac{19\ 318{,}52}{9{,}8} - 800$$
$$= 1171{,}28 \text{ kg}$$

d) There will be no changes to the forces since the acceleration is 0.

33. **Car in Tow**

Car A is towing Car B with a light tow rope. The cars move along a straight, horizontal road.

a) Write down a statement of Newton's second law of Motion (in words).

b) As they start off, Car A exerts a forwards force of 600 N at its end of the tow rope. The force of friction on Car B when it starts to move is 200 N. The mass of Car B is 1200 kg. Calculate the acceleration of Car B.

c) After a while, the cars travel at constant velocity. The force exerted on the tow rope is now 300 N while the force of friction on Car B increases. What is the magnitude and direction of the force of friction on Car B now?

d) Towing with a rope is very dangerous. A solid bar should be used in preference to a tow rope. This is especially true should Car A suddenly apply brakes. What would be the advantage of the solid bar over the tow rope in such a situation?

e) The mass of Car A is also 1200 kg. Car A and Car B are now joined by a solid tow bar and the total braking force is 9600 N. Over what distance could the cars stop from a velocity of 20 m·s^{-1}?

[IEB 2002/11 HG1]

Solution:

a) If a resultant force acts on a body, it will cause the body to accelerate in the direction of the resultant force. The acceleration of the body will be directly proportional to the resultant force and inversely proportional to the mass of the body.

b) The total force on car B is 600 N − 200 N = 400 N.

$$F = ma$$
$$a = \frac{400}{1200}$$
$$= 0{,}33 \text{ m·s}^{-2}$$

c) The frictional force is equal to the force exerted on the tow rope but in the opposite direction. $F_f = 300$ N.

d) Newton's first law states that an object will keep on moving in a straight line unless acted on by a force. A solid bar will stop car B from moving if car A breaks while a rope will not.

e) The acceleration is:

$$F = ma$$
$$a = \frac{9600}{2(1200)}$$
$$= 4$$

So the stopping distance is:

$$v_f^2 = v_i^2 + 2a\Delta x$$
$$0 = (20)^2 + 2(-4)\Delta x$$
$$8\Delta x = 400$$
$$\Delta x = 50 \text{ m}$$

34. **Testing the Brakes of a Car**

A braking test is carried out on a car travelling at 20 m·s^{-1}. A braking distance of 30 m is measured when a braking force of 6000 N is applied to stop the car.

a) Calculate the acceleration of the car when a braking force of 6000 N is applied.

b) **Show** that the mass of this car is 900 kg.

c) How long (in s) does it take for this car to stop from 20 m·s^{-1} under the braking action described above?

d) A trailer of mass 600 kg is attached to the car and the braking test is repeated from 20 m·s^{-1} using the same braking force of 6000 N. How much longer will it take to stop the car with the trailer in tow?

[IEB 2001/11 HG1]

Solution:

a)

$$v_f^2 = v_i^2 + 2a\Delta x$$
$$0 = (20)^2 + 2(30)a$$
$$60a = -400$$
$$a = -6{,}67 \text{ m·s}^{-2}$$

b)

$$F = ma$$
$$6000 = (6{,}67)m$$
$$m = 900 \text{ kg}$$

c)

$$v_f = v_i + at$$
$$0 = 20 + (-6{,}67)t$$
$$6{,}67t = 20$$
$$t = 3{,}0 \text{ s}$$

d)

$$F = ma$$
$$6000 = (900 + 600)a$$
$$a = 4$$

$$t = \frac{-20}{4}$$
$$t = 5 \text{ s}$$

It will take 2 s longer.

35. A box is held stationary on a smooth plane that is inclined at angle θ to the horizontal.
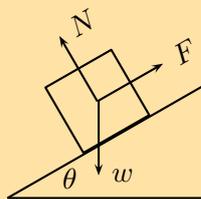
F is the force exerted by a rope on the box. w is the weight of the box and N is the normal force of the plane on the box. Which of the following statements is correct?

a) $\tan \theta = \frac{F}{w}$

b) $\tan \theta = \frac{F}{N}$

c) $\cos \theta = \frac{F}{w}$

d) $\sin \theta = \frac{N}{w}$

[IEB 2005/11 HG]

Solution:

$\sin \theta = \frac{N}{w}$

36. As a result of three forces F_1, F_2 and F_3 acting on it, an object at point P is in equilibrium.
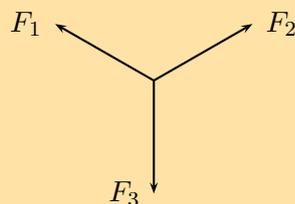
Which of the following statements is **not true** with reference to the three forces?

a) The resultant of forces F_1, F_2 and F_3 is zero.

b) Force F_1, F_2 and F_3 lie in the same plane.

c) Force F_3 is the resultant of forces F_1 and F_2.

d) The sum of the components of all the forces in any chosen direction is zero.

Solution:

Force F_3 is the resultant of forces F_1 and F_2.

37. A block of mass M is held stationary by a rope of negligible mass. The block rests on a frictionless plane which is inclined at 30° to the horizontal.
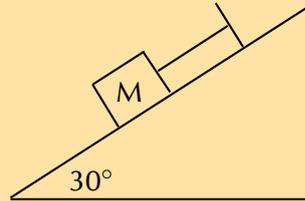
   a) Draw a labelled force diagram which shows all the forces acting on the block.

   b) Resolve the force due to gravity into components that are parallel and perpendicular to the plane.

   c) Calculate the weight of the block when the force in the rope is 8 N.

   Solution:

a)
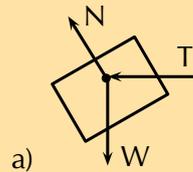
b)

$$W = Mg$$
$$W_{\parallel} = Mg\sin\theta$$
$$W_{\perp} = Mg\cos\theta$$

c)

$$W_{\parallel} = Mg\sin\theta$$
$$8 = M(9,8)\sin(30)$$
$$M = 1,63 \text{ N}$$

38. A heavy box, mass m, is lifted by means of a rope R which passes over a pulley fixed to a pole. A second rope S, tied to rope R at point P, exerts a horizontal force and pulls the box to the right. After lifting the box to a certain height, the box is held stationary as shown in the sketch below. Ignore the masses of the ropes. The tension in rope R is 5850 N.
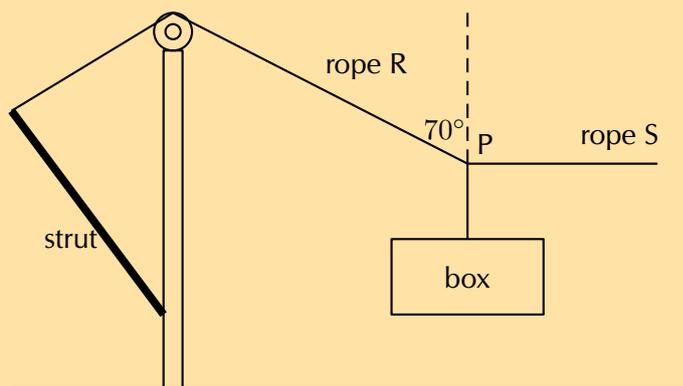
a) Draw a diagram (with labels) of all the forces acting at the point P, when P is in equilibrium.

b) By resolving the force exerted by rope R into components, calculate the...

   i. magnitude of the force exerted by rope S.
   ii. mass, m, of the box.

[SC 2003/11]

Solution:

a)

b)   i.

$$S = R\cos\theta$$
$$= (5850)\cos(20)$$
$$= 5497,2 \text{ N}$$

ii.
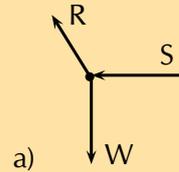
$$mg = R\sin\theta$$
$$9,8m = (5850)\sin(20)$$
$$m = 204,17 \text{ kg}$$

39. A tow truck attempts to tow a broken down car of mass 400 kg. The coefficient of static friction is 0,60 and the coefficient of kinetic (dynamic) friction is 0,4. A rope connects the tow truck to the car. Calculate the force required:

   a) to just move the car if the rope is parallel to the road.

   b) to keep the car moving at constant speed if the rope is parallel to the road.

   c) to just move the car if the rope makes an angle of 30° to the road.

   d) to keep the car moving at constant speed if the rope makes an angle of 30° to the road.

Solution:

a)

$$F_s = \mu_s F$$
$$F_s = \mu_s mg$$
$$= (0,6)(400)(9,8)$$
$$F_s = 2352 \text{ N}$$

b)

$$F_f = \mu_k F$$
$$= \mu_k mg$$
$$= (0,4)(400)(9,8)$$
$$= 1568 \text{ N}$$

c)

$$F_s \cos \theta = \mu_s F$$
$$F_s = \frac{2352}{\cos 30}$$
$$= 2715{,}9 \text{ N}$$

d)

$$F_f \cos \theta = \mu_f F$$
$$F_f = \frac{1568}{\cos 30}$$
$$= 1810{,}6 \text{ N}$$

Exercise 2 – 11: Gravitation

1. An object attracts another with a gravitational force F. If the distance between the centres of the two objects is now decreased to a third ($\frac{1}{3}$) of the original distance, the force of attraction that the one object would exert on the other would become...

   a) $\frac{1}{9}F$

   b) $\frac{1}{3}F$

   c) $3F$

   d) $9F$

   [SC 2003/11]

   Solution:

   $9F$

2. An object is dropped from a height of 1 km above the Earth. If air resistance is ignored, the acceleration of the object is dependent on the...

   a) mass of the object

   b) radius of the earth

   c) mass of the earth

   d) weight of the object

   [SC 2003/11]

   Solution:

   weight of the object

3. A man has a mass of 70 kg on Earth. He is walking on a new planet that has a mass four times that of the Earth and the radius is the same as that of the Earth ($M_E = 6 \times 10^{24}$ kg, $r_E = 6 \times 10^6$ m)

   a) Calculate the force between the man and the Earth.

b) What is the man's weight on the new planet?

c) Would his weight be bigger or smaller on the new planet? Explain how you arrived at your answer.

Solution:

a)

$$F_E = \frac{Gm_1m_2}{d^2}$$
$$= \frac{(6{,}67 \times 10^{-11})(70)(6 \times 10^{24})}{(6 \times 10^6)^2}$$
$$= 778{,}2 \text{ N}$$

b) The mass of the new planet is four times the mass of the Earth so we get:

$$F_{\text{planet}} = \frac{Gm_1 4m_2}{d^2}$$
$$= 4F_E$$
$$= (778{,}2)(4)$$
$$= 3112{,}8 \text{ N}$$

c) His weight is bigger on the new planet. The new planet has the same radius as the Earth, but a larger mass. Since the mass of the planet is proportional to the force of gravity (or the weight) the man's weight must be larger.

4. Calculate the distance between two objects, 5000 kg and 6×10^{12} kg respectively, if the magnitude of the force between them is 3×10^8 N

Solution:

$$F = \frac{Gm_1m_2}{d^2}$$
$$3 \times 10^8 = \frac{(6{,}67 \times 10^{-11})(5000)(6 \times 10^{12})}{d^2}$$
$$3 \times 10^8 d^2 = 2{,}001 \times 10^6$$
$$d^2 = 6{,}67 \times 10^{-3}$$
$$= 8{,}2 \times 10^{-2} \text{ m}$$

5. An astronaut in a satellite 1600 km above the Earth experiences a gravitational force of magnitude 700 N on Earth. The Earth's radius is 6400 km. Calculate:

a) The magnitude of the gravitational force which the astronaut experiences in the satellite.

b) The magnitude of the gravitational force on an object in the satellite which weighs 300 N on Earth.

Solution:

a) We first work out the mass of the astronaut:

$$F = \frac{Gm_1m_2}{d^2}$$

$$700 = \frac{(6{,}67 \times 10^{-11})(6 \times 10^{24})m_A}{(6400 \times 10^3)^2}$$

$$9{,}77m_A = 700$$

$$m_A = 71{,}6 \text{ kg}$$

Now we can work out the gravitational force in the satellite:

$$F_S = \frac{Gm_1m_2}{d^2}$$

$$= \frac{(6{,}67 \times 10^{-11})(6 \times 10^{24})(71{,}6)}{(6400 \times 10^3 + 1600 \times 10^3)^2}$$

$$= 448 \text{ N}$$

b) We first work out the mass of the object:

$$F = \frac{Gm_1m_2}{d^2}$$

$$300 = \frac{(6{,}67 \times 10^{-11})(6 \times 10^{24})m_O}{(6400 \times 10^3)^2}$$

$$9{,}77m_O = 300$$

$$m_O = 30{,}7 \text{ kg}$$

Now we can work out the gravitational force in the satellite:

$$F_S = \frac{Gm_1m_2}{d^2}$$

$$= \frac{(6{,}67 \times 10^{-11})(6 \times 10^{24})(30{,}7)}{(6400 \times 10^3 + 1600 \times 10^3)^2}$$

$$= 192 \text{ N}$$

6. An astronaut of mass 70 kg on Earth lands on a planet which has half the Earth's radius and twice its mass. Calculate the magnitude of the force of gravity which is exerted on him on the planet.

Solution:

The gravitational force on Earth is:

$$F_E = \frac{Gm_E m_A}{r_E^2}$$

On the planet we have twice the Earth's mass and half the Earth's radius:

$$F_P = \frac{Gm_P m_A}{r_P^2}$$

$$= \frac{2Gm_E m_A}{\frac{r_E^2}{4}}$$

$$= 8F_E$$

$$= 8(70)(9{,}8)$$

$$= 5488 \text{ N}$$

7. Calculate the magnitude of the gravitational force of attraction between two spheres of lead with a mass of 10 kg and 6 kg respectively if they are placed 50 mm apart.

   Solution:

   $$F = \frac{Gm_1 m_2}{d^2}$$
   $$= \frac{(6{,}67 \times 10^{-11})(10)(6)}{(50 \times 10^{-3})^2}$$
   $$= 1{,}6 \times 10^6 \text{ N}$$

8. The gravitational force between two objects is 1200 N. What is the gravitational force between the objects if the mass of each is doubled and the distance between them halved?

   Solution:

   The gravitational force is:

   $$F_1 = \frac{Gm_1 m_2}{d^2}$$

   If we double each mass and halve the distance between them we now have:

   $$F_2 = \frac{G(2m_1)(2m_2)}{(0{,}5d)^2}$$
   $$= \frac{4Gm_1 m_2}{0{,}25d^2}$$
   $$= 16F_1$$

   So the force will be 16 times as much.

9. Calculate the gravitational force between the Sun with a mass of 2×10^{30} kg and the Earth with a mass of 6×10^{24} kg if the distance between them is $1{,}4 \times 10^8$ km.

   Solution:

   $$F = \frac{Gm_1 m_2}{d^2}$$
   $$= \frac{(6{,}67 \times 10^{-11})(2 \times 10^{30})(6 \times 10^{24})}{(1{,}4 \times 10^{11})^2}$$
   $$= 4{,}1 \times 10^{22} \text{ N}$$

10. How does the gravitational force of attraction between two objects change when

    a) the mass of each object is doubled.

    b) the distance between the centres of the objects is doubled.

    c) the mass of one object is halved, and the distance between the centres of the objects is halved.

    Solution:

a) The gravitational force will be four times as much.

b) The gravitational force will be one fourth as much or four times smaller.

c) The gravitational force will be twice as much.

11. Read each of the following statements and say whether you agree or not. Give reasons for your answer and rewrite the statement if necessary:

    a) The gravitational acceleration g is constant.

    b) The weight of an object is independent of its mass.

    c) G is dependent on the mass of the object that is being accelerated.

    Solution:

    a) Agree

    b) Disagree. Weight is related to mass via the gravitational acceleration

    c) Disagree. G is a universal constant

12. An astronaut weighs 750 N on the surface of the Earth.

    a) What will his weight be on the surface of Saturn, which has a mass 100 times greater than the Earth, and a radius 5 times greater than the Earth?

    b) What is his mass on Saturn?

    Solution:

    a) On Earth we have:

    $$a_E = G\frac{M_E}{R_E^2}$$

    On Saturn we note that $M_S = 100M_E$ and $R_S = 5R_E$. So the gravitational acceleration on Saturn is:

    $$a_S = G\frac{M_S}{R_S^2}$$
    $$= G\frac{100M_E}{25R_E^2}$$
    $$= 4G\frac{M_E}{R_E^2}$$
    $$\therefore a_S = 4a_E$$

    So the weight of the astronaut on Saturn is:

    $$a_S = 4a_E$$
    $$= 4(750)$$
    $$= 3000 \text{ N}$$

    b) On Earth his mass is:

$$a_E = mg_E$$
$$m = \frac{a_E}{g_E}$$
$$= \frac{750}{9{,}8}$$
$$= 76{,}53 \text{ kg}$$

His mass on Saturn is the same as his mass on Earth, it is only his weight that is different.

13. Your mass is 60 kg in Paris at ground level. How much less would you weigh after taking a lift to the top of the Eiffel Tower, which is 405 m high? Assume the Earth's mass is $6{,}0 \times 10^{24}$ kg and the Earth's radius is 6400 km.

    Solution:

    We start with your weight on the surface of the Earth. The gravitational acceleration at the surface of the Earth is $9{,}8$ m·s^{-2} and so your weight is:

$$F_g = mg$$
$$= (60)(9{,}8)$$
$$= 588 \text{ N}$$

At the top of the Eiffel Tower the gravitational acceleration is:

$$a_o = G\frac{M_{\text{Earth}}}{d^2}$$
$$= \frac{(6{,}67 \times 10^{-11})(6{,}0 \times 10^{24})}{(6400 \times 10^3 + 405)^2}$$
$$= 9{,}77 \text{ m·s}^{-2}$$

Your weight is:

$$F_g = mg$$
$$= (60)(9{,}77)$$
$$= 586{,}2 \text{ N}$$

So you would weigh $1{,}8$ N less.

14. a) State Newton's law of universal gravitation.

    b) Use Newton's law of universal gravitation to determine the magnitude of the acceleration due to gravity on the Moon.
       The mass of the Moon is $7{,}4 \times 10^{22}$ kg.
       The radius of the Moon is $1{,}74 \times 10^6$ m.

    c) Will an astronaut, kitted out in his space suit, jump higher on the Moon or on the Earth? Give a reason for your answer.

    Solution:

a) Every body in the universe exerts a force on every other body. The force is directly proportional to the product of the masses of the bodies and inversely proportional to the square of the distance between them.

b)

$$\begin{aligned} g &= \frac{F}{m_o} \\ &= \frac{G \frac{m_{moon} m_o}{r^2}}{m_2} \\ &= G \frac{m_{moon}}{r^2} \\ &= \frac{(6{,}67 \times 10^{-11})(7{,}4 \times 10^{22})}{(1{,}74 \times 10^6)^2} \\ &= 1{,}63 \end{aligned}$$

c) He will be able to jump higher on the moon. The acceleration due to gravity is lower on the moon than on the Earth and so there is less gravitational force pulling him down on the moon.

CHAPTER 3

Atomic combinations

In this chapter learners will explore the concept of a covalent bond in greater detail. In grade ten learners learnt about the three types of chemical bond (ionic, covalent and metallic). A great video to introduce this topic is: Veritasium chemical bonding song. In this chapter the focus is on the covalent bond. A short breakdown of the topics in this chapter follows.

- **Electron structure and Lewis diagrams (from grade 10)**

  As revision you can ask learners to draw Lewis diagrams for the first 20 elements and give the electronic structure (this was covered in grade 10). This then leads into thinking how the elements can share electrons in a bond. Learners should recognise that there are unpaired electrons in atoms that can be shared to form the bonds.

  It is important to note that when drawing Lewis diagrams, we first place single electrons around the central atom and only once four electrons have been placed, do we pair electrons up. This will avoid the need to explain hybridisation. It is also important for learners to realise that the placement of electrons is arbitrary and the electrons can be placed anywhere around the atom.

- **Why hydrogen is a diatomic molecule but helium is a monatomic molecule**

  This part of the chapter is interleaved with electron structure and Lewis diagrams as these two concepts play a key role in understanding why hydrogen is diatomic and helium is monatomic. In this part learners are introduced to the idea that when two atoms come close together there is a change in the potential energy. This forms a strong foundation for explaining the energy changes that occur in chemical reactions and will be seen again in chapter 12 (energy changes in chemical reactions).

- **Deducing simple rules about bond formation (and drawing Lewis diagrams for these molecules)**

  Four cases are looked at to try to understand why bonds form. This is all about the covalent bond, so all the examples you use must be of covalent molecules (and you must also only pick examples of covalent molecular structures as covalent network structures are more like ionic networks and do not form simple molecular units). It is also important to help learners realise that a lone pair of electrons is very much dependant on the molecule that they are looking at. Lone pairs of electrons can be used under special circumstances to form dative (or coordinate) covalent bonds.

- **The basic principles of VSEPR and predicting molecular shape**

  You can build the different molecular shapes before starting to teach VSEPR from large polystyrene balls and kebab sticks or you can give your learners jellytots or marshmallows and toothpicks and get them to build the molecular shapes. Remember that the shapes with lone pairs need more space for the lone pairs and so it is not as simple as just removing the toothpick for the lone pair.

  This topic covers the shapes that molecules have. This is only the shapes of covalent molecular compounds, covalent network structures, ionic compounds and metals have very different three dimensional forms. This topic is important to help learners determine polarity of molecules. Two approaches are used to determine the shape of a molecule. The first one looks at molecules matching

up to a general formula while the second one considers how many electron pairs are around a central atom. These two approaches can be used together to help learners fully understand this topic.

- **Electronegativity and polarity of bonds**

  It is important to note that CAPs does not give a definitive source for electronegativity values. You should use the ones found on the periodic table in the matric exams (these are the same values as the ones on the periodic table at the front of this book). Learners should be aware that they may see different values on other periodic tables. Learners must not think of the different types of bonding as being exactly defined. Also, the values for where the types of bonding transition are not exact and different sources quote different cut-off points.

  The simplest examples of polarity are the ideal shapes with all the end atoms the same and so you should stick to this in your explanation. You can explain this for trigonal planar molecules by using your learners. Get three girls or three boys to link hands (they all put their right hand into the centre and hold the other two learners right hands). Then they try to pull away (all learners pull equally). This is the even sharing of electrons. Now replace a girl with a boy (or vice versa) and tell the new learner to pull a bit less. This shows the uneven sharing of electrons.

- **Bond length and bond energy**

  In this final part of the chapter we return to our energy diagram and add two pieces of information: bond energy and bond length. The bond length is the distance between the two atoms when they are at their minimum energy, while the bond energy is this minimum energy. The bond energy comes up again in chapter 12 (energy and chemical change) when the topic of exothermic and endothermic reactions is covered.

Coloured text has been used as a tool to highlight different parts of Lewis diagrams. Ensure that learners understand that the coloured text does not mean there is anything special about that part of the diagram, this is simply a teaching tool to help them identify the important aspects of the diagram, in particular the unpaired electrons.

3.1 Chemical bonds

Valence electrons and Lewis diagrams

Exercise 3 – 1: Lewis diagrams

Give the spectroscopic notation and draw the Lewis diagram for:

1. magnesium

   Solution:

   $[Ne]3s^2$

$$\overset{\bullet}{\text{Mg}} \, \bullet$$

2. sodium

   Solution:

   $[Ne]3s^1$

$$Na \cdot$$

3. chlorine

   Solution:

   $[Ne]3s^2 3p^5$

$$\cdot \overset{\displaystyle \cdot \cdot}{\underset{\displaystyle \cdot \cdot}{Cl}} :$$
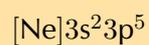
4. aluminium

   Solution:

   $[Ne]3s^2 3p^1$

$$\overset{\displaystyle \cdot}{\underset{\displaystyle \cdot}{Al}} \cdot$$

5. argon

   Solution:

   $[Ne]3s^2 3p^6$

$$: \overset{\displaystyle \cdot \cdot}{\underset{\displaystyle \cdot \cdot}{Ar}} :$$
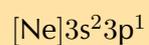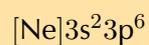
Covalent bonds and bond formation

Case 1: Two atoms that each have an unpaired electron

Exercise 3 – 2:

Represent the following molecules using Lewis diagram:

1. chlorine (Cl_2)

   Solution:

$$: \overset{\displaystyle \cdot \cdot}{\underset{\displaystyle \cdot \cdot}{Cl}} - \overset{\displaystyle \cdot \cdot}{\underset{\displaystyle \cdot \cdot}{Cl}} :$$

2. boron trifluoride (BF_3)

Solution:
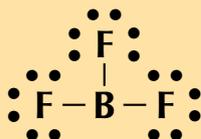
Case 2: Atoms with lone pairs

Exercise 3 – 3:

Represent the following molecules using Lewis diagrams:

1. ammonia (NH_3)

   Solution:
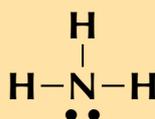
2. oxygen difluoride (OF_2)

   Solution:
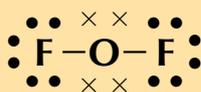
Case 3: Atoms with multiple bonds

Exercise 3 – 4:

Represent the following molecules using Lewis diagrams:

1. acetylene (C_2H_2)

   Solution:

2. formaldehyde (CH_2O)

   Solution:

$$\overset{\overset{\displaystyle ..}{\displaystyle O:}}{\underset{\displaystyle H-C-H}{\|}}$$
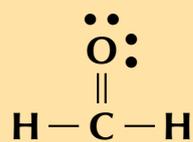
Case 4: Co-ordinate or dative covalent bonds

1. Represent each of the following *atoms* using Lewis diagrams:

    a) calcium

    b) lithium

    c) phosphorous

    d) potassium

    e) silicon

    f) sulfur

    Solution:

    a) Ca•

    b) Li •

    c) • P :

    d) K •

    e) • Si •

    f) • S :
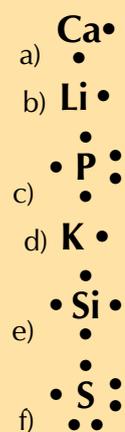
2. Represent each of the following *molecules* using Lewis diagrams:

    a) bromine (Br_2)

    b) carbon dioxide (CO_2)

    c) nitrogen (N_2)

    d) hydronium ion (H_3O^+)

    e) sulfur dioxide (SO_2)

    Solution:

    a) :Br ×Br×

    b) :O:× C ×:O:

c) $\overset{\times}{\underset{\times}{\times}} N \overset{\times}{\underset{\times}{:}} N \textbf{:}$

d) $\begin{bmatrix} & H & \\ & \overset{\bullet\bullet}{} & \\ H & \overset{\times}{:} O & \textbf{:} \\ & \overset{\times\bullet}{} & \\ & H & \end{bmatrix}^{+}$

e) $\overset{\bullet\bullet}{:} \overset{\times}{\underset{\times}{O}} \overset{\times}{:} S \overset{\times}{\underset{\times}{:}} \overset{\bullet\bullet}{O} \textbf{:}$

3. Two chemical reactions are described below.

- nitrogen and hydrogen react to form NH_3
- carbon and hydrogen bond to form CH_4

For each reaction, give:

a) the number of valence electrons for each of the atoms involved in the reaction

b) the Lewis diagram of the product that is formed

c) the name of the product

Solution:

a) Nitrogen: 5, hydrogen: 1
Carbon: 4, hydrogen: 1

b) NH_3

$$H \overset{\times}{\underset{\bullet\bullet}{:}} \overset{\overset{H}{\bullet\times}}{N} \overset{\bullet}{\underset{\times}{:}} H$$

CH_4

$$H \overset{\times}{\underset{\times\bullet}{:}} \overset{\overset{H}{\bullet\times}}{\underset{H}{C}} \overset{\bullet}{\underset{\times}{:}} H$$

c) NH_3: ammonia
CH_4: methane

4. A chemical compound has the following Lewis diagram:

$$X \overset{\times}{\underset{\bullet}{:}} \overset{\overset{\bullet\bullet}{}}{\underset{\underset{H}{\times\bullet}}{Y}} \textbf{:}$$

a) How many valence electrons does element Y have?

b) How many valence electrons does element X have?

c) How many covalent bonds are in the molecule?

d) Suggest a name for the elements X and Y.

Solution:

a) 6. There are 6 dots around element Y and from our knowledge of Lewis diagrams we know that these represent the valence electrons.

b) 1. X contributes one electron (represented by a cross) to the bond and X has no other electrons.

c) 2 single bonds. From our knowledge of Lewis diagrams we look at how many cross and dot pairs there are in the molecule and that gives us the number of covalent bonds.

These are single bonds since there is only one dot and cross pair between adjacent atoms.

d) The most likely atoms are: Y: oxygen and X: hydrogen.

Note that Y could also be sulfur and X hydrogen and the molecule would then be hydrogen sulfide (sulfur dihydride).

5. Complete the following table:

Compound	CO_2	CF_4	HI	C_2H_2
Lewis diagram				
Total number of bonding pairs				
Total number of non-bonding pairs				
Single, double or triple bonds				

Solution:

Compound	CO_2	CF_4
Lewis diagram		
Total number of bonding pairs	4	4
Total number of non-bonding pairs	4	12
Single, double or triple bonds	Two double bonds	Four single bonds

Compound	HI	C_2H_2
Lewis diagram		H—C≡C—H
Total number of bonding pairs	1	5
Total number of non-bonding pairs	3	0
Single, double or triple bonds	One single bond	One triple bond and two single bonds

Exercise 3 – 6: Molecular shape

Determine the shape of the following molecules.

1. $BeCl_2$

   Solution:

   The central atom is beryllium (draw the molecules Lewis structure to see this).

   There are two electron pairs around beryllium and no lone pairs.

   There are two bonding electron pairs and no lone pairs. The molecule has the general formula AX_2. Using this information and Table **??** we find that the molecular shape is linear.

2. F_2

   Solution:

   The molecular shape is linear. (This is a diatomic molecule)

3. PCl_5

   Solution:

   The central atom is phosphorous.

   There are five electron pairs around phosphorous and no lone pairs.

   The molecule has the general formula AX_5. Using this information and Table **??** we find that the molecular shape is trigonal pyramidal.
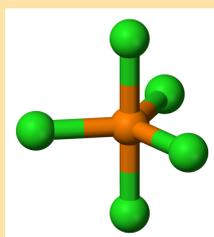
4. SF_6

Solution:

The central atom is sulfur.

There are six electron pairs around phosphorous and no lone pairs.

The molecule has the general formula AX_6. Using this information and Table **??** we find that the molecular shape is octahedral.
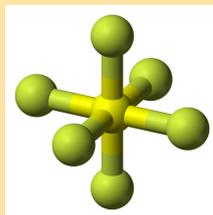
5. CO_2

Solution:

The central atom is carbon. (Draw the molecules Lewis diagram to see this.)

There are four electron pairs around carbon. These form two double bonds. There are no lone pairs around carbon.

The molecule has the general formula AX_2. Using this information and Table **??** we find that the molecular shape is linear.
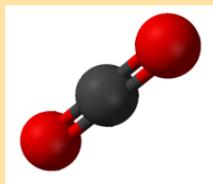
6. CH_4

Solution:

The central atom is carbon. (Draw the molecules Lewis diagram to see this).

There are four electron pairs around carbon and no lone pairs.

The molecule has the general formula AX_4. Using this information and Table **??** we find that the molecular shape is tetrahedral.
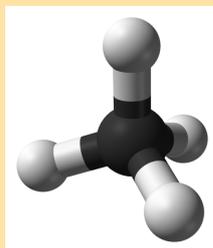
7. H_2O

Solution:

The central atom is oxygen (you can see this by drawing the molecules Lewis diagram).

There are two electron pairs around oxygen and two lone pairs.

The molecule has the general formula AX_2E_2. Using this information and Table **??** we find that the molecular shape is bent or angular.
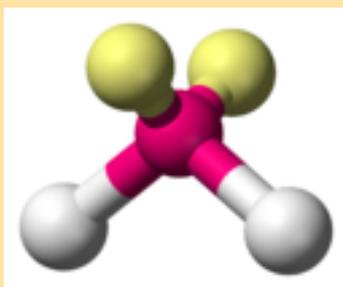
8. COH_2

Solution:

The central atom is carbon.

There are four electron pairs around carbon, two of which form a double bond to the oxygen atom. There are no lone pairs.

The molecule has the general formula AX_3. Using this information and Table **??** we find that the molecular shape is trigonal planar.
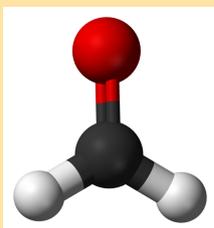
3.3 Electronegativity

Exercise 3 – 7:

1. Calculate the electronegativity difference between: Be and C, H and C, Li and F, Al and Na, C and O.

   Solution:

   Be and C: $2,5 - 1,5 = 1,0$.

   H and C: $2,5 - 2,1 = 0,4$.

   Li and F: $4,0 - 1,0 = 3,0$.

   Al and Na: $1,5 - 0,9 = 0,6$.

   C and O: $3,5 - 2,5 = 1,0$.

Exercise 3 – 8: Electronegativity

1. In a molecule of beryllium chloride ($BeCl_2$):

   a) What is the electronegativity of beryllium?

   b) What is the electronegativity of chlorine?

   c) Which atom will have a slightly positive charge and which will have a slightly negative charge in the molecule? Represent this on a sketch of the molecule using partial charges.

   d) Is the bond a non-polar or polar covalent bond?

   e) Is the molecule polar or non-polar?

   Solution:

   a) 2,1

   b) 3,0

   c) Hydrogen will have a slightly positive charge and chlorine will have a slightly negative charge.

   $$\overset{\delta^-}{Cl}\!-\!\overset{\delta^+}{Be}\!-\!\overset{\delta^-}{Cl}$$
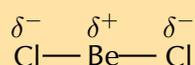
   d) Polar covalent bond. The electronegativity difference is: $3,0 - 2,1 = 0,9$. The bond is weakly polar.

   e) Hydrogen chloride is linear and therefore is a polar molecule.

2. Complete the table below:

Molecule	Difference in electronegativity between atoms	Non-polar/polar covalent bond	Polar/non-polar molecule
H_2O			
HBr			
F_2			
CH_4			
PF_5			
$BeCl_2$			
CO			
C_2H_2			
SO_2			
BF_3			

   Solution:

Molecule	Difference in electronegativity between atoms	Non-polar/polar covalent bond	Polar/non-polar molecule
H_2O	$3,5 - 2,1 = 1,4$	Polar covalent bond	Polar molecule. Water has a bent or angular shape.
HBr	$2,8 - 2,1 = 0,7$	Polar covalent bond	Polar molecule. Hydrogen bromide is linear.
F_2	$4,0 - 4,0 = 0$	Non-polar covalent bond	Non-polar molecule.
CH_4	$2,5 - 2,1 = 0,4$	Polar covalent bond	Non-polar molecule. Methane is tetrahedral.
PF_5	$4,0 - 2,1 = 1,9$	Polar covalent bond	Non-polar molecule. Phosphorous pentafluoride is trigonal bypramidal and symmetrical.
$BeCl_2$	$3,0 - 1,5 = 1,5$	Polar covalent bond	Non-polar molecule. Beryllium chloride is linear and symmetrical.
CO	$3,5 - 2,5 = 1,0$	Polar covalent bond	Polar molecule. Carbon monoxide is linear, but not symmetrical.
C_2H_2	$2,5 - 2,1 = 0,4$	Polar covalent bond	Non-polar molecule. Acetylene is linear and symmetrical.
SO_2	$3,5 - 2,5 = 1,0$	Polar covalent bond	Polar molecule. Sulfur dioxide is bent or angular and is not symmetrical.
BF_3	$4,0 - 2,0 = 2,0$	Polar covalent bond	Non-polar molecule. Boron trifluoride is trigonal pyramidal and not symmetrical.

3.5 Chapter summary

Exercise 3 – 9:

1. Give **one word/term** for each of the following descriptions.

    a) The distance between two adjacent atoms in a molecule.

    b) A type of chemical bond that involves the sharing of electrons between two atoms.

c) A measure of an atom's ability to attract electrons to itself in a chemical bond.

Solution:

a) Bond length

b) Covalent bond

c) Electronegativity

2. Which ONE of the following best describes the bond formed between an H^+ ion and the NH_3 molecule?

a) Covalent bond

b) Dative covalent (co-ordinate covalent) bond

c) Ionic Bond

d) Hydrogen Bond

Solution:

Dative covalent (co-ordinate covalent) bond

3. Explain the meaning of each of the following terms:

a) valence electrons

b) bond energy

c) covalent bond

Solution:

a) The number of electrons in the outermost shell of an atom that are available for use in bonding either by sharing, donating or accepting.

b) The amount of energy needed for a bond to break.

c) A type of bond that occurs between two atoms with a difference in electronegativity between 0 and 2,1.

4. Which of the following reactions will *not* take place? Explain your answer.

a) $H + H \rightarrow H_2$

b) $Ne + Ne \rightarrow Ne_2$

c) $Cl + Cl \rightarrow Cl_2$

Solution:

$Ne + Ne \rightarrow Ne_2$ will not take place as neon does not have electrons available for bonding. Neon is a noble gas and has a full outer shell of electrons.

5. Draw the Lewis diagrams for each of the following:

a) An atom of strontium (Sr). (Hint: Which group is it in? It will have an identical Lewis diagram to other elements in that group).

b) An atom of iodine.

c) A molecule of hydrogen bromide (HBr).

d) A molecule of nitrogen dioxide (NO_2). (Hint: There will be a single unpaired electron).

Solution:

a) Sr ×
   ×

b) :I:
   (iodine with lone pairs)

c) H ×   •Br:
                (bromine with lone pairs)

d) :O: N :O:
   (with × and • electrons shown)
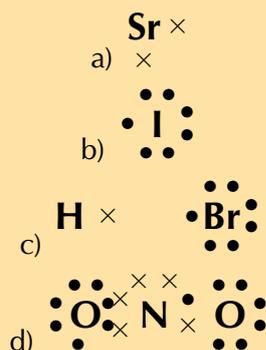
6. Given the following Lewis diagram, where X and Y each represent a different element:

$$X \overset{\times \times}{\underset{\times}{\cdot} Y} \overset{\cdot}{\underset{\cdot}{\times}} X$$
$$X$$

a) How many valence electrons does X have?

b) How many valence electrons does Y have?

c) Which elements could X and Y represent?

Solution:

a) 1

b) 5

c) X could be hydrogen and Y could be nitrogen.

7. Determine the shape of the following molecules:

a) O_2

b) MgI_2

c) BCl_3

d) CS_2

e) CCl_4

f) CH_3Cl

g) Br_2

h) SCl_5F

Solution:

a) This is a diatomic molecule and so the molecular shape is linear.

b) The central atom is magnesium (draw the molecules Lewis structure to see this).

   There are two electron pairs around magnesium and no lone pairs.

   There are two bonding electron pairs and no lone pairs. The molecule has the general formula AX_2. Using this information and Table **??** we find that the molecular shape is linear.

c) The central atom is boron (draw the molecules Lewis structure to see this).

There are three electron pairs around boron and no lone pairs.

There are three bonding electron pairs and no lone pairs. The molecule has the general formula AX_3. Using this information and Table **??** we find that the molecular shape is trigonal planar.

d) The central atom is carbon (draw the molecules Lewis structure to see this).

There are four electron pairs around carbon forming two double bonds. There are no lone pairs.

The molecule has the general formula AX_2. Using this information and Table **??** we find that the molecular shape is linear.

e) The central atom is carbon (draw the molecules Lewis structure to see this).

There are four bonding electron pairs and no lone pairs. The molecule has the general formula AX_4. Using this information and Table **??** we find that the molecular shape is tetrahedral.

f) The central atom is carbon (draw the molecules Lewis structure to see this).

There are four bonding electron pairs and no lone pairs. The molecule has the general formula AX_4. Using this information and Table **??** we find that the molecular shape is tetrahedral.

g) This is a diatomic molecule and so the molecular shape is linear.

h) The central atom is sulfur

There are six electron pairs around beryllium and no lone pairs.

The molecule has the general formula AX_6. Using this information and Table **??** we find that the molecular shape is octahedral.

8. Complete the following table.

Element pair	Electronegativity difference	Type of bond that could form
Hydrogen and lithium		
Hydrogen and boron		
Hydrogen and oxygen		
Hydrogen and sulfur		
Magnesium and nitrogen		
Magnesium and chlorine		
Boron and fluorine		
Sodium and fluorine		
Oxygen and nitrogen		
Oxygen and carbon		

Solution:

Element pair	Electronegativity difference	Type of bond that could form
Hydrogen and lithium	1,22	Strong polar covalent bond
Hydrogen and boron	0,16	Weak polar covalent bond
Hydrogen and oxygen	1,24	Strong polar covalent bond
Hydrogen and sulfur	0,38	Weak polar covalent bond
Magnesium and nitrogen	1,73	Strong polar covalent bond
Magnesium and chlorine	1,73	Strong polar covalent bond

9. Are the following molecules polar or non-polar?

   a) O_2

   b) $MgBr_2$

   c) BF_3

   d) CH_2O

   Solution:

   a) The molecule is linear. There are two bonding pairs forming a double bond and two lone pairs on each oxygen atom.

   There is one bond. The electronegativity difference between oxygen and oxygen is 0. The bond is non-polar.

   The molecule is symmetrical and is non-polar.

   b) The molecule is linear. There are two bonding pairs forming two single bonds and three lone pairs on each bromine atom.

   There are two bonds, both of which are between magnesium and bromine. The electronegativity difference between magnesium and bromine is $1,6$. The bonds are polar.

   The molecule is symmetrical and is non-polar.

   c) The molecule is trigonal planar. There are three bonding pairs forming three single bonds and three lone pairs on each fluorine atom.

   There are three bonds, all of which are between boron and fluorine. The electronegativity difference between boron and fluorine is $2,0$. The bonds are polar.

   The molecule is symmetrical and is non-polar.

   d) The molecule is trigonal planar. There are four bonding pairs forming two single bonds and one double bond. There are two lone pairs on the oxygen atom.

   There are three bonds, two of which are between carbon and hydrogen. The electronegativity difference between carbon and hydrogen is $0,4$. The other bond is between carbon and oxygen. The electronegativity difference between carbon and oxygen is $1,0$. All the bonds are polar.

   The molecule is not symmetrical and is polar.

10. Given the following graph for hydrogen:
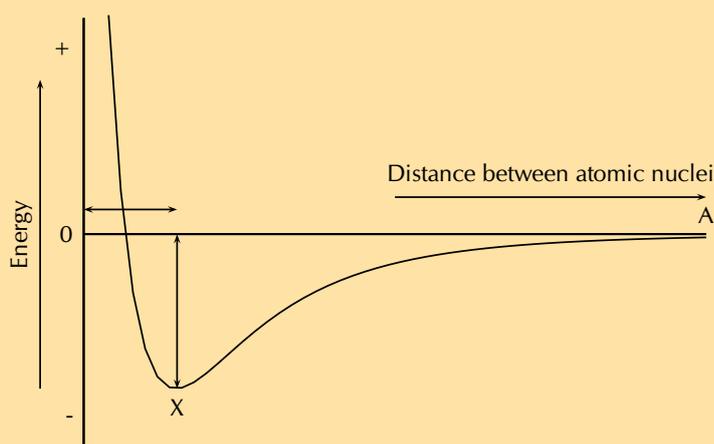
a) The bond length for hydrogen is 74 pm. Indicate this value on the graph. (Remember that pm is a picometer and means 74×10^{-12} m).

The bond energy for hydrogen is 436 kJ·mol^{-1}. Indicate this value on the graph.

b) What is important about point X?

Solution:

a)
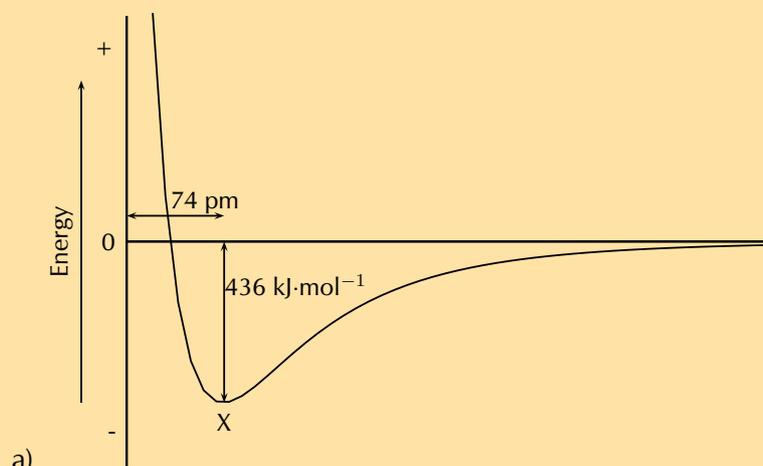
b) At point X the attractive and repulsive forces acting on the two hydrogen atoms are balanced. The energy is at a minimum.

11. Hydrogen chloride has a bond length of 127 pm and a bond energy of 432 kJ·mol^{-1}. Draw a graph of energy versus distance and indicate these values on your graph. The graph does not have to be accurate, a rough sketch graph will do.

Solution:

CHAPTER 4

Intermolecular forces

4   Intermolecular forces

If there were no intermolecular forces than all matter would exist as gases and we would not be here. This chapter introduces learners to a new concept called an intermolecular force. It is ea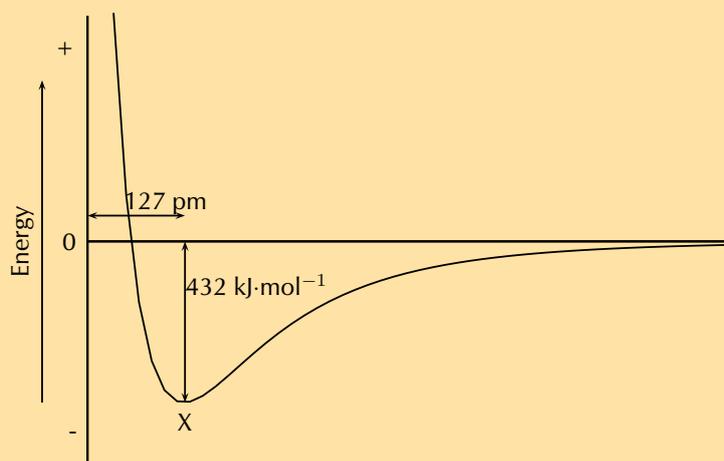sy for learners to become confused as to whether they are talking about bonds or about intermolecular forces, particularly when the intermolecular forces in the noble gases are discussed. For this reason you should try and use the word bond or bonding to refer to the interatomic forces (the things holding the atoms together) and intermolecular forces for the things holding the molecules together. Getting learners to label the bonds and intermolecular forces on diagrams of molecules will help them to come to grips with the terminology.

This topic comes right after learners have learnt about electronegativity and polarity so this is a good chapter to reinforce those concepts and help learners see the use of electronegativity and polarity. Learners need to be very comfortable with determining the polarity and shape of molecules as this will help them determine the kinds of intermolecular forces that occur.

A brief overview of the topics covered in this chapter follows.

- **What are intermolecular and how do they differ from bonds (interatomic forces).**

  This topic introduces learners to the concept of intermolecular forces. The five different types of intermolecular forces are introduced. Intermolecular forces are one of the main reason that matter exists in different states (solids, liquids and gases). Gases have no intermolecular forces between particles. For this reason you should either choose examples that are all in the liquid or solid state at room temperature (this temperature is the most familiar to learner) or remind learners that although the examples may be gases, we can consider the intermolecular forces between gases when they are cooled down and become liquids. It is also important to take care if you use the noble gases to explain induced dipole forces since technically these forces are not between molecules and so may confuse learners.

- **Physical state and density.**

  Although this is listed as a separate point in CAPs, in this book it has been worked into the explanation of intermolecular forces. Solids have the strongest intermolecular forces between molecules and it is these forces which hold the molecules in a rigid shape. In a liquid the intermolecular forces are continuously breaking and reforming as the molecules move and slide over each other.

- **Particle kinetic energy and temperature.**

  This topic is also listed as a separate point in CAPs and is worked into the explanation of intermolecular forces. This topic links back to grade 10 (states of matter and the kinetic molecular theory) and also links to chapter 7 (ideal gases).

- **The chemistry of water.**

  The second half of this chapter is devoted to understanding more about water. Water is a unique liquid in many aspects. Some of these properties of water are explained in this part of the text. It is important to link this into intermolecular forces and in particular the strong hydrogen bonds that are found between water molecules.

The worked examples on intermolecular forces condense a lot of information into the first step. You may need to remind learners how to determine molecular polarity. To do this, you can use the worked examples in atomic combinations as a quick refresher of the topic. In tests and exams learners need to be able to quickly identify a polar or non-polar molecule and so need to be very comfortable with this skill.

A formal experiment on the effects of intermolecular forces is included in this chapter. In this experiment learners will investigate how intermolecular forces affect evaporation, surface tension, solubility, boiling points and capillarity. Some of the substances that are used (nail polish remover (mainly acetone if you use the non acetone free variety), methylated spirits (a mixture of methanol and ethanol), oil (a mostly non-polar hydrocarbon), glycerin (a fairly complex organic molecule)) are quite complex substances and learners may not have the skills needed to determine the types of intermolecular forces at work here. You should guide learners in this and tell them the intermolecular forces for these substances.

You can help learners work out the strength of the intermolecular forces by telling them that larger molecules have stronger intermolecular forces than smaller molecules. This is often a big factor in determining which substance has the strongest intermolecular forces.

This experiment is split into five experiments. Each experiment focuses on a different property and sees how that property relates to intermolecular forces. It will often not be easy for learners to see the small differences between some of the molecules chosen and so they need to use a combination of experimental results and knowledge about the strength of the intermolecular force to try and predict what may happen. Each experiment ends with a conclusion about what should be found to guide learners.

It is very important to work in a well ventilated room (one with lots of air flow) particularly when working with methanol and ethanol. Many of the substances used (particularly nail polish remover, ethanol and methylated spirits) are highly flammable and so care must be taken when heating these substances. It is recommended that learners use a hot plate rather than a Bunsen burner to heat these substances as this reduces the risk of fire. When doing chemistry experiments it is also extra important to ensure that your learners do not run around, do not try to drink the chemicals, do not eat and drink in the lab, do not throw chemicals on the other learners and in general do act in a responsible and safe way. The guidelines for safe experimental work can be found in the science skills chapter from grade 10.

An experiment for informal assessment is included in this chapter. This experiment is very similar to the one on intermolecular forces. In this experiment learners focus on the properties of water. This is a good experiment to do to guide learners in understanding the properties of water.

When working with Bunsen burners learners should ensure that loose clothing is tucked away and long hair is tied back. As always with chemistry experiments you should open all the windows to ensure a well ventilated room. At the end of the experiment check that all Bunsen burners are turned off.

If learners leave the beaker of water on the Bunsen burner for to long and the water starts to boil or steam is observed, then make sure the learners do not touch the beaker as they will be burn themselves.

4.1 Intermolecular and interatomic forces

Types of intermolecular forces

The difference between intermolecular and interatomic forces

Exercise 4 – 1:

Which intermolecular forces are found in:

1. hydrogen fluoride (HF)

   Solution:

   Hydrogen fluoride is a polar covalent molecule. (It is linear and not symmetrical.) So the type of intermolecular force is dipole-dipole forces.

2. methane (CH_4)

   Solution:

   Methane is a non-polar covalent molecule. (It is tetrahedral and symmetrical.) So the type of intermolecular force is induced dipole forces.

3. potassium chloride in ammonia (KCl in NH_3)

   Solution:

   Potassium chloride is an ionic compound. Ammonia is a polar covalent molecule. (It is trigonal pyramidal and not symmetrical.) So the type of intermolecular force is ion-dipole forces.

4. krypton (Kr)

   Solution:

   Krypton is a noble gas. So the type of intermolecular force is induced dipole forces.

Understanding intermolecular forces

Exercise 4 – 2: Types of intermolecular forces
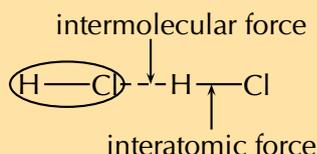
1. Given the following diagram:

$$H\!-\!\!-\!Cl\text{-}\text{-}\text{-}H\!-\!\!-\!Cl$$

   a) Name the molecule and circle it on the diagram

   b) Label the interatomic forces (covalent bonds)

   c) Label the intermolecular forces

Solution:

The molecule is hydrogen chloride.

2. Given the following molecules and solutions:

   HCl, CO_2, I_2, H_2O, KI(aq), NH_3, NaCl(aq), HF, $MgCl_2$ in CCl_4, NO, Ar, SiO_2

   Complete the table below by placing each molecule next to the correct type of intermolecular force.

Ion-dipole	
Ion-induced-dipole	
Dipole-dipole (no hydrogen bonding)	
Dipole-dipole (hydrogen bonding)	
Induced dipole	
Dipole-induced-dipole	

   In which one of the substances listed above are the intermolecular forces:

   a) strongest

   b) weakest

Solution:

Ion-dipole	KI(aq), NaCl(aq), HF(aq)
Ion-induced-dipole	$MgCl_2$ in CCl_4
Dipole-dipole (no hydrogen bonding)	HCl, NO
Dipole-dipole (hydrogen bonding)	H_2O, NH_3
Induced dipole	CO_2, I_2, Ar
Dipole-induced-dipole	SiO_2 in water

Water or ammonia are likely to have the strongest forces, while argon, iodine and carbon dioxide are likely to have the weakest forces. Induced dipole forces are the weakest intermolecular forces and hydrogen bonding is the strongest.

3. Use your knowledge of different types of intermolecular forces to explain the following statements:

   a) The boiling point of F_2 is much lower than the boiling point of NH_3

   b) Water evaporates slower than carbon tetrachloride (CCl_4).

   c) Sodium chloride is likely to dissolve in methanol (CH_3OH) .

Solution:

   a) NH_3 has hydrogen bonds which are much stronger than the induced dipole forces in F_2. In order for a liquid to boil the intermolecular forces must be broken and if the intermolecular forces are very strong then it will take a lot of energy to overcome these forces and so the boiling point will be higher.

b) Water has strong intermolecular forces (hydrogen bonds) while carbon tetrachloride only has weaker induced dipole forces. (Carbon tetrachloride is non-polar). Substances with stronger intermolecular forces take longer to evaporate than substances with weaker intermolecular forces.

c) Sodium chloride is ionic. Methanol is polar. The type of intermolecular force that can exist when sodium chloride dissolves in methanol is ion-dipole forces. The formation of these forces helps to disrupt the ionic bonds in sodium chloride and so sodium chloride can dissolve in methanol.

4. Tumi and Jason are helping their dad tile the bathroom floor. Their dad tells them to leave small gaps between the tiles. Why do they need to leave these small gaps?

Solution:

Materials (such as tiles) expand on heating and so small gaps need to be left between the tiles to allow for this expansion. If Tumi and Jason did not leave these gaps between the tiles, the tiles would soon lift up.

4.2 The chemistry of water

The unique properties of water

Exercise 4 – 3: The properties of water

1. Hope returns home from school on a hot day and pours herself a glass of water. She adds ice cubes to the water and notices that they float on the water.

   a) What property of ice cubes allows them to float in the water?

   b) Briefly describe how this property affects the survival of aquatic life during winter.

   Solution:

   a) Ice cubes are less dense than liquid water. Water has a less dense solid phase than solid water.

   b) If the ice did not float on top of the water then all bodies of water would freeze from the bottom up. This would mean that aquatic life would not be able to survive through the cold winters as there would be no habitat for them.

2. Which properties of water allow it to remain in its liquid phase over a large temperature range? Explain why this is important for life on earth.

   Solution:

   High boiling point and low melting point. Water has strong hydrogen bonds between molecules. These bonds require a lot of energy before they will break. This leads to water having a higher boiling point than if there were only weaker dipole-dipole forces. Water also has a high specific heat. If water did not have

such a large range in which it is a liquid, bodies of water would freeze over faster, destroying the life in them. Also if the boiling point of water was lower, then all the water could evaporate on a hot day, which would cause all life to die.

3. Which property of water allows the oceans to act as heat reservoirs? What effect does this have on the Earths climate?

Solution:

Water is able to absorb infrared radiation (heat) from the sun. This heat energy is stored in the oceans. Without this effect, the heat from the sun would cause the daytime temperatures on the Earth to become unbearably hot.

4.3   Chapter summary

Exercise 4 – 4:

1. Give one word or term for each of the following descriptions:

   a) The attractive force that exists between molecules.
   b) A molecule that has an unequal distribution of charge.
   c) The amount of heat energy that is needed to increase the temperature of a unit mass of a substance by one degree.

   Solution:

   a) Intermolecular force
   b) Polar molecule
   c) Specific heat

2. Refer to the list of substances below:

   HCl, Cl_2, H_2O, NH_3, N_2, HF

   Select the true statement from the list below:

   a) NH_3 is a non-polar molecule
   b) The melting point of NH_3 will be higher than for Cl_2
   c) Ion-dipole forces exist between molecules of HF
   d) At room temperature N_2 is usually a liquid

   Solution:

   The melting point of NH_3 will be higher than for Cl_2

3. The following table gives the melting points of various hydrides:

Hydride	Melting point (°C)
HI	−34
NH_3	−33
H_2S	−60
CH_4	−164

a) In which of these hydrides does hydrogen bonding occur?

    i. HI only

    ii. NH_3 only

    iii. HI and NH_3 only

    iv. HI, NH_3 and H_2S

b) Draw a graph to show the melting points of the hydrides.

c) Explain the shape of the graph.

(IEB Paper 2, 2003)

Solution:

a) HI and NH_3 only

b) Hydrides are numbered as: HI is 4, NH_3 is 3, H_2S is 2 and CH_4 is 1.
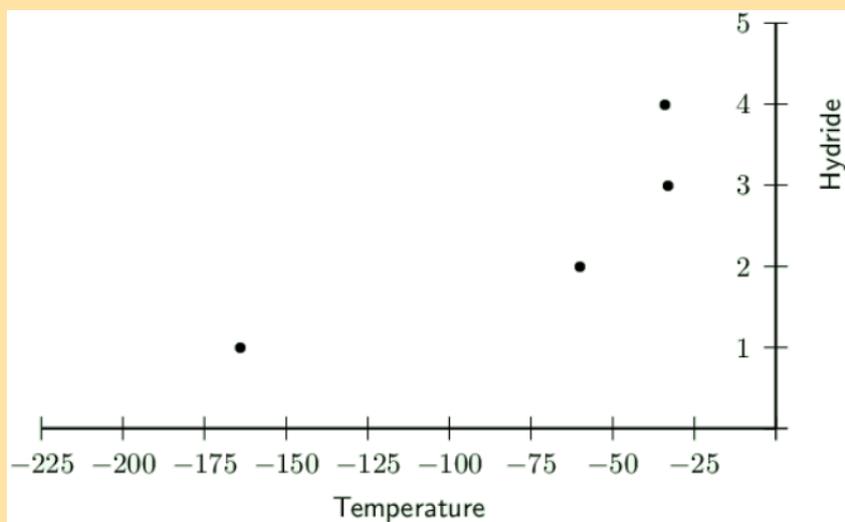
c) There is a decrease in the temperature from the first hydride (number 4 on the graph) to the last one (number 1 on the graph). The shape of the graph shows the decreasing intermolecular forces between the molecules in the compounds.

4. The respective boiling points for four chemical substances are given below:

Hydrogen sulphide	$-60\ °C$
Ammonia	$-33\ °C$
Hydrogen fluoride	$20\ °C$
Water	$100\ °C$

a) Which one of the substances exhibits the strongest forces of attraction between its molecules in the liquid state?

b) Give the name of the force responsible for the relatively high boiling points of hydrogen fluoride and water and explain how this force originates.

c) The shapes of the molecules of hydrogen sulfide and water are similar, yet their boiling points differ. Explain.

(IEB Paper 2, 2002)

Solution:

a) Water

b) Hydrogen bonding. This force occurs between the hydrogen atoms of one molecule and a high electronegativity atom of another molecule. The relatively positive hydrogen atom is attracted to the relatively negative atom (e.g. oxygen, nitrogen, fluorine).

c) Hydrogen sulfide does not have hydrogen bonding since sulfur has a low electronegativity. This reduces the boiling point of hydrogen sulfide since it is easier to break the intermolecular forces between molecules of hydrogen sulfide.

5. Susan states that van der Waals forces include ion-dipole forces, dipole-dipole forces and induced dipole forces.

Simphiwe states that van der Waals forces include ion-dipole forces, ion-induced dipole forces and induced dipole forces.

Thembile states that van der Waals forces include dipole-induced dipole forces, dipole-dipole forces and induced dipole forces.

Who is correct and why?

Solution:

Thembile is correct. Van der Waals forces are the only forces that can exist with covalent molecules and so including either ion-dipole or ion-induced dipole forces is not correct.

6. Jason and Bongani are arguing about which molecules have which intermolecular forces. They have drawn up the following table:

Compound	Type of force
Potassium iodide in water (KI(aq))	dipole-induced dipole forces
Hydrogen sulfide (H_2S)	induced dipole forces
Helium (He)	ion-induced dipole forces
Methane (CH_4)	induced dipole forces

a) Jason says that hydrogen sulfide (H_2S) is non-polar and so has induced dipole forces. Bongani says hydrogen sulfide is polar and has dipole-dipole forces. Who is correct and why?

b) Bongani says that helium (He) is an ion and so has ion-induced dipole forces. Jason says helium is non-polar and has induced dipole forces. Who is correct and why?

c) They both agree on the rest of the table. However, they have not got the correct force for potassium iodide in water (KI(aq)). What type of force actually exists in this compound?

Solution:

a) Bongani is correct. NH_3 is polar. It has one lone pair of electrons and so is trigonal pyramidal. The three polar bonds do not cancel each other out, since the molecule is not symmetrical. This makes ammonia a dipole molecule. So the type of intermolecular force that exists is dipole-dipole forces.

b) Jason is correct. Helium is a noble gas and so exists as single atoms, not as a compound. Helium is non-polar and so has induced-dipole forces.

c) KI(aq) has potassium and iodine ions in water. Water is a polar molecule. So the type of force must be ion-dipole.

7. Khetang is looking at power lines around him for a school project. He notices that they sag slightly between the pylons. Why do power lines need to sag slightly?

Solution:

Materials expand on heating and contract on cooling. If the power lines were strung tightly and did not sag then every time the weather got cold the power lines would contract and break.

8. Briefly describe how the properties of water make it a good liquid for life on Earth.

Solution:

Water has a high heat of vaporisation which means that it has to be heated to a high temperature before it will evaporate. This helps to ensure that all the water in our bodies does not evaporate on a warm day. This also keeps the oceans and other water bodies liquid.

Water has a high specific heat which means that it takes a long time for the temperature of the water to increase by one degree. Water is also able to absorb infrared radiation from the sun. These two properties help to regulate the climate of the Earth. Without these properties of water, the Earth would be much hotter.

Water also has a less dense solid phase. This means that when water freezes only the top layer freezes. If this did not happen then the Earth would freeze over. This property also ensures that life can exist beneath the ice surface during cold winters.

CHAPTER 5

Geometrical optics

This chapter introduces the concepts of geometrical optics. In earlier grades learners were introduced to reflection and the laws of reflection. This work and the concept of a light ray are revised in this chapter. Learners will also learn about the speed of light and how to sketch ray diagrams. Snell's law and how total internal reflection arises are covered.

The topics covered in this chapter are briefly summarised below.

- **Reflection and the law of reflection**

  The concept of a light ray is first introduced. This imaginary construct is a useful tool to help learners understand how light reflects and refracts. The concepts of reflection and the law of reflection are revised from earlier grades. Although this topic is included under refraction in CAPs it is treated separately in this book.

- **The speed of light**

  Light has a set speed in different mediums. The maximum speed of light occurs for light travelling in a vacuum. In all other media light travels slightly slower. This changing of the speed of light is what gives rise to refraction.

- **Refraction and refractive index**

  Refraction is the bending of light as it moves from one medium to another. It is best seen when looking at a straw in water. Each medium has a refractive index associated with it and this is a measure of how much light will refract when entering that medium. The refractive index is closely linked to the optical density (or optical absorbance) of a medium.

- **Sketching ray diagrams for the path of light through different media**

  This topic teaches learners how to sketch the path of light as it moves through different media and helps to consolidate the concept of refractive indices. Learners need to understand that light bends to different degrees and the direction in which it bends depends on the refractive index of the medium.

- **Snell's law**

  This topic shows learners how to calculate a precise angle for the degree of bending of light, as well as how to calculate the refractive index of a medium. Snell's law relates the refractive index of one medium to the refractive index of another medium using the sine of the angle of incidence and the sine of the angle of refraction.

- **Total internal reflection**

  Snell's law leads into the idea of a critical angle for light travelling from a more optically dense to a less optically dense medium. This concept is then expanded on to total internal reflection. Total internal reflection is important in communications and medicine, among other fields.

A recommended experiment for informal assessment is also included. This investigates the propagation of light from one medium into another. Learners will need glass blocks of various shapes, a transparent container, water, plain paper, a pencil, a ruler,

a protractor and a ray box. The questions for discussion at the end of the experiment get learners thinking about what they found in the experiment.

A recommended project for formal assessment on verifying Snell's laws and determining the refractive index of an unknown transparent solid should be done. Learners will need a glass block, ray box, 0-360 protractor, a glass block, a transparent block of an unknown material and A4 paper. This project is given as two formal experiments, which learners can complete and then write up as a project. The project should include some background information on Snell's law, all the steps taken in the experiments, key experimental results and a conclusion about the experiment (including identification of the unknown solid from the refractive index).

A recommended experiment for informal assessment is also included. This covers determining the critical angle for light travelling through a rectangular glass block. Learners will need a rectangular glass block, a 360° protractor, pencil, paper, ruler and a ray box. Learners should all get similar results at the end of the experiment.

5.3 Properties of light: revision

Reflection

**Exercise 5 – 1: Rays and Reflection**

1. Are light rays real? Explain.
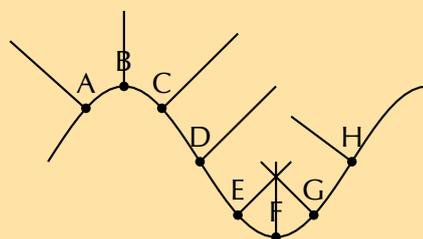
   Solution:

   Light rays are not real.

   In physics we use the idea of a light ray to indicate the direction in which light travels. In geometrical optics, we represent light rays with straight arrows to show how light propagates. Light rays are not an exact description of the light; rather they show the direction in which the light wavefronts are travelling.
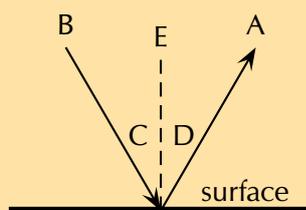
2. The diagram shows a curved surface. Draw normals to the surface at the marked points.
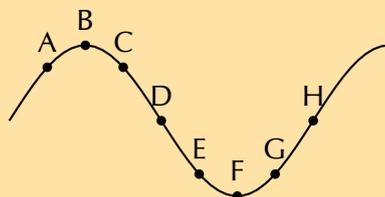
   Solution:

3. Which of the labels, A–H, in the diagram, correspond to the following:

B   E   A

C ¦ D

surface

a) normal

b) angle of incidence

c) angle of reflection

d) incident ray

e) reflected ray

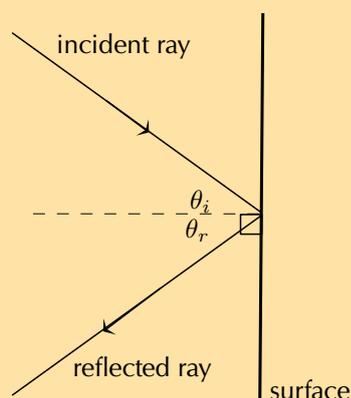**Solution:**

a) E

b) C

c) D

d) B

e) A

4. State the Law of Reflection. Draw a diagram, label the appropriate angles and write a mathematical expression for the Law of Reflection.
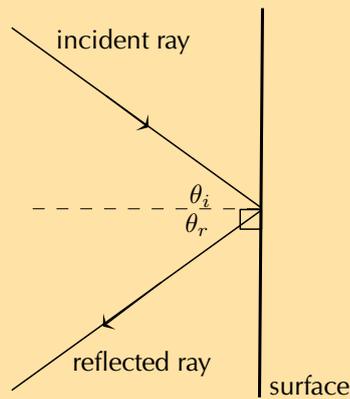
Solution:

The law of reflection states that the angle of incidence is equal to the angle of reflection and the incident ray, reflected ray, and the normal, all lie in the same plane.

incident ray

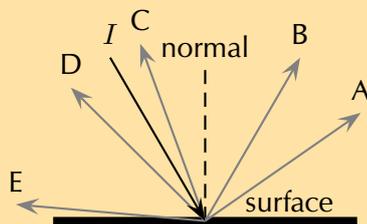θ_i
θ_r

reflected ray

surface

We can write this mathematically as: $\theta_i = \theta_r$.

5. Draw a ray diagram to show the relationship between the angle of incidence and the angle of reflection.

Solution:

6. The diagram shows an incident ray I. Which of the other 5 rays (A, B, C, D, E) best represents the reflected ray of I?
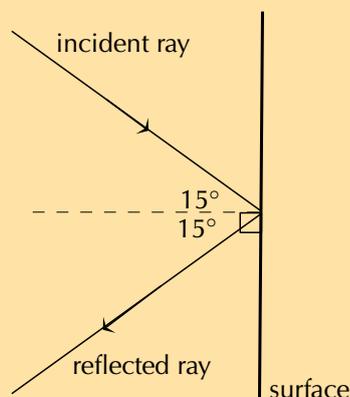

Solution:

Ray B looks like it is reflected at the same angle as the incident ray.

7. A ray of light strikes a surface at $15°$ to the normal to the surface. Draw a ray diagram showing the incident ray, reflected ray and surface normal. Calculate the angles of incidence and reflection and fill them in on your diagram.

Solution:

We are told that the ray of light strikes the surface at $15°$ to the normal to the surface, so the angle of incidence is $15°$. Since the angle of incidence is equal to the angle of reflection, the angle of reflection is also $15°$.

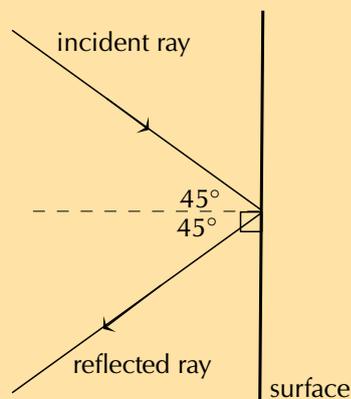Putting this onto a ray diagram (not to scale!):

8. A ray of light leaves a surface at $45°$ to the normal to the surface. Draw a ray diagram showing the incident ray, reflected ray and surface normal. Calculate the angles of incidence and reflection and fill them in on your diagram.

Solution:

We are told that the ray of light leaves the surface at $45°$ to the normal to the surface, so the angle of reflection is $45°$. Since the angle of reflection is equal to the angle of incidence, the angle of incidence is also $15°$.

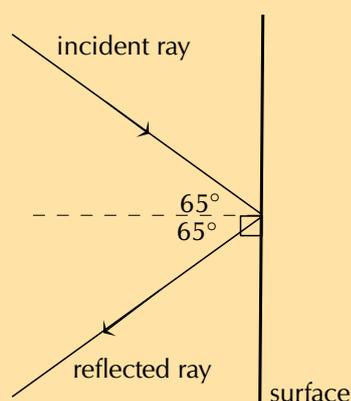Putting this onto a ray diagram (not to scale!):

9. A ray of light strikes a surface at $25°$ to the surface. Draw a ray diagram showing the incident ray, reflected ray and surface normal. Calculate the angles of incidence and reflection and fill them in on your diagram.

Solution:

We are told that the ray of light strikes the surface at $25°$ to the surface. To find the angle of incidence we note that the normal is drawn at $90°$ to the surface. So the angle of incidence is $90° - 25° = 65°$. Since the angle of incidence is equal to the angle of reflection, the angle of reflection is also $65°$.

Putting this onto a ray diagram (not to scale!):

10. A ray of light leaves a surface at $65°$ to the surface. Draw a ray diagram showing the incident ray, reflected ray and surface normal. Calculate the angles of incidence and reflection and fill them in on your diagram.

Solution:

We are told that the ray of light leaves the surface at $65°$ to the surface. To find the angle of reflection we note that the normal is drawn at $90°$ to the surface. So the angle of reflection is $90° - 65° = 25°$. Since the angle of reflection is equal to the angle of incidence, the angle of incidence is also $25°$.

Putting this onto a ray diagram (not to scale!):

incident ray

25°
25°

reflected ray

surface
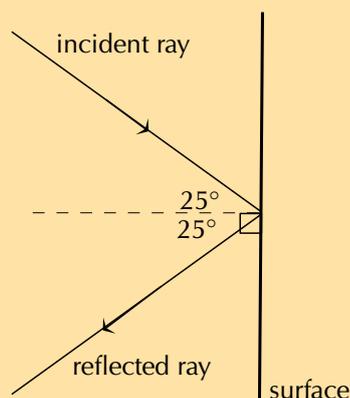
11. A beam of light (for example from a torch) is generally not visible at night, as it travels through air. Try this for yourself. However, if you shine the torch through dust, the beam is visible. Explain why this happens.

Solution:

To see the beam from a torch the light rays emitted by the torch need to be reflected off something for our eye to see it. At night the light rays are not being reflected off anything and so we do not see the beam of the torch. If we shine the torch through dust, then we can see the beam as the light rays reflect off the dust particles.

12. If a torch beam is shone across a classroom, only students in the direct line of the beam would be able to see that the torch is shining. However, if the beam strikes a wall, the entire class will be able to see the spot made by the beam on the wall. Explain why this happens.

Solution:

To see the beam from a torch the light rays emitted by the torch need to be reflected off something for our eye to see it. So only the students in the direct path of the light will see that the torch is shining. However as soon as the torch beam is reflected off the wall, the learners are all able to see the spot where the beam reflects.

13. A scientist looking into a flat mirror hung perpendicular to the floor cannot see her feet but she can see the hem of her lab coat. Draw a ray diagram to help explain the answers to the following questions:

 a) Will she be able to see her feet if she backs away from the mirror?

 b) What if she moves towards the mirror?

Solution:

 a) She will not be able to see her feet. Your eye sights along a line to see your feet. This line must intersect the mirror.

 The solid lines show the path of light from her labcoat hem to her eyes, while the dotted line shows the path that light would have to take to reach her eyes from her feet. As she moves further away the only thing that changes is the angles of incidence and reflection.
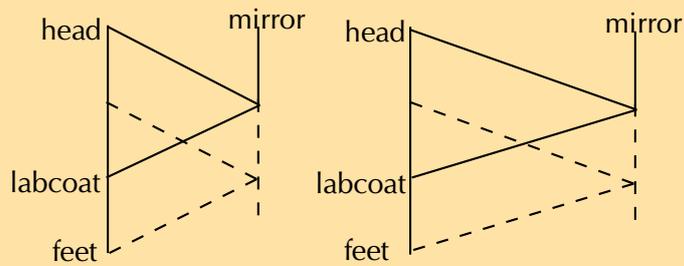
b) She still will not be able to see her feet. Your eye sights along a line to see your feet. This line must intersect the mirror.

The solid lines show the path of light from her labcoat hem to her eyes, while the dotted line shows the path that light would have to take to reach her eyes from her feet. As she moves closer the only thing that changes is the angles of incidence and reflection.
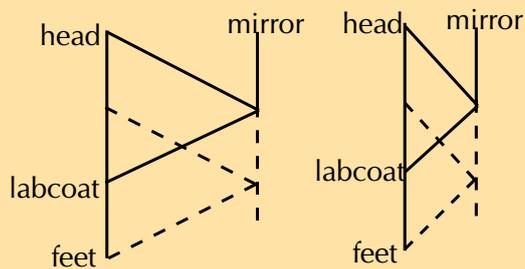
5.5 Refraction

Refractive index

Exercise 5 – 2: Refractive index

1. Use the values given in Table **??**, and the definition of refractive index to calculate the speed of light in water (ice).

   Solution:

   We look in Table **??** and find that the refractive index of water (ice) is $1,31$. So the speed of light in ice is:

   $$
   \begin{aligned}
   v &= \frac{c}{n} \\
   &= \frac{3 \times 10^8 \text{ m·s}^{-1}}{1,31} \\
   &= 2,29 \times 10^8 \text{ m·s}^{-1}
   \end{aligned}
   $$

2. Calculate the refractive index of an unknown substance where the speed of light through the substance is $1,974 \times 10^8$ m·s^{-1}. Round off your answer to 2 decimal places. Using Table **??**, identify what the unknown substance is.

Solution:

We are told that the speed of light in the unknown substance is: $1{,}974 \times 10^8$ m·s^{-1}. We use this to find the refractive index of the substance:

$$v = \frac{c}{n}$$
$$n = \frac{c}{v}$$
$$= \frac{3 \times 10^8 \text{ m·s}^{-1}}{1{,}974 \times 10^8 \text{ m·s}^{-1}}$$
$$= 1{,}52$$

We look on Table **??** for the substance that has this refractive index and find that the substance is crown glass.

Representing refraction with ray diagrams

Exercise 5 − 3: Refraction

1. Explain refraction in terms of a change of wave speed in different media.
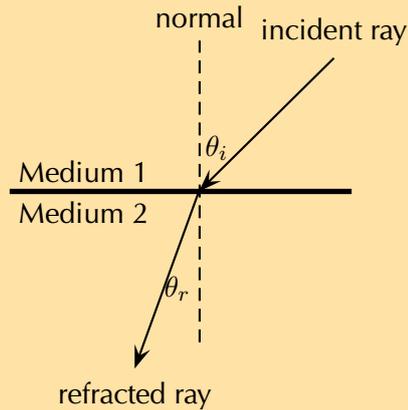
   Solution:

   Refraction occurs at the boundary of two media when light travels from one medium into the other and its speed changes but its frequency remains the same. If the light ray hits the boundary at an angle which is not perpendicular to or parallel to the surface, then it will change direction and appear to 'bend'.

   We can also say that refraction is the bending of light caused by a change in the speed of light as it moves from medium to another.

2. In the diagram, label the following:

   a) angle of incidence
   b) angle of refraction
   c) incident ray
   d) refracted ray
   e) normal
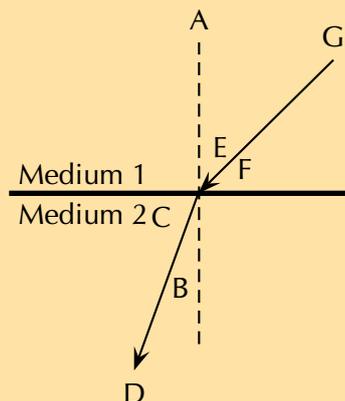
Solution:

3. What is *the angle of refraction*?

   Solution:

   The angle of refraction is the angle defined between the normal to a surface and the refracted light ray.

4. Describe what is meant by *the refractive index of a medium*.

   Solution:

   The refractive index (symbol n) of a material is the ratio of the speed of light in a vacuum to its speed in the material and gives an indication of how difficult it is for light to get through the material.
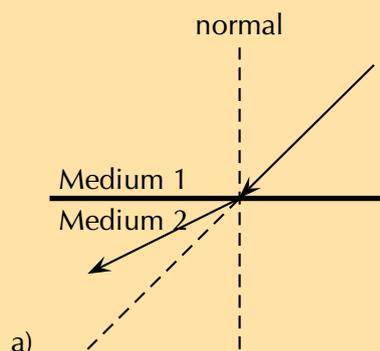
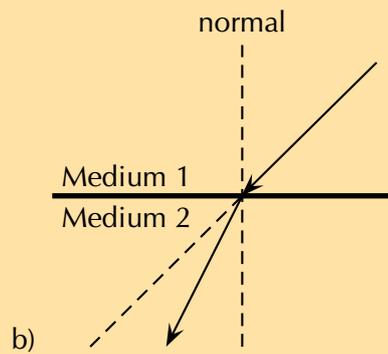5. In the diagram, a ray of light strikes the interface between two media.
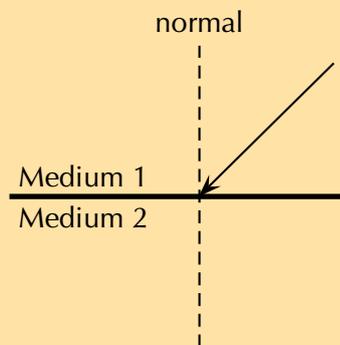
   Draw what the refracted ray would look like if:

   a) medium 1 had a higher refractive index than medium 2.
   b) medium 1 had a lower refractive index than medium 2.

   Solution:

b)

6. **Challenge question:** What values of n are physically impossible to achieve? Explain your answer. The values provide the limits of possible refractive indices.

   Solution:

   A refractive index of less than 1 is impossible. This would mean that light in this medium was travelling at a speed faster than the speed of light in a vacuum. However scientists have produced materials with negative refractive indices. So the values of n that are impossible to achieve physically would be: $0 \leqslant n < 1$.

7. **Challenge question:** You have been given a glass beaker full of an unknown liquid. How would you identify what the liquid is? You have the following pieces of equipment available for the experiment: a laser or ray box, a protractor, a ruler, a pencil, and a reference guide containing optical properties of various liquids.

   Solution:

   The answer to this question is covered in the experiment on using Snell's law to determine the refractive index of an unknown liquid.

5.6   Snell's Law

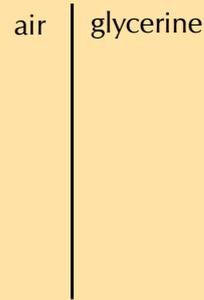**Exercise 5 – 4: Snell's Law**

1. State Snell's Law.

   Solution:

   Snell's law states that the angle of incidence times the refractive index of medium 1 is equal to the angle of refraction times the refractive index of medium 2. Mathematically this is: $n_1 \sin \theta_1 = n_2 \sin \theta_2$

2. Light travels from a region of glass into a region of glycerine, making an angle of incidence of $40°$.

air | glycerine

a) Draw the incident and refracted light rays on the diagram and label the angles of incidence and refraction.

b) Calculate the angle of refraction.

Solution:

a)
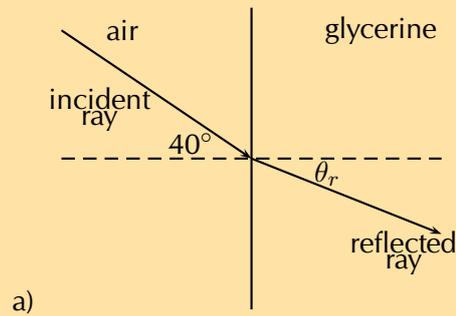
b) The refractive index of air is 1 and the refractive index of glycerine is $1{,}4729$.

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$
$$1 \sin 40° = 1{,}4729 \sin \theta_2$$
$$\sin \theta_2 = 0{,}436$$
$$\theta_2 = 25{,}88°$$

3. A ray of light travels from silicon to water. If the ray of light in the water makes an angle of $69°$ to the normal to the surface, what is the angle of incidence in the silicon?

Solution:

The refractive index of silicon is $4{,}01$ and the refractive index of water is $1{,}333$.

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$
$$1{,}333 \sin 69° = 4{,}01 \sin \theta_2$$
$$\sin \theta_2 = 0{,}310$$
$$\theta_2 = 18{,}08°$$

4. Light travels from a medium with $n = 1{,}25$ into a medium of $n = 1{,}34$, at an angle of $27°$ from the normal.

a) What happens to the speed of the light? Does it increase, decrease, or remain the same?

b) What happens to the wavelength of the light? Does it increase, decrease, or remain the same?

c) Does the light bend towards the normal, away from the normal, or not at all?

Solution:

a) The speed of light decreases as it enters the second medium.

b) The wavelength of the light remains the same. Wavelength is related to frequency and the frequency of light does not change as it moves from one medium to another.

c) Towards the normal.

5. Light travels from a medium with $n = 1,63$ into a medium of $n = 1,42$.

a) What happens to the speed of the light? Does it increase, decrease, or remain the same?

b) What happens to the wavelength of the light? Does it increase, decrease, or remain the same?

c) Does the light bend towards the normal, away from the normal, or not at all?

Solution:

a) The speed of light increases as it enters the second medium.

b) The wavelength of the light remains the same. Wavelength is related to frequency and the frequency of light does not change as it moves from one medium to another.

c) Away from the normal

6. Light is incident on a rectangular glass prism. The prism is surrounded by air. The angle of incidence is $23°$. Calculate the angle of reflection and the angle of refraction.

Solution:

The angle of reflection is the same as the angle of incidence: $23°$.

To find the angle of refraction we need the indices of refraction for glass and air. The index of refraction for air is 1 and for the prism it is $1,5 - 1,9$ (we assume that the prism is made of typical glass). Since typical glass has a range of refractive indices we will calculate the maximum and minimum possible angles of refraction.

The maximum possible angle of refraction is:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$
$$1 \sin 23° = 1,5 \sin \theta_2$$
$$\sin \theta_2 = 0,2604$$
$$\theta_2 = 15,10°$$

The minimum possible angle of refraction is:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$
$$1 \sin 23° = 1,9 \sin \theta_2$$
$$\sin \theta_2 = 0,2056$$
$$\theta_2 = 11,87°$$

7. Light is refracted at the interface between air and an unknown medium. If the angle of incidence is $53°$ and the angle of refraction is $37°$, calculate the refractive index of the unknown, second medium.

   Solution:

   The index of refraction for air is 1. The refractive index of the unknown medium is:

   $$n_1 \sin \theta_1 = n_2 \sin \theta_2$$
   $$1 \sin 53° = n_2 \sin 37°$$
   $$n_2 = 1{,}327$$

8. Light is refracted at the interface between a medium of refractive index $1{,}5$ and a second medium of refractive index $2{,}1$. If the angle of incidence is $45°$, calculate the angle of refraction.

   Solution:

   The angle of refraction is:

   $$n_1 \sin \theta_1 = n_2 \sin \theta_2$$
   $$1{,}5 \sin 45° = 2{,}1 \sin \theta_2$$
   $$\sin \theta_2 = 0{,}5051$$
   $$\theta_2 = 30{,}34°$$

9. A ray of light strikes the interface between air and diamond. If the incident ray makes an angle of $30°$ with the interface, calculate the angle made by the refracted ray with the interface.

   Solution:

   The refractive index of air is 1 and the refractive index of diamond is $2{,}419$. The angle of refraction is:

   $$n_1 \sin \theta_1 = n_2 \sin \theta_2$$
   $$1 \sin 30° = 2{,}419 \sin \theta_2$$
   $$\sin \theta_2 = 0{,}20669$$
   $$\theta_2 = 11{,}93°$$

   This is the angle between the normal and the refracted ray. The question actually wants the angle between the refracted ray and the interface. This angle is:

   $$90° - 11{,}93° = 78{,}07°$$

10. The angles of incidence and refraction were measured in five unknown media and recorded in the table below. Use your knowledge about Snell's Law to identify each of the unknown media A–E. Use Table **??** to help you.

Medium 1	n_1	θ_1	θ_2	n_2	Unknown Medium
Air	1,0002926	38	27	?	A
Air	1,0002926	65	38,4	?	B
Vacuum	1	44	16,7	?	C
Air	1,0002926	15	6,9	?	D
Vacuum	1	20	13,3	?	E

Solution:

For substance A, the refractive index is:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$
$$1{,}0002926 \sin 38° = n_2 \sin 27°$$
$$n_2 = 1{,}36$$

Substance A is acetone.

For substance B, the refractive index is:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$
$$1{,}0002926 \sin 65° = n_2 \sin 38{,}4°$$
$$n_2 = 1{,}459$$

Substance B is fused quartz.

For substance C, the refractive index is:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$
$$1 \sin 44° = n_2 \sin 16{,}7°$$
$$n_2 = 2{,}419$$

Substance C is diamond.

For substance D, the refractive index is:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$
$$1{,}0002926 \sin 15° = n_2 \sin 6{,}9°$$
$$n_2 = 2{,}15$$

Substance D is cubic zirconia.

For substance E, the refractive index is:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$
$$1 \sin 20° = n_2 \sin 13{,}3°$$
$$n_2 = 1{,}49$$

Substance E is an 80% sugar solution.

11. Zingi and Tumi performed an investigation to identify an unknown liquid. They shone a ray of light into the unknown liquid, varying the angle of incidence and recording the angle of refraction. Their results are recorded in the following table:

Angle of incidence	Angle of refraction
0,0°	0,00°
5,0°	3,76°
10,0°	7,50°
15,0°	11,2°
20,0°	14,9°
25,0°	18,5°
30,0°	22,1°
35,0°	25,5°
40,0°	28,9°
45,0°	32,1°
50,0°	35,2°
55,0°	38,0°
60,0°	40,6°
65,0°	43,0°
70,0°	?
75,0°	?
80,0°	?
85,0°	?

a) Write down an aim for the investigation.

b) Make a list of all the apparatus they used.

c) Identify the unknown liquid.

Solution:

a) To use refractive indices and Snell's law to determine an unknown substance.

b) Protractor, ruler, ray box, paper, pencils, beakers.

c) We can use any of the pairs of data to find the refractive index of the unknown substance. We will use the second pair of readings.

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$
$$1 \sin 5° = n_2 \sin 3{,}76°$$
$$n_2 = 1{,}329$$

The substance is water.

12. Predict what the angle of refraction will be for 70°, 75°, 80° and 85°.

Solution:

The angle of refraction at 70° is:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$
$$1 \sin 70° = 1{,}329 \sin \theta_2$$
$$\sin \theta_2 = 0{,}707$$
$$n_2 = 45{,}0°$$

The angle of refraction at 75° is:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$
$$1 \sin 75° = 1{,}329 \sin \theta_2$$
$$\sin \theta_2 = 0{,}7268$$
$$n_2 = 46{,}6°$$

The angle of refraction at 80° is:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$
$$1 \sin 80° = 1{,}329 \sin \theta_2$$
$$\sin \theta_2 = 0{,}741$$
$$n_2 = 47{,}8°$$

The angle of refraction at 85° is:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$
$$1 \sin 85° = 1{,}329 \sin \theta_2$$
$$\sin \theta_2 = 0{,}75$$
$$n_2 = 48{,}55°$$

5.7 Critical angles and total internal reflection

Total internal reflection

Exercise 5 – 5: Total internal reflection and fibre optics

1. Describe total internal reflection by using a diagram and referring to the conditions that must be satisfied for total internal reflection to occur.
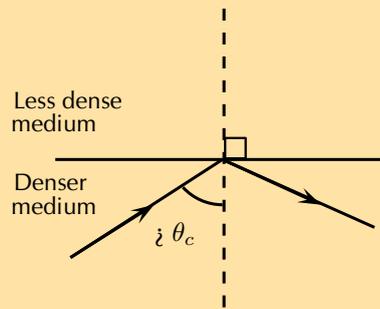
   Solution:

   If the angle of incidence is bigger than the critical angle, the refracted ray will not emerge from the medium, but will be reflected back into the medium. This is called total internal reflection.

   The critical angle occurs when the angle of incidence where the angle of refraction is 90°. The light must travel from an optically more dense medium to an optically less dense medium.

   The conditions for total internal reflection are the the light is travelling from an optically denser medium (higher refractive index) to an optically less dense medium (lower refractive index) and that the angle of incidence is greater than the critical angle.
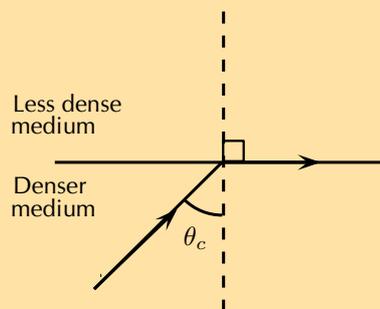
   Representing this on a diagram gives:

2. Define what is meant by the *critical angle* when referring to total internal reflection. Include a ray diagram to explain the concept.

   Solution:

   The critical angle occurs when the angle of incidence where the angle of refraction is $90°$. The light must travel from an optically more dense medium to an optically less dense medium.

3. Will light travelling from diamond to silicon ever undergo total internal reflection?

   Solution:

   Diamond (index of refraction is about 3) is less optically dense than silicon (index of refraction is about 4) and so total internal reflection cannot occur.

4. Will light travelling from sapphire to diamond undergo total internal reflection?

   Solution:

   Sapphire (index of refraction is 1,77) is less optically dense than diamond (index of refraction is about 3) and so total internal reflection cannot occur.

5. What is the critical angle for light travelling from air to acetone?

   Solution:

   The critical angle is:

$$n_1 \sin \theta_c = n_2 \sin 90°$$
$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$
$$= \sin^{-1}\left(\frac{1}{1,36}\right)$$
$$= 47,33°$$

6. Light travelling from diamond to water strikes the interface with an angle of incidence of $86°$ as shown in the picture. Calculate the critical angle to determine whether the light be totally internally reflected and so be trapped within the diamond.
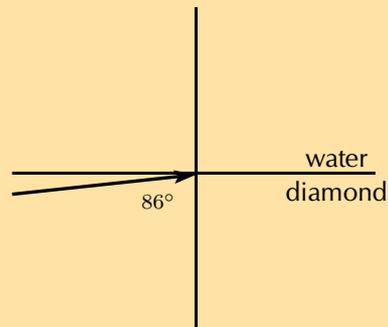
Solution:

The critical angle is:

$$n_1 \sin \theta_c = n_2 \sin 90°$$

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

$$= \sin^{-1}\left(\frac{1{,}33}{2{,}419}\right)$$

$$= 33{,}35°$$

The angle of incidence is greater than the critical angle and so the light will be trapped within the diamond.

7. Which of the following interfaces will have the largest critical angle?

   a) a glass to water interface

   b) a diamond to water interface

   c) a diamond to glass interface

Solution:

a glass to water interface

8. If a fibre optic strand is made from glass, determine the critical angle of the light ray so that the ray stays within the fibre optic strand.

Solution:

The critical angle is:

$$n_1 \sin \theta_c = n_2 \sin 90°$$

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

$$= \sin^{-1}\left(\frac{1}{1{,}5}\right)$$

$$= 41{,}8°$$

9. A glass slab is inserted in a tank of water. If the refractive index of water is $1{,}33$ and that of glass is $1{,}5$, find the critical angle.

Solution:

The critical angle is:

$$n_1 \sin \theta_c = n_2 \sin 90°$$
$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$
$$= \sin^{-1}\left(\frac{1{,}33}{1{,}5}\right)$$
$$= 62{,}46°$$

Note that the light must be travelling from the glass into the water for total internal reflection to occur.

10. A diamond ring is placed in a container full of glycerin. If the critical angle is found to be $37{,}4°$ and the refractive index of glycerin is given to be $1{,}47$, find the refractive index of diamond.

Solution:

The refractive index is:

$$n_1 \sin \theta_c = n_2 \sin 90°$$
$$n_1 = \frac{n_1}{\sin \theta_c}$$
$$= \frac{1{,}47}{\sin 37{,}4°}$$
$$= 2{,}42°$$

11. An optical fibre is made up of a core of refractive index $1{,}9$, while the refractive index of the cladding is $1{,}5$. Calculate the maximum angle which a light pulse can make with the wall of the core. NOTE: The question does not ask for the angle of incidence but for the angle made by the ray with the wall of the core, which will be equal to $90° -$ angle of incidence.
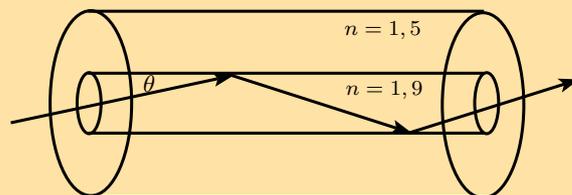
Solution:

The critical angle is:

$$n_1 \sin \theta_c = n_2 \sin 90°$$
$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$
$$= \sin^{-1}\left(\frac{1{,}5}{1{,}9}\right)$$
$$= 52{,}14°$$

The maximum angle is: $90° - 52,14° = 37,86°$

5.8 Summary

1. Give one word for each of the following descriptions:

   a) The perpendicular line that is drawn at right angles to a reflecting surface at the point of incidence.

   b) The bending of light as it travels from one medium to another.

   c) The bouncing of light off a surface.

   Solution:

   a) normal

   b) refraction

   c) reflection

2. State whether the following statements are **true** or **false**. If they are false, rewrite the statement correcting it.

   a) The refractive index of a medium is an indication of how fast light will travel through the medium.

   b) Total internal refraction takes place when the incident angle is larger than the critical angle.

   c) The speed of light in a vacuum is about 3×10^8 m·s^{-1}.

   Solution:

   a) False. The refractive index is the ratio of the speed of light in a vacuum to its speed in the material and gives an indication of how difficult it is for light to get through the material.

   b) True

   c) True

3. Complete the following ray diagrams to show the path of light.

a)

b)

c)

d)

e)

f)

Solution:

a)

b)

c)

d)

e)

f)
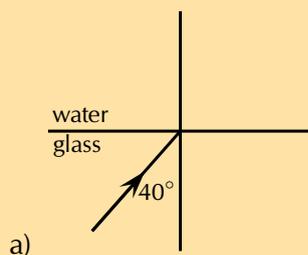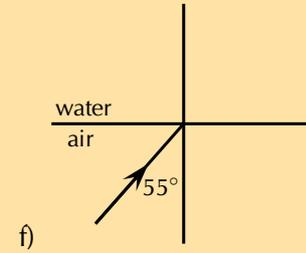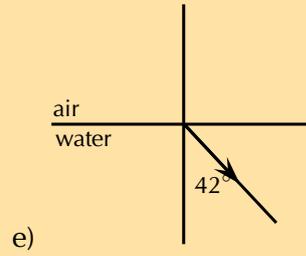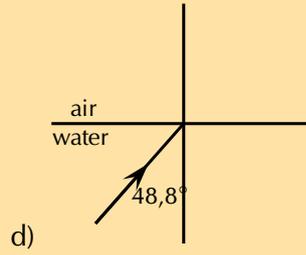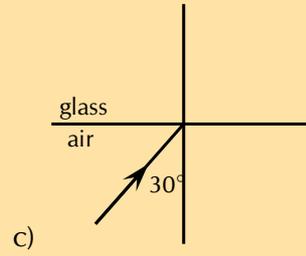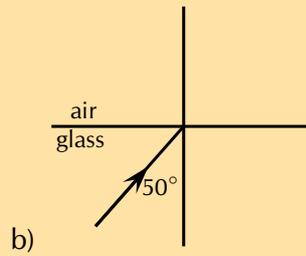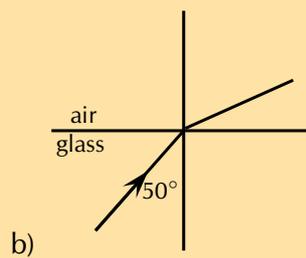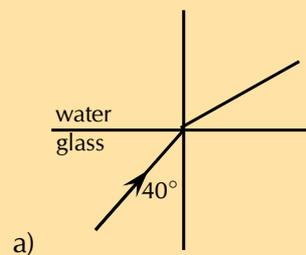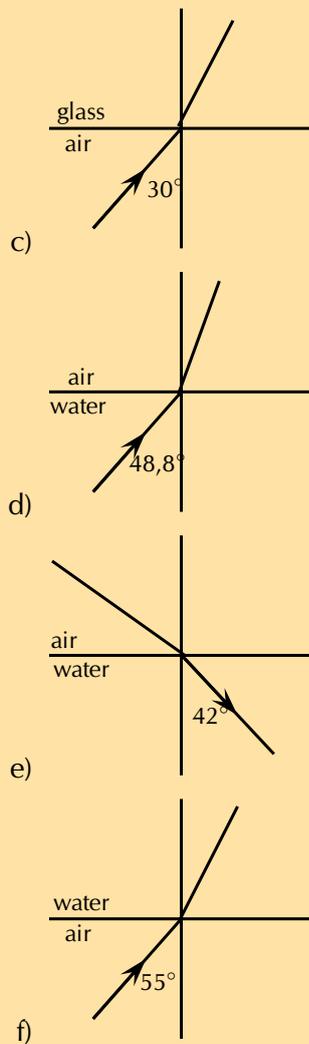
4. A ray of light strikes a surface at $35°$ to the normal to the surface. Draw a ray diagram showing the incident ray, reflected ray and surface normal. Calculate the angles of incidence and reflection and fill them in on your diagram.

Solution:
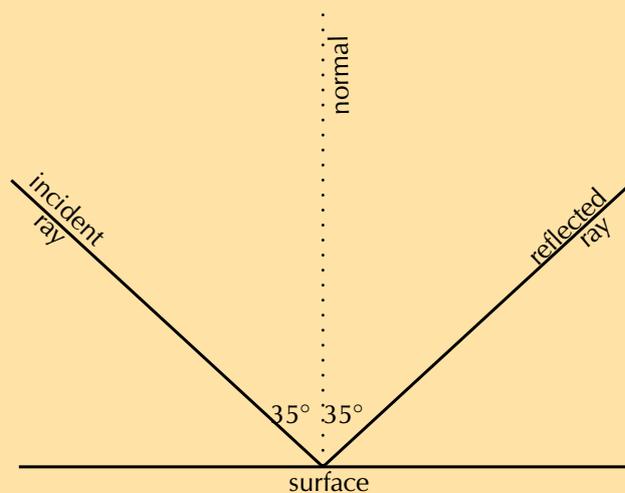
The angle of incidence equals the angle of reflection. This angle is $35°$.

5. Light travels from glass ($n = 1{,}5$) to acetone ($n = 1{,}36$). The angle of incidence is $25°$.

a) Describe the path of light as it moves into the acetone.

b) Calculate the angle of refraction.

c) What happens to the speed of the light as it moves from the glass to the acetone?

d) What happens to the wavelength of the light as it moves into the acetone?

Solution:

a) The light bends towards the normal.

b)

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$
$$1{,}5 \sin 25° = 1{,}36 \sin \theta_2$$
$$\sin \theta_2 = 0{,}466$$
$$\theta_2 = 27{,}78°$$

c) The speed of light increases.

d) The wavelength of the light stays the same.

6. Light strikes the interface between diamond and an unknown medium with an incident angle of $32°$. The angle of refraction is measured to be $46°$. Calculate the refractive index of the medium and identify the medium.

Solution:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$
$$2{,}419 \sin 32° = n_2 \sin 46°$$
$$n_2 = 1{,}78$$

The substance is sapphire.

7. Explain what total internal reflection is and how it is used in medicine and telecommunications. Why is this technology much better to use?

Solution:

Total internal reflection takes place when light travels from one medium to another of lower optical density. If the angle of incidence is greater than the critical angle for the medium, the light will be reflected back into the medium. No refraction takes place.

Optical fibres are most common in telecommunications, because information can be transported over long distances, with minimal loss of data. This gives optical fibres an advantage over conventional cables. Signals are transmitted from one end of the fibre to another in the form of laser pulses. A single strand of fibre optic cable is capable of handling over 3000 transmissions at the same time which is a huge improvement over the conventional co-axial cables. Multiple signal transmission is achieved by sending individual light pulses at slightly different angles. The transmitted signal is received almost instantaneously at the other end of the cable since the information coded onto the laser travels at the speed of light! During transmission over long distances repeater stations are used to amplify the signal which has weakened by the time it reaches the station. The amplified signals are then relayed towards their destination and may encounter several other repeater stations on the way.

Optic fibres are used in medicine in endoscopes. The main part of an endoscope is the optical fibre. Light is shone down the optical fibre and a medical doctor can use the endoscope to look inside the body of a patient. Endoscopes can be used to examine the inside of a patient's stomach, by inserting the endoscope down the patient's throat. Endoscopes also allow minimally invasive surgery. This means that a person can be diagnosed and treated through a small incision (cut). This has advantages over open surgery because endoscopy is quicker and cheaper and the patient recovers more quickly. The alternative is open surgery which is expensive, requires more time and is more traumatic for the patient.

CHAPTER 6

2D and 3D wavefronts

6   2D and 3D wavefronts

This chapter looks at different kinds of wavefronts. Learners should recall electromagnetic radiation from grade 10 as well as different wave properties from transverse waves and longitudinal waves. The following provides a summary of the topics covered in this chapter.

- **Definition of a wavefront.**

  Wavefronts are imaginary lines joining waves that are in phase. A short activity in which learners join up the points in phase helps them to come to grips with what a wavefront is.

- **Huygen's principle.**

  Every point of a wave front serves as a point source of spherical, secondary waves. After a time t, the new position of the wave front will be that of a surface tangent to the secondary waves. This principle describes how waves propagate and how waves interfere with other waves.

- **Definition of diffraction.**

  Diffraction is the ability of a wave to spread out in wavefronts as the wave passes through a small aperture or around a sharp edge. This is most easily seen with water waves in a ripple tank. As the water waves reach the slit or barrier they spread out into the calm area beyond the slit or barrier.

- **Sketching the diffraction pattern for a slit.**

  In this topic learners are shown how to make simple sketches of diffraction patterns as well as learning about how waves interfere to form crests and troughs.

- **The degree of diffraction and calculating this.**

  The degree of diffraction is proportional to the wavelength and inversely proportional to the width of the slit. The degree of diffraction is explored in more detail and some simple calculations of this are given.

- **Diffraction of light and how this relates to the wave nature of light.**

  All the above concepts are applied to how light diffracts when moving through a single slit. Light shone through a diffraction grating emphasises the wave nature of light.

It is important to note that this chapter contains a section on calculating the maxima and minima of diffraction. This topic is not in CAPs and is included for learners enrichment only. This topic should not be included in tests and exams.

Wave nature of light

Exercise 6 – 1:

1. As the width of the slit producing a single-slit diffraction pattern is reduced, how will the diffraction pattern produced change?

   Solution:

   More diffraction is observed as the slit width is reduced.

2. A water break at the entrance to a harbour consists of a rock barrier with a 50 m wide opening. Ocean waves of 20 m wavelength approach the opening straight on. Light with a wavelength of 500×10^{-9} m strikes a single slit of width 30×10^{-9} m. Which waves are diffracted to a greater extent?

   Solution:

   We need to calculate the diffraction for each type of wave. We start by calculating the diffraction of the water waves:

   $$\begin{aligned} \text{diffraction} &\propto \frac{\lambda}{w} \\ &\propto \frac{20 \text{ m}}{50 \text{ m}} \\ &\propto 0{,}4 \end{aligned}$$

   The diffraction of the light waves is:

   $$\begin{aligned} \text{diffraction} &\propto \frac{\lambda}{w} \\ &\propto \frac{500 \times 10^{-9} \text{ m}}{30 \times 10^{-9} \text{ m}} \\ &\propto 16{,}67 \end{aligned}$$

   The light waves are diffracted more.

3. For the diffraction pattern below, sketch what you expect to change if:

   a) the wavelength gets larger

   b) the wavelength gets smaller

   c) the slit width gets larger

   d) the slit width gets smaller

   e) the frequency of the wave gets smaller

   f) the frequency of the wave gets larger

   Solution:

a) More diffraction would occur. The resulting diffraction pattern is wider:

b) Less diffraction would occur. The resulting diffraction pattern is narrower:

c) Less diffraction would occur. The resulting diffraction pattern is narrower:

d) More diffraction would occur. The resulting diffraction pattern is wider:

e) Frequency is inversely related to wavelength. So the wavelength gets longer and more diffraction would occur. The resulting diffraction pattern is wider:

f) Frequency is inversely related to wavelength. The wavelength gets shorter and less diffraction would occur. The resulting diffraction pattern is narrower:

6.6   Chapter summary

Exercise 6 – 2:

1. In the diagram below the peaks of wavefronts are shown by black lines and the troughs by grey lines. Mark all the points where constructive interference between two waves is taking place and where destructive interference is taking place. Also note whether the interference results in a peak or a trough.
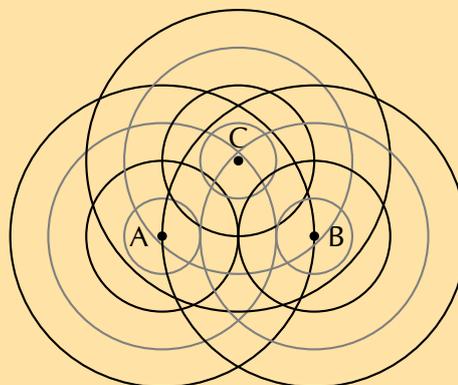
Solution:

The solid diamonds represent constructive interference (crests) and the open diamonds represent destructive interference (troughs).
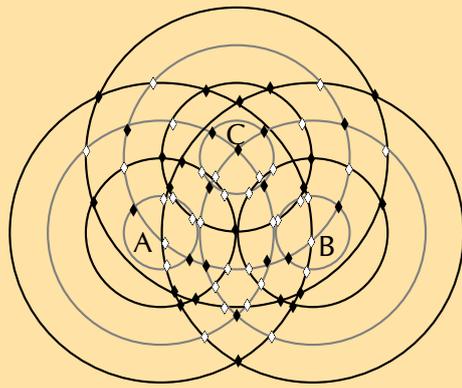
2. For a slit of width 1300 nm, order the following EM waves from least to most diffracted:

   a) green at 510 nm

   b) blue at 475 nm

   c) red at 650 nm

   d) yellow at 570 nm

   Solution:

   blue, green, yellow, red

3. For light of wavelength 540 nm, determine which of the following slits widths results in the maximum and which results in the minimum amount of diffraction

   a) 323×10^{-9} m

   b) $12{,}47$ nm

   c) $21{,}1$ pm

   Solution:

   We know that narrower slits give more diffraction. So we need to determine which is the narrowest slit and which is the widest. $21{,}1$ pm is the narrowest slit (A pm is 10^{-12} m). The widest slit is 323×10^{-9} m (this is the same as 323 nm).

   So the slit with the maximum diffraction is $21{,}1$ pm.

   The slit with the minimum diffraction is 323 nm.

4. For light of wavelength 635 nm, determine what the width of the slit needs to be to have the diffraction be less than the angle of diffraction in each of these cases:

   a) Water waves at the entrance to a harbour which has a rock barrier with a 3 m wide opening. The waves have a wavelength of 16 m wavelength approach the opening straight on.

   b) Light with a wavelength of 786×10^{-9} m strikes a single slit of width 30×10^{-7} m.

   Solution:

a) The angle of diffraction for the water waves is:

$$\sin\theta = \frac{m\lambda}{w}$$
$$= \frac{3 \text{ m}}{16 \text{ m}}$$
$$\sin\theta = 0{,}1875$$
$$\theta = 10{,}81$$

We need to find a slit width that will give a smaller angle of diffraction for light of 635 nm:

$$\sin\theta = \frac{m\lambda}{w}$$
$$0{,}1875 = \frac{635 \times 10^{-9} \text{ m}}{w}$$
$$w = \frac{635 \times 10^{-9} \text{ m}}{0{,}1875}$$
$$= 3{,}4 \times 10^{-6} \text{ m}$$

The slit must be less than $3{,}4 \times 10^{-6}$ m.

b) The angle of diffraction is:

$$\sin\theta = \frac{m\lambda}{w}$$
$$= \frac{786 \times 10^{-9} \text{ m}}{30 \times 10^{-7} \text{ m}}$$
$$\sin\theta = 0{,}262$$
$$\theta = 15{,}18$$

We need to find a slit width that will give a smaller angle of diffraction for light of 635 nm:

$$\sin\theta = \frac{m\lambda}{w}$$
$$0{,}262 = \frac{635 \times 10^{-9} \text{ m}}{w}$$
$$w = \frac{635 \times 10^{-9} \text{ m}}{0{,}262}$$
$$= 2{,}4 \times 10^{-6} \text{ m}$$

The slit must be less than $2{,}4 \times 10^{-6}$ m.

CHAPTER 7

Ideal gases

In this chapter learners are introduced to the concept of ideal gases. They will explore the different gas laws and learn about the motion of gas particles. The following list provides a summary of the topics covered in this chapter.

- **The motion of particles**

  In grade 10 learners were introduced to the kinetic molecular theory and the idea that all particles in a substance are constantly moving. In this chapter the motion of particles is applied to gases and is used to help distinguish between real gases and ideal gases.

- **Real gases and ideal gases**

  A real gas is very similar to an ideal gas except at high pressures and low temperatures. The main distinguishing characteristics of a real gas are that the particles have volume, the particles in the gas have an average speed (since each particle is moving at a different speed) and that forces of attraction exist between particles.

- **The kinetic theory of gases**

  The kinetic theory of gases is similar to the kinetic theory of matter that learners learnt about in grade 10 (states of matter and the kinetic molecular theory). The kinetic theory of gases helps to explain all the gas laws and learners are encouraged to think about what the different laws mean rather than just learning the laws.

- **Boyle's law, Charles' law, the pressure-temperature relation (Gay-Lussac's or Amonton's law), the general gas equation and the ideal gas equation**

  In this book we talk about the pressure-temperature relation. CAPs refers to this as Gay-Lussac's law, while other sources call it Amonton's law. At the time when this law was discovered many scientists were working on the same problems and often it is hard now to say who actually discovered what. Learners should know that different names for the gas laws exist and they should be aware of the other names that exist.

  Boyle's law, Charles' law and the pressure-temperature relation all require very specific conditions to be true. These laws are then expanded into the general gas equation and finally the ideal gas equation is introduced. All the gas laws except the ideal gas law compare a sample of gas at two different sets of readings (e.g. two different sets of pressure and temperature readings, while the volume and amount of gas remains the same).

- **Temperature and pressure**

  Although temperature and pressure are given last in CAPs, in this book they are placed with the kinetic theory of gases and the explanation of real and ideal gases since temperature and pressure are key to understanding these topics.

  The pressure is a result of the motion of particles in the gas and is a measure of how many times the particles in the gas collide with each other and with the walls of the container. Temperature is a measure of the kinetic energy that the particles have.

An informal experiment is included here. This is to verify Boyle's law and to verify Charles' law. This experiment is split into two parts and each part is given at the relevant section of the book. For the experiment on Boyle's law you will need a pressure gauge, 10 ml syringe, 3 cm silicon tubing and water bowl or Boyle's law apparatus. Learners will plot a graph of their results and use these to determine if Boyle's law has been verified. They should get a curved line for plotting pressure (x-axis) vs. volume (y-axis) and a straight line when they plot pressure against the inverse of volume ($\frac{1}{V}$).

An informal experiment is included here. This is to verify Boyle's law and to verify Charles' law (see notes under Boyle's law). In this second part of the experiment learners will verify Charles' law. You will need glass bottles, balloons, beakers or pots, water, hot plates. Learners place a balloon over the mouth of the bottle and then place it in a beaker or pot of water and see what happens. They can also place a bottle with a balloon into a freezer and see what happens.

7.1 Motion of particles

Ideal gases and non-ideal gas behaviour

Exercise 7 – 1:

1. Summarise the difference between a real gas and an ideal gas in the following table:

Property	Ideal gas	Real gas
Size of particles		
Attractive forces		
Speed of molecules		

Solution:

Property	Ideal gas	Real gas
Size of particles	No volume and can be ignored	Non-negligible volume
Attractive forces	No attractive forces	Attractive forces
Speed of molecules	All molecules at the same speed.	Molecules move at different speeds and we use the average speed.

7.2 Ideal gas laws

Boyle's law: Pressure and volume of an enclosed gas

Exercise 7 – 2: Boyle's law

1. An unknown gas has an initial pressure of 150 kPa and a volume of 1 L. If the volume is increased to 1,5 L, what will the pressure be now?

   Solution:

   $$p_2 V_2 = p_1 V_1$$
   $$1,5 \text{ L} p_2 = (150 \text{ kPa})(1 \text{ L})$$
   $$p_2 = \frac{150}{1,5}$$
   $$= 100 \text{ kPa}$$

2. A bicycle pump contains 250 cm^3 of air at a pressure of 90 kPa. If the air is compressed, the volume is reduced to 200 cm^3. What is the pressure of the air inside the pump?

   Solution:

   $$p_2 V_2 = p_1 V_1$$
   $$200 \text{ cm}^3 p_2 = (90 \text{ kPa})(250 \text{ cm}^3)$$
   $$200 \text{ cm}^3 p_2 = 22 \ 500 \text{ kPa}$$
   $$p_2 = \frac{22 \ 500}{200}$$
   $$= 112,5 \text{ kPa}$$

3. The air inside a syringe occupies a volume of 10 cm^3 and exerts a pressure of 100 kPa. If the end of the syringe is sealed and the plunger is pushed down, the pressure increases to 120 kPa. What is the volume of the air in the syringe?

   Solution:

   $$p_2 V_2 = p_1 V_1$$
   $$120 \text{ kPa} V_2 = (100 \text{ kPa})(10 \text{ cm}^3)$$
   $$V_2 = \frac{1000}{120}$$
   $$= 8,33 \text{ cm}^3$$

4. During an investigation to find the relationship between the pressure and volume of an enclosed gas at constant temperature, the following results were obtained.
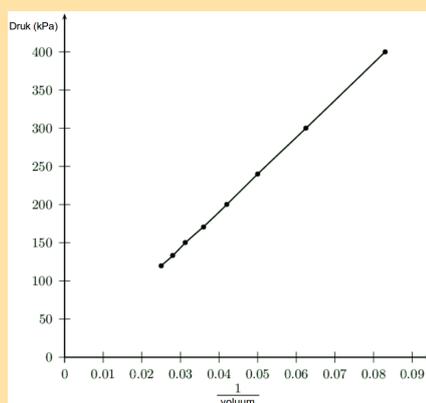
Volume (dm^3)	12	16	20	24	28	32	36	40
Pressure (kPa)	400	300	240	200	171	150	133	120

a) Plot a graph of pressure (p) against volume (V). Volume will be on the x-axis and pressure on the y-axis. Describe the relationship that you see.

b) Plot a graph of p against $\frac{1}{V}$. Describe the relationship that you see.

c) Do your results support Boyle's Law? Explain your answer.

Solution:
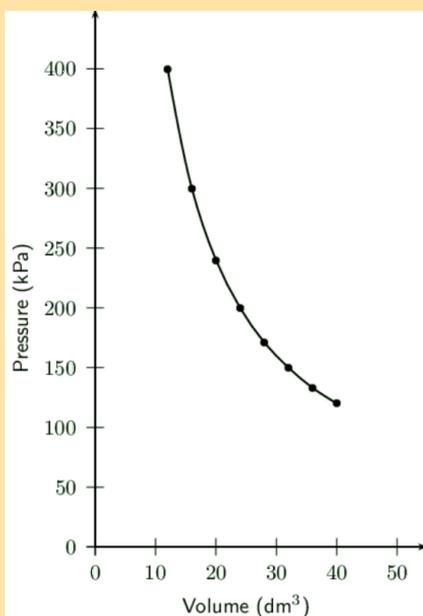
a)

b)

The graph is a straight line. This straight line shows the inverse relation between pressure and volume.

c) The graph of p against $\frac{1}{V}$ gives a straight line which shows that pressure is inversely proportional to volume. This is the expected result from Boyle's law.

5. Masoabi and Justine are experimenting with Boyle's law. They both used the same amount of gas. Their data is given in the table below:

	Masoabi		Justine	
	Initial	Final	Initial	Final
Temperature (K)	325	350	325	325
Volume (dm³)	1	3	1	3
Pressure (Pa)	650	233	650	217

Masoabi and Justine argue about who is correct.

a) Calculate the final pressure that would be expected using the initial pressure and volume and the final volume.

b) Who correctly followed Boyle's law and why?

Solution:

a)

$$p_2 V_2 = p_1 V_1$$
$$(3)p_2 = (650)(1)$$
$$3p_2 = 650$$
$$p_2 = 217 \text{ Pa}$$

b) From the above calculation we see that Justine has the correct pressure as predicted by Boyle's law. Masoabi did not get the expected reading.

Justine kept the temperature constant and so has correctly repeated Boyle's law. Masoabi changed the temperature for the second part of the experiment and so did not repeat Boyle's law (she actually repeated another gas law).

## Charles' law: Volume and temperature of an enclosed gas

Exercise 7 – 3: Charles' law

1. The table below gives the temperature (in °C) of helium gas under different volumes at a constant pressure.
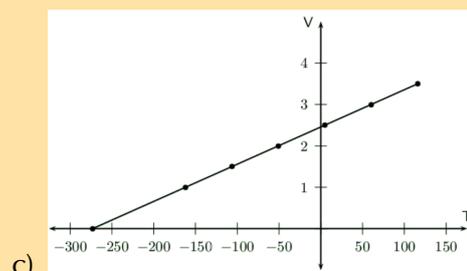
Volume (L)	Temperature (°C)
1,0	−161,9
1,5	−106,7
2	−50,8
2,5	4,8
3,0	60,3
3,5	115,9

a) Draw a graph to show the relationship between temperature and volume.

b) Describe the relationship you observe.

c) If you extrapolate the graph (in other words, extend the graph line even though you may not have the exact data points), at what temperature does it intersect the x-axis?

d) What is significant about this temperature?

e) What conclusions can you draw? Use Charles' law to help you.

Solution:

a)
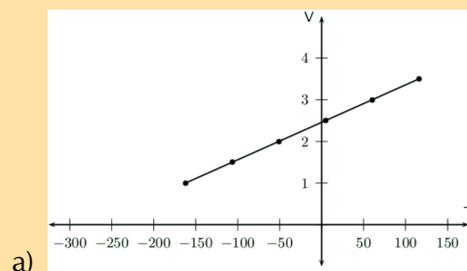
b) As the volume increases so does the temperature.

c)

The graph intersects the x-axis at $-273\,°\mathrm{C}$

d) This point corresponds to absolute zero. At absolute zero all gases have no volume. This is the lowest temperature that it is possible to achieve.

e) We can conclude that volume and temperature are directly proportional.

2. A sample of nitrogen monoxide (NO) gas is at a temperature of $8\,°\mathrm{C}$ and occupies a volume of $4{,}4\ \mathrm{dm}^3$. What volume will the sample of gas have if the temperature is increased to $25\,°\mathrm{C}$?

Solution:

First convert the temperature to Kelvin:

$$T_1 = 8 + 273 = 281$$
$$T_2 = 25 + 273 = 298$$

$$\frac{V_2}{T_2} = \frac{V_1}{T_1}$$
$$\frac{V_2}{298} = \frac{4{,}4}{281}$$
$$\frac{V_2}{298} = 0{,}01565\ldots$$
$$= 4{,}67\ \mathrm{dm}^3$$

3. A sample of oxygen (O_2) gas is at a temperature of 340 K and occupies a volume of $1{,}2\ \mathrm{dm}^3$. What temperature will the sample of gas be at if the volume is decreased to 200 cm^3?

Solution:

Note that the two volumes are not in the same units. So we convert the second volume to dm^3.

$$V_2 = \frac{200}{1000} = 0{,}2\ \mathrm{dm}^3$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$
$$\frac{1{,}2}{340} = \frac{0{,}2}{T_2}$$
$$0{,}003529\ldots = \frac{0{,}2}{T_2}$$
$$(0{,}003529\ldots)T_2 = 0{,}2$$
$$= 56{,}67 \text{ K}$$

4. Explain what would happen if you were verifying Charles' law and you let some of the gas escape.

Solution:

If some of the gas escaped, the volume of the gas would decrease. This would lead to an decrease in the temperature. So the final temperature that you read would be lower than it should be. This will lead to the incorrect conclusion that Charles' law is not correct.

Pressure-temperature relation

Exercise 7 – 4: Pressure-temperature relation

1. The table below gives the temperature (in °C) of helium under different pressures at a constant volume.

Pressure (atm)	Temperature (°C)	Temperature (K)
1,0	20	
1,2	78,6	
1,4	137,2	
1,6	195,8	
1,8	254,4	
2,0	313	

a) Convert all the temperature data to Kelvin.

b) Draw a graph to show the relationship between temperature and pressure.

c) Describe the relationship you observe.

Solution:

	Pressure (atm)	Temperature (°C)	Temperature (K)
	1,0	20	293
	1,2	78,6	351,6
a)	1,4	137,2	410,2
	1,6	195,8	468,8
	1,8	254,4	527,4
	2,0	313	586
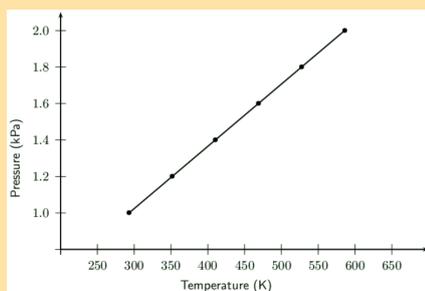
b)

c) There is a linear relationship. Pressure and temperature are directly proportional. The graph is a straight line.

2. A cylinder that contains methane gas is kept at a temperature of $15\,°C$ and exerts a pressure of 7 atm. If the temperature of the cylinder increases to $25\,°C$, what pressure does the gas now exert?

Solution:

We need to convert the given temperatures to Kelvin temperature.

$$
\begin{aligned}
T_1 &= 15 + 273 \\
&= 288 \text{ K} \\
T_2 &= 25 + 273 \\
&= 298 \text{ K}
\end{aligned}
$$

$$
\begin{aligned}
\frac{p_2}{T_2} &= \frac{p_1}{T_1} \\
\frac{p_2}{298} &= \frac{7}{288} \\
\frac{p_2}{298} &= 0{,}024\ldots \\
p_2 &= 7{,}24 \text{ atm}
\end{aligned}
$$

The pressure will be $7{,}24$ atm.

3. A cylinder of propane gas at a temperature of $20\,°C$ exerts a pressure of 8 atm. When a cylinder has been placed in sunlight, its temperature increases to $25\,°C$. What is the pressure of the gas inside the cylinder at this temperature?

Solution:

We first convert the temperature to Kelvin:

$$
\begin{aligned}
T_1 &= 20 + 273 = 293 \text{ K} \\
T_2 &= 25 + 273 = 298 \text{ K}
\end{aligned}
$$

And then we can find the pressure:

$$
\begin{aligned}
\frac{p_2}{T_2} &= \frac{p_1}{T_1} \\
\frac{p_2}{298} &= \frac{8}{293} \\
\frac{p_2}{298} &= 0{,}0273\ldots \\
p_2 &= 8{,}14 \text{ atm}
\end{aligned}
$$

4. A hairspray can is a can that contains a gas under high pressures. The can has the following warning written on it: "Do not place near open flame. Do not dispose of in a fire. Keep away from heat." Use what you know about the pressure and temperature of gases to explain why it is dangerous to not follow this warning.

Solution:

The pressure of a gas is directly proportional to its temperature at a fixed volume. Since the volume of gas in an aerosol can does not change, this relation holds. If the temperature of the can is increased by placing near an open flame or in a hot place, the pressure will increase. This may lead to the can exploding. The same is true if you dispose of in a fire.

5. A cylinder of acetylene gas is kept at a temperature of 291 K. The pressure in the cylinder is 5 atm. This cylinder can withstand a pressure of 8 atm before it explodes. What is the maximum temperature that the cylinder can safely be stored at?

Solution:

$$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$
$$\frac{5}{291} = \frac{8}{T_2}$$
$$0{,}017\ldots = \frac{8}{T_2}$$
$$(0{,}017\ldots)T_2 = 8$$
$$T_2 = 465{,}6 \text{ K}$$

The general gas equation

Exercise 7 – 5: The general gas equation

1. A closed gas system initially has a volume of 8 L and a temperature of 100 °C. The pressure of the gas is unknown. If the temperature of the gas decreases to 50 °C, the gas occupies a volume of 5 L and the pressure of the gas is 1,2 atm. What was the initial pressure of the gas?

Solution:

We first convert the temperature to Kelvin:

$$T_1 = 100 + 273 = 373 \text{ K}$$
$$T_2 = 50 + 273 = 323 \text{ K}$$

Now we can use the gas equation:

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$\frac{(p_1)(8)}{373} = \frac{(1,2)(5)}{323}$$

$$\frac{(p_1)(8)}{373} = 0,0185\ldots$$

$$(p_1)(8) = 6,928\ldots$$

$$p_1 = 0,87 \text{ atm}$$

2. A balloon is filled with helium gas at 27 °C and a pressure of 1,0 atm. As the balloon rises, the volume of the balloon increases by a factor of 1,6 and the temperature decreases to 15 °C. What is the final pressure of the gas (assuming none has escaped)?

   Solution:

   We first convert the temperature to Kelvin:

   $$T_1 = 27 + 273 = 300 \text{ K}$$

   $$T_2 = 15 + 273 = 288 \text{ K}$$

   Let the initial volume be V_1 and the final volume be $1,6V_1$

   Now we can use the gas equation:

   $$\frac{p_2 V_2}{T_2} = \frac{p_1 V_1}{T_1}$$

   $$\frac{p_2(1,6V_1)}{288} = \frac{(1)V_1}{300}$$

   $$480V_1 p_2 = 288V_1$$

   $$p_2 = 0,6 \text{ atm}$$

3. 25 cm^3 of gas at 1 atm has a temperature of 25 °C. When the gas is compressed to 20 cm^3, the temperature of the gas increases to 28 °C. Calculate the final pressure of the gas.

   Solution:

   We first convert the temperature to Kelvin:

   $$T_1 = 25 + 273 = 298 \text{ K}$$

   $$T_2 = 28 + 273 = 301 \text{ K}$$

   Now we can use the gas equation:

   $$\frac{p_2 V_2}{T_2} = \frac{p_1 V_1}{T_1}$$

   $$\frac{(p_2)(20)}{301} = \frac{(1)(25)}{298}$$

   $$\frac{(p_2)(20)}{301} = 0,08389\ldots$$

   $$(p_2)(20) = 25,516\ldots$$

   $$p_1 = 1,26 \text{ atm}$$

1. An unknown gas has a pressure of 0,9 atm, a temperature of 120 °C and the number of moles is 0,28 mol. What is the volume of the sample?

   Solution:

   First convert all units to SI units:

   $$T = 273 + 120 = 393 \text{ K}$$
   $$V = ?$$
   $$p = \frac{101\ 325}{0,9} = 112\ 583,33 \text{ Pa}$$

   Now we can use the ideal gas equation to find the volume of gas:

   $$pV = nRT$$
   $$(112\ 583,33)V = (0,28)(8,314)(393)$$
   $$112\ 583,33V = 895,7256$$
   $$V = 0,008 \text{ m}^3$$
   $$V = 8 \text{ dm}^3$$

2. 6 g of chlorine (Cl_2) occupies a volume of 0,002 m^3 at a temperature of 26 °C. What is the pressure of the gas under these conditions?

   Solution:

   First find the number of moles of chlorine gas:

   $$n = \frac{m}{M}$$
   $$= \frac{6}{70}$$
   $$= 0,0857 \text{ mol}$$

   Next convert all units to SI units:

   $$T = 273 + 26 = 299 \text{ K}$$

   Now we can use the ideal gas equation to find the pressure:

   $$pV = nRT$$
   $$(p)(0,002) = (0,0857)(8,314)(299)$$
   $$(p)(0,002) = 213,075\ldots$$
   $$= 106\ 537,9714 \text{ Pa}$$
   $$= 106,54 \text{ kPa}$$

3. An average pair of human lungs contains about 3,5 L of air after inhalation and about 3,0 L after exhalation. Assuming that air in your lungs is at 37 °C and 1,0 atm, determine the number of moles of air in a typical breath.

Solution:

One breath has a volume of:

$$3,5 - 3,0 = 0,5 \text{ L}$$
$$= 0,0005 \text{ m}^3$$

Convert all units to SI units:

$$T = 273 + 37 = 310 \text{ K}$$
$$p = 101\ 325 \text{ Pa}$$

Now we can use the ideal gas equation to find the number of moles:

$$pV = nRT$$
$$(101\ 325)(0,0005) = (n)(8,314)(310)$$
$$50,6625 = (n)(2577,34)$$
$$= 0,02 \text{ mol}$$

4. A learner is asked to calculate the answer to the problem below:

   Calculate the pressure exerted by 1,5 moles of nitrogen gas in a container with a volume of 20 dm³ at a temperature of 37 °C.

   The learner writes the solution as follows:

   $$V = 20 \text{ dm}^3$$
   $$n = 1,5 \text{ mol}$$
   $$R = 8,314 \text{ J·K}^{-1}\text{·mol}^{-1}$$
 $$T = 37 + 273 = 310 \text{ K}$$

   $$pT = nRV$$
   $$p(310) = (1,5)(8,314)(20)$$
   $$p(310) = (249,42)$$
   $$= 0,8 \text{ kPa}$$

   a) Identify 2 mistakes the learner has made in the calculation.
   b) Are the units of the final answer correct?
   c) Rewrite the solution, correcting the mistakes to arrive at the right answer.

   Solution:

   a) The learner has not used the correct equation. The ideal gas equation is $pV = nRT$. The learner did not convert the volume to SI units.

b) The pressure should be given in pascals (Pa) first and then converted to kiloPascals (kPa).

c)

$$V = 0{,}02 \text{ m}^3$$
$$n = 1{,}5 \text{ mol}$$
$$R = 8{,}314 \text{ J·K}^{-1}\text{·mol}^{-1}$$
$$T = 37 + 273 = 310 \text{ K}$$

$$pV = nRT$$
$$p(0{,}020 \text{ m}^3) = (1{,}5 \text{ mol})(8{,}314 \text{ J·K}^{-1}\text{·mol}^{-1})(310 \text{ K})$$
$$p(0{,}020 \text{ m}^3) = 3866{,}01 \text{ J}$$
$$p = 193\ 300{,}5 \text{ Pa}$$
$$= 193{,}3 \text{ kPa}$$

5. Most modern cars are equipped with airbags for both the driver and the passenger. An airbag will completely inflate in $0{,}05$ s. This is important because a typical car collision lasts about $0{,}125$ s. The following reaction of sodium azide (a compound found in airbags) is activated by an electrical signal:

$$2\text{NaN}_3(\text{s}) \rightarrow 2\text{Na (s)} + 3\text{N}_2(\text{g})$$
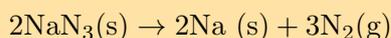
a) Calculate the mass of $\text{N}_2(\text{g})$ needed to inflate a sample airbag to a volume of 65 dm^3 at 25 °C and $99{,}3$ kPa. Assume the gas temperature remains constant during the reaction.

b) The above reaction produces heat, which raises the temperature in the airbag. Describe, in terms of the kinetic theory of gases, how the pressure in the sample airbag will change, if at all, as the gas temperature returns to 25 °C.

Solution:

a) We first convert all units to SI units:

$$P = 99{,}3 \times 1000 = 99\ 300 \text{ Pa}$$
$$V = \frac{65}{1000} = 0{,}065 \text{ m}^3$$
$$T = 25 + 273 = 298 \text{ K}$$

$$pV = nRT$$
$$(99\ 300)(0{,}065) = n(8{,}314)(298)$$
$$6464{,}5 = n(2477{,}572)$$
$$n = 2{,}61 \text{ mol}$$

Next we convert the number of moles to grams:

$$n = \frac{m}{M}$$
$$2{,}61 = \frac{m}{28}$$
$$m = 73{,}1 \text{ g}$$

b) When the temperature decreases the intensity of collisions with the walls of the airbag and between particles decreases. Therefore pressure decreases.

Exercise 7 – 7:

1. Give one word or term for each of the following definitions.

   a) A gas is that has identical particles of zero volume, with no intermolecular forces between the particles.

   b) The law that states that the volume of a gas is directly proportional to the temperature of the gas, provided that the pressure and the amount of the gas remain constant.

   c) A measure of the average kinetic energy of gas particles.

   Solution:

   a) Ideal gas

   b) Charles' law

   c) Temperature

2. Which one of the following properties of a fixed quantity of a gas must be kept constant during an investigation of Boyle's law?

   a) density

   b) pressure

   c) temperature

   d) volume

   (*IEB 2003 Paper 2*)

   Solution:

   Temperature must be kept constant.

3. Three containers of *equal volume* are filled with *equal masses* of helium, nitrogen and carbon dioxide gas respectively. The gases in the three containers are all at the same *temperature*. Which one of the following statements is correct regarding the pressure of the gases?

   a) All three gases will be at the same pressure

   b) The helium will be at the greatest pressure

   c) The nitrogen will be at the greatest pressure

   d) The carbon dioxide will be at the greatest pressure

   (*IEB 2004 Paper 2*)

   Solution:

   The sample of helium gas will be at the greatest pressure.

4. The ideal gas equation is given by **pV = nRT**. Which one of the following conditions is true according to Avogadro's hypothesis?

a	$p \propto \frac{1}{V}$	(T = constant)
b	$V \propto T$	(p = constant)
c	$V \propto n$	(p, T = constant)
d	$p \propto T$	(n = constant)

(*DoE Exemplar paper 2, 2007*)

Solution:

$V \propto n$, with p, T constant.

5. Complete the following table by stating whether or not the property is constant or variable for the given gas law.

Law	Pressure (p)	Volume (V)	Temperature (T)	Moles (n)
Boyle's law				
Charles' law				
Gay-Lussac's law				
General gas equation				
Ideal gas equation				

Solution:

Law	Pressure (p)	Volume (V)	Temperature (T)	Moles (n)
Boyle's law	variable	variable	constant	constant
Charles' law	constant	variable	variable	constant
Gay-Lussac's law	variable	constant	variable	constant
General gas equation	variable	variable	variable	constant
Ideal gas equation	variable	variable	variable	variable

6. Use your knowledge of the gas laws to explain the following statements.

   a) It is dangerous to put an aerosol can near heat.

   b) A pressure vessel that is poorly designed and made can be a serious safety hazard (a pressure vessel is a closed, rigid container that is used to hold gases at a pressure that is higher than the normal air pressure).

   c) The volume of a car tyre increases after a trip on a hot road.

Solution:

   a) As the temperature of a gas increases the pressure also increases (provided the volume stays constant). Since the volume of an aerosol can stays constant, the pressure will increase and cause the can to explode.

b) As the temperature of a gas increases the pressure also increases (provided the volume stays constant). Since the volume of a pressure vessel stays constant, the pressure will increase if the temperature increases and cause the vessel to explode. Also since the vessel keeps the gas at a higher pressure than normal, the gas is more likely to burst out of the container.

c) Volume is proportional to temperature at a fixed pressure. The pressure of a car tyre is fixed. After a long trip, the temperature of the tyre increases which leads to an increase in the volume.

7. Copy the following set of labelled axes and answer the questions that follow:
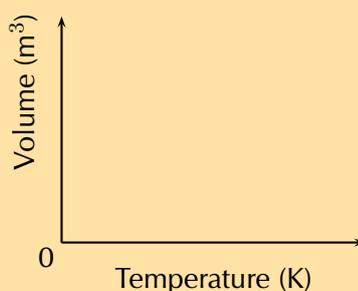
a) On the axes, **using a solid line**, draw the graph that would be obtained for a fixed mass of an ideal gas if the pressure is kept constant.

b) If the gradient of the above graph is measured to be $0{,}008 \text{ m}^3 \cdot \text{K}^{-1}$, calculate the pressure that $0{,}3$ mol of this gas would exert.

(*IEB 2002 Paper 2*)

Solution:

a)
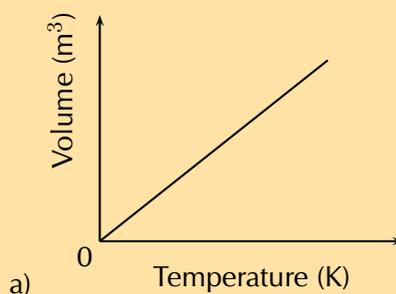
b) The equation of a straight line graph is $y = mx + x$. Since the above graph intersects the y-axis at 0, $c = 0$. We are told that the gradient is $0{,}008$. So the equation for the above graph is $y = 0{,}008x$.

The ideal gas equation is $pV = nRT$. Since the above graph plots temperature on the x-axis and volume on the y-axis, we rearrange the ideal gas equation to find: $V = \frac{nR}{p}T$.

Therefore, the gradient of the graph is equal to $\frac{nR}{p}$.

$$0{,}008 = \frac{(0{,}3)(8{,}314)}{p}$$
$$p = 311{,}775 \text{ Pa}$$

8. Two gas cylinders, A and B, have a volume of $0{,}15 \text{ m}^3$ and $0{,}20 \text{ m}^3$ respectively. Cylinder A contains 35 mol He gas at pressure p and cylinder B contains 40 mol He gas at 5 atm. The ratio of the Kelvin temperatures A:B is $1{,}80{:}1{,}00$. Calculate the pressure of the gas (in kPa) in cylinder A.

(*IEB 2002 Paper 2*)

Solution:

Let the temperature of gas B be T_B and the temperature of gas A is then $1{,}8T_B$.

We convert the pressure of gas B to SI units:

$$p_B = (5)(101\ 325) = 506\ 625$$

For gas B we have:

$$pV = nRT$$
$$(506\ 625)(0{,}2) = (40)(8{,}314)(T_B)$$
$$101\ 325 = (332{,}56)(T_B)$$
$$T_B = 304{,}68 \text{ K}$$

So the temperature of gas A is:

$$T_A = 1{,}8T_B$$
$$= (1{,}8)(304{,}68)$$
$$= 548{,}43 \text{ K}$$

For gas A we have:

$$pV = nRT$$
$$p(0{,}15) = (35)(8{,}314)(548{,}43)$$
$$p(0{,}15) = (159\ 586{,}875)$$
$$p = 106\ 391{,}25 \text{ Pa}$$
$$p = 106{,}39 \text{ kPa}$$

9. A learner investigates the relationship between the Celsius temperature and the pressure of a fixed amount of helium gas in a 500 cm^3 closed container. From the results of the investigation, she draws the graph below:
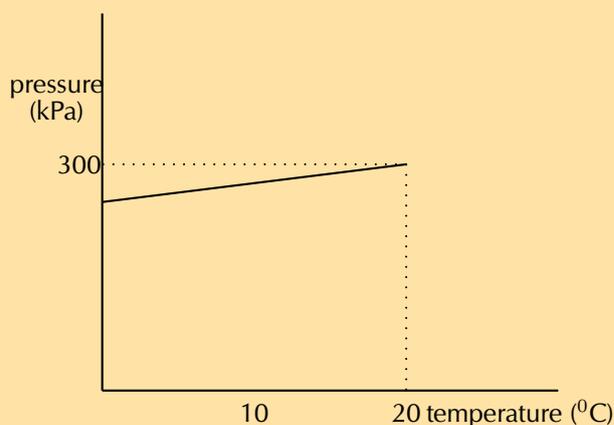
a) Under the conditions of this investigation, helium gas behaves like an ideal gas. Explain briefly why this is so.

b) From the shape of the graph, the learner concludes that the pressure of the helium gas is directly proportional to the Celsius temperature. Is her conclusion correct? Briefly explain your answer.

c) Calculate the pressure of the helium gas at 0 °C.

d) Calculate the mass of helium gas in the container.

(IEB 2003 Paper 2)

Solution:

a) The pressure is not high enough to cause molecular volume to start taking effect. The temperature range is well above heliums boiling point to prevent helium becoming a liquid.

b) No. The pressure is directly proportional to the Kelvin temperature. The conversion between Kelvin and Celsius relies on addition and subtraction. This conversion affects the proportionality constant differently depending on whether Celsius or Kelvin is used. If the conversion used multiplication or division then the learners conclusion would be correct.

c) At 20 °C the pressure is 300 kPa. Using the relationship between pressure and temperature we find the pressure:

$$\frac{p_2}{T_2} = \frac{p_1}{T_1}$$
$$\frac{p_2}{(273)} = \frac{(300 \times 10^3)}{(293)}$$
$$\frac{p_2}{(273)} = 87\ 900\ 000$$
$$p_2 = 321\ 978{,}022\ \text{Pa}$$
$$= 322{,}0\ \text{kPa}$$

d) We can use either of the temperature-pressure readings.
The number of moles is:

$$pV = nRT$$
$$(300 \times 10^3)(500 \times 10^{-3}) = n(8{,}314)(293)$$
$$150\ 000 = (2436{,}002)n$$
$$n = 61{,}58\ \text{mol}$$

And the mass is:

$$m = nM$$
$$= (61{,}58)(4)$$
$$= 246{,}31\ \text{g}$$

10. One of the cylinders of a motor car engine, before compression contains 450 cm^3 of a mixture of air and petrol in the gaseous phase, at a temperature of 30 °C and a pressure of 100 kPa. If the volume of the cylinder after compression decreases to one tenth of the original volume, and the temperature of the gas mixture rises to 140 °C, calculate the pressure now exerted by the gas mixture.

Solution:

First convert the temperature to Kelvin and the volume to m³:

$$T_1 = 30 + 273 = 303 \text{ K}$$
$$T_2 = 140 + 273 = 413 \text{ K}$$
$$V_1 = \frac{450}{1000} = 0{,}450 \text{ m}^3$$
$$V_2 = \frac{1}{10}V_1 = 0{,}0450 \text{ m}^3$$

Now we can find the pressure:

$$\frac{p_2 V_2}{T_2} = \frac{p_1 V_1}{T_1}$$
$$\frac{p_2(0{,}045)}{413} = \frac{(100\ 000)(0{,}450)}{303}$$
$$\frac{p_2(0{,}045)}{413} = 148{,}52$$
$$p_2(0{,}045) = 61\ 336{,}6$$
$$p_2 = 1\ 363\ 036{,}3 \text{ Pa}$$
$$= 1363{,}0 \text{ kPa}$$

11. A gas of unknown volume has a temperature of $14 \ ^\circ\text{C}$. When the temperature of the gas is increased to $100 \ ^\circ\text{C}$, the volume is found to be $5{,}5$ L. What was the initial volume of the gas?

   Solution:

   First convert the temperature to Kelvin:

$$T_1 = 14 + 273 = 287 \text{ K}$$
$$T_2 = 100 + 273 = 373 \text{ K}$$

   Next we an find the initial volume:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$
$$\frac{V_1}{287} = \frac{5{,}5}{373}$$
$$\frac{V_1}{287} = 0{,}020\ldots$$
$$V_1 = 5{,}78 \text{ L}$$

12. A gas has an initial volume of 2600 mL and a temperature of 350 K.

   a) If the volume is reduced to 1500 mL, what will the temperature of the gas be in Kelvin?

   b) Has the temperature *increased* or *decreased*?

   c) Explain this change, using the kinetic theory of gases.

Solution:

a)

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$
$$\frac{2,6}{350} = \frac{1,5}{T_2}$$
$$0,007428\ldots = \frac{1,5}{T_2}$$
$$(0,007428\ldots)T_2 = 1,5$$
$$T_2 = 201,9 \text{ K}$$

b) Decreased.

c) When the temperature of a gas decreases, so does the average speed of its molecules. The molecules collide with the walls of the container less often and with lesser impact. These collisions will not push back the walls, so that the gas occupies a lesser volume than it did at the start.

13. In an experiment to determine the relationship between pressure and temperature of a fixed mass of gas, a group of learners obtained the following results:

Pressure (kPa)	101	120	130,5	138
Temperature (°C)	0	50	80	100
Total gas volume (cm³)	250	250	250	250

a) Draw a straight-line graph of pressure (on the dependent, y-axis) versus temperature (on the independent, x-axis) on a piece of graph paper. Plot the points. Give your graph a suitable heading.

b) *A straight-line graph passing through the origin is essential to obtain a mathematical relationship between pressure and temperature..*

   Extrapolate (extend) your graph and determine the temperature (in °C) at which the graph will pass through the temperature axis.

c) Write down, in words, the relationship between pressure and Kelvin temperature.

d) From your graph, determine the pressure (in kPa) at 173 K. Indicate on your graph how you obtained this value.

e) How would the gradient of the graph be affected (if at all) if a larger mass of the gas is used? Write down ONLY **increases**, **decreases** or **stays the same**.

(DoE Exemplar Paper 2, 2007)

Solution:

a)
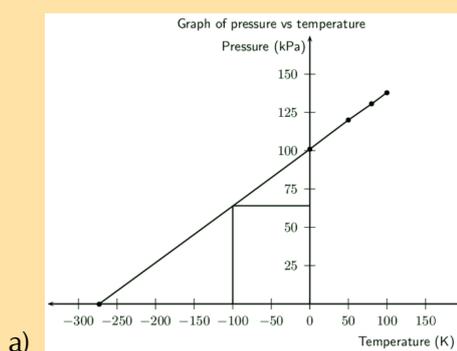
b) See above.

c) The pressure for a fixed mass of gas is directly proportional to the Kelvin temperature.

d) 173 K is $-100\,°C$. So we draw a line from $-100\,°C$ to the graph and then draw a line across to the y-axis. (See graph above).

This gives a pressure of about 64 kPa (you can check this by calculation).

e) Increases.

CHAPTER 8

Quantitative aspects of chemical change

This topic builds on the gr10 chapter of the same title. A quick revision (no more than 15 minutes) of the gr10 work would greatly help learners to recall how to calculate molar masses, number of moles and concentration. In this chapter learners will learn about molar volumes of gases, concentration of solutions, more on stoichiometry and volume relationships in gaseous reactions. Topics covered in this chapter are:

- **1 mole of gas occupies 22,4 dm^3 and expansion to any number of moles of gas**

  In grade 10 learners learnt about the molar volume of gases. This concept is refreshed here and then expanded on to cover any number of moles of gases. This is a very important result that is used in stoichiometric calculations to go from volume of a gas to number of moles and from there via the balanced equation to the desired quantity of one of the products or reactants in that reaction.

- **Interpretation of chemical equations**

  Learners should be able to understand that chemical equations can be interpreted in words or in symbols (as seen in grade 10) or in terms of the quantities of the substances involved in the reaction. This second interpretation is a valuable skill for stoichiometric calculations.

- **Calculation of molar concentration of solutions and expanding this to titration calculations**

  CAPs lists an activity on titration calculations which is worked into the text with concentration calculations. In grade 10 learners learnt about calculating concentration, in this chapter concentration calculations are expanded to cover titrations.

- **Limiting reagents**

  In grade 10 learners saw how to use the balanced chemical equation to move from products to reactants and vice versa. This is now expanded on to include information about both reactants where learners have to determine which reactant is limiting and which is in excess before calculating the amount of product formed.

- **Percent yield**

  In this section learners are introduced to percent yield. Care must be taken in this section to ensure that learners do not mix up theoretical yield and percent yield. The percent yield can only be determined from experiment, while the theoretical yield comes from stoichiometric calculations.

- **Empirical and molecular formula**

  This is revision from grade 10. Learners are reminded how to calculate the empirical and molecular formula of a substance and are reminded about percent composition.

- **Percent purity**

  The concept of percent purity is the last concept that is covered. This looks at how pure a sample of a compound is. It is important to note that CAPs refers to percent purity as being interchangeable with percent composition which is not strictly true. Percent composition deals with how much of a particular element is in a substance while percent purity deals with how much of a compound is in a sample or how pure your final product in a reaction is.

- **Calculations involving gases, liquids and solids**

  CAPs also lists an activity on precipitation calculations which can be included in the final topic.

  At the end of the chapter learners bring together all the concepts that they have learnt into this topic. Learners are introduced to the idea that the balanced chemical equation can be used to calculate any quantity of a substance from any of the other products or reactions in that reaction. This is particularly important as reactions do not occur in just one state but occur in multiple states and being able to move freely between, for example, calculating moles of a gas to calculating mass of a substance, is an important skill.

This chapter includes a recommended experiment for informal assessment. It is recommended that you do this experiment as a demonstration for your class. In this experiment you will show your learners what happens when a small sample (about $0{,}5$ g) of lead(II) nitrate is heated. You must work outside or in a well ventilated space and remember that lead(II) nitrate makes a crackling sound on heating.

A second experiment to determine the percent yield of magnesium carbonate from magnesium sulfate (Epsom salts) and sodium carbonate (washing soda) has also been included. You will need magnesium sulfate, sodium carbonate, mass meter, hot plate or Bunsen burner, small heat resistant beakers, funnel and filter paper. If time and space permits, learners can write their names on their pieces of filter paper before filtering and leave these in an undisturbed place (such as a back room) overnight to dry before weighing them. If this cannot be done remind learners that due to their samples containing some water the sample may not be completely dry when weighing. This makes it slightly harder to take a mass reading and may lead to a percent yield of greater than 100%.

When working with a Bunsen burner you must ensure that the room is well ventilated by opening all windows. Also ensure that learners tie long hair up and tuck in any loose clothing.

8.1 Gases and solutions

Molar volumes of gases

Reactions and gases

Exercise 8 – 1:

1. Methane burns in oxygen, forming water and carbon dioxide according to the following equation:

$$CH_4(g) + 2O_2(g) \rightarrow 2H_2O(g) + CO_2(g)$$
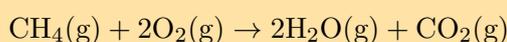
If 4 dm^3 of methane is used, what volume of water is produced?

Solution:

$$V_A = \frac{a}{b}V_B$$

$$V_{H_2O} = \frac{1}{2}V_{CH_4}$$

$$= \frac{1}{2}(4)$$

$$= 2 \text{ dm}^3$$

Solutions

Exercise 8 – 2: Gases and solutions

1. Acetylene (C_2H_2) burns in oxygen according to the following reaction:

$$2C_2H_2(g) + 5O_2(g) \rightarrow 4CO_2(g) + 2H_2O\ (g)$$
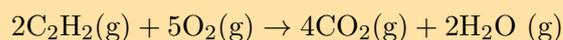
   If 3,5 dm^3 of acetylene gas is burnt, what volume of carbon dioxide will be produced?

   Solution:

$$V_A = \frac{a}{b}V_B$$

$$V_{CO_2} = \frac{4}{2}(3,5)$$

$$= 7 \text{ dm}^3$$

2. 130 g of magnesium chloride ($MgCl_2$) is dissolved in 300 mL of water.

   a) Calculate the concentration of the solution.

   b) What mass of magnesium chloride would need to be added for the concentration to become 6,7 mol·dm^{-3}?

   Solution:

   a) The number of moles of magnesium chloride is:

$$n = \frac{m}{M}$$

$$= \frac{130}{95}$$

$$= 1,37 \text{ mol}$$

   To convert from mL to dm^3 we divide by 1000 (1000 mL = 1 L = 1 dm^3):

$$\frac{300 \text{ mL}}{1000} = 0,3 \text{ dm}^3$$

The concentration is:

$$C = \frac{n}{V}$$
$$= \frac{1,37}{0,3}$$
$$= 4,56 \text{ mol·dm}^{-3}$$

b) We first work out what mass of magnesium chloride is needed to make a solution with a concentration of 6,7 mol·dm^{-3}. The volume will still be 0,3 dm^3.

$$n = CV$$
$$= (6,7)(0,3)$$
$$= 2,01 \text{ mol}$$

This gives a mass of:

$$m = nM$$
$$= (2,01)(95)$$
$$= 190,95 \text{ g}$$

To get from a concentration of 4,56 mol·dm^{-3} to a concentration of 6,7 mol·dm^{-3} we must add:

$$190,95 - 130 = 60,95 \text{ g}$$

of magnesium chloride.

3. Given the equation:

$$KOH \text{ (aq)} + HNO_3(aq) \rightarrow KNO_3(aq) + H_2O \text{ (l)}$$
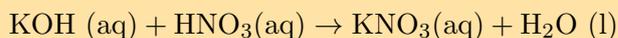
20 cm^3 of a 1,3 mol·dm^{-3} potassium hydroxide (KOH) solution was pipetted into a conical flask and titrated with nitric acid (HNO$_3$). It was found that 17 cm^3 of the nitric acid was needed to neutralise the base. Calculate the concentration of the nitric acid.

Solution:

Write down all the information you know about the reaction, and make sure that the equation is balanced.

KOH: $V = 20$ cm^3; $C = 1,3$ mol·dm^{-3}

HNO$_3$: $V = 17$ cm^3

The equation is already balanced.

Next convert the volumes to dm^3

$$V_{KOH} = \frac{20}{1000}$$
$$= 0,020 \text{ dm}^3$$

$$V_{HNO_3} = \frac{17}{1000}$$
$$= 0,017 \text{ dm}^3$$

And now we can work out the concentration of the nitric acid:

$$\frac{C_1 V_1}{n_1} = \frac{C_2 V_2}{n_2}$$

$$\frac{(1,3)(0,017)}{1} = \frac{(C_{HNO_3})(0,020)}{2}$$

$$0,0221 = (0,020)C_{HNO_3}$$

$$C_{HNO_3} = 1,105 \text{ mol·dm}^{-3}$$

4. Given the equation:

$$3Ca(OH)_2(aq) + 2H_3PO_4(aq) \rightarrow Ca_3(PO_4)_2(aq) + 6H_2O \text{ (l)}$$
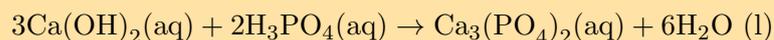
10 cm^3 of a 0,4 mol·dm^{-3} calcium hydroxide (Ca(OH)$_2$) solution was pipetted into a conical flask and titrated with phosphoric acid (H$_3$PO$_4$). It was found that 11 cm^3 of the phosphoric acid was needed to neutralise the base. Calculate the concentration of the phosphoric acid.

Solution:

Write down all the information you know about the reaction, and make sure that the equation is balanced.

Ca(OH)$_2$: $V = 10$ cm^3; $C = 0,4$ mol·dm^{-3}

H$_3$PO$_4$: $V = 11$ cm^3
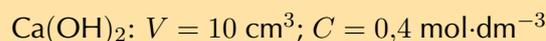
The equation is already balanced.

Next convert the volumes to dm^3

$$V_{Ca(OH)_2} = \frac{10}{1000}$$
$$= 0,010 \text{ dm}^3$$

$$V_{H_3PO_4} = \frac{11}{1000}$$
$$= 0,011 \text{ dm}^3$$

And now we can work out the concentration of the nitric acid:

$$\frac{C_1 V_1}{n_1} = \frac{C_2 V_2}{n_2}$$

$$\frac{(0,4)(0,010)}{3} = \frac{(C_{H_3PO_4})(0,011)}{2}$$

$$0,001333 = (0,0055)C_{H_3PO_4}$$

$$C_{H_3PO_4} = 0,24 \text{ mol·dm}^{-3}$$

5. A 3,7 g sample of an antacid (which contains mostly calcium carbonate) is dissolved in water. The final solution has a volume of 500 mL. 25 mL of this solution is then pipetted into a conical flask and titrated with hydrochloric acid. It is found that 20 mL of the hydrochloric acid completely neutralises the antacid solution. What is the concentration of the hydrochloric acid?

The equation for this reaction is:

$$CaCO_3(aq) + 2HCl \text{ (aq)} \rightarrow CaCl_2(aq) + H_2O \text{ (l)} + CO_2(g)$$

Solution:

First convert the volume into dm³:

$$V = \frac{500}{1000} = 0,5 \text{ L}$$
$$= 0,5 \text{ dm}^3$$

Then calculate the number of moles of calcium carbonate:

$$n = \frac{m}{M}$$
$$= \frac{3,7}{100}$$
$$= 0,037 \text{ mol}$$

Now we can calculate the concentration of the calcium carbonate:

$$C = \frac{n}{V}$$
$$= \frac{0,037}{0,5}$$
$$= 0,074 \text{ mol·dm}^{-3}$$

Now calculate the concentration of the hydrochloric acid.

Remember that only 25 mL or 0,025 dm³ of the calcium carbonate solution is used.

$$\frac{C_1 V_1}{n_1} = \frac{C_2 V_2}{n_2}$$
$$\frac{(0,074)(0,025)}{1} = \frac{(C_{\text{HCl}})(0,02)}{2}$$
$$0,00185 = (0,01)C_{\text{HCl}}$$
$$C_{\text{HCl}} = 0,185 \text{ mol·dm}^{-3}$$

8.2   Stoichiometric calculations

Limiting reagents

Exercise 8 – 3:

1. When an electrical current is passed through a sodium chloride solution, sodium hydroxide can be produced according to the following equation:

$$2NaCl \text{ (aq)} + H_2O \text{ (l)} \rightarrow Cl_2(g) + H_2(g) + 2NaOH \text{ (aq)}$$
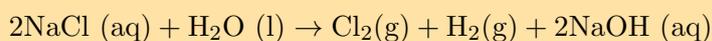
What is the maximum mass of sodium hydroxide that can be obtained from 4,0 kg of sodium chloride and 3,0 kg of water?

Solution:

Moles of sodium chloride:

$$n = \frac{m}{M}$$
$$= \frac{4000}{58,4}$$
$$= 68,49 \text{ mol}$$

Moles of water:

$$n = \frac{m}{M}$$
$$= \frac{3000}{18}$$
$$= 166,67 \text{ mol}$$

Now we look at the number of moles of product that each reactant can form.

The mole ratio of H_2O to NaOH is $1 : 2$. So the number of moles of NaOH that can be produced from water is:

$$n_{\text{NaOH}} = n_{H_2O} \times \frac{\text{stoichiometric coefficient NaOH}}{\text{stoichiometric coefficient } H_2O}$$
$$= 166,67 \text{ mol } H_2O \times \frac{1 \text{ molNaOH}}{2 \text{ mol } H_2O}$$
$$= 83,33 \text{ molNaOH}$$

The mole ratio of NaCl to NaOH is $2 : 2$ (or $1 : 1$). So the number of moles of NaOH that can be produced from sodium chloride is:

$$n_{\text{NaOH}} = n_{\text{NaCl}} \times \frac{\text{stoichiometric coefficient NaOH}}{\text{stoichiometric coefficient NaCl}}$$
$$= 68,49 \text{ molNaCl} \times \frac{2 \text{ molNaOH}}{2 \text{ mol NaCl}}$$
$$= 68,49 \text{ molNaOH}$$

Since NaCl produces less NaOH than is produced from H_2O, the sodium chloride is the limiting reagent.

We have $68,49$ mol of NaOH.

The maximum mass of sodium hydroxide that can be produced is calculated as follows:

$$m = nM$$
$$= (68,49)(40)$$
$$= 2739,6 \text{ g}$$
$$= 2,7396 \text{ kg}$$

The maximum amount of sodium hydroxide that can be produced is $2,74$ kg.

Exercise 8 – 4:

1. When an electrical current is passed through a sodium chloride solution, sodium hydroxide can be produced according to the following equation:

$$2NaCl + H_2O \rightarrow Cl_2 + H_2 + 2NaOH$$

A chemist carries out the above reaction using $4,0$ kg of sodium chloride and $3,0$ kg of water. The chemist finds that they get $1,8$ kg of sodium hydroxide. What is the percentage yield?

Solution:

We first find the limiting reagent.

Moles of sodium chloride:

$$
\begin{aligned}
n &= \frac{m}{M} \\
&= \frac{4000}{58,4} \\
&= 68,49 \text{ mol}
\end{aligned}
$$

Moles of water:

$$
\begin{aligned}
n &= \frac{m}{M} \\
&= \frac{3000}{18} \\
&= 166,67 \text{ mol}
\end{aligned}
$$

Now we look at the number of moles of product that each reactant can form.

The mole ratio of H_2O to NaOH is $1 : 2$. So the number of moles of NaOH that can be produced from water is:

$$
\begin{aligned}
n_{NaOH} &= n_{H_2O} \times \frac{\text{stoichiometric coefficient NaOH}}{\text{stoichiometric coefficient } H_2O} \\
&= 166,67 \text{ mol } H_2O \times \frac{1 \text{ mol NaOH}}{2 \text{ mol } H_2O} \\
&= 83,33 \text{ mol NaOH}
\end{aligned}
$$

The mole ratio of NaCl to NaOH is $2 : 2$ (or $1 : 1$). So the number of moles of NaOH that can be produced from sodium chloride is:

$$
\begin{aligned}
n_{NaOH} &= n_{NaCl} \times \frac{\text{stoichiometric coefficient NaOH}}{\text{stoichiometric coefficient NaCl}} \\
&= 68,49 \text{ mol NaCl} \times \frac{2 \text{ mol NaOH}}{2 \text{ mol NaCl}} \\
&= 68,49 \text{ mol NaOH}
\end{aligned}
$$

Since NaCl produces less NaOH than is produced from H_2O, the sodium chloride is the limiting reagent.

We have $68,49$ mol of NaOH.

The maximum mass of sodium hydroxide that can be produced is calculated as follows:

$$m = nM$$
$$= (68{,}49)(40)$$
$$= 2739{,}6 \text{ g}$$
$$= 2{,}7396 \text{ kg}$$

The maximum amount (theoretical yield) of sodium hydroxide that can be produced is 2,74 kg.

The percent yield is:

$$\%\text{yield} = \frac{\text{actual yield}}{\text{theoretical yield}} \times 100$$
$$= \frac{1{,}8}{2{,}74}(100)$$
$$= 65{,}69\%$$

Molecular and empirical formulae

Exercise 8 – 5:

1. A sample of oxalic acid has the following percentage composition: 26,7% carbon, 2,2% hydrogen and 71,1% oxygen.

   Determine the molecular formula of oxalic acid if the molar mass of oxalic acid is 90 g·mol^{-1}.

   Solution:

   In 100 g of oxalic acid, there is: 26,7 g C, 2,2 g H and 71,1 g O.

   $n = \frac{m}{M}$

   $$n_{\text{C}} = \frac{26{,}7}{12} = 2{,}225 \text{ mol}$$
   $$n_{\text{H}} = \frac{2{,}2}{1{,}01} = 2{,}18 \text{ mol}$$
   $$n_{\text{O}} = \frac{71{,}1}{16} = 4{,}44 \text{ mol}$$

   To find the empirical formula we first note how many moles of each element we have. Then we divide by the smallest number to get the ratios of each element. This ratio is rounded off to the nearest whole number.

C	H	O
2,225	2,18	4,44
$\frac{2{,}225}{2{,}18}$	$\frac{2{,}18}{2{,}18}$	$\frac{4{,}44}{2{,}18}$
1	1	2

The empirical formula is CHO_2.

The molar mass of oxalic acid using the empirical formula is 45 g·mol^{-1}. However the question gives the molar mass as 90 g·mol^{-1}. Therefore the actual number of moles of each element must be double what it is in the empirical formula ($\frac{90}{45} = 2$). The molecular formula is therefore $C_2H_2O_4$.

Percent purity

Exercise 8 – 6:

1. Hematite contains iron oxide (Fe_2O_3) as well as other compounds. Thembile wants to know how much iron oxide is in a sample of hematite. He finds that the sample of hematite weighs $6,2$ g. After performing some experiments he finds that the mass of iron oxide and the crucible (a container that is used to heat compounds in) is $4,8$ g. The mass of the crucible is $0,5$ g. How much iron oxide is in the sample of hematite?

   Solution:

   Percent purity is given by:

   $$\%\text{purity} = \frac{\text{mass of compound}}{\text{mass of sample}} \times 100$$

   We are given the mass of the product and the mass of the crucible. We need to subtract the mass of the crucible from this to obtain just the mass of the product.

   $$\text{Mass product} = 4,8 \text{ g} - 0,5 \text{ g}$$
   $$= 4,3 \text{ g}$$

   Substituting the calculated mass into the equation for percent purity gives:

   $$\%\text{purity} = \frac{\text{mass of compound}}{\text{mass of sample}} \times 100$$
   $$= \frac{4,3}{6,2}(100)$$
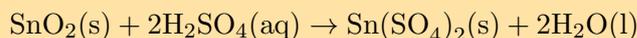 $$= 69\%$$

Exercise 8 – 7: Stoichiometry

1. Given the following reaction:

$$3Fe_2O_3(s) + CO\ (g) \rightarrow 2Fe_2O_4(s) + CO_2(g)$$
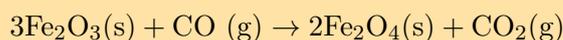
If 2,3 kg of Fe_2O_3 and 1,7 kg of CO is used, what is the maximum mass of Fe_2O_4 that can be produced?

Solution:

Moles of Fe_2O_3 (the molar mass of iron is 56 g·mol^{-1}):

$$\begin{aligned} n &= \frac{m}{M} \\ &= \frac{2300}{160} \\ &= 14{,}375 \text{ mol} \end{aligned}$$

Moles of water:

$$\begin{aligned} n &= \frac{m}{M} \\ &= \frac{1700}{28} \\ &= 60{,}71 \text{ mol} \end{aligned}$$

Now we look at the number of moles of product that each reactant can form.

The mole ratio of Fe_2O_3 to Fe_2O_4 is 3 : 2. So the number of moles of Fe_2O_4 that can be produced from Fe_2O_3 is:

$$\begin{aligned} n_{Fe_2O_4} &= n_{Fe_2O_4} \times \frac{\text{stoichiometric coefficient } Fe_2O_4}{\text{stoichiometric coefficient } Fe_2O_4} \\ &= 14{,}375 \text{ mol } Fe_2O_3 \times \frac{2 \text{ mol} Fe_2O_4}{3 \text{ mol } Fe_2O_3} \\ &= 9{,}58 \text{ mol} Fe_2O_4 \end{aligned}$$

The mole ratio of CO to Fe_2O_4 is or 1 : 2. So the number of moles of Fe_2O_3 that can be produced from carbon monoxide is:

$$\begin{aligned} n_{Fe_2O_4} &= n_{CO} \times \frac{\text{stoichiometric coefficient } Fe_2O}{\text{stoichiometric coefficient } CO} \\ &= 60{,}71 \text{ mol} CO \times \frac{2 \text{ mol} Fe_2O_4}{1 \text{ mol } CO} \\ &= 121{,}42 \text{ mol} Fe_2O_4 \end{aligned}$$

Since Fe_2O_3 produces less Fe_2O_4 than is produced from CO, the Fe_2O_3 is the limiting reagent.

We have 9,58 mol of Fe_2O_4.

The maximum mass of Fe_2O_4 that can be produced is calculated as follows:

$$\begin{aligned} m &= nM \\ &= (9{,}58)(176) \\ &= 1686{,}08 \text{ g} \\ &= 1{,}686 \text{ kg} \end{aligned}$$

The maximum amount (theoretical yield) of Fe_2O_4 that can be produced is 1,69 kg.

2. Sodium nitrate decomposes on heating to produce sodium nitrite and oxygen according to the following equation:

$$2NaNO_3(s) \rightarrow 2NaNO_2(s) + O_2(g)$$
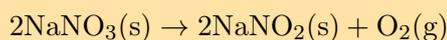
Nombusa carries out the above reaction using 50 g of sodium nitrate. Nombusa finds that they get 36 g of sodium nitrite. What is the percentage yield?

Solution:

There is only one reactant and so we do not need to find the limiting reagent.

We find the number of moles of sodium nitrate:

$$\begin{aligned} n &= \frac{m}{M} \\ &= \frac{50}{85} \\ &= 0{,}588 \text{ mol} \end{aligned}$$

Now we find the number of moles of sodium nitrite.

The mole ratio of $NaNO_3$ to $NaNO_2$ is 2 : 2 (or 1 : 1). So the number of moles of $NaNO_2$ is also 0,588 mol.

The maximum mass of sodium nitrite that can be produced is:

$$\begin{aligned} m &= nM \\ &= (0{,}588)(69) \\ &= 40{,}588 \text{ g} \end{aligned}$$

The maximum amount (theoretical yield) of sodium nitrite that can be produced is 40,588 g.

The percent yield is:

$$\begin{aligned} \%\text{yield} &= \frac{\text{actual yield}}{\text{theoretical yield}} \times 100 \\ &= \frac{36}{40{,}588}(100) \\ &= 88{,}69\% \end{aligned}$$

3. Benzene has the following percentage composition: 92,31% carbon and 7,69% hydrogen

Determine the molecular formula of benzene if the molar mass of benzene is 78 g·mol^{-1}.

Solution:

In 100 g of benzene, there is: 92,31 g C and 7,69 g H.

$n = \frac{m}{M}$

$$n_C = \frac{92{,}31}{12} = 7{,}6925 \text{ mol}$$

$$n_H = \frac{7{,}69}{1{,}01} = 7{,}614 \text{ mol}$$

We note that the two values are approximately equal.

The empirical formula is CH.

The molar mass of benzene using the empirical formula is 13 g·mol^{-1}. However the question gives the molar mass as 78 g·mol^{-1}. We divide the given molar mass by the calculated molar mass to find the molecular formula: $\frac{78}{13} = 6$. Therefore the molecular formula is: C_6H_6.

4. Cuprite is a minor ore of copper. Cuprite is mainly composed of copper(I) oxide (Cu_2O). Jennifer wants to know how much copper oxide is in a sample of cuprite. She has a sample of cuprite that weighs 7,7 g. She performs some experiments and finds that the mass of iron oxide and crucible (a container that is used to heat compounds in) is 7,4 g. The mass of the crucible is 0,2 g. What is the percent purity of the sample of cuprite?

Solution:

Percent purity is given by:

$$\%\text{purity} = \frac{\text{mass of compound}}{\text{mass of sample}} \times 100$$

We are given the mass of the product and the mass of the crucible. We need to subtract the mass of the crucible from this to obtain just the mass of the product.

$$\text{Mass product} = 7,4 \text{ g} - 0,2 \text{ g}$$
$$= 7,2 \text{ g}$$

Substituting the calculated mass into the equation for percent purity gives:

$$\%\text{purity} = \frac{\text{mass of compound}}{\text{mass of sample}} \times 100$$
$$= \frac{7,2}{7,7}(100)$$
$$= 93,5\%$$

5. A sample containing tin dioxide (SnO_2) is to be tested to see how much tin dioxide it contains. The sample weighs 6,2 g. Sulfuric acid (H_2SO_4) is added to the sample and tin sulfate ($Sn(SO_4)_2$) forms. The equation for this reaction is:

$$SnO_2(s) + 2H_2SO_4(aq) \rightarrow Sn(SO_4)_2(s) + 2H_2O(l)$$

If the mass of tin sulfate produced is 4,7 g, what is the percent purity of the sample?

Solution:

The number of moles of tin sulfate is (the molar mass of tin is (119 g·mol^{-1})):

$$n = \frac{m}{M}$$
$$= \frac{6,2}{311}$$
$$= 0,0199 \text{ mol}$$

The molar ratio of tin sulfate to tin dioxide is 1:1. Therefore the number of moles of tin dioxide is 0,0199 mol.

The mass of tin dioxide is:

$$\begin{aligned} m &= nM \\ &= (0,0199)(151) \\ &= 3,01 \text{ g} \end{aligned}$$

Substituting the calculated mass into the equation for percent purity gives:

$$\begin{aligned} \%\text{purity} &= \frac{\text{mass of compound}}{\text{mass of sample}} \times 100 \\ &= \frac{3,01}{6,2} \times (100) \\ &= 48,6\% \end{aligned}$$

8.3   Volume relationships in gaseous reactions

Exercise 8 – 8: Gases 2

1. What volume of oxygen is needed for the complete combustion of 5 g of magnesium to form magnesium oxide?

   Solution:

   The balanced equation for this reaction is:

   $$2\text{Mg (s)} + \text{O}_2\text{(g)} \rightarrow 2\text{MgO (s)}$$
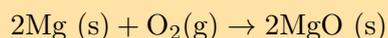
   The number of moles of magnesium used is:

   $$\begin{aligned} n &= \frac{m}{M} \\ &= \frac{5}{24} \\ &= 0,2083 \text{ mol} \end{aligned}$$

   The mole ratio of Mg to O_2 is 2 : 1. So the number of moles of O_2 is:

   $$\begin{aligned} n_{\text{O}_2} &= n_{\text{Mg}} \times \frac{\text{stoichiometric coefficient O}_2}{\text{stoichiometric coefficient Mg}} \\ &= 0,2083 \text{ molMg} \times \frac{1 \text{ molO}_2}{2 \text{ mol Mg}} \\ &= 0,1042 \text{ molO}_2 \end{aligned}$$

   The volume of oxygen is:

$$V = (22,4)n$$
$$= (22,4)(0,1042)$$
$$= 2,33 \text{ dm}^3$$

2. Annalize is making a mini volcano for her science project. She mixes baking soda (mostly $NaHCO_3$) and vinegar (mostly CH_3COOH) together to make her volcano erupt. The reaction for this equation is:

$$NaHCO_3(s) + CH_3COOH \text{ (aq)} \rightarrow CH_3COONa \text{ (aq)} + H_2O \text{ (l)} + CO_2(g)$$

What volume of carbon dioxide is produced if Annalize uses 50 ml of $0,2$ mol·dm^3 acetic acid?

Solution:

The number of moles of acetic acid used is:

$$C = \frac{n}{V}$$
$$0,2 \text{ mol·dm}^3 = \frac{n}{0,05 \text{ dm}^3}$$
$$n = 0,01 \text{ mol}$$

The mole ratio of CH_3COOH to CO_2 is 1 : 1. So the number of moles of CO_2 is $0,01$ mol.

$$V = (22,4)n$$
$$= (22,4)(0,01)$$
$$= 0,224 \text{ dm}^3$$

8.4   Chapter summary

Exercise 8 – 9:

1. Write only the word/term for each of the following descriptions:

    a) A reagent that is completely used up in a chemical reaction.

    b) The simplest formula of a compound.

    c) The amount of product that is calculated for a reaction using stoichiometric methods.

    d) A technique for determining the concentration of an unknown solution.

    Solution:

    a) Limiting reagent

b) Empirical formula

c) Theoretical yield

d) Titration

2. What is the volume of 3 mol of N_2 gas at STP?

    a) 67,2 dm^3

    b) 22,4 dm^3

    c) 33,6 dm^3

    d) 63,2 dm^3

Solution:

67,2 dm^3

3. Given the following reaction:

$$3NO_2(g) + H_2O\ (g) \rightarrow 2HNO_3(g) + NO\ (g)$$
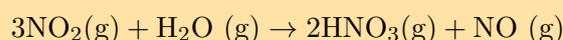
If 2,7 dm^3 of NO_2 is used, what volume of HNO_3 is produced?

    a) 4,1 dm^3

    b) 2,7 dm^3

    c) 1,8 dm^3

    d) 3,4 dm^3

Solution:

1,8 dm^3

$$V_A = \frac{a}{b} V_B$$
$$V_{HNO_3} = \frac{2}{3} V_{NO_2}$$
$$= \frac{2}{3}(2,7)$$
$$= 1,8 \text{ dm}^3$$

4. If 4,2 g of magnesium sulfate is dissolved in 350 cm^3 of water, what is the concentration of the solution?

    a) 0,1 mol·dm^{-3}

    b) 0,05 mol·dm^{-3}

    c) 0,003 mol·dm^{-3}

    d) 0,0001 mol·dm^{-3}

Solution:

The concentration is: 0,1 mol·dm^{-3}

The number of moles of magnesium sulfate is:

$$n = \frac{m}{M}$$
$$= \frac{4,2}{120}$$
$$= 0,035 \text{ mol}$$

To convert from cm^3 to dm^3 we divide by 1000:

$$\frac{350 \text{ cm}^3}{1000} = 0{,}35 \text{ dm}^3$$

The concentration is:

$$C = \frac{n}{V}$$
$$= \frac{0{,}035}{0{,}35}$$
$$= 0{,}1 \text{ mol·dm}^{-3}$$

5. Gold is occasionally found as the mineral calaverite. Calaverite contains a tel-
luride of gold ($AuTe_2$). Phumza wants to know how much calaverite is in a
sample of rock. She finds that the rock sample weighs 3,6 g. After performing
some experiments she finds that the mass of calaverite and the crucible is 2,4 g.
The mass of the crucible is 0,3 g. What is the percent purity of calaverite in the
sample?

   a) 63%

   b) 54%

   c) 58%

   d) 67%

Solution:

The percent purity is 58%.

Percent purity is given by:

$$\%\text{purity} = \frac{\text{mass of compound}}{\text{mass of sample}} \times 100$$

We are given the mass of the product and the mass of the crucible. We need to
subtract the mass of the crucible from this to obtain just the mass of the product.

$$\text{Mass product} = 2{,}4 \text{ g} - 0{,}3 \text{ g}$$
$$= 2{,}1 \text{ g}$$

Substituting the calculated mass into the equation for percent purity gives:

$$\%\text{purity} = \frac{\text{mass of compound}}{\text{mass of sample}} \times 100$$
$$= \frac{2{,}1}{3{,}6}(100)$$
$$= 58\%$$

6. 300 cm^3 of a 0,1 mol·dm^{-3} solution of sulfuric acid is added to 200 cm^3 of a
0,5 mol·dm^{-3} solution of sodium hydroxide.

   a) Write down a balanced equation for the reaction which takes place when
   these two solutions are mixed.

b) Calculate the number of moles of sulfuric acid which were added to the sodium hydroxide solution.

c) Is the number of moles of sulfuric acid enough to fully neutralise the sodium hydroxide solution? Support your answer by showing all relevant calculations.

(IEB Paper 2 2004)

Solution:

a) $2NaOH + H_2SO_4 \rightarrow Na_2SO_4 + 2H_2O$
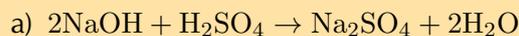
b)

$$C = \frac{n}{V}$$
$$0,1 = \frac{n}{0,3}$$
$$n = 0,03 \text{ mol}$$

We need $0,03$ mol.

c) We calculate the number of moles of sodium hydroxide that are added

$$C = \frac{n}{V}$$
$$0,5 = \frac{n}{0,2}$$
$$n = 0,1 \text{ mol}$$

From the molar ratio of sulfuric acid to sodium hydroxide we find that we can neutralise

$$n_{NaOH} = n_{H_2SO_4} \times \frac{\text{stoichiometric coefficient NaOH}}{\text{stoichiometric coefficient H}_2\text{SO}_4}$$
$$= 0,03 \text{ molH}_2\text{SO}_4 \times \frac{2 \text{ molNaOH}}{1 \text{ mol H}_2\text{SO}_4}$$
$$= 0,015 \text{ mol}$$

of sodium hydroxide. We have $0,01$ mol of sodium hydroxide and so the sodium hydroxide will be fully neutralised. The sulfuric acid is slightly in excess.

7. Given the equation:

$2NaOH \text{ (aq)} + H_2SO_4\text{(aq)} \rightarrow Na_2SO_4\text{(aq)} + 2H_2O \text{ (l)}$
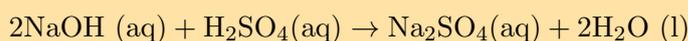
25 cm^3 of a $0,7$ mol·dm^{-3} sulfuric acid (H_2SO_4) solution was pipetted into a conical flask and titrated with sodium hydroxide (NaOH). It was found that 23 cm^3 of the sodium hydroxide was needed to neutralise the acid. Calculate the concentration of the sodium hydroxide.

Solution:

Write down all the information you know about the reaction, and make sure that the equation is balanced.

H_2SO_4: $V = 25 \text{ cm}^3$; $C = 0,7$ mol·dm^{-3}

NaOH: $V = 23 \text{ cm}^3$

The equation is already balanced.

Next convert the volumes to dm^3

$$V_{\text{H}_2\text{SO}_4} = \frac{25}{1000}$$
$$= 0{,}025 \text{ dm}^3$$

$$V_{\text{NaOH}} = \frac{23}{1000}$$
$$= 0{,}023 \text{ dm}^3$$

And now we can work out the concentration of the sodium hydroxide:

$$\frac{C_1 V_1}{n_1} = \frac{C_2 V_2}{n_2}$$
$$\frac{(0{,}7)(0{,}025)}{1} = \frac{(C_{\text{NaOH}})(0{,}023)}{2}$$
$$0{,}0175 = (0{,}0115)C_{\text{NaOH}}$$
$$C_{\text{NaOH}} = 1{,}52 \text{ mol·dm}^{-3}$$

8. Ozone (O_3) reacts with nitrogen monoxide gas (NO) to produce NO_2 gas. The NO gas forms largely as a result of emissions from the exhausts of motor vehicles and from certain jet planes. The NO_2 gas also contributes to the brown smog (smoke and fog), which is seen over most urban areas. This gas is also harmful to humans, as it causes breathing (respiratory) problems. The following equation indicates the reaction between ozone and nitrogen monoxide:

$$O_3(g) + NO\ (g) \rightarrow O_2(g) + NO_2(g)$$
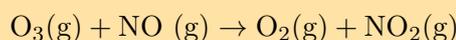
In one such reaction $0{,}74$ g of O_3 reacts with $0{,}67$ g NO.

   a) Calculate the number of moles of O_3 and of NO present at the start of the reaction.

   b) Identify the limiting reagent in the reaction and justify your answer.

   c) Calculate the mass of NO_2 produced from the reaction.

(DoE Exemplar Paper 2, 2007)

Solution:

   a) Moles of O_3:

$$n = \frac{m}{M}$$
$$= \frac{0{,}74}{48}$$
$$= 0{,}0154 \text{ mol}$$

Moles of NO:

$$n = \frac{m}{M}$$
$$= \frac{0{,}67}{30}$$
$$= 0{,}0223 \text{ mol}$$

b) Now we look at the number of moles of product that each reactant can form.

The mole ratio of O_3 to NO_2 is $1 : 1$. So the number of moles of O_3 that can be produced from O_3 is $0,0154$ mol

The mole ratio of NO to NO_2 is $1 : 1$. So the number of moles of Fe_2O_3 that can be produced from carbon monoxide is $0,0223$ mol

Since O_3 produces less NO_2 than is produced from NO, the O_3 is the limiting reagent.

c) We have $0,0154$ mol of NO_2.

The maximum mass of NO_2 that can be produced is calculated as follows:

$$m = nM$$
$$= (0,0154)(46)$$
$$= 0,71 \text{ g}$$

The maximum amount (theoretical yield) of NO_2 that can be produced is $0,71$ g.

9. Calcium carbonate decomposes on heating to produce calcium oxide and oxygen according to the following equation:

$$CaCO_3(s) \rightarrow CaO\ (s) + O_2(g)$$

Thabang carries out the above reaction using 127 g of calcium carbonate. He finds that he gets $68,2$ g of calcium oxide. What is the percentage yield?

Solution:

There is only one reactant and so we do not need to find the limiting reagent.

We find the number of moles of calcium carbonate:

$$n = \frac{m}{M}$$
$$= \frac{127}{100}$$
$$= 1,27 \text{ mol}$$

Now we find the number of moles of calcium oxide.

The mole ratio of $CaCO_3$ to CaO is $1 : 1$. So the number of moles of CaO is also $1,27$ mol.

The maximum mass of calcium oxide that can be produced is:

$$m = nM$$
$$= (1,27)(56)$$
$$= 71,12 \text{ g}$$

The maximum amount (theoretical yield) of calcium oxide that can be produced is $71,12$ g.

The percent yield is:

$$\%\text{yield} = \frac{\text{actual yield}}{\text{theoretical yield}} \times 100$$
$$= \frac{68,2}{71,12} \times 100$$
$$= 96,9\%$$

10. Some airbags contain a mixture of sodium azide (NaN_3) and potassium nitrate (KNO_3). When a car crash is detected by the signalling system, the sodium azide is heated until it decomposes to form nitrogen gas and sodium metal:

$$2NaN_3(s) \rightarrow 2Na\ (s) + 3N_2(g)$$

The potassium nitrate then reacts with the sodium metal forming more nitrogen:

$$10Na\ (s) + 2KNO_3(s) \rightarrow K_2O\ (s) + 5Na_2O\ (s) + N_2(g)$$

A typical passenger side airbag contains 250 g of sodium azide.

   a) What mass of sodium metal is formed in the first reaction?

   b) What is the total volume of nitrogen gas formed from both reactions?

   c) How much potassium nitrate (in g) is needed for all the sodium to be used up in the second reaction?

Solution:

   a) To find the amount of sodium metal we first calculate how many moles of sodium azide is in the airbag:

$$n = \frac{250}{65}$$
$$= 3,846 \text{ mol}$$

Next we find the number of moles of sodium. The molar ratio of sodium azide to sodium is 2 : 2 (or 1 : 1) so the number of moles of sodium is: 3,846 mol

And the mass of sodium metal is:

$$m = (3,846)(23)$$
$$= 88,461 \text{ g}$$

   b) To find the total volume of nitrogen produced we need to calculate the volume of nitrogen produced in each reaction and then add these two numbers together.

For the first reaction we have 3,846 mol of sodium azide. The mole ratio of sodium azide to nitrogen is 2 : 3, so the number of moles of nitrogen is:

$$n_{N_2} = n_{NaN_3} \times \frac{\text{stoichiometric coefficient } N_2}{\text{stoichiometric coefficient } NaN_3}$$
$$= 3,846 \text{ molNaN}_3 \times \frac{2 \text{ molN}_2}{3 \text{ mol NaN}_3}$$
$$= 2,564 \text{ molN}_2$$

Now we find the volume:

$$V = 22,4n$$
$$= (22,4)(2,564)$$
$$= 57,436 \text{ dm}^3$$

For the second reaction we have $3,846$ mol of sodium (the sodium from the first reaction is used up in the second reaction). The mole ratio of sodium to nitrogen is $10 : 1$, so the number of moles of nitrogen is:

$$n_{N_2} = n_{Na} \times \frac{\text{stoichiometric coefficient } N_2}{\text{stoichiometric coefficient Na}}$$
$$= 3,846 \text{ molNa} \times \frac{1 \text{ molN}_2}{10 \text{ mol Na}}$$
$$= 0,385 \text{ molN}_2$$

Now we find the volume:

$$V = 22,4n$$
$$= (22,4)(0,385)$$
$$= 8,615 \text{ dm}^3$$

And the total volume of nitrogen is:

$$V_T = V_1 + V_2$$
$$= 57,436 + 8,615$$
$$= 66,051 \text{ dm}^3$$

c) The number of moles of sodium is: $3,846$ mol. The molar ratio of sodium to potassium nitrate is: $10 : 2$ or $5 : 1$. So the number of moles of potassium nitrate is:

$$n_{KNO_3} = n_{Na} \times \frac{\text{stoichiometric coefficient } KNO_3}{\text{stoichiometric coefficient Na}}$$
$$= 3,846 \text{ molNa} \times \frac{1 \text{ molKNO}_3}{5 \text{ mol Na}}$$
$$= 0,7692 \text{ mol}$$

And the mass of potassium nitrate needed is:

$$m = (0,7692)(101)$$
$$= 77,69 \text{ g}$$

To ensure a complete reaction we could use 78 g of potassium nitrate.

11. Chlorofluorocarbons (CFC's) are a class of compounds that have a long history of use in refrigerators. CFC's are slowly being phased out as they deplete the amount of ozone in the ozone layer. Jabu has a sample of a CFC that has the following percentage composition: $14,05\%$ carbon, $41,48\%$ chlorine and $44,46\%$ fluorine.

Determine the molecular formula of this CFC if the molar mass is 171 g·mol^{-1}.

Solution:

In 100 g of the CFC, there is: 14,05 g C, 41,48 g Cl, and 44,46 g F.

$n = \frac{m}{M}$

$$n_C = \frac{14,05}{12} = 1,17 \text{ mol}$$

$$n_{Cl} = \frac{41,48}{35} = 1,19 \text{ mol}$$

$$n_F = \frac{44,46}{19} = 2,34 \text{ mol}$$

To find the empirical formula we first note how many moles of each element we have. Then we divide by the smallest number to get the ratios of each element. This ratio is rounded off to the nearest whole number.

C	Cl	F
1,17	1,19	2,34
$\frac{1,17}{1,17}$	$\frac{1,19}{1,17}$	$\frac{2,34}{1,17}$
1	1	2

The empirical formula is $CClF_2$.

The molar mass of the CFC using the empirical formula is 85 g·mol^{-1}. However the question gives the molar mass as 171 g·mol^{-1}. We divide the given molar mass by the calculated molar mass to find the molecular formula: $\frac{171}{85} = 2$. Therefore the molecular formula is: $C_2Cl_2F_4$.

12. A sample containing tin dioxide (SnO_2) is to be tested to see how much tin dioxide it contains. The sample weighs 6,2 g. Sulfuric acid (H_2SO_4) is added to the sample and tin sulfate ($Sn(SO_4)_2$) forms. The equation for this reaction is:

$$SnO_2(s) + 2H_2SO_4(aq) \rightarrow Sn(SO_4)_2(s) + 2H_2O \ (l)$$

If the mass of tin sulfate produced is 4,7 g, what is the percent purity of the sample?

Solution:

The number of moles of tin sulfate is (the molar mass of tin is (119 g·mol^{-1})):

$$n = \frac{m}{M}$$
$$= \frac{6,2}{311}$$
$$= 0,0199 \text{ mol}$$

The molar ratio of tin sulfate to tin dioxide is 1:1. Therefore the number of moles of tin dioxide is 0,0199 mol.

The mass of tin dioxide is:

$$m = nM$$
$$= (0,0199)(151)$$
$$= 3,01 \text{ g}$$

Substituting the calculated mass into the equation for percent purity gives:

$$\%\text{purity} = \frac{\text{mass of compound}}{\text{mass of sample}} \times 100$$
$$= \frac{3{,}01}{6{,}2} \times (100)$$
$$= 48{,}6\%$$

13. Syngas (synthesis gas) is a mixture of carbon monoxide and hydrogen. Syngas can be produced from methane using:

$$CH_4(g) + H_2O\ (g) \rightarrow CO\ (g) + 3H_2(g)$$

Neels wants to make a mixture of syngas that has three times the volume of hydrogen gas.

   a) If the volume of methane used is 4 dm^3, what volume of carbon monoxide and hydrogen will be produced?

   b) Will this amount of methane produce the correct mixture?

Solution:

   a) The volume of carbon monoxide produced is:

$$V_{CO_2} = \frac{a}{b}V_{CH_4}$$
$$= \frac{1}{1}(4)$$
$$= 4\ \text{dm}^3$$

The volume of hydrogen produced is:

$$V_{H_2} = \frac{a}{b}V_{CH_4}$$
$$= \frac{1}{3}(4)$$
$$= 1{,}33\ \text{dm}^3$$

   b) Yes. Since the two gases have a ratio of $1 : 3$ we will always have three times as much carbon monoxide as hydrogen gas.

CHAPTER 9

Electrostatics

9.1   Introduction

This chapter builds on the work covered in electrostatics in grade 10. Learners should be familiar with the two types of charges and with simple calculations of amount of charge. The following list summarises the topics covered in this chapter.

- **Coulomb's law**

  In this part of the topic learners are introduced to Coulomb's law. This is an inverse square law and has a similar form to Newton's law of Universal Gravitation.

- **Electric fields**

  The concept of an electric field is introduced in this part of the chapter. Learners will see how to draw the electric field lines for different configurations of charges and will learn how to determine the magnitude of the electric field. Learners will also learn how to calculate the electric field at a point due to a number of point charges.

9.2   Coulomb's law

Exercise 9 – 1: Electrostatic forces

1. Calculate the electrostatic force between two charges of $+6$ nC and $+1$ nC if they are separated by a distance of 2 mm.
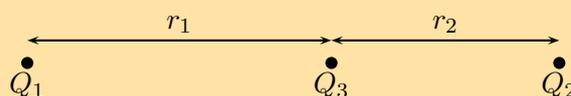
   Solution:

   $$
   \begin{aligned}
   F_e &= \frac{kQ_1Q_2}{r^2} \\
   &= \frac{(9{,}0 \times 10^9)(6 \times 10^{-9})(1 \times 10^{-9})}{(2 \times 10^{-3})^2} \\
   &= 1{,}35 \times 10^{-2} \text{ N}
   \end{aligned}
   $$

   This force is repulsive since it is between two like charges.

2. What is the magnitude of the repulsive force between two pith balls (a pith ball is a small, light ball that can easily be charged) that are 8 cm apart and have equal charges of -30 nC?

   Solution:

$$F_e = \frac{kQ_1Q_2}{r^2}$$
$$= \frac{(9{,}0 \times 10^9)(30 \times 10^{-9})(30 \times 10^{-9})}{(8 \times 10^{-2})^2}$$
$$= 1{,}27 \times 10^{-3} \text{ N}$$

This force is repulsive since it is between two like charges.

3. How strong is the attractive force between a glass rod with a $0{,}7\ \mu$C charge and a silk cloth with a $-0{,}6\ \mu$C charge, which are 12 cm apart, using the approximation that they act like point charges?

Solution:

$$F_e = \frac{kQ_1Q_2}{r^2}$$
$$= \frac{(9{,}0 \times 10^9)(0{,}6 \times 10^{-6})(0{,}7 \times 10^{-6})}{(12 \times 10^{-2})^2}$$
$$= 0{,}26 \text{ N}$$

This force is attractive since it is between two unlike charges.

4. Two point charges exert a 5 N force on each other. What will the resulting force be if the distance between them is increased by a factor of three?

Solution:

Let the charges be Q_1 and Q_2. For the first situation we have:

$$F_{e1} = \frac{kQ_1Q_2}{r^2}$$

Now we increase the distance by a factor of three so we get:

$$F_{e2} = \frac{kQ_1Q_2}{9r^2}$$

We now note the following:

$$(F_{e1})(r^2) = kQ_1Q_2$$

and:

$$(F_{e2})(9r^2) = kQ_1Q_2$$

Therefore:

$$(F_{e1})(r^2) = (F_{e2})(9r^2)$$
$$F_{e1} = (F_{e2})(9)$$
$$\therefore F_{e2} = 0{,}56 \text{ N}$$

5. Two point charges are brought closer together, increasing the force between them by a factor of 25. By what factor was their separation decreased?

Solution:

Let the charges be Q_1 and Q_2. For the first situation we have:

$$F_{e1} = \frac{kQ_1Q_2}{r^2}$$

Now we decrease by an unknown factor, x, the distance and we get:

$$F_{e2} = \frac{kQ_1Q_2}{(xr)^2}$$

We now note the following:

and:

$$(F_{e2}) = 25\frac{kQ_1Q_2}{r^2}$$

Therefore:

$$25(\frac{kQ_1Q_2}{r^2}) = \frac{kQ_1Q_2}{(xr)^2}$$
$$25 = \frac{1}{x^2}$$
$$\therefore x = 0,2$$

6. If two equal charges each of 1 C each are separated in air by a distance of 1 km, what is the magnitude of the force acting between them?

   Solution:

   $$F_e = \frac{kQ_1Q_2}{r^2}$$
   $$= \frac{(9,0 \times 10^9)(1)(1)}{(1 \times 10^3)^2}$$
   $$= 9000 \text{ N}$$

   This force is repulsive since it is between two like charges.

   You should notice that even at a distance as large as 1 km, the repulsive force is substantial because 1 C is a very significant amount of charge.

7. Calculate the distance between two charges of $+4$ nC and -3 nC if the electro-static force between them is 0,005 N.

   Solution:

   $$F_e = \frac{kQ_1Q_2}{r^2}$$
   $$0,005 = \frac{(9,0 \times 10^9)(4 \times 10^{-9})(3 \times 10^{-9})}{(d)^2}$$
   $$0,005d^2 = 1,07 \times 10^{-9}$$
   $$d^2 = 2,1576 \times 10^{-5}$$
   $$= 4,6 \times 10^{-3} \text{ m}$$

8. For the charge configuration shown, calculate the resultant force on Q_2 if:

- $Q_1 = 2,3 \times 10^{-7}$ C
- $Q_2 = 4 \times 10^{-6}$ C
- $Q_3 = 3,3 \times 10^{-7}$ C
- $r_1 = 2,5 \times 10^{-1}$ m
- $r_2 = 3,7 \times 10^{-2}$ m
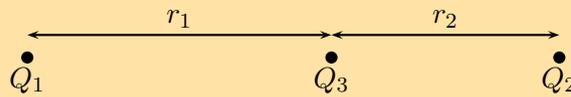
Solution:

We first calculate the force on Q_2 from Q_3:

$$
\begin{aligned}
F_{e1} &= \frac{kQ_1Q_2}{r^2} \\
&= \frac{(9,0 \times 10^9)(2,3 \times 10^{-7})(4 \times 10^{-6})}{(3,7 \times 10^{-2} + 2,5 \times 10^{-1})^2} \\
&= 1,00 \times 10^{-1} \text{ N}
\end{aligned}
$$

And then we calculate the force of Q_1 on Q_2. Note that for this force we must add r_1 and r_2.

$$
\begin{aligned}
F_{e2} &= \frac{kQ_2Q_3}{r^2} \\
&= \frac{(9,0 \times 10^9)(4 \times 10^{-6})(3,3 \times 10^{-7})}{(2,5 \times 10^{-1} + 3,7 \times 10^{-2})^2} \\
&= 8,67 \text{ N}
\end{aligned}
$$

Next we note that the force of Q_3 on Q_2 is repulsive and the force of Q_1 on Q_2 is also repulsive. So these two forces act in the same direction (towards the right). The resultant force is:

$$
\begin{aligned}
F_{eR} &= F_{e1} + F_{e2} \\
&= 8,67 \text{ N} + 0,1 \text{ N} \\
&= 8,77 \text{ N } \textit{to the right.}
\end{aligned}
$$

9. For the charge configuration shown, calculate the charge on Q_3 if the resultant force on Q_2 is $6,3 \times 10^{-1}$ N to the right and:

- $Q_1 = 4,36 \times 10^{-6}$ C
- $Q_2 = -7 \times 10^{-7}$ C
- $r_1 = 1,85 \times 10^{-1}$ m
- $r_2 = 4,7 \times 10^{-2}$ m

Solution:

We are told that the resultant force is $6,3 \times 10^{-5}$ N to the right. Since the force of Q_1 on Q_2 is attractive, the force of Q_3 on Q_2 must be repulsive to cause a resultant force to the right (if it was also attractive, the resultant force would be to the left). So we know that Q_3 must be negative.

We first calculate the force on Q_2 from Q_1:

$$
\begin{aligned}
F_{e1} &= \frac{kQ_1Q_2}{r^2} \\
&= \frac{(9,0 \times 10^9)(4,36 \times 10^{-6})(7 \times 10^{-7})}{(1,85 \times 10^{-1} + 4,7 \times 10^{-2})^2} \\
&= 0,51 \text{ N}
\end{aligned}
$$

Next we use this and the resultant force to find the force on Q_2 from Q_3

$$
\begin{aligned}
F_{eR} &= F_{e1} + F_{e2} \\
F_{e2} &= 6,3 \times 10^{-1} \text{ N} - 0,51 \text{ N} \\
&= 0,12 \text{ N}
\end{aligned}
$$

And then we calculate the charge on Q_3:

$$
\begin{aligned}
F_{e2} &= \frac{kQ_2Q_3}{r^2} \\
0,12 &= \frac{(9,0 \times 10^9)(7 \times 10^{-7})(Q_3)}{(4,7 \times 10^{-2})^2} \\
2,6 \times 10^{-4} &= (6,293 \times 10^3)(Q_3) \\
Q_3 &= 4,2 \times 10^{-8} \text{ C}
\end{aligned}
$$

10. Calculate the resultant force on Q_1 given this charge configuration:
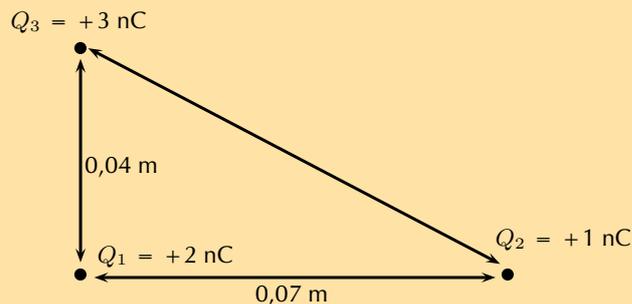
Solution:

We first calculate the force on Q_1 from Q_2:

$$
\begin{aligned}
F_e &= \frac{kQ_1Q_2}{r^2} \\
&= \frac{(9,0 \times 10^9)(1 \times 10^{-9})(2 \times 10^{-9})}{(0,07)^2} \\
&= 3,7 \times 10^{-6} \text{ N}
\end{aligned}
$$

And then we calculate the force of Q_3 on Q_1:

$$F_e = \frac{kQ_1Q_3}{r^2}$$
$$= \frac{(9{,}0 \times 10^9)(3 \times 10^{-9})(2 \times 10^{-9})}{(0{,}04)^2}$$
$$= 3{,}4 \times 10^{-5} \text{ N}$$

The magnitude of the resultant force acting on Q_1 can be calculated from the forces using Pythagoras' theorem because there are only two forces and they act in the x- and y-directions:

$$F_R^2 = F_x^2 + F_y^2$$
$$F_R = \sqrt{(3{,}7 \times 10^{-6})^2 + (3{,}4 \times 10^{-5})^2}$$
$$= 3{,}42 \times 10^{-5} \text{ N}$$

We can find the angle using trigonometry:

$$\tan \theta_R = \frac{y\text{-component}}{x\text{-component}}$$
$$= \frac{3{,}42 \times 10^{-5}}{3{,}7 \times 10^{-6}}$$
$$= 9{,}2432\ldots$$
$$\theta_R = 83{,}8°$$

The final resultant force acting on Q_1 is $3{,}42 \times 10^{-5}$ N acting at an angle of $83{,}8°$ to the negative x-axis.

11. Calculate the resultant force on Q_1 given this charge configuration:

Solution:

We first calculate the force on Q_1 from Q_2:

$$F_e = \frac{kQ_1Q_2}{r^2}$$
$$= \frac{(9{,}0 \times 10^9)(1 \times 10^{-9})(9 \times 10^{-9})}{(0{,}65)^2}$$
$$= 1{,}92 \times 10^{-7} \text{ N}$$

And then we calculate the force of Q_3 on Q_1:

$$F_e = \frac{kQ_1Q_3}{r^2}$$
$$= \frac{(9{,}0 \times 10^9)(3 \times 10^{-9})(9 \times 10^{-9})}{(0{,}4)^2}$$
$$= 1{,}52 \times 10^{-6} \text{ N}$$

The magnitude of the resultant force acting on Q_1 can be calculated from the forces using Pythagoras' theorem because there are only two forces and they act in the x- and y-directions:

$$F_R^2 = F_x^2 + F_y^2$$
$$F_R = \sqrt{(1{,}92 \times 10^{-7})^2 + (1{,}52 \times 10^{-6})^2}$$
$$= 1{,}52 \times 10^{-6} \text{ N}$$

We can find the angle using trigonometry:

$$\tan \theta_R = \frac{y\text{-component}}{x\text{-component}}$$
$$= \frac{1{,}52 \times 10^{-6}}{1{,}92 \times 10^{-7}}$$
$$= 7{,}91666\ldots$$
$$\theta_R = 82{,}80°$$

The final resultant force acting on Q_1 is $1{,}52 \times 10^{-6}$ N acting at an angle of $82{,}80°$ to the positive x-axis.

12. Calculate the resultant force on Q_2 given this charge configuration:
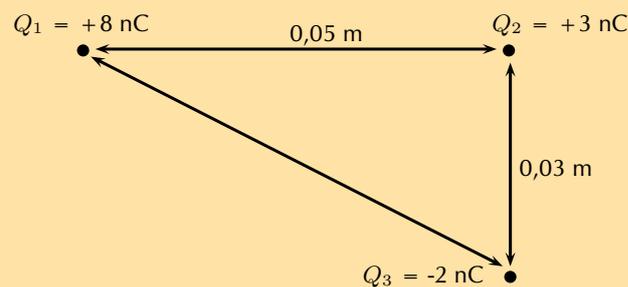

Solution:

We first calculate the force on Q_2 from Q_1:

$$F_e = \frac{kQ_1Q_2}{r^2}$$
$$= \frac{(9{,}0 \times 10^9)(8 \times 10^{-9})(3 \times 10^{-9})}{(0{,}05)^2}$$
$$= 8{,}63 \times 10^{-5} \text{ N}$$

And then we calculate the force of Q_3 on Q_2:

$$F_e = \frac{kQ_1Q_3}{r^2}$$
$$= \frac{(9{,}0 \times 10^9)(3 \times 10^{-9})(2 \times 10^{-9})}{(0{,}03)^2}$$
$$= 5{,}99 \times 10^{-5} \text{ N}$$

The magnitude of the resultant force acting on Q_2 can be calculated from the forces using Pythagoras' theorem because there are only two forces and they act in the x- and y-directions:

$$F_R^2 = F_x^2 + F_y^2$$
$$F_R = \sqrt{(8{,}63 \times 10^{-5})^2 + (5{,}99 \times 10^{-5})^2}$$
$$= 1{,}05 \times 10^{-4} \text{ N}$$

We can find the angle using trigonometry:

$$\tan \theta_R = \frac{y\text{-component}}{x\text{-component}}$$
$$= \frac{5{,}99 \times 10^{-5}}{8{,}63 \times 10^{-5}}$$
$$= 0{,}694\ldots$$
$$\theta_R = 34{,}76°$$

The final resultant force acting on Q_1 is $1{,}05 \times 10^{-4}$ N acting at an angle of $34{,}76°$ to the positive x-axis.

9.3   Electric field

Electric field strength

Exercise 9 – 2: Electric fields

1. Calculate the electric field strength 20 m from a 7 nC charge.

+7 nC

•   x

20 m
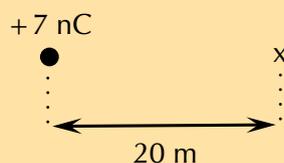
Solution:

We need to calculate the electric field a distance from a given charge. We are given the magnitude of the charge and the distance from the charge. We will use the equation: $E = k\frac{Q}{r^2}$.

$$E = \frac{kQ}{r^2}$$
$$= \frac{(9{,}0 \times 10^9)(7 \times 10^{-9})}{20^2}$$
$$= 0{,}15 \text{ N·C}^{-1}$$

2. Two charges of $Q_1 = -6$ pC and $Q_2 = -8$ pC are separated by a distance of 3 km. What is the electric field strength at a point that is 2 km from Q_1 and 1 km from Q_2? The point lies between Q_1 and Q_2.

Solution:

We need to calculate the electric field a distance from two given charges. We are given the magnitude of the charges and the distances from the charges.

We will use the equation: $E = k\frac{Q}{r^2}$.

We need to calculate the electric field for each charge separately and then add them to determine the resultant field.

We first solve for Q_1:

$$E = \frac{kQ}{r^2}$$
$$= \frac{(9{,}0 \times 10^9)(6 \times 10^{-12})}{(2 \times 10^3)^2}$$
$$= 1{,}3 \times 10^{-8} \text{ N·C}^{-1}$$

Then for Q_2:

$$E = \frac{kQ}{r^2}$$
$$= \frac{(9{,}0 \times 10^9)(8 \times 10^{-12})}{(1 \times 10^3)^2}$$
$$= 7{,}1 \times 10^{-8} \text{ N·C}^{-1}$$

We need to subtract the two electric fields because they are in opposite directions. The electric fields due to each charge will be towards the charge causing it. Therefore, $E_{\text{total}} = 1{,}3 \times 10^{-8} - 7{,}1 \times 10^{-8} = -5{,}8 \times 10^{-8}$ N·C^{-1} in the direction of the -8 pC charge.

9.4 Chapter summary

1. Two charges of $+3$ nC and -5 nC are separated by a distance of 40 cm. What is the electrostatic force between the two charges?

   Solution:

   $$
   \begin{aligned}
   F &= \frac{kQ_1Q_2}{r^2} \\
   &= \frac{(9{,}0 \times 10^9)(3 \times 10^{-9})(5 \times 10^{-9})}{(40 \times 10^{-2})^2} \\
   &= 8{,}44 \times 10^{-7} \text{ N·C}^{-1}
   \end{aligned}
   $$

2. Two conducting metal spheres carrying charges of $+6$ nC and -10 nC are separated by a distance of 20 mm.

   a) What is the electrostatic force between the spheres?

   b) The two spheres are touched and then separated by a distance of 60 mm. What are the new charges on the spheres?

   c) What is new electrostatic force between the spheres at this distance?

   Solution:

   a)

   $$
   \begin{aligned}
   F &= \frac{kQ_1Q_2}{r^2} \\
   &= \frac{(9{,}0 \times 10^9)(6 \times 10^{-9})(10 \times 10^{-9})}{(20 \times 10^{-3})^2} \\
   &= 1{,}35 \times 10^{-3} \text{ N·C}^{-1} \text{ towards the other sphere.}
   \end{aligned}
   $$

   b) The spheres are conducting and so when they touch the charge is spread equally across the two spheres. So the new charge on each sphere is:

   $$
   \frac{Q_1 + Q_2}{2} = \frac{6 + (-10)}{2} = -2 \text{ nC}
   $$

   c)

   $$
   \begin{aligned}
   F &= \frac{kQ_1Q_2}{r^2} \\
   &= \frac{(9{,}0 \times 10^9)(2 \times 10^{-9})(2 \times 10^{-9})}{(60 \times 10^{-3})^2} \\
   &= 1{,}00 \times 10^{-5} \text{ N·C}^{-1} \text{ towards the other sphere.}
   \end{aligned}
   $$

3. The electrostatic force between two charged spheres of $+3$ nC and $+4$ nC respectively is $0{,}4$ N. What is the distance between the spheres?

Solution:

$$F_e = \frac{kQ_1Q_2}{r^2}$$

$$0{,}4 = \frac{(9{,}0 \times 10^9)(4 \times 10^{-9})(3 \times 10^{-9})}{(d)^2}$$

$$0{,}4d^2 = 1{,}07 \times 10^{-9}$$

$$d^2 = 1{,}07 \times 10^{-7}$$

$$= 3{,}3 \times 10^{-4} \text{ m}$$

4. Draw the electric field pattern lines between:

   a) two equal positive point charges.

   b) two equal negative point charges.

   Solution:

a)

b)
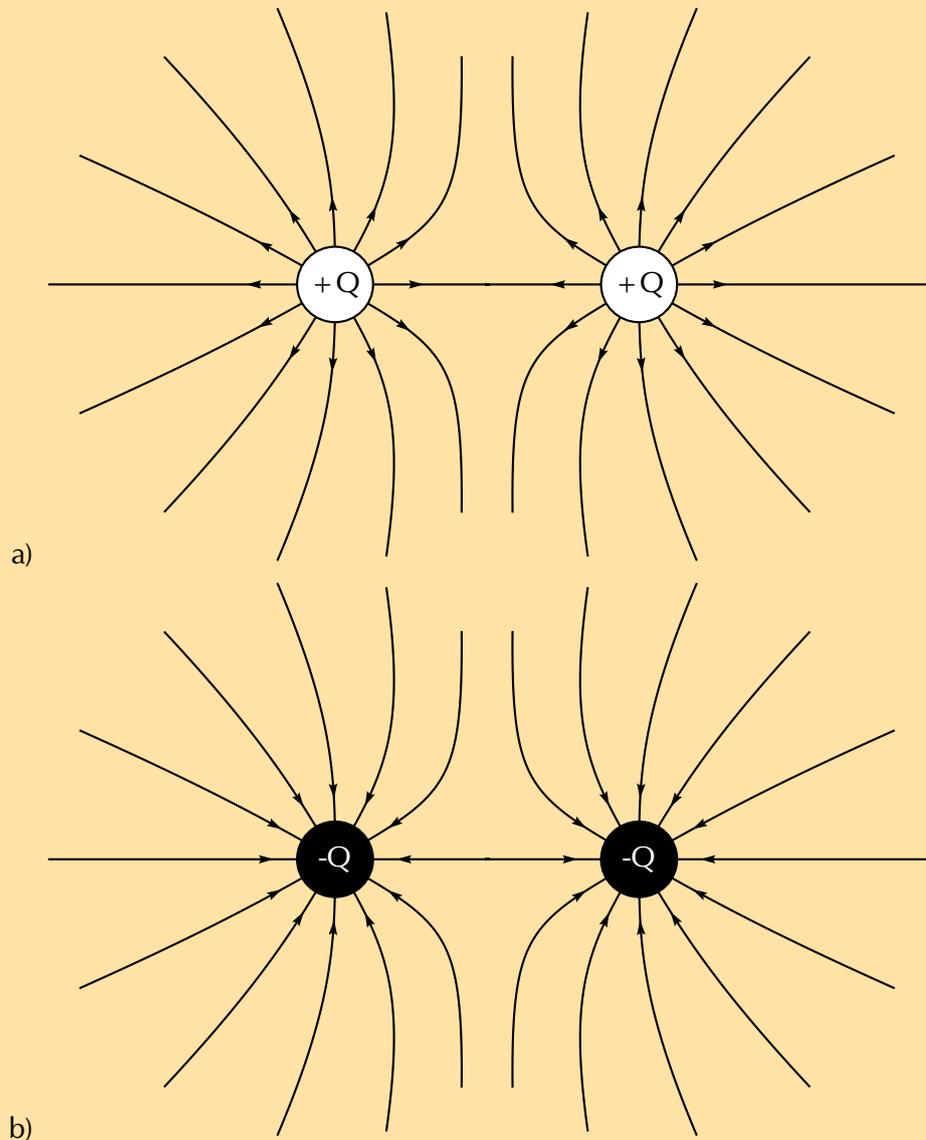
5. Two small identical metal spheres, on insulated stands, carry charges $-q$ and $+3q$ respectively. When the centres of the spheres are separated by a distance d the one exerts an electrostatic force of magnitude F on the other.
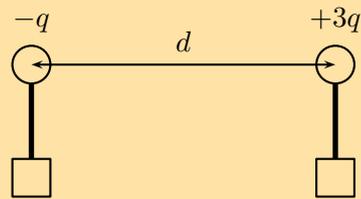
The spheres are now made to touch each other and are then brought back to the same distance d apart. What will be the magnitude of the electrostatic force which one sphere now exerts on the other?

   a) $\frac{1}{4}F$
   b) $\frac{1}{3}F$
   c) $\frac{1}{2}F$
   d) $3F$

[SC 2003/11]

Solution:

$\frac{1}{3}F$

6. Three point charges of magnitude $+1$ C, $+1$ C and -1 C respectively are placed on the three corners of an equilateral triangle as shown.
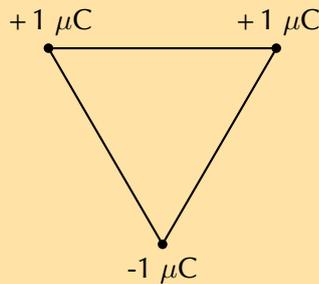
Which vector best represents the direction of the resultant force acting on the -1 C charge as a result of the forces exerted by the other two charges?
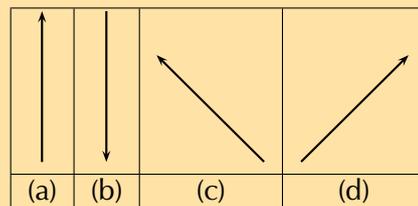
[SC 2003/11]

Solution:

a

7.   a) Write a statement of Coulomb's law.

   b) Calculate the magnitude of the force exerted by a point charge of $+2$ nC on another point charge of -3 nC separated by a distance of 60 mm.

   c) Sketch the electric field between two point charges of $+2$ nC and -3 nC, respectively, placed 60 mm apart from each other.

[IEB 2003/11 HG1 - Force Fields]

Solution:

a) The magnitude of the electrostatic force between two point charges is directly proportional to the product of the magnitudes of the charges and inversely proportional to the square of the distance between them.

b)

$$F = \frac{kQ_1Q_2}{r^2}$$
$$= \frac{(9{,}0 \times 10^9)(2 \times 10^{-9})(3 \times 10^{-9})}{(60 \times 10^{-3})^2}$$
$$= 1{,}5 \times 10^{-5} \text{ N·C}^{-1}$$
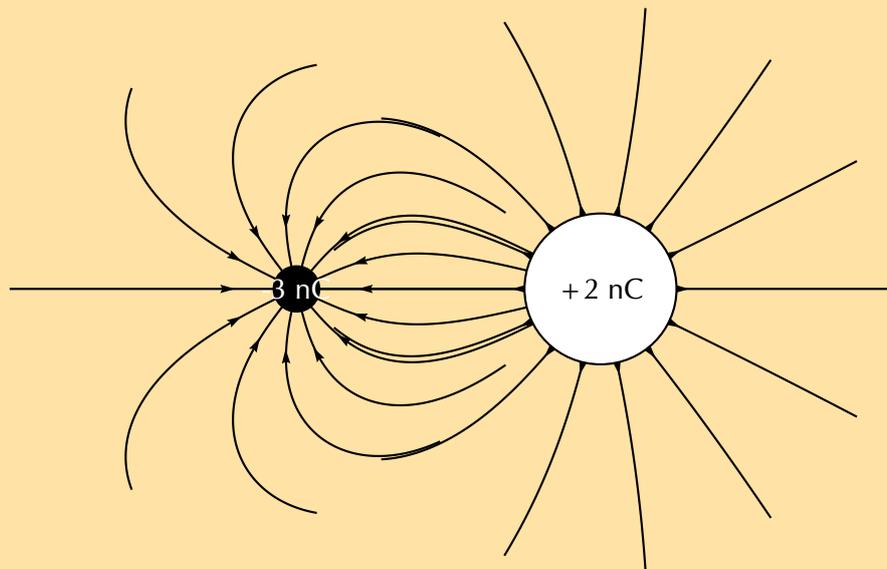
c)

8. The electric field strength at a distance x from a point charge is E. What is the magnitude of the electric field strength at a distance $2x$ away from the point charge?

   a) $\frac{1}{4}E$

   b) $\frac{1}{2}E$

   c) $2E$

   d) $4E$

[SC 2002/03 HG1]

Solution:

$2E$

CHAPTER 10

Electromagnetism

10.1   Introduction

This chapter takes the ideas of magnetism and the ideas of electricity and combines them into one. Learners should know about magnetism from grade 10. The following list summarises the topics covered in this chapter.

- **Magnetic fields are associated with current carrying wires**

  The idea that magnetism and electricity are linked is introduced. The concept of a magnetic field around a current carrying wire is explored and a simple method to determine the direction of the magnetic field is explained.

- **Faraday's law**

  Faraday's law relates the emf produced to the magnetic flux around a loop of conductor. The concept of electromagnetic induction is introduced and some simple calculations using Faraday's law are shown.

10.2   Magnetic field associated with a current

Exercise 10 – 1: Magnetic Fields

1. Give evidence for the existence of a magnetic field near a current carrying wire.

   Solution:

   If you hold a compass near a wire through which current is flowing, the needle on the compass will be deflected. Since compasses work by pointing along magnetic field lines, this means that there must be a magnetic field near the wire through which the current is flowing. If the current stops flowing the compass returns to its original direction. If the current starts to flow again then the deflection happens again.

2. Describe how you would use your right hand to determine the direction of a magnetic field around a current carrying conductor.

   Solution:

   We use the right hand rule which says that the magnetic field lines produced by a current-carrying wire will be oriented in the same direction as the curled fingers of a person's right hand (in the "hitchhiking" position), with the thumb pointing in the direction of the current flow:
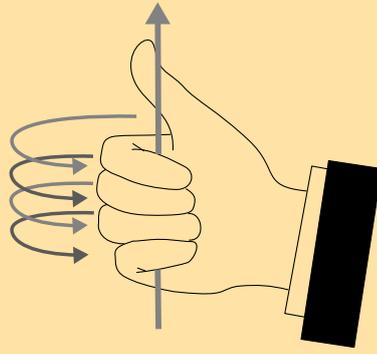
3. Use the Right Hand Rule to determine the direction of the magnetic field for the following situations:

current flow

a)

current flow

b)
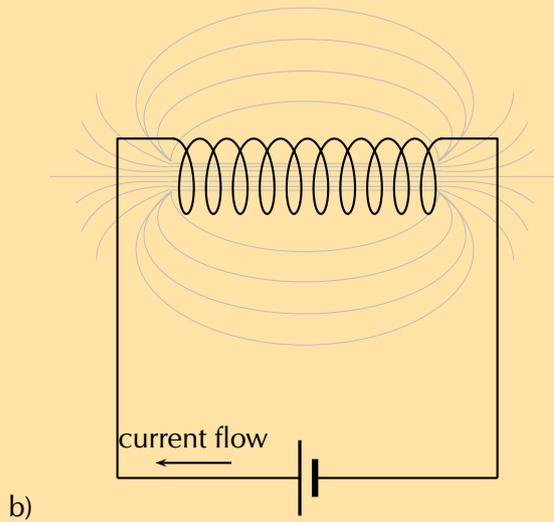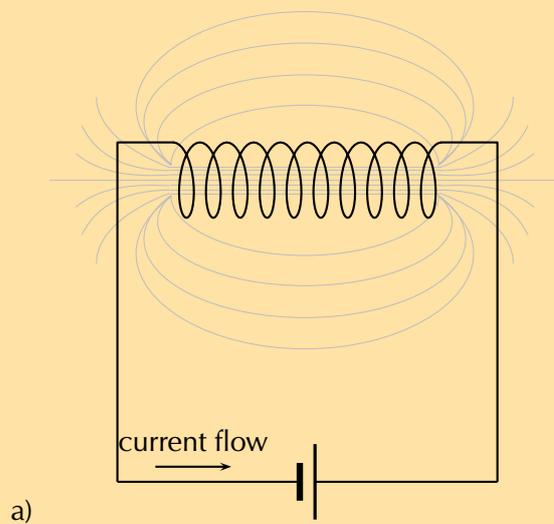
Solution:

a) Out of the page

b) into the page

4. Use the Right Hand Rule to find the direction of the magnetic fields at each of the points labelled A - H in the following diagrams.
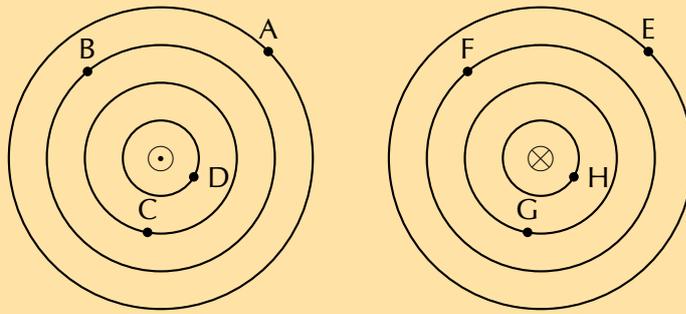
Solution:

- A: counterclockwise
- B: counterclockwise
- C: counterclockwise
- D: counterclockwise
- E: clockwise
- F: clockwise
- G: clockwise
- H: clockwise

10.3   Faraday's law of electromagnetic induction

Direction of induced current in a solenoid

Exercise 10 – 2: Faraday's Law

1. State Faraday's Law of electromagnetic induction in words and write down a mathematical relationship.

   Solution:

   The emf, $\mathcal{E}$, produced around a loop of conductor is proportional to the rate of change of the magnetic flux, ϕ, through the area, A, of the loop. This can be stated mathematically as:

   $$\mathcal{E} = -N\frac{\Delta\phi}{\Delta t}$$

   where $\phi = B \cdot A$ and B is the strength of the magnetic field. N is the number of circuit loops. A magnetic field is measured in units of teslas (T). The minus sign indicates direction and that the induced emf tends to oppose the change in the magnetic flux. The minus sign can be ignored when calculating magnitudes.

2. Describe what happens when a bar magnet is pushed into or pulled out of a solenoid connected to an ammeter. Draw pictures to support your description.

Solution:

In the case where a north pole is brought towards the solenoid the current will flow so that a north pole is established at the end of the solenoid closest to the approaching magnet to repel it (verify using the Right Hand Rule):
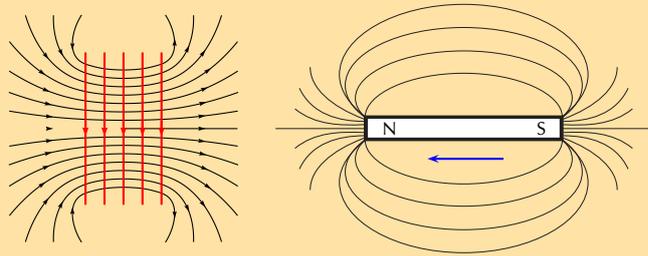
In the case where a north pole is moving away from the solenoid the current will flow so that a south pole is established at the end of the solenoid closest to the receding magnet to attract it:
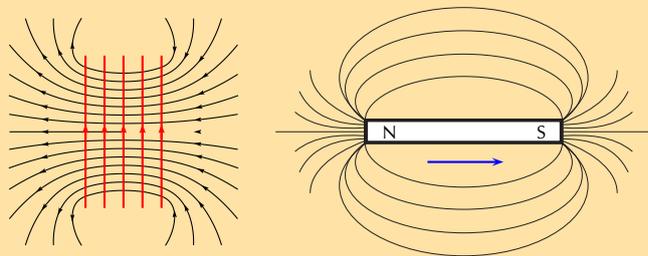
In the case where a south pole is moving away from the solenoid the current will flow so that a north pole is established at the end of the solenoid closest to the receding magnet to attract it:
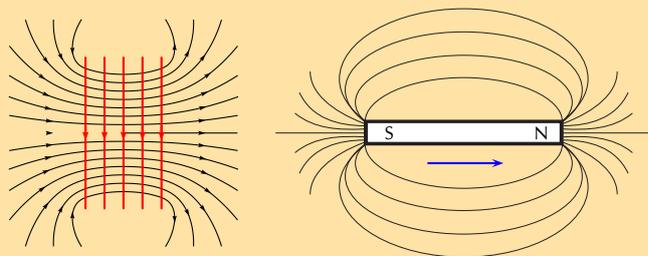
In the case where a south pole is brought towards the solenoid the current will flow so that a south pole is established at the end of the solenoid closest to the approaching magnet to repel it:
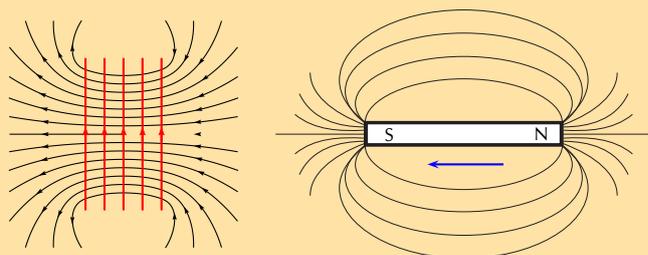
3. Explain how it is possible for the magnetic flux to be zero when the magnetic field is not zero.

Solution:

The flux is related to the magnetic field:

$$\phi = BA\cos\theta$$

If $\cos\theta$ is 0, then the magnetic flux will be 0 even if there is a magnetic field. In this case the magnetic field is parallel to the surface and does not pass through it.

4. Use the Right Hand Rule to determine the direction of the induced current in the solenoid below.

coil with N turns and cross-sectional area, A
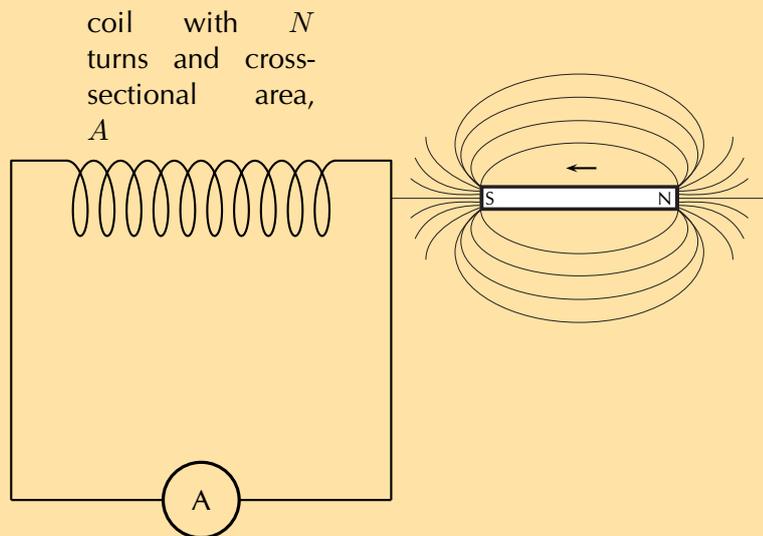
Solution:

A south pole of a magnet is approaching the solenoid. Lenz's law tells us that the current will flow so as to oppose the change. A south pole at the end of the solenoid would oppose the approaching south pole. The current will circulate into the page at the top of the coil so that the thumb on a right hand points to the left.

5. Consider a circular coil of 5 turns with radius 1,73 m. The coil is subjected to a varying magnetic field that changes uniformly from 2,18 T to 12,7 T in an interval of 3 minutes. The axis of the solenoid makes an angle of $27°$ to the magnetic field. Find the induced emf.

Solution:

We know that the magnetic field is at an angle to the surface normal. This means we must account for the angle that the field makes with the normal and $\phi = BA\cos(\theta)$. The starting or initial magnetic field, B_i, is given as is the final field magnitude, B_f. We want to determine the magnitude of the emf so we can ignore the minus sign. The area, A, will be πr^2 .

$$\mathcal{E} = N\frac{\Delta\phi}{\Delta t}$$

$$= N\frac{\phi_f - \phi_i}{\Delta t}$$

$$= N\frac{B_f A \cos\theta - B_i A \cos\theta}{\Delta t}$$

$$= N\frac{A \cos\theta (B_f - B_i)}{\Delta t}$$

$$= 5\left(\frac{\pi(1.73)^2 \cos(27)(12{,}7 - 2{,}18)}{3 \times 60}\right)$$

$$= 2{,}45 \text{ V}$$

6. Consider a solenoid coil of 11 turns with radius $13{,}8 \times 10^{-2}$ m. The solenoid is subjected to a varying magnetic field that changes uniformly from $5{,}34$ T to $2{,}7$ T in an interval of 12 s. The axis of the solenoid makes an angle of $13°$ to the magnetic field.

   a) Find the induced emf.

   b) If the angle is changed to $67{,}4°$, what would the radius need to be for the emf to remain the same?

   Solution:

   a)

$$\mathcal{E} = N\frac{\Delta\phi}{\Delta t}$$

$$= N\frac{\phi_f - \phi_i}{\Delta t}$$

$$= N\frac{B_f A \cos\theta - B_i A \cos\theta}{\Delta t}$$

$$= N\frac{A \cos\theta (B_f - B_i)}{\Delta t}$$

$$= 11\left(\frac{\pi(13{,}8 \times 10^{-2})^2 \cos(13)(2{,}7 - 5{,}34)}{12}\right)$$

$$= -0{,}14 \text{ V}$$

   b)

$$\mathcal{E} = N\frac{\Delta\phi}{\Delta t}$$

$$= N\frac{\phi_f - \phi_i}{\Delta t}$$

$$= N\frac{B_f A \cos\theta - B_i A \cos\theta}{\Delta t}$$

$$= N\frac{A \cos\theta (B_f - B_i)}{\Delta t}$$

$$-0{,}14 = 11\left(\frac{\pi(r)^2 \cos(67{,}4)(2{,}7 - 5{,}34)}{12}\right)$$

$$-1{,}68 = -35{,}06(r)^2$$

$$r^2 = 0{,}0479$$

$$r = 0{,}22 \text{ m}$$

7. Consider a solenoid with 5 turns and a radius of 11×10^{-2} m. The axis of the solenoid makes an angle of $23°$ to the magnetic field.

   a) Find the change in flux if the emf is 12 V over a period of 12 s.

   b) If the angle is changed to $45°$, what would the time interval need to change to for the induced emf to remain the same?

   Solution:

   a)

   $$\mathcal{E} = N\frac{\Delta\phi}{\Delta t}$$

   $$12 = 5\left(\frac{\Delta\phi}{12}\right)$$

   $$\Delta\phi = 28{,}8 \text{ Wb}$$

   b)

   $$\mathcal{E} = N\frac{\Delta\phi}{\Delta t}$$

   $$= N\frac{\phi_f - \phi_i}{\Delta t}$$

   $$= N\frac{B_f A \cos\theta - B_i A \cos\theta}{\Delta t}$$

   $$= \cos\theta \times N\frac{B_f A - B_i A}{\Delta t}$$

   All the values remain the same between the two situations described except for the angle and the time. We can equate the equations for the two scenarios:

   $$\mathcal{E}_1 = \mathcal{E}_2$$

   $$\cos\theta_1 \times N\frac{B_f A - B_i A}{\Delta t_1} = \cos\theta_2 \times N\frac{B_f A - B_i A}{\Delta t_2}$$

   $$\cos\theta_1 \frac{1}{\Delta t_1} = \cos\theta_2 \frac{1}{\Delta t_2}$$

   $$\Delta t_2 = \frac{\Delta t_1 \cos\theta_2}{\cos\theta_1}$$

   $$\Delta t_2 = \frac{(12\cos(45)}{\cos(23)}$$

   $$\Delta t_2 = 9{,}22 \text{ s}$$

10.4   Chapter summary

1. What did Hans Oersted discover about the relationship between electricity and

magnetism?

Solution:

He discovered that electricity and magnetism were related to one another. By passing an electric current through a metal wire suspended above a magnetic compass, Oersted was able to produce a definite motion of the compass needle in response to the current.

2. List two uses of electromagnetism.

Solution:

cellular telephones, microwave ovens, radios, televisions, etc.

3. a) A uniform magnetic field of $0{,}35$ T in the vertical direction exists. A piece of cardboard, of surface area $0{,}35$ m^2 is placed flat on a horizontal surface inside the field. What is the magnetic flux through the cardboard?

   b) The one edge is then lifted so that the cardboard is now inclined at $17°$ to the positive x-direction. What is the magnetic flux through the cardboard? What will the induced emf be if the field drops to zero in the space of 3 s? Why?

   Solution:

   a)

$$\phi = BA\cos\theta$$
$$= (0{,}35)(0{,}35)$$
$$= 0{,}1225 \text{ Wb}$$

   b) When the piece of cardboard is inclined the flux is:

$$\phi = BA\cos\theta$$
$$= (0{,}35)(0{,}35)\cos(17)$$
$$= 0{,}117$$

   The change in flux is $0{,}117$ Wb and the induced emf will be zero because the cardboard is not a conductor.

4. A uniform magnetic field of 5 T in the vertical direction exists. What is the magnetic flux through a horizontal surface of area $0{,}68$ m^2? What is the flux if the magnetic field changes to being in the positive x-direction?

   Solution:

   For the first case:

$$\phi = BA\cos\theta$$
$$= (5)(0{,}68)$$
$$= 3{,}4 \text{ Wb}$$

   If the flux is in the x-direction the flux will be 0.

   $3{,}4$ Wb and 0 Wb

5. A uniform magnetic field of 5 T in the vertical direction exists. What is the magnetic flux through a horizontal circle of radius 0,68 m?

   Solution:

$$\phi = BA\cos\theta$$
$$= (5)\pi(0{,}68)^2$$
$$= 7{,}26 \text{ Wb}$$

6. Consider a square coil of 3 turns with a side length of 1,56 m. The coil is subjected to a varying magnetic field that changes uniformly from 4,38 T to 0,35 T in an interval of 3 minutes. The axis of the solenoid makes an angle of 197° to the magnetic field. Find the induced emf.

   Solution:

$$\mathcal{E} = N\frac{\Delta\phi}{\Delta t}$$
$$= N\frac{\phi_f - \phi_i}{\Delta t}$$
$$= N\frac{B_f A\cos\theta - B_i A\cos\theta}{\Delta t}$$
$$= N\frac{A\cos\theta(B_f - B_i)}{\Delta t}$$
$$= 3\left(\frac{(1{,}56)^2\cos(197)(0{,}35 - 4{,}38)}{3}\right)$$
$$= 9{,}34 \text{ V}$$

7. Consider a solenoid coil of 13 turns with radius $6{,}8 \times 10^{-2}$ m. The solenoid is subjected to a varying magnetic field that changes uniformly from -5 T to 1,8 T in an interval of 18 s. The axis of the solenoid makes an angle of 88° to the magnetic field.

   a) Find the induced emf.

   b) If the angle is changed to 39°, what would the radius need to be for the emf to remain the same?

   Solution:

   a)

$$\mathcal{E} = N\frac{\Delta\phi}{\Delta t}$$
$$= N\frac{\phi_f - \phi_i}{\Delta t}$$
$$= N\frac{B_f A\cos\theta - B_i A\cos\theta}{\Delta t}$$
$$= N\frac{A\cos\theta(B_f - B_i)}{\Delta t}$$
$$= 13\left(\frac{\pi(6{,}8 \times 10^{-2})^2\cos(88)(1{,}8 - (-5))}{18}\right)$$
$$= 0{,}00249 \text{ V}$$

b)

$$\mathcal{E} = N\frac{\Delta\phi}{\Delta t}$$
$$= N\frac{\phi_f - \phi_i}{\Delta t}$$
$$= N\frac{B_f A\cos\theta - B_i A\cos\theta}{\Delta t}$$
$$= N\frac{A\cos\theta(B_f - B_i)}{\Delta t}$$
$$0,00249 = 13\left(\frac{\pi(r)^2\cos(39)(1,8 - (-5))}{18}\right)$$
$$0,04482 = 215,83(r)^2$$
$$r^2 = 0,00002077$$
$$r = 0,014 \text{ m}$$

8. Consider a solenoid with 5 turns and a radius of $4,3 \times 10^{-1}$ mm. The axis of the solenoid makes an angle of $11°$ to the magnetic field.

   Find the change in flux if the emf is $0,12$ V over a period of $0,5$ s.

   Solution:

$$\mathcal{E} = N\frac{\Delta\phi}{\Delta t}$$
$$0,12 = 5\frac{\Delta\phi}{0,5}$$
$$\Delta\phi = 0,012 \text{ Wb}$$

9. Consider a rectangular coil of area $1,73$ m^2. The coil is subjected to a varying magnetic field that changes uniformly from 2 T to 10 T in an interval of 3 ms. The axis of the solenoid makes an angle of $55°$ to the magnetic field. Find the induced emf.

   Solution:

$$\mathcal{E} = N\frac{\Delta\phi}{\Delta t}$$
$$= N\frac{\phi_f - \phi_i}{\Delta t}$$
$$= N\frac{B_f A\cos\theta - B_i A\cos\theta}{\Delta t}$$
$$= N\frac{A\cos\theta(B_f - B_i)}{\Delta t}$$
$$= N\left(\frac{(1,73)\cos(55)(10 - 2)}{3}\right)$$
$$= 2,65 \text{ V}$$

CHAPTER

11

Electric circuits

In grade 10 learners learnt about current, voltage and resistance. In this chapter they will learn about Ohm's law and power and energy. They will see how to apply the concepts learnt in grade 10 and series and parallel circuits to more complex circuit problems. The following list provides a summary of the concepts covered in this chapter.

- **Ohm's law**

  In grade 10 learners learnt about parallel and series circuits, as well as the concepts of voltage, current and resistance. These concepts are now brought together in Ohm's law. Ohm's law relates voltage, current and resistance. Ohm's law only applies to Ohmic resistors (such as most resistors) and does not apply to non Ohmic resistors (such as light bulbs). Ohm's law is introduced and then learners get to see it in action in various circuits.

  The first circuits covered are simple series or simple parallel circuits. Once learners are comfortable handling calculations for these circuits, series and parallel networks are introduced. In these circuits learners need to carefully work their way through the circuit calculating the equivalent resistances for each separate part of the circuit. It is advised for learners to circle the different parts of the circuit to help them in calculations. In the first example that they do they can redraw the circuit after each simplification replacing the part of the circuit with a resistor of the calculated equivalent resistance as the part of the circuit.

- **Electrical power**

  The concept of electrical power is introduced. A source of energy is required to drive current round a complete circuit. This is provided by batteries in the circuits you have been looking at. The batteries convert chemical potential energy into electrical energy. The energy is used to do work on the electrons in the circuit.

  Power is a measure of how rapidly *work* is done. Power is the **rate** at which the work is done, work done per unit time. Work is measured in joules (J) and time in seconds (s) so power will be $\frac{J}{s}$ which we call a watt (W).

- **Electrical energy**

  The final part of this chapter deals with electrical energy. This has real world applications in teaching learners about how much electricity various appliances use around their home. Learners should be encouraged to make a list of as many appliances as possible that they use and find the power rating for the appliance. This is usually given on the back of the appliances. This part of the chapter helps learners understand some of the power saving tips that they are always told and helps to rationalise these tips. For example, learners can calculate how much it costs to leave a 100 W bulb burning all night compared to leaving it on for just an hour.

A recommended experiment for informal assessment is included. In this experiment learners will obtain current and voltage data for a resistor and light bulb and determine which obeys Ohm's law. You will need light bulbs, resistors, connecting wires, power source, ammeter and voltmeter. Learners should find that the resistor obeys Ohm's law, while the light bulb does not.

Exercise 11 – 1: Ohm's Law

1. Use the data in the table below to answer the following questions.

Voltage, V (V)	Current, I (A)
3,0	0,4
6,0	0,8
9,0	1,2
12,0	1,6

a) Plot a graph of voltage (on the x-axis) and current (on the y-axis).

b) What type of graph do you obtain (straight-line, parabola, other curve)

c) Calculate the gradient of the graph.

d) Do your experimental results verify Ohm's Law? Explain.

e) How would you go about finding the resistance of an unknown resistor using only a power supply, a voltmeter and a known resistor R_0?

Solution:

a)
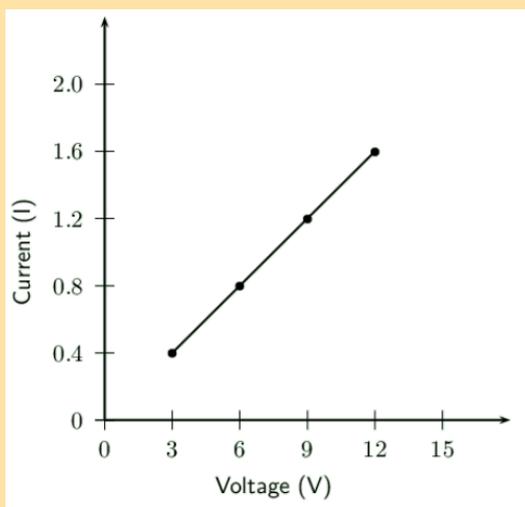
b) straight-line

c) The gradient of the graph (m) is the change in the current divided by the change in the voltage:

$$m = \frac{\Delta I}{\Delta V}$$
$$= \frac{(1,6) - (0,4)}{(12) - (3)}$$
$$= 0,13$$

d) Yes. A straight line graph is obtained when we plot a graph of voltage vs. current.

e) You start by connecting the known resistor in a circuit with the power supply. Now you read the voltage of the power supply and note this down.

Next you connect the two resistors in series. You can now take the voltage measurements for each of resistors.

So we can find the voltages for the two resistors. Now we note that:

$$V = IR$$

So using this and the fact that for resistors in series, the current is the same everywhere in the circuit we can find the unknown resistance.

$$V_0 = IR_0$$
$$I = \frac{V_0}{R_0}$$
$$V_U = IR_U$$
$$I = \frac{V_U}{R_U}$$
$$\frac{V_U}{R_U} = \frac{V_0}{R_0}$$
$$\therefore R_U = \frac{V_U R_0}{V_0}$$

Ohmic and non-ohmic conductors

Using Ohm's Law

Exercise 11 – 2: Ohm's Law

1. Calculate the resistance of a resistor that has a potential difference of 8 V across it when a current of 2 A flows through it. Draw the circuit diagram before doing the calculation.

Solution:

The resistance of the unknown resistor is:

$$R = \frac{V}{I}$$
$$= \frac{8}{2}$$
$$= 4 \ \Omega$$
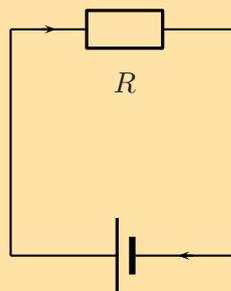
2. What current will flow through a resistor of 6 Ω when there is a potential difference of 18 V across its ends? Draw the circuit diagram before doing the calculation.

   Solution:

   The resistance of the unknown resistor is:

$$I = \frac{V}{R}$$
$$= \frac{18}{6}$$
$$= 3 \ \text{A}$$
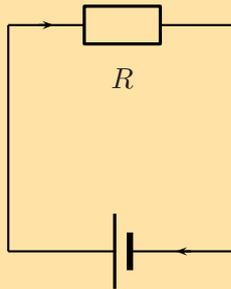
3. What is the voltage across a 10 Ω resistor when a current of 1,5 A flows though it? Draw the circuit diagram before doing the calculation.

   Solution:

   The resistance of the unknown resistor is:

$$V = I \cdot R$$
$$= (1{,}5)(10)$$
$$= 15 \ \text{V}$$

11.2. Ohm's Law

1. Two 10 kΩ resistors are connected in series. Calculate the equivalent resistance.

   Solution:

   Since the resistors are in series we can use:

   $$R_s = R_1 + R_2$$

   The equivalent resistance is:

   $$\begin{aligned}
   R_s &= R_1 + R_2 \\
   &= 10 \text{ k}\Omega + 10 \text{ k}\Omega \\
   &= 20 \text{ k}\Omega
   \end{aligned}$$

2. Two resistors are connected in series. The equivalent resistance is 100 Ω. If one resistor is 10 Ω, calculate the value of the second resistor.

   Solution:

   Since the resistors are in series we can use:

   $$R_s = R_1 + R_2$$

   The equivalent resistance is:

   $$\begin{aligned}
   R_s &= R_1 + R_2 \\
   R_2 &= R_s - R_1 \\
   &= 100 \ \Omega - 10 \ \Omega \\
   &= 90 \ \Omega
   \end{aligned}$$

3. Two 10 kΩ resistors are connected in parallel. Calculate the equivalent resistance.

   Solution:

   Since the resistors are in parallel we can use:

   $$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$

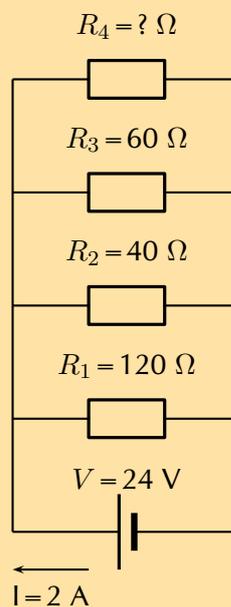   The equivalent resistance is:

   $$\begin{aligned}
   \frac{1}{R_p} &= \frac{1}{R_1} + \frac{1}{R_2} \\
   &= \frac{1}{100} + \frac{1}{10} \\
   &= \frac{1 + 10}{100} \\
   &= \frac{11}{100} \\
   R_p &= 9{,}09 \text{ k}\Omega
   \end{aligned}$$

4. Two resistors are connected in parallel. The equivalent resistance is $3{,}75$ Ω. If one resistor has a resistance of 10 Ω, what is the resistance of the second resistor?

Solution:

Since the resistors are in parallel we can use:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$

The equivalent resistance is:

$$
\begin{aligned}
\frac{1}{R_p} &= \frac{1}{R_1} + \frac{1}{R_2} \\
\frac{1}{R_2} &= \frac{1}{R_p} - \frac{1}{R_1} \\
&= \frac{1}{3{,}75} - \frac{1}{10} \\
&= \frac{10 - 3{,}75}{37{,}5} \\
&= \frac{6{,}25}{37{,}5} \\
R_2 &= 6 \ \Omega
\end{aligned}
$$

5. Calculate the equivalent resistance in each of the following circuits:

a)

$R_1 = 3\Omega$

$R_2 = 2\Omega$

b)

$R_1 = 2\Omega$

$R_2 = 3\Omega$

$R_3 = 4\Omega$

$R_4 = 1\Omega$

c)

$R_2 = 2\Omega$

$R_1 = 3\Omega$

d)

$R_1 = 2\Omega$

$R_4 = 1\Omega$

$R_2 = 3\Omega$ $R_3 = 4\Omega$
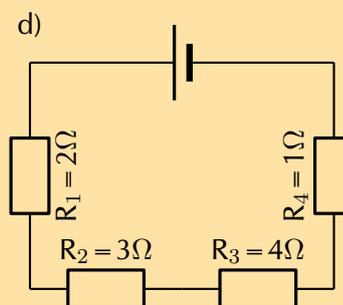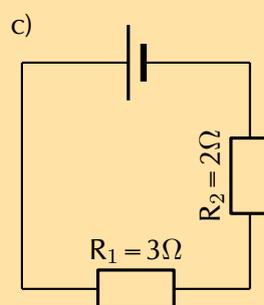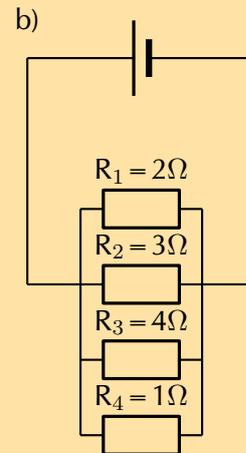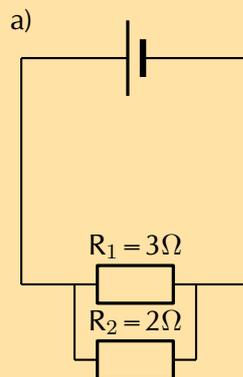
Solution:

a) The resistors are in parallel and so we use:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$

The equivalent resistance is:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$
$$= \frac{1}{3} + \frac{1}{2}$$
$$= \frac{2+3}{6}$$
$$= \frac{5}{6}$$
$$R = 1{,}2 \ \Omega$$

b) The resistors are in parallel and so we use:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

The equivalent resistance is:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$
$$= \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{1}$$
$$= \frac{6+4+3+12}{12}$$
$$= \frac{25}{12}$$
$$R = 0{,}48 \ \Omega$$

c) The resistors are in series and so we use:

$$R_s = R_1 + R_2$$

The equivalent resistance is:

$$R_s = R_1 + R_2$$
$$= 2 \ \Omega + 3 \ \Omega$$
$$= 5 \ \Omega$$

d) The resistors are in series and so we use:

$$R_s = R_1 + R_2 + R_3 + R_4$$

The equivalent resistance is:

$$R_s = R_1 + R_2 + R_3 + R_4$$
$$= 2 \ \Omega + 3 \ \Omega + 4 \ \Omega + 1 \ \Omega$$
$$= 10 \ \Omega$$

Exercise 11 – 4: Ohm's Law in series and parallel circuits

1. Calculate the value of the unknown resistor in the circuit:
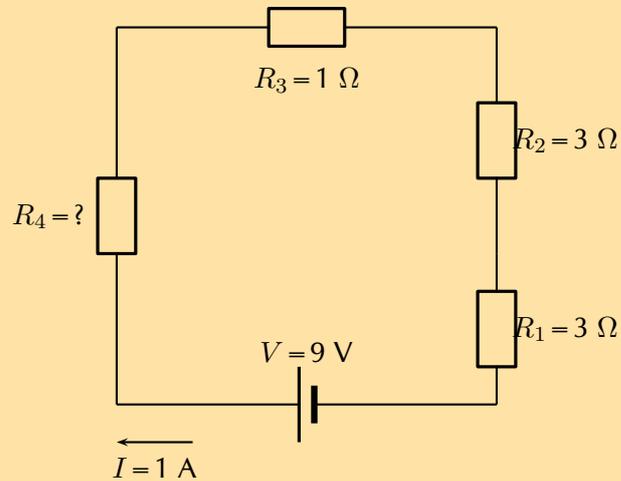
Solution:

We first use Ohm's law to calculate the total series resistance:

$$R = \frac{V}{I}$$
$$= \frac{9}{1}$$
$$= 9 \ \Omega$$

Now we can find the unknown resistance:

$$R_s = R_1 + R_2 + R_3 + R_4$$
$$R_4 = R_s - R_1 - R_2 - R_3$$
$$= 9 - 3 - 3 - 1$$
$$= 2 \ \Omega$$

2. Calculate the value of the current in the following circuit:

Solution:

We first find the total resistance:

$$R_s = R_1 + R_2 + R_3$$
$$= 1 + 2{,}5 + 1{,}5$$
$$= 5 \; \Omega$$

Now we can calculate the current:

$$I = \frac{V}{R}$$
$$= \frac{9}{5}$$
$$= 1{,}8 \text{ A}$$

3. Three resistors with resistance 1 Ω, 5 Ω and 10 Ω respectively, are connected in series with a 12 V battery. Calculate the value of the current in the circuit.

   Solution:

   We draw the circuit diagram:

   We now find the total resistance:

$$R_s = R_1 + R_2 + R_3$$
$$= 1 + 5 + 10$$
$$= 16 \; \Omega$$

   Now we can calculate the current:

$$I = \frac{V}{R}$$
$$= \frac{12}{16}$$
$$= 0{,}75 \text{ A}$$

4. Calculate the current through the cell if the resistors are both ohmic in nature.
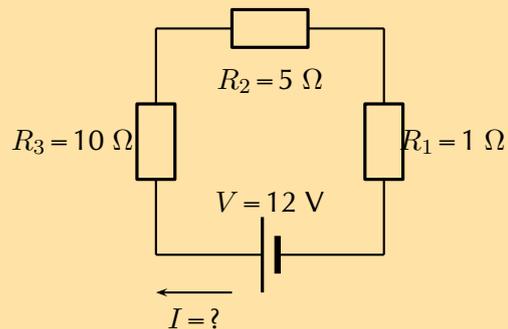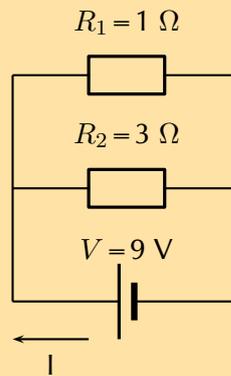
Solution:

We first find the total resistance:

$$
\begin{aligned}
\frac{1}{R_p} &= \frac{1}{R_1} + \frac{1}{R_2} \\
&= \frac{1}{1} + \frac{1}{3} \\
&= \frac{3+1}{3} \\
&= \frac{4}{3} \\
&= 0{,}75 \ \Omega
\end{aligned}
$$

Now we can calculate the current:

$$
\begin{aligned}
I &= \frac{V}{R} \\
&= \frac{9}{0{,}75} \\
&= 12 \ \text{A}
\end{aligned}
$$

5. Calculate the value of the unknown resistor R_4 in the circuit:

Solution:

We first find the total resistance:

$$R = \frac{V}{I}$$
$$= \frac{24}{2}$$
$$= 12 \ \Omega$$

Now we can calculate the unknown resistance:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$
$$\frac{1}{R_4} = \frac{1}{R_p} - \frac{1}{R_1} - \frac{1}{R_2} - \frac{1}{R_3}$$
$$= \frac{1}{12} - \frac{1}{120} - \frac{1}{40} - \frac{1}{60}$$
$$= \frac{10 - 1 - 3 - 2}{120}$$
$$= \frac{4}{120}$$
$$= 30 \ \Omega$$

6. Three resistors with resistance 1 Ω, 5 Ω and 10 Ω respectively, are connected in parallel with a 20 V battery. All the resistors are ohmic in nature. Calculate:

    a) the value of the current through the battery

    b) the value of the current in each of the three resistors.

Solution:

    a) We draw a circuit diagram:

$R_3 = 10 \ \Omega$

$R_2 = 5 \ \Omega$

$R_1 = 1 \ \Omega$

$V = 20$ V

$I = 2$ A
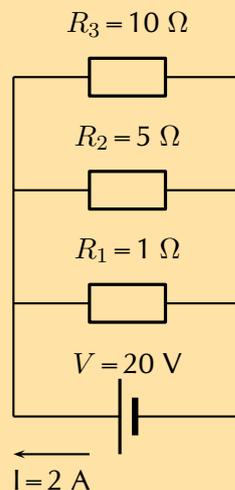
    To calculate the value of the current through the battery we first need to calculate the equivalent resistance:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$= \frac{1}{1} + \frac{1}{5} + \frac{1}{10}$$

$$= \frac{10 + 2 + 1}{10}$$

$$= \frac{13}{10}$$

$$= 0{,}77 \ \Omega$$

Now we can calculate the current through the battery:

$$I = \frac{V}{R}$$

$$= \frac{20}{0{,}77}$$

$$= 26 \ \text{A}$$

b) For a parallel circuit the voltage across cell is the same as the voltage across each of the resistors. For this circuit:

$$V = V_1 = V_2 = V_3 = 20 \ \text{V}$$

Now we can calculate the current through each resistor. We will start with R_1:

$$I = \frac{V}{R}$$

$$= \frac{20}{1}$$

$$= 20 \ \text{A}$$

Next we calculate the current through R_2:

$$I = \frac{V}{R}$$

$$= \frac{20}{5}$$

$$= 4 \ \text{A}$$

And finally we calculate the current through R_3:

$$I = \frac{V}{R}$$

$$= \frac{20}{10}$$

$$= 2 \ \text{A}$$

You can check that these add up to the total current.

1. Determine the equivalent resistance of the following circuits:

a)

b)

c)

Solution:

a) We start by determining the equivalent resistance of the parallel combination:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$
$$= \frac{1}{4} + \frac{1}{2}$$
$$= \frac{3}{4}$$
$$R_p = 1{,}33 \ \Omega$$

Now we have a circuit with two resistors in series so we can calculate the equivalent resistance:

$$R_s = R_3 + R_p$$
$$= 2 + 1{,}33$$
$$= 3{,}33 \ \Omega$$

b) We start by determining the equivalent resistance of the parallel combination:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$
$$= \frac{1}{1} + \frac{1}{2}$$
$$= \frac{3}{2}$$
$$R_p = 0{,}67 \ \Omega$$

Now we have a circuit with three resistors in series so we can calculate the equivalent resistance:

$$R_s = R_3 + R_4 + R_p$$
$$= 4 + 6 + 0{,}67$$
$$= 10{,}67 \ \Omega$$

c) We start by determining the equivalent resistance of the parallel combination:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$
$$= \frac{1}{3} + \frac{1}{5} + \frac{1}{1}$$
$$= \frac{23}{15}$$
$$R_p = 0{,}652 \ \Omega$$

Now we have a circuit with two resistors in series so we can calculate the equivalent resistance:

$$R_s = R_4 + R_p$$
$$= 2 + 0{,}652$$
$$= 2{,}652 \ \Omega$$

2. Examine the circuit below:

If the potential difference across the cell is 12 V, calculate:

a) the current I through the cell.

b) the current through the $5 \ \Omega$ resistor.

Solution:

a) To find the current I we first need to find the equivalent resistance. We start by calculating the equivalent resistance of the parallel combination:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$
$$= \frac{1}{3} + \frac{1}{5} + \frac{1}{1}$$
$$= \frac{23}{15}$$
$$R_p = 0{,}652 \ \Omega$$

Now we have a circuit with two resistors in series so we can calculate the equivalent resistance:

$$R_s = R_4 + R_p$$
$$= 2 + 0{,}652$$
$$= 2{,}652 \ \Omega$$

So the current through the cell is:

$$I = \frac{V}{R}$$
$$= \frac{12}{2{,}652}$$
$$= 4{,}52 \ \text{A}$$

b) The current through the parallel combination of resistors is $4{,}52$ A. (The current is the same through series combinations of resistors and we can consider the entire parallel set of resistors as one series resistor.)

Using this we can find the voltage through the parallel combination of resistors (remember to use the equivalent parallel resistance and not the equivalent resistance of the circuit):

$$V = I \cdot R$$
$$= (4{,}52)(0{,}652)$$
$$= 2{,}95 \ \text{V}$$

Since the voltage across each resistor in the parallel combination is the same, this is also the voltage across the $5 \ \Omega$ resistor.

So now we can calculate the current through the resistor:

$$I = \frac{V}{R}$$
$$= \frac{2{,}95}{5}$$
$$= 0{,}59 \ \text{A}$$

3. If current flowing through the cell is 2 A, and all the resistors are ohmic, calculate the voltage across the cell and each of the resistors, R_1, R_2, and R_3 respectively.
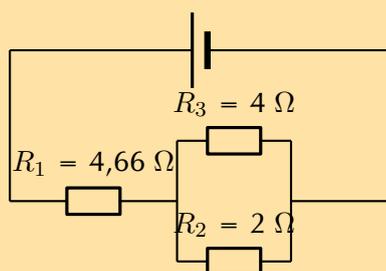
Solution:

To find the voltage we first need to find the equivalent resistance. We start by calculating the equivalent resistance of the parallel combination:

$$\begin{aligned}
\frac{1}{R_p} &= \frac{1}{R_2} + \frac{1}{R_3} \\
&= \frac{1}{2} + \frac{1}{4} \\
&= \frac{3}{4} \\
R_p &= 1{,}33 \ \Omega
\end{aligned}$$

Now we have a circuit with two resistors in series so we can calculate the equivalent resistance:

$$\begin{aligned}
R_s &= R_1 + R_p \\
&= 4{,}66 + 1{,}33 \\
&= 5{,}99 \ \Omega
\end{aligned}$$

So the voltage across the cell is:

$$\begin{aligned}
V &= I \cdot R \\
&= (2)(5{,}99) \\
&= 12 \ \text{V}
\end{aligned}$$

The current through the parallel combination of resistors is 2 A. (The current is the same through series combinations of resistors and we can consider the entire parallel set of resistors as one series resistor.)

Using this we can find the voltage through the each of the resistors. We start by finding the voltage across R_1:

$$\begin{aligned}
V &= I \cdot R \\
&= (2)(4{,}66) \\
&= 9{,}32 \ \text{V}
\end{aligned}$$

Now we find the voltage across the parallel combination:

$$V = I \cdot R$$
$$= (2)(1{,}33)$$
$$= 2{,}66 \text{ V}$$

Since the voltage across each resistor in the parallel combination is the same, this is also the voltage across resistors R_2 and R_3.

4. For the following circuit, calculate:
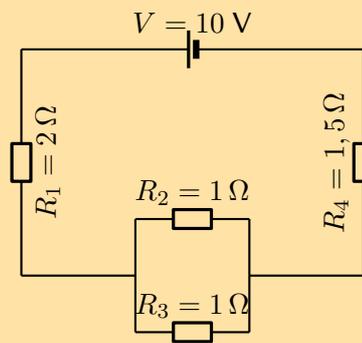
a) the current through the cell

b) the voltage drop across R_4

c) the current through R_2

Solution:

a) To find the current we first need to find the equivalent resistance. We start by calculating the equivalent resistance of the parallel combination:

$$\frac{1}{R_p} = \frac{1}{R_2} + \frac{1}{R_3}$$
$$= \frac{1}{1} + \frac{1}{1}$$
$$= 2$$
$$R_p = 0{,}5 \ \Omega$$

Now we have a circuit with two resistors in series so we can calculate the equivalent resistance:

$$R_s = R_1 + R_4 + R_p$$
$$= 2 + 1{,}5 + 0{,}5$$
$$= 4 \ \Omega$$

So the current through the cell is:

$$I = \frac{V}{R}$$
$$= \frac{10}{4}$$
$$= 2{,}5 \text{ A}$$

b) The current through all the resistors is 2,5 A. (The current is the same through series combinations of resistors and we can consider the entire parallel set of resistors as one series resistor.)

Using this we can find the voltage through R_4:

$$\begin{aligned} V &= I \cdot R \\ &= (2,5)(1,5) \\ &= 3,75 \text{ V} \end{aligned}$$

c) The current through all the resistors is 2,5 A. (The current is the same through series combinations of resistors and we can consider the entire parallel set of resistors as one series resistor.)

Using this we can find the current through R_2.

We first need to find the voltage across the parallel combination:

$$\begin{aligned} V &= I \cdot R \\ &= (2,5)(0,5) \\ &= 1,25 \text{ V} \end{aligned}$$

Now we can find the current through R_2 using the fact that the voltage is the same across each resistor in the parallel combination:

$$\begin{aligned} I &= \frac{V}{R} \\ &= \frac{1,25}{1} \\ &= 1,25 \text{ A} \end{aligned}$$

11.3   Power and energy

Electrical power

Exercise 11 – 6:

1. What is the power of a $1,00 \times 10^8$ V lightning bolt having a current of $2,00 \times 10^4$ A?

   Solution:

   $$\begin{aligned} P &= VI \\ &= (1,00 \times 10^8)(2,00 \times 10^4) \\ &= 2,00 \times 10^{12} \text{ W} \end{aligned}$$

2. How many watts does a torch that has $6{,}00 \times 10^2$ C pass through it in $0{,}50$ h use if its voltage is $3{,}00$ V?

Solution:

We first need to find the current. Recall from grade 10 that current is charge divided by total time (in seconds):

$$\begin{aligned}
I &= \frac{C}{\Delta t} \\
&= \frac{6{,}00 \times 10^2}{1800} \\
&= 0{,}333 \text{ A}
\end{aligned}$$

$$\begin{aligned}
P &= VI \\
&= (3{,}00)(0{,}333) \\
&= 0{,}99 \text{ W}
\end{aligned}$$

3. Find the power dissipated in each of these extension cords:

   a) an extension cord having a $0{,}06$ Ω resistance and through which $5{,}00$ A is flowing

   b) a cheaper cord utilising (using) thinner wire and with a resistance of $0{,}30$ Ω, through which $5{,}00$ A is flowing

Solution:

   a)
   $$\begin{aligned}
   P &= I^2 R \\
   &= (5{,}00)^2(0{,}06) \\
   &= 1{,}5 \text{ W}
   \end{aligned}$$

   b)
   $$\begin{aligned}
   P &= I^2 R \\
   &= (5{,}00)^2(0{,}30) \\
   &= 7{,}5 \text{ W}
   \end{aligned}$$

4. Determine the power dissipated by each the resistors in the following circuits, if the batteries are 6 V:

   a)

b)
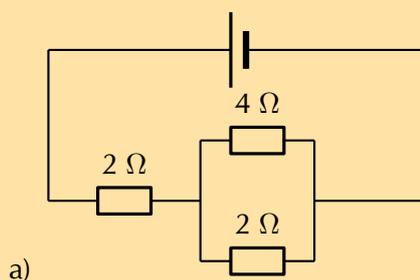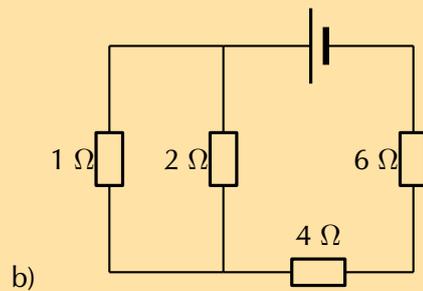
c) Also determine the value of the unknown resistor if the total power dissipated is 9,8 W
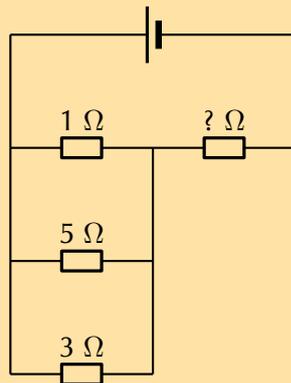
Solution:

a) We start by determining the equivalent resistance of the parallel combination:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$
$$= \frac{1}{4} + \frac{1}{2}$$
$$= \frac{3}{4}$$
$$R_p = 1,33 \ \Omega$$

Now we have a circuit with two resistors in series so we can calculate the equivalent resistance:

$$R_s = R_3 + R_p$$
$$= 2 + 1,33$$
$$= 3,33 \ \Omega$$

Now we can calculate the total current:

$$I = \frac{V}{R}$$
$$= \frac{6}{3,33}$$
$$= 1,8 \ A$$

This is the current in the 2 Ω resistor and through the entire parallel connection. Using this we can find the power dissipated in the 2 Ω resistor:

$$P = I^2 R$$
$$= (1{,}8)^2 (2)$$
$$= 6{,}48 \text{ W}$$

Next we find the voltage across this resistor and use this to find the voltage across the parallel combination:

$$V = IR$$
$$= (1{,}8)(2)$$
$$= 3{,}6 \text{ V}$$

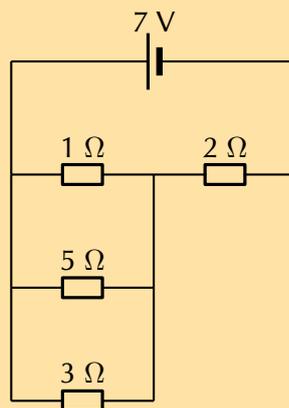$$V_T = V_1 + V_2$$
$$V_2 = V_T - V_1$$
$$= 6 - 3{,}6$$
$$= 2{,}4 \text{ V}$$

This is the voltage across each of the parallel resistors. So we can find the power dissipated by each of these resistors:

$$P = \frac{V^2}{R}$$
$$= \frac{(2{,}4)^2}{4}$$
$$= 1{,}44 \text{ W}$$

$$P = \frac{V^2}{R}$$
$$= \frac{(2{,}4)^2}{2}$$
$$= 2{,}88 \text{ W}$$

b) We start by determining the equivalent resistance of the parallel combination:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$
$$= \frac{1}{1} + \frac{1}{2}$$
$$= \frac{3}{2}$$
$$R_p = 0{,}67 \ \Omega$$

Now we have a circuit with three resistors in series so we can calculate the equivalent resistance:

$$R_s = R_3 + R_4 + R_p$$
$$= 4 + 6 + 0{,}67$$
$$= 10{,}67 \ \Omega$$

Now we can calculate the total current:

$$I = \frac{V}{R}$$
$$= \frac{6}{10{,}67}$$
$$= 0{,}56 \text{ A}$$

This is the current in the two series resistor and through the entire parallel connection. Using this we can find the power dissipated in the two series resistors:

$$P = I^2 R$$
$$= (0{,}56)^2 (6)$$
$$= 1{,}88 \text{ W}$$

$$P = I^2 R$$
$$= (0{,}56)^2 (4)$$
$$= 1{,}25 \text{ W}$$

Next we find the voltage across each of these resistors and use this to find the voltage across the parallel combination:

$$V = IR$$
$$= (0{,}56)(6)$$
$$= 3{,}36 \text{ V}$$

$$V = IR$$
$$= (0{,}56)(4)$$
$$= 2{,}24 \text{ V}$$

$$V_T = V_1 + V_2 + V_p$$
$$V_p = V_T - V_1 - V_2$$
$$= 6 - 3{,}36 - 2{,}24$$
$$= 0{,}4 \text{ V}$$

This is the voltage across each of the parallel resistors. So we can find the power dissipated by each of these resistors:

$$P = \frac{V^2}{R}$$
$$= \frac{(0{,}4)^2}{1}$$
$$= 0{,}16 \text{ W}$$

$$P = \frac{V^2}{R}$$
$$= \frac{(0{,}4)^2}{2}$$
$$= 0{,}32 \text{ W}$$

c) We start by calculating the equivalent resistance of the resistors. We know the total power and the total voltage, so we use that to find the total resistance.

$$P = \frac{V^2}{R_T}$$
$$R_T = \frac{V^2}{P}$$
$$= \frac{6^2}{9,8}$$
$$= 3,67\ \Omega$$

We can now find the unknown resistance by first calculating the equivalent parallel resistance:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$
$$= \frac{1}{1} + \frac{1}{5} + \frac{1}{3}$$
$$= \frac{23}{15}$$
$$R_p = 0,65\ \Omega$$

$$R_s = R_4 + R_p$$
$$R_4 = R_s - R_p$$
$$= 3,67 - 0,65$$
$$= 3,02\ \Omega$$

Now we can calculate the total current:

$$I = \frac{V}{R}$$
$$= \frac{6}{3,67}$$
$$= 1,63\ \text{A}$$

This is the current in the series resistor and through the entire parallel connection. Using this we can find the power dissipated in the series resistor:

$$P = I^2 R$$
$$= (1,63)^2(3,02)$$
$$= 0,89\ \text{W}$$

Next we find the voltage across this resistors and use this to find the voltage across the parallel combination:

$$V = IR$$
$$= (1,63)(3,02)$$
$$= 4,92\ \text{V}$$

$$V_T = V_1 + V_p$$
$$V_p = V_T - V_1$$
$$= 6 - 4,92$$
$$= 1,08 \text{ V}$$

This is the voltage across each of the parallel resistors. So we can find the power dissipated by each of these resistors:

$$P = \frac{V^2}{R}$$
$$= \frac{(1,08)^2}{1}$$
$$= 1,17 \text{ W}$$

$$P = \frac{V^2}{R}$$
$$= \frac{(1,08)^2}{5}$$
$$= 5,83 \text{ W}$$

$$P = \frac{V^2}{R}$$
$$= \frac{(1,08)^2}{3}$$
$$= 3,5 \text{ W}$$

5. Examine the circuit below:

If the potential difference across the cell is 7 V, calculate:

a) the current I through the cell.

b) the current through the 5 Ω resistor

c) the power dissipated in the 5 Ω resistor

Solution:

a) We can find the equivalent parallel resistance:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$
$$= \frac{1}{1} + \frac{1}{5} + \frac{1}{3}$$
$$= \frac{23}{15}$$
$$R_p = 0{,}65 \ \Omega$$

And now we can find the equivalent resistance:

$$R_s = R_4 + R_p$$
$$= 2 + 0{,}65$$
$$= 2{,}65 \ \Omega$$

Now we can calculate the total current:

$$I = \frac{V}{R}$$
$$= \frac{7}{2{,}65}$$
$$= 2{,}64 \ \text{A}$$

b) For a series circuit the total current is equal to the current in each of the resistors. Since we can consider the total parallel combination as one series connection the total current through the connection is 2,64 A.

Using this and the total parallel resistance we can calculate the voltage across each resistor:

$$V = IR$$
$$= (2{,}64)(0{,}65)$$
$$= 1{,}72 \ \text{V}$$

Now we can calculate the current through the 5 Ω resistor:

$$I = \frac{V}{R}$$
$$= \frac{1{,}72}{5}$$
$$= 0{,}34 \ \text{A}$$

c) We know the resistance, the voltage and the current for this resistor so we can use any two of these to find the power. We will use the resistance and the current:

$$P = I^2 R$$
$$= (0{,}34)(5)$$
$$= 1{,}7 \ \text{W}$$

6. If current flowing through the cell is 2 A, and all the resistors are ohmic, calculate the power dissipated in each of the resistors, R_1, R_2, and R_3 respectively.
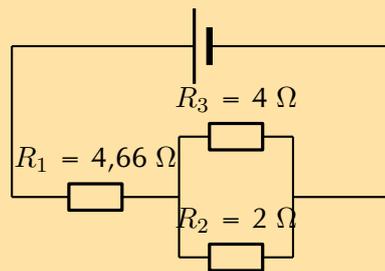
Solution:

We start by determining the equivalent resistance of the parallel combination:

$$\begin{aligned}
\frac{1}{R_p} &= \frac{1}{R_2} + \frac{1}{R_3} \\
&= \frac{1}{4} + \frac{1}{2} \\
&= \frac{3}{4} \\
R_p &= 1{,}33 \ \Omega
\end{aligned}$$

Now we have a circuit with two resistors in series so we can calculate the equivalent resistance:

$$\begin{aligned}
R_s &= R_1 + R_p \\
&= 4{,}66 + 1{,}33 \\
&= 5{,}99 \ \Omega
\end{aligned}$$

We are given the total current and this is the current in the series resistor and through the entire parallel connection. Using this we can find the power dissipated in the series resistor:

$$\begin{aligned}
P &= I^2 R \\
&= (2)^2(4{,}66) \\
&= 18{,}64 \ \text{W}
\end{aligned}$$

Next we find the voltage across this resistor:

$$\begin{aligned}
V &= IR \\
&= (2)(4{,}66) \\
&= 9{,}32 \ \text{V}
\end{aligned}$$

And we use the total resistance and the total current to find the total voltage:

$$V = IR$$
$$= (2)(5{,}99)$$
$$= 12 \text{ V}$$

Now we can find the voltage across the parallel combination of resistors:

$$V_T = V_1 + V_p$$
$$V_p = V_T - V_1$$
$$= 12 - 9{,}32$$
$$= 2{,}68 \text{ V}$$

This is the voltage across each of the parallel resistors. So we can find the power dissipated by each of these resistors:

$$P = \frac{V^2}{R}$$
$$= \frac{(2{,}68)^2}{4}$$
$$= 10{,}72 \text{ W}$$

$$P = \frac{V^2}{R}$$
$$= \frac{(2{,}68)^2}{2}$$
$$= 5{,}36 \text{ W}$$

11.4 Chapter summary

Exercise 11 – 7:

1. Give one word or term for each of the following definitions:

   a) The amount of energy per unit charge needed to move that charge between two points in a circuit.

   b) The rate at which electrical energy is converted in an electric circuit.

   c) A law that states that the amount of current through a conductor, at constant temperature, is proportional to the voltage across the resistor.

   Solution:

   a) Voltage

   b) Electrical power

c) Ohm's law

2. A 10 Ω has a voltage of 5 V across it. What is the current through the resistor?

   a) 50 A

   b) 5 A

   c) 0,5 A

   d) 7 A

   Solution:

   0,5 A

3. Three resistors are connected in series. The resistances of the three resistors are: 10 Ω, 4 Ω and 3 Ω. What is the equivalent series resistance?

   a) 1,5 Ω

   b) 17 Ω

   c) 0,68 Ω

   d) 8 Ω

   Solution:

   17 Ω

4. Three resistors are connected in parallel. The resistances of the three resistors are: 5 Ω, 4 Ω and 2 Ω. What is the equivalent parallel resistance?

   a) 1,05 Ω

   b) 11 Ω

   c) 0,95 Ω

   d) 3 Ω

   Solution:

   1,05 Ω

5. A circuit consists of a 6 Ω resistor. The voltage across the resistor is 12 V. How much power is dissipated in the circuit?

   a) 864 W

   b) 3 W

   c) 2 W

   d) 24 W

   Solution:

   24 W

6. Calculate the current in the following circuit and then use the current to calculate the voltage drops across each resistor.
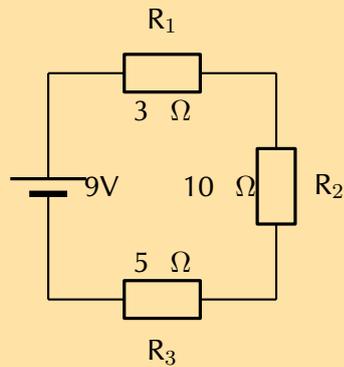
Solution:

We first calculate the equivalent series resistance:

$$R_s = R_1 + R_2 + R_3$$
$$= 3 + 10 + 5$$
$$= 18 \ \Omega$$

Now we can use Ohm's law to calculate the current:

$$I = \frac{V}{R}$$
$$= \frac{9}{18}$$
$$= 0{,}5 \text{ A}$$

Now we can use the fact that this is a series circuit and that the current is the same everywhere in the circuit to find the voltage drops across each resistor:

$$V = IR$$
$$= (0{,}5)(3)$$
$$= 1{,}5 \text{ V}$$

$$V = IR$$
$$= (0{,}5)(10)$$
$$= 5 \text{ V}$$

$$V = IR$$
$$= (0{,}5)(5)$$
$$= 2{,}5 \text{ V}$$

7. A battery is connected to this arrangement of resistors. The power dissipated in the 100 Ω resistor is 0,81 W. The resistances of voltmeters V_1 and V_2 are so high that they do not affect the current in the circuit.
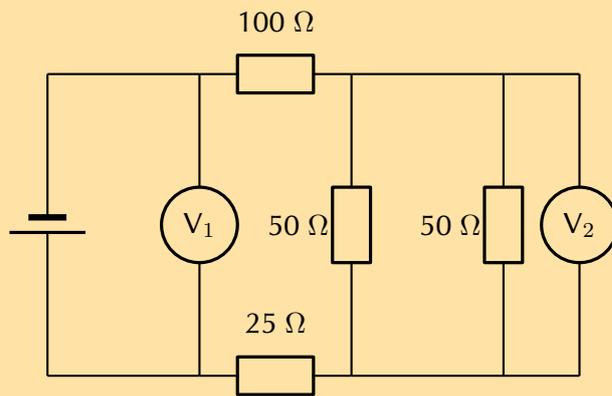
a) Calculate the current in the 100 Ω resistor.

b) Calculate the reading on voltmeter V_2.

c) Calculate the reading on voltmeter V_1.

Solution:

a)

$$P = I^2 R$$
$$I^2 = \frac{P}{R}$$
$$= \frac{0{,}81}{100}$$
$$= 0{,}0081$$
$$I = 0{,}09 \text{ A}$$

b) This voltmeter is reading the voltage across the one 50 Ω resistor so we need to determine what this voltage is.

We know that the current through the parallel combination of resistors is 0,09 A. We need to calculate the equivalent parallel resistance and then use this with the current to find the voltage across the parallel combination.

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$
$$= \frac{1}{50} + \frac{1}{50}$$
$$= \frac{2}{50}$$
$$R_p = 25 \text{ }\Omega$$

$$V = IR$$
$$= (0{,}09)(25)$$
$$= 2{,}25 \text{ V}$$

This is the reading on V_2 since the voltage across a parallel combination of resistors is equal to the voltage across each resistor.

c) We need to calculate the equivalent resistance of the circuit first. We already know the equivalent resistance across the parallel combination. So the equivalent resistance is:

$$R = R_1 + R_2 + R_p$$
$$= 100 + 25 + 25$$
$$= 150 \ \Omega$$

Now we can use Ohm's law to calculate the voltage, since we know the current in the circuit:

$$V = IR$$
$$= (0{,}09)(150)$$
$$= 13{,}5 \ \text{V}$$

8. A kettle is marked 240 V; 1500 W.

   a) Calculate the resistance of the kettle when operating according to the above specifications.

   b) If the kettle takes 3 minutes to boil some water, calculate the amount of electrical energy transferred to the kettle.

   [SC 2003/11]

   Solution:

   a)

$$P = \frac{V^2}{R}$$
$$R = \frac{V^2}{P}$$
$$= \frac{240^2}{1500}$$
$$= 38{,}4 \ \Omega$$

   b) We first convert the time to seconds:

$$(3)(60) = 180 \ \text{s}$$

   And now we calculate the energy:

$$E = Pt$$
$$= (1500)(180)$$
$$= 270 \ 000 \ \text{J}$$
$$= 270 \ \text{kJ}$$

9. **Electric Eels**

   Electric eels have a series of cells from head to tail. When the cells are activated by a nerve impulse, a potential difference is created from head to tail. A healthy electric eel can produce a potential difference of 600 V.

   a) What is meant by "a potential difference of 600 V"?

   b) How much energy is transferred when an electron is moved through a potential difference of 600 V?

[IEB 2001/11 HG1]

Solution:

a) Potential difference or voltage, V, is the amount of energy per unit charge needed to move that charge between two points in a circuit. In the example of the electric eel this is the amount of energy per unit charge needed to move that charge from the head to the tail.

b) An electron has a charge of $-1{,}69 \times 10^{-19}$ C. So we can find the amount of energy needed to move this charge through the potential difference:

$$V = \frac{E}{C}$$
$$600 = \frac{E}{-1{,}69 \times 10^{-19}}$$
$$E = -1{,}014 \times 10^{-16} \text{ J}$$

CHAPTER

12

Energy and chemical change

In grade 10 learners learnt about physical and chemical changes. In this chapter learners will learn about the energy changes that occur in chemical reactions. The concepts of exothermic and endothermic reactions are introduced. Learners will also learn about activation energy. The following list summarises the concepts covered in this chapter.

- **Bond energy and how it leads to the energy changes in reactions.**

  In chapter 3 (atomic combinations) the concept of bond energy and graphs of potential energy versus atomic distance were covered. These two concepts form the cornerstone to understanding the energy changes in chemical reactions. In this topic the fact that bond forming requires energy and bond breaking releases energy is introduced. These two concepts are linked to the potential energy graph for bonding.

- **Exothermic and endothermic reactions.**

  Two classes of chemical reaction exist: exothermic and endothermic. It is important to note that these are not different types of reactions. For example an acid-base reaction can be exothermic or endothermic. All chemical reactions will either be exothermic or endothermic and this is determined by the energy of bond formation and bond breaking.

- **Energy diagrams for reactions with and without activation energy.**

  We can draw energy diagrams to show how a reaction proceeds. These diagrams give the reactants energy and the products energy. These diagrams show the energy of the system as a whole and are not concerned with just one reactant or one product. It is important for learners to draw these diagrams as a curve linking the reactants energy to the product energy as this shows the activation energy of the reaction. Learners first draw these curves without knowing about activation energy and then towards the end of the chapter they start adding the activation energy.

- **Activation energy.**

  A practical demonstration that can be done is to burn magnesium ribbon in air and in oxygen to investigate the concept of activation energy.

  All reactions (even the exothermic ones) need something to get them going. This may be very small or may be very large. At the maximum energy of the reaction the transition state or activated complex occurs. This is the point at which the reaction is somewhere between forming the products and breaking apart the reactants.

A recommended project for formal assessment is included. In this experiment learners will investigate an exothermic reaction and an endothermic reaction. This experiment is split into two parts. The first part looks at an endothermic reaction, while the second part looks at an exothermic reaction. You will need polystyrene or cardboard cups, plastic lids, thermometers, vinegar, steel wool, citric acid, sodium bicarbonate and stirring rods. There is also a further investigation on exothermic and endothermic reactions that learners can complete. All these experiments can be combined into one project in which learners investigate several different reactions and classify these reactions as exothermic or endothermic.

This series of experiments starts with an endothermic reaction between citric acid and sodium bicarbonate. The second experiment in the series looks at the exothermic reaction between steel and oxygen in the air. The final part of the series is given as an investigation into various exothermic and endothermic reactions.

Examples of endothermic and exothermic reactions

In the investigation on exothermic and endothermic reactions learners work with concentrated sulfuric acid. They must work in a well ventilated room or, if possible, in a fume cupboard. This is a highly corrosive substance and learners must handle it with care. If they spill any on themselves they must immediately wash the affected area with plenty of running water. Either the learner or their friend should inform you as soon as possible so you can ensure that the learner is ok. If necessary the learner may need to go to the bathroom to remove and rinse clothing that is affected. Either you or another learner should accompany them.

Most of the salts that the learners will work with are hygroscopic and will quickly absorb water from the air. These salts can cause chemical burns and should be handled with care. If possible learners should wear gloves to protect their hands.

12.1   Energy changes in chemical reactions

Exercise 12 − 1: Exothermic and endothermic reactions 1

1. State whether energy is taken in or released in each of the following situations:

   a) The bond between hydrogen and chlorine in a molecule of hydrogen chloride breaks.
   b) A bond is formed between hydrogen and fluorine to form a molecule of hydrogen fluoride.
   c) A molecule of nitrogen (N_2) is formed.
   d) A molecule of carbon monoxide breaks apart.

   Solution:

   a) This is bond breaking and so energy is taken in.
   b) This is bond forming and so energy is released.
   c) A bond is formed and so energy is released.
   d) A bond is broken and so energy is taken in.

2. State whether the following descriptions are used to describe an endothermic or an exothermic reaction:

   a) Reactants react to give products and energy.
   b) The energy that must be absorbed to break the bonds in the reactants is greater than the energy that is released when the products form.
   c) The energy of the products is found to be greater than the energy of the reactants for this type of reaction.

d) Heat or light must be absorbed from the surroundings before this type of reaction takes place.

Solution:

a) Exothermic

b) Endothermic

c) Exothermic

d) Endothermic

12.2 Exothermic and endothermic reactions

The heat of reaction

Exercise 12 − 2: Endothermic and exothermic reactions

1. In each of the following reactions, say whether the reaction is endothermic or exothermic, and give a reason for your answer. Draw the resulting energy graph for each reaction.

   a) $H_2(g) + I_2(g) \rightarrow 2HI\ (g) + 21$ kJ·mol^{-1}

   b) $CH_4(g) + 2O_2(g) \rightarrow CO_2(g) + 2H_2O\ (g)$ $\Delta H = -802$ kJ·mol^{-1}
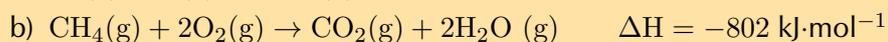
   c) The following reaction takes place in a flask:
   $Ba(OH)_2.8H_2O\ (s) + 2NH_4NO_3(aq) \rightarrow Ba(NO_3)_2(aq) + 2NH_3(aq) + 10H_2O\ (l)$
   Within a few minutes, the temperature of the flask drops by approximately 20°C.
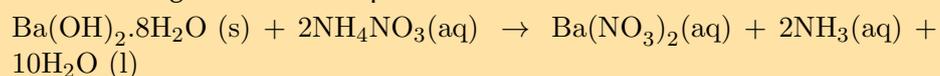
   d) $2Na\ (aq) + Cl_2(aq) \rightarrow 2NaCl\ (aq)$ $\Delta H = -411$ kJ·mol^{-1}

   e) $C\ (s) + O_2(g) \rightarrow CO_2(g)$
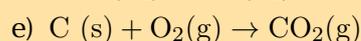
Solution:

   a) Exothermic. Heat is given off and this is represented by showing + energy on the right hand side of the equation.
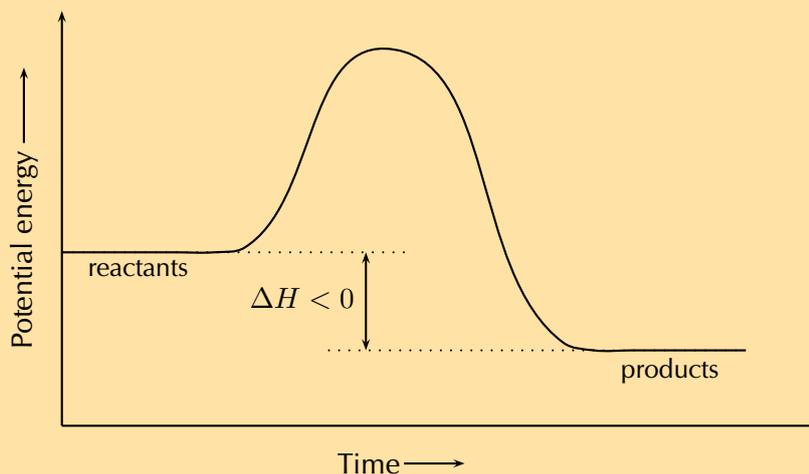
b) Exothermic. ΔH is negative for this reaction.
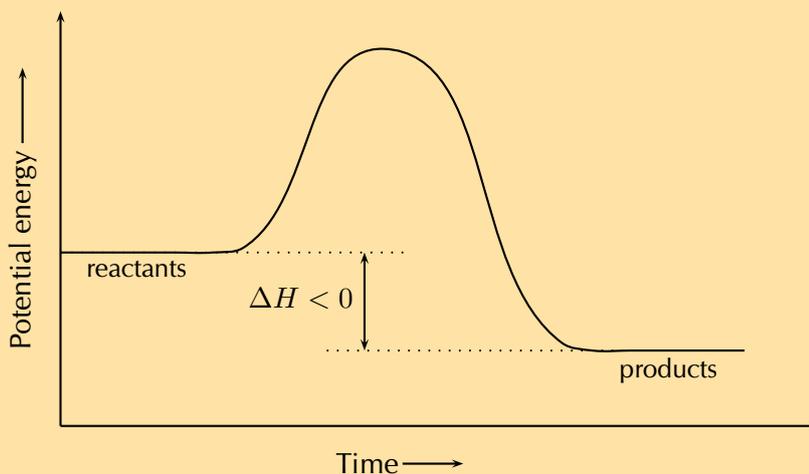
c) Endothermic. The temperature of the reaction vessel decreases indicating that heat was needed for the reaction.
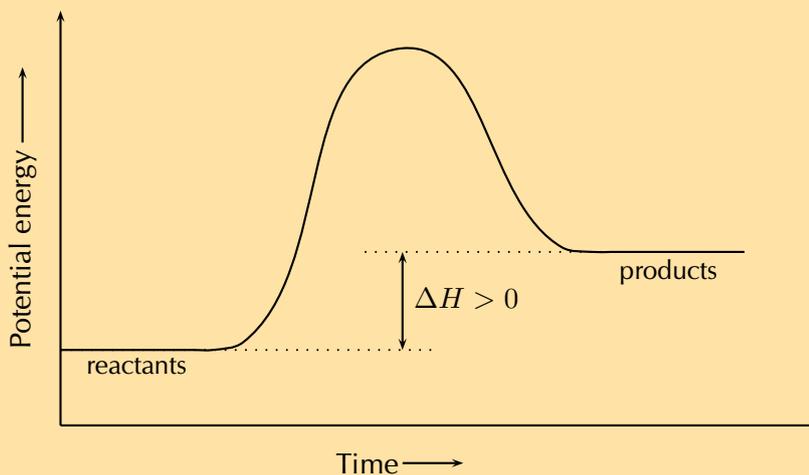
d) Exothermic. $\Delta H < 0$
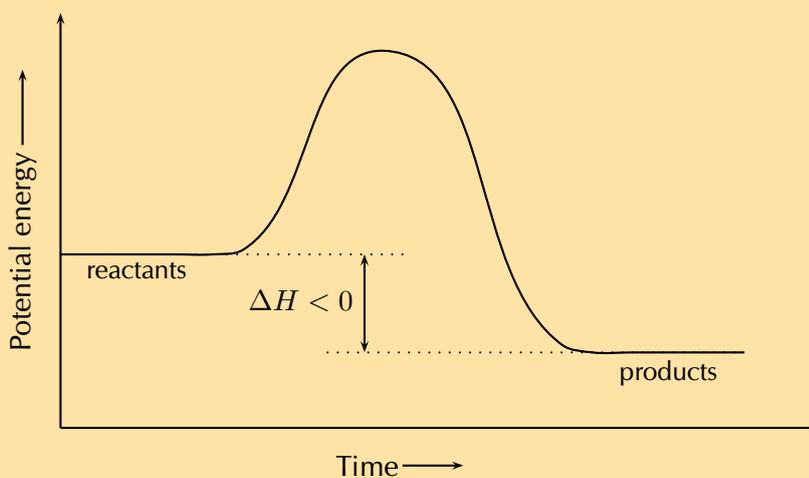
e) Exothermic. This reaction is the combustion of wood or coal to form carbon dioxide and gives off heat.
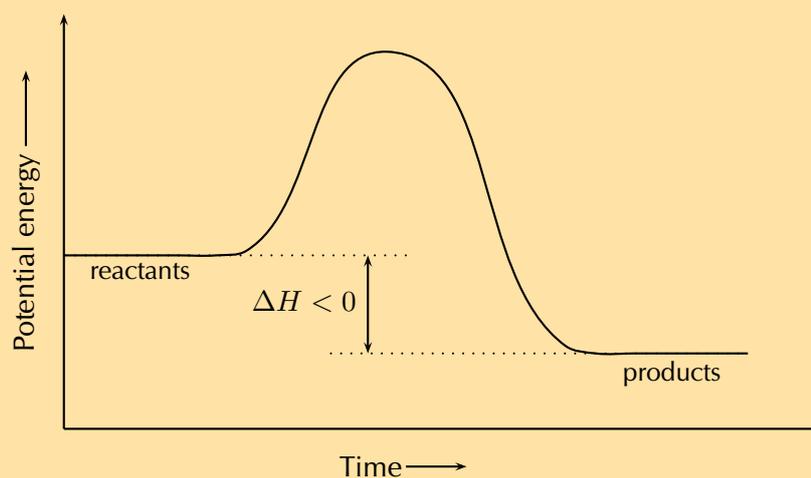
2. For each of the following descriptions, say whether the process is endothermic or exothermic and give a reason for your answer.

   a) evaporation

   b) the combustion reaction in a car engine

   c) bomb explosions

   d) melting ice

   e) digestion of food

   f) condensation

Solution:

   a) Endothermic. Energy is needed to break the intermolecular forces.

   b) Exothermic. Energy is released.

   c) Exothermic. In an explosion a large amount of energy is released.

   d) Endothermic. Energy is needed to break the intermolecular forces.

   e) Exothermic. Digestion of food involves the release of energy that your body can then use.

   f) Exothermic. Energy is given off as the particles are going from a higher energy state to a lower energy state.

3. When you add water to acid the resulting solution splashes up. The beaker also gets very hot. Explain why.

Solution:

The reaction between acid and water is an exothermic reaction. This reaction produces a lot of heat and energy which causes the resulting solution to splash. Since the acid is usually more dense than the water adding water to the acid causes the reaction to happen in a small area and on the surface. This leads to a more vigorous (fast) reaction. If you add the acid to the water then there is a larger volume of water to absorb the heat of the reaction and so the reaction proceeds more slowly.

Exercise 12 – 3: Energy and reactions

1. Carbon reacts with water according to the following equation:

$$C\ (s) + H_2O\ (g) \rightarrow CO\ (g) + H_2(g) \qquad \Delta H > 0$$
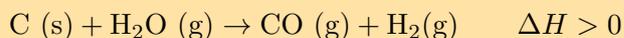
Is this reaction endothermic or exothermic? Give a reason for your answer.

Solution:

Endothermic $\Delta H > 0$

2. Refer to the graph below and then answer the questions that follow:
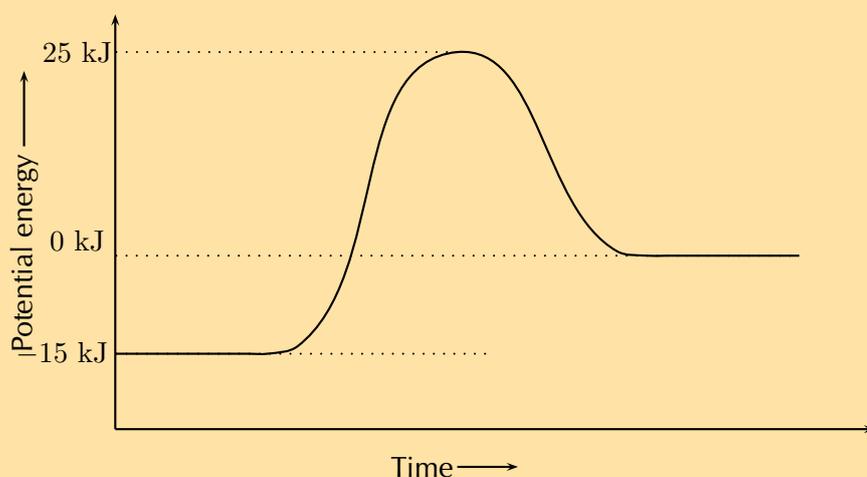
a) What is the energy of the reactants?

b) What is the energy of the products?

c) Calculate ΔH.

d) What is the activation energy for this reaction?

Solution:

a) -15 kJ

b) 0 kJ

c) We find ΔH using:

$$\begin{aligned}
\Delta H &= \text{energy of products} - \text{energy of reactants} \\
&= 0\text{ kJ} - (-15\text{ kJ}) \\
&= 15\text{ kJ}
\end{aligned}$$

d)

$$\begin{aligned}
\text{activation energy} &= \text{energy of activated complex} - \text{energy of reactants} \\
&= 25\text{ kJ} - (-15\text{ kJ}) \\
&= 40\text{ kJ}
\end{aligned}$$

12.4 Chapter summary

1. For each of the following, give one word or term for the description.

    a) The minimum amount of energy that is needed for a reaction to proceed.

    b) A measure of the bond strength in a chemical bond.

    c) A type of reaction where ΔH is less than zero.

    d) A type of reaction that requires heat or light to proceed.

    Solution:

    a) Activation energy

    b) Bond energy

    c) Exothermic reaction

    d) Endothermic reaction

2. For the following reaction:

$$HCl\ (aq) + NaOH\ (aq) \rightarrow NaCl\ (aq) + H_2O\ (l)$$
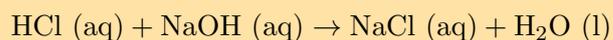
    choose the correct statement from the list below.

    a) Energy is taken in when the new bonds in NaCl are formed.

    b) Energy is released when the bonds in HCl break.

    c) Energy is released when the bonds in H_2O form.

    d) Energy is released when the bonds in NaOH break.

    Solution:

    Energy is released when the bonds in H_2O form.

3. For the following reaction:

$$A + B \rightarrow AB \qquad \Delta H = -129\ kJ{\cdot}mol^{-1}$$

    choose the correct statement from the list below.

    a) The energy of the reactants is less than the energy of the product.

    b) The energy of the product is less than the energy of the reactants.

    c) The reaction needs energy to occur.

    d) The overall energy of the system increases during the reaction.

    Solution:

    The energy of the product is less than the energy of the reactants. This is an exothermic reaction ($\Delta H > 0$).

4. Consider the following chemical reaction:

$$2NO_2(g) \rightarrow N_2O_4(g) \qquad \Delta H < 0$$

Which one of the following graphs best represents the changes in potential energy that take place during the production of N_2O_4?

(a)   (b)   (c)   (d)
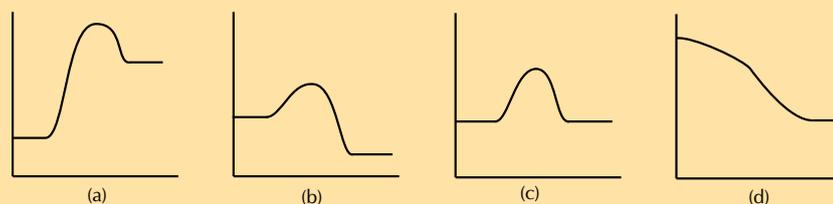
Solution:

The second graph (b) is correct. The reaction is exothermic ($\Delta H > 0$), so the energy of the reactants must be greater than the energy of the products. Graphs (c) and (d) are not valid energy graphs.

5. In each of the following reactions, say whether the reaction is endothermic or exothermic, and give a reason for your answer. Draw the resulting energy graph for each reaction.

a) $Fe_2O_3(s) + 2Al\ (s) \rightarrow 2Fe\ (s) + Al_2O_3(s) +$ heat

b) $NH_4Cl\ (s) +$ heat $\rightarrow NH_3(g) + HCl\ (g)$

Solution:

a) Exothermic. Heat is given off and this is represented by showing $+$ heat on the right hand side of the equation.
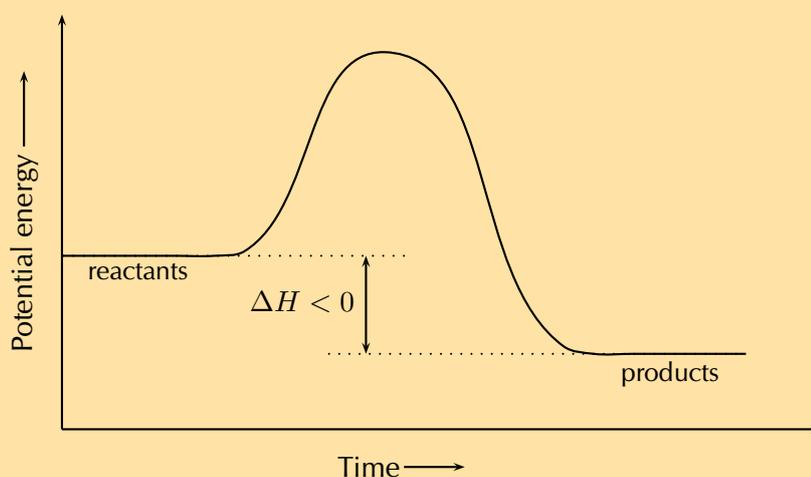
b) Endothermic. Energy is needed (shown by $+$ energy on the left hand side of the reaction equation) for the reaction to proceed.
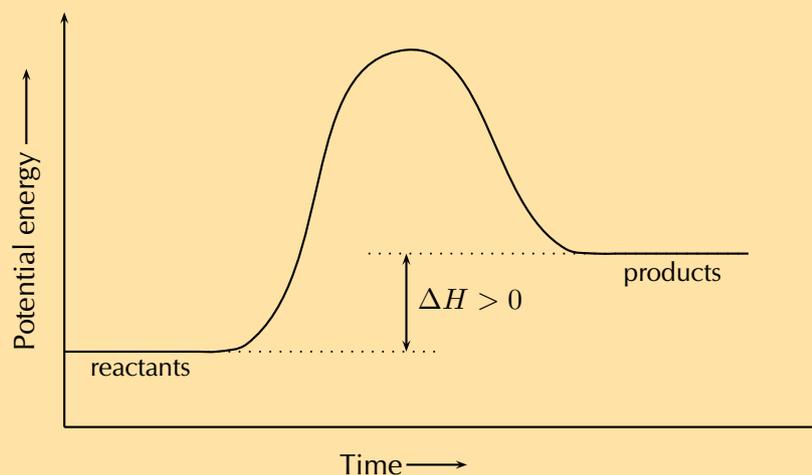
6. The cellular respiration reaction is catalysed by enzymes. The equation for the reaction is:

$$C_6H_{12}O_6(s) + 6O_2(g) \rightarrow 6CO_2(g) + 6H_2O \ (l)$$
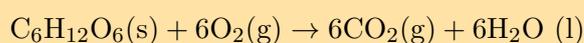
The change in potential energy during this reaction is shown below:
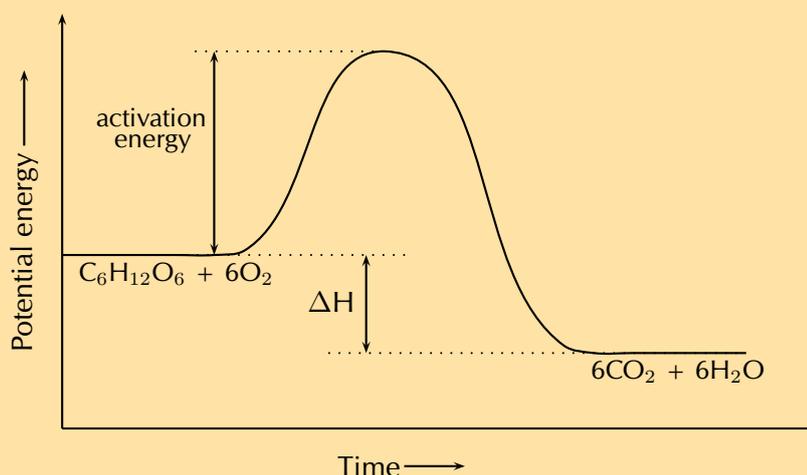
a) Will the value of ΔH be positive or negative? Give a reason for your answer.

b) Explain what is meant by activation energy.

c) Glucose is one of the reactants in cellular respiration. What important chemical reaction produces glucose?

d) Is the reaction in your answer above an endothermic or an exothermic one? Explain your answer.

e) Draw the energy graph for the reaction that produces glucose.

Solution:

a) Negative. The energy of the products is less than the energy of the reactants.

b) The activation energy is the minimum energy that must be overcome to enable the reaction to proceed.

c) Photosynthesis

d) Endothermic. Photosynthesis needs energy (light) to proceed.
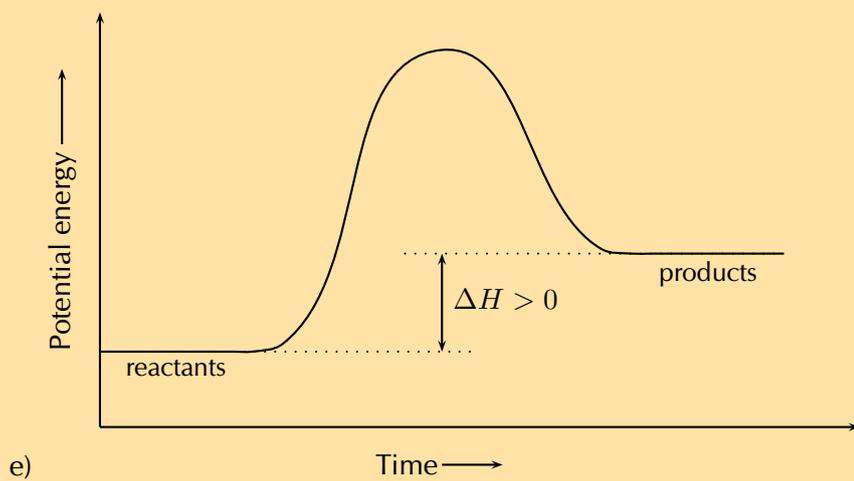
e)

CHAPTER 13

Types of reactions

In this chapter learners will explore acid-base reactions and redox reactions. Redox reactions were briefly introduced in gr10. The concepts of acids, bases, reduction, oxidation and oxidation numbers are all introduced here. The following list provides a summary of the topics covered in this chapter.

- **Acids and bases.**

  This chapter begins by revising all the concepts done on acids and bases up to this point. Learners are reminded what an acid and a base are (in particular the Bronsted-Lowry definition) and how the definition and concept have changed over time. Although the most recent definition of an acid and a base is the Lowry definition this is not covered at school level and the Bronsted-Lowry definition serves as a good working model for the acids and bases that learners encounter at school.

  The concept of a polyprotic acid is introduced although it is not in CAPS. This is done to help learners understand how to handle acids such as sulfuric acid in reactions. You should try to use polyprotic acids sparingly in your examples.

- **Conjugate acids and bases and amphoteric (amphiprotic) substances.**

  The concept of conjugate acids and bases requires learners to think about reactions going in reverse. By writing the equation in reverse, learners can see how the acid becomes a base. This base is said to be the conjugate base of the acid since it is conjugated (linked) to the acid.

- **Acid-hydroxide, acid-oxide and acid-carbonate reactions.**

  Three different types of bases are examined in detail to see how they react with acids. Several examples of each type are given and the general equation for the reaction is also given.

- **Oxidation numbers for compounds.**

  This topic is placed after redox reactions in CAPS but must be taught before redox reactions and so is placed before redox reactions in this book. This topic provides the tools needed to understand redox reactions.

- **Balancing redox reactions.**

  In grade 10 learners learnt how to balance chemical equations by inspection. In this topic they will learn how to balance redox reactions which often cannot be balanced by inspection. The simpler examples can be balanced by inspection and this can be used as a comparison for the two techniques. Learners need to be able to break a reaction up into two parts and follow different chemical species through an equation. This skill starts with conjugate acids and bases and carries over into this topic.

Coloured text has been used as a tool to highlight different parts of reactions. Ensure that learners understand that the coloured text does not mean there is anything special about that part of the reaction, this is simply a teaching tool to help them identify the important parts of the reaction.

It is also important to note that this chapter is split across term 3 and term 4. Acids and bases should be completed in term 3 and redox reactions are done in term 4.

A recommended experiment for informal assessment on discovering natural indicators is included. Learners can test a variety of colourful plants to see what happens to each plant when mixed with an acid or a base. The basic idea is for learners to extract the colour of the plant by boiling the plant matter and then draining the liquid. For substances such as the curry powder, learners can dissolve this in water and for the tea they can brew a cup of tea and then remove the teabag before testing. The resulting liquid can then be tested to see if it is an indicator. An alternative to mixing the acid or base into the liquid is to soak strips of paper in the liquid and then place a drop of the acid or base onto the paper. The experiment below also covers some other substances such as baking powder, vanilla essence and onions. Baking powder fizzes in acids but not in bases. Onions and vanilla essence lose their characteristic smell when in a basic solution.

It is important that learners do not place their faces or noses directly over or into the beaker when smelling the onions and vanilla essence. They must hold the beaker in one hand and use the other hand to waft (i.e. wave their hand back and forth) the smell towards their face.

Acids and bases are corrosive and can cause serious burns so must be handled with care.

A recommended experiment for informal assessment on redox reactions is also included. This experiment is split into three parts. Each part looks at a different type of redox reaction (displacement, synthesis and decomposition). Learners looked at the second two reactions in grade 10.

When burning magnesium ribbon remind learners to not look directly at the flame. Also, since this experiment involves Bunsen burners, learners must work in a well ventilated room and take the usual care with loose scarves and long hair.

Hydrogen peroxide can cause serious chemical burns. Learners must work carefully with this substance. If they spill any on themselves then they must rinse the affected area with plenty of water and call you. If necessary they may need to go to the bathroom to rinse affected clothing off. You or a learner should accompany them.

13.1 Acids and bases

Defining acids and bases

Exercise 13 – 1: Acids and bases

1. Identify the Bronsted-Lowry acid and the Bronsted-Lowry base in the following reactions:
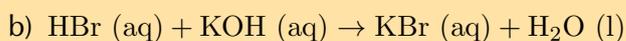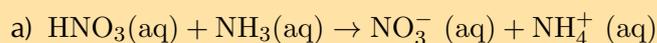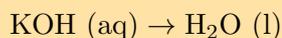
   a) $HNO_3(aq) + NH_3(aq) \rightarrow NO_3^-\ (aq) + NH_4^+\ (aq)$
   b) $HBr\ (aq) + KOH\ (aq) \rightarrow KBr\ (aq) + H_2O\ (l)$

   Solution:

   a) We break the reaction into two parts:

HNO_3 (aq) $\rightarrow NO_3^-$(aq) and

NH_3(aq) $\rightarrow NH_4^+$(aq)
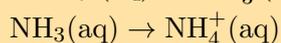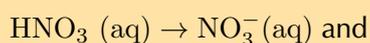
From this we see that the Bronsted-Lowry acid is HNO_3 and the Bronsted-Lowry base is NH_3.

b) We break the reaction into two parts:

HBr (aq) $\rightarrow KBr$ (aq) and
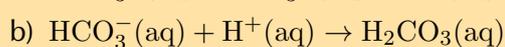
KOH (aq) $\rightarrow H_2O$ (l)

From this we see that the Bronsted-Lowry acid is HBr and the Bronsted-Lowry base is KOH.

2. a) Write a reaction equation to show HCO_3^- acting as an acid.

b) Write a reaction equation to show HCO_3^- acting as an base.

c) Compounds such as HCO_3^- are ...

Solution:

a) HCO_3^- (aq) $\rightarrow CO_3^{2-}$(aq) $+ H^+$(aq)

b) HCO_3^-(aq) $+ H^+$(aq) $\rightarrow H_2CO_3$(aq)

c) Amphoteric

Conjugate acid-base pairs

Exercise 13 – 2: Acids and bases

1. In each of the following reactions, label the conjugate acid-base pairs.

a) H_2SO_4(aq) $+ H_2O$ (l) $\rightarrow H_3O^+$(aq) $+ HSO_4^-$(aq)

b) NH_4^+(aq) $+ F^-$(aq) $\rightarrow HF$(aq) $+ NH_3$(aq)

c) H_2O (l) $+ CH_3COO^-$(aq) $\rightarrow CH_3COOH$ (aq) $+ OH^-$(aq)

d) H_2SO_4(aq) $+ Cl^-$(aq) $\rightarrow HCl$ (aq) $+ HSO_4^-$(aq)

Solution:

conjugate pair

$\mathbf{H_2SO_4}$ + $\mathbf{H_2O}$ $\longrightarrow$ $\mathbf{H_3O^+}$ + $\mathbf{HSO_4^-}$
acid 1   base 2   acid 2   base 1

conjugate pair

a)

conjugate pair

$\mathbf{NH_4^+}$ + $\mathbf{F^-}$ $\longrightarrow$ $\mathbf{HF}$ + $\mathbf{NH_3}$
acid 1 base 2   acid 2   base 1

conjugate pair

b)

$$\text{conjugate pair}$$

$$H_2O + CH_3COO^- \longrightarrow CH_3COOH + OH^-$$

acid 1   base 2   acid 2   base 1

conjugate pair

c)

conjugate pair

$$H_2SO_4 + Cl^- \longrightarrow HCl + HSO_4^-$$

acid 1   base 2   acid 2   base 1

conjugate pair

d)

2. Given the following reaction:

$$H_2O\ (l) + NH_3(aq) \rightarrow NH_4^+(aq) + OH^-(aq)$$

a) Write down which reactant is the base and which is the acid.

b) Label the conjugate acid-base pairs.

c) In your own words explain what is meant by the term conjugate acid-base pair.

Solution:

a) We break the reaction into two parts:

$H_2O\ (aq) \rightarrow OH^-(aq)$ and

$NH_3(aq) \rightarrow NH_4^+(aq)$
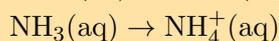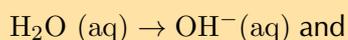
From this we see that the Bronsted-Lowry acid is H_2O and the Bronsted-Lowry base is NH_3.

conjugate pair

$$H_2O + NH_3 \longrightarrow NH_4^+ + OH^+$$

acid 1   base 2   acid 2   base 1

conjugate pair

b)
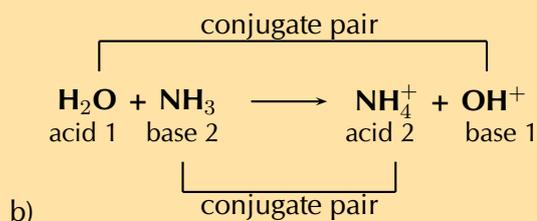
c) A conjugate acid-base pair is a reactant and product pair that is transformed into each other through the loss or gain of a proton. So for example an acid loses a proton to form a base. The acid and the resulting base are said to be a conjugate acid-base pair.

13.2 Acid-base reactions

Acid and metal hydroxide

Exercise 13 – 3:

1. Write a balanced equation for the reaction between HNO_3 and KOH.

   Solution:

   $HNO_3(aq) + KOH\ (aq) \rightarrow KNO_3\ (aq) + H_2O\ (l)$
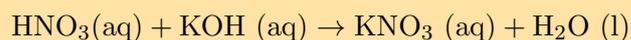
Acid and metal oxide

Exercise 13 – 4:

1. Write a balanced equation for the reaction between HBr and K_2O.

   Solution:

   $2HBr\ (aq) + K_2O\ (aq) \rightarrow 2KBr\ (aq) + H_2O\ (l)$
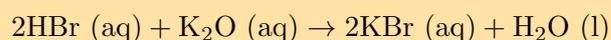
Acid and a metal carbonate

Exercise 13 – 5:

1. Write a balanced equation for the reaction between HCl and K_2CO_3.

   Solution:

   $2HCl\ (aq) + K_2CO_3(aq) \rightarrow 2KCl\ (aq) + H_2O\ (l) + CO_2(g)$

Exercise 13 – 6: Acids and bases

For each of the following reactants state what type of acid-base reaction the pair of reactants undergoes and write the balanced reaction equation.

1. HNO_3 and $Ca(OH)_2$

Solution:

Acid and metal hydroxide

$$2HNO_3(aq) + Ca(OH)_2(aq) \rightarrow Ca(NO_3)_2(aq) + 2H_2O \text{ (l)}$$
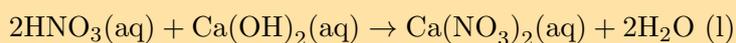
2. HCl and BeO

Solution:

Acid and metal oxide

$$2HCl \text{ (aq)} + BeO \text{ (aq)} \rightarrow BeCl_2(aq) + H_2O \text{ (l)}$$
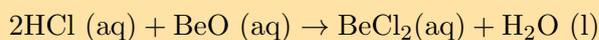
3. HI and K_2CO_3

Solution:

Acid and carbonate

$$2HI \text{ (aq)} + K_2CO_3(aq) \rightarrow 2KI \text{ (aq)} + H_2O \text{ (l)} + CO_2(g)$$

4. H_3PO_4 and KOH

Solution:

Acid and metal hydroxide

$$H_3PO_4(aq) + 3KOH \text{ (aq)} \rightarrow K_3PO_4(aq) + 3H_2O \text{ (l)}$$
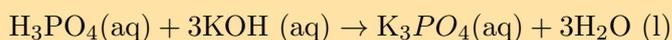
5. HCl and $MgCO_3$

Solution:

Acid and carbonate

$$2HCl \text{ (aq)} + MgCO_3(aq) \rightarrow MgCl_2(aq) + H_2O \text{ (l)} + CO_2(g)$$

6. HNO_3 and Al_2O_3

Solution:

Acid and metal oxide

$$6HNO_3(aq) + Al_2O_3(aq) \rightarrow 2Al(NO_3)_3(aq) + 3H_2O \text{ (l)}$$
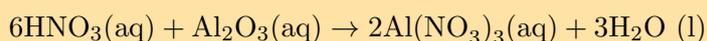
13.3   Redox reactions

Oxidation numbers

Exercise 13 – 7: Oxidation numbers

1. Give the oxidation numbers for each element in the following chemical compounds:

   a) MgF_2

   b) $CaCl_2$

   c) CH_4

d) $MgSO_4$

Solution:

a) In the compound MgF_2, the oxidation number of fluorine is -1 (rule 7).

Let the oxidation number of magnesium be x. We know that fluorine has an oxidation number of -1 and since there are two fluorine atoms in the compound, then the sum of the oxidation numbers of these two fluorine atoms is -2.

Putting this together gives:

$$x + (-2) = 0$$
$$= +2$$

So the oxidation number of magnesium is $+2$.

Magnesium has an oxidation number of $+2$ and fluorine has an oxidation number of -1.

b) This is an ionic compound composed of Ca^{2+} and Cl^- ions. Using rule 2 the oxidation number for the calcium ion is $+2$ and for the chlorine ion it is -1.

This then gives us a sum of 0 for the compound.

Calcium has an oxidation number of $+2$ and chlorine has an oxidation number of -1.

c) In the compound CH_4, the sum of the oxidation numbers must be 0 (rule 3).

Let the oxidation number of carbon be x. We know that hydrogen has an oxidation number of $+1$ (this is not a metal hydride) and since there are four hydrogen atoms in the molecule, then the sum of the oxidation numbers of these four hydrogen atoms is $+4$.

Putting this together gives:

$$x + (+4) = 0$$
$$= -4$$

So the oxidation number of carbon is -4.

Hydrogen has an oxidation number of $+1$ and carbon has an oxidation number of -4.

d) This is an ionic compound composed of Mg^{2+} and SO_4^{2-} ions. Using rule 2 the oxidation number for the magnesium ion is $+2$. In the polyatomic SO_4^{2-} ion, the sum of the oxidation numbers must be -2 (rule 4).

Let the oxidation number of sulfur be x. We know that oxygen has an oxidation number of -2 (it is not in a peroxide) and since there are four oxygen atoms in the sulfate ion, then the sum of the oxidation numbers of these four oxygen atoms is -8.

Putting this together gives:

$$x + (-8) = -2$$
$$x = -2 + 8$$
$$= +6$$

So the oxidation number of sulfur is $+6$.

Putting all the information together we find that magnesium has an oxidation number of $+2$, oxygen has an oxidation number of -2 and sulfur has an oxidation number of $+6$.

2. Compare the oxidation numbers of:

   a) nitrogen in:

   NO_2 and NO

   b) carbon in:

   CO_2 and CO

   c) chromium in:

   $Cr_2O_7^{2-}$ and CrO_4^{-}

   d) oxygen in:

   H_2O and H_2O_2

   e) hydrogen in:

   NaH and H_2O

   Solution:

   a) In the compound NO_2, the sum of the oxidation numbers must be 0 (rule 3).

   Let the oxidation number of nitrogen be x. We know that oxygen has an oxidation number of -2 (this is not a peroxide) and since there are two oxygen atoms in the molecule, then the sum of the oxidation numbers of these two oxygen atoms is -4.

   Putting this together gives:

   $$x + (-4) = 0$$
   $$= +4$$

   So the oxidation number of nitrogen is $+4$ in NO_2.

   In the compound NO, the sum of the oxidation numbers must be 0 (rule 3).

   We know that oxygen has an oxidation number of -2 (this is not a peroxide) and since there is only one oxygen atom in the molecule, then the nitrogen atom must have an oxidation number of $+2$.

   So the oxidation number of nitrogen is $+2$ in NO.

   Nitrogen has an oxidation number of $+4$ in NO_2 and $+2$ in NO.

   b) In the compound CO_2, the sum of the oxidation numbers must be 0 (rule 3).

   Let the oxidation number of carbon be x. We know that oxygen has an oxidation number of -2 (this is not a peroxide) and since there are two oxygen atoms in the molecule, then the sum of the oxidation numbers of these two oxygen atoms is -4.

   Putting this together gives:

   $$x + (-4) = 0$$
   $$= +4$$

   So the oxidation number of carbon is $+4$ in CO_2.

   In the compound CO, the sum of the oxidation numbers must be 0 (rule 3).

   We know that oxygen has an oxidation number of -2 (this is not a peroxide) and since there is only one oxygen atom in the molecule, then the carbon atom must have an oxidation number of $+2$.

   So the oxidation number of carbon is $+2$ in CO.

   Carbon has an oxidation number of $+4$ in CO_2 and $+2$ in CO.

c) In the compound $Cr_2O_7^{2-}$, the sum of the oxidation numbers must be -2 (rule 4).

Let the oxidation number of chromium be x. We know that oxygen has an oxidation number of -2 (this is not a peroxide) and since there are seven oxygen atoms in the molecule, then the sum of the oxidation numbers of these seven oxygen atoms is -14.

Putting this together gives:

$$2x + (-14) = -2$$
$$2x = -2 + 14$$
$$x = +6$$

So the oxidation number of chromium is $+6$ in $Cr_2O_7^{2-}$.

In the compound CrO_4^-, the sum of the oxidation numbers must be -1 (rule 4).

Let the oxidation number of chromium be x. We know that oxygen has an oxidation number of -2 (this is not a peroxide) and since there are four oxygen atoms in the molecule, then the sum of the oxidation numbers of these four oxygen atoms is -8.

Putting this together gives:

$$x + (-8) = -1$$
$$x = +7$$

So the oxidation number of chromium is $+7$ in CrO_4^-.

Chromium has an oxidation number of $+6$ in $Cr_2O_7^{2-}$ and $+7$ in CrO_4^-.

d) In the compound H_2O, the sum of the oxidation numbers must be 0 (rule 3).

This compound is not a metal hydride, so the oxidation number of hydrogen is $+1$ (rule 6). This compound is also not a peroxide, so the oxidation number of oxygen is -2 (rule 5).

We confirm that this gives us a sum of 0: $2(+1)+(-2) = 0$. So the oxidation number of oxygen is -2 in H_2O.

In the compound H_2O_2, the sum of the oxidation numbers must be 0 (rule 4).

Let the oxidation number of oxygen be x (this is a peroxide and so oxygen does not have an oxidation number of -2). We know that hydrogen has an oxidation number of $+1$ (this is not a metal hydride) and since there are two hydrogen atoms in the molecule, the sum of the oxidation numbers of these two hydrogen atoms is $+2$.

Putting this together gives:

$$2x + (+2) = 0$$
$$x = -1$$

So the oxidation number of oxygen is -1 in H_2O_2. (Note that this confirms what has been stated about peroxides (see rule 5).)

Oxygen has an oxidation number of -2 in H_2O and -1 in H_2O_2.

e) In the compound NaH, the sum of the oxidation numbers must be 0 (rule 3).

This compound is a metal hydride, so the oxidation number of hydrogen is -1 (rule 6). This compound is also an ionic compound with sodium ions and so sodium must have an oxidation number of $+1$ (rule 2).

We confirm that this gives us a sum of 0: $+1 + (-1) = 0$ So the oxidation number of hydrogen is -1 in NaH.

In the compound H_2O, the sum of the oxidation numbers must be 0 (rule 4).

This compound is not a metal hydride, so the oxidation number of hydrogen is $+1$ (rule 6). This compound is also not a peroxide, so the oxidation number of oxygen is -2 (rule 5).

So the oxidation number of hydrogen is $+2$ in H_2O.

Hydrogen has an oxidation number of -1 in NaH and $+1$ in H_2O.

3. Give the oxidation numbers for each of the elements in all the compounds. State if there is any difference between the oxidation number of the element in the reactant and the element in the product.

   a) $C\ (s) + O_2(g) \rightarrow CO_2(g)$
   b) $N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$
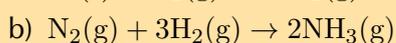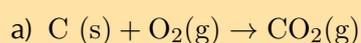
Solution:

   a) In the compound C, the oxidation number of carbon is 0 (rule 1).

   In the compound O_2, the oxidation number of oxygen is 0 (rule 1).

   In the compound CO_2, the sum of the oxidation numbers must be 0 (rule 3).

   Let the oxidation number of carbon be x. We know that oxygen has an oxidation number of -2 (this is not a peroxide) and since there are two oxygen atoms in the molecule, then the sum of the oxidation numbers of these two oxygen atoms is -4.

   Putting this together gives:

   $$x + (-4) = 0$$
   $$x = +4$$

   So the oxidation number of carbon is $+4$.

   The oxidation number of carbon in the products is 0 and in the reactants it is $+4$ The oxidation number has increased (become more positive).

   The oxidation number of oxygen in the products is 0 and in the reactants it is -2 The oxidation number has decreased (become more negative).

   b) In the compound N_2, the oxidation number of nitrogen is 0 (rule 1).

   In the compound H_2, the oxidation number of hydrogen is 0 (rule 1).

   In the compound NH_3, the sum of the oxidation numbers must be 0 (rule 3).

   Let the oxidation number of nitrogen be x. We know that hydrogen has an oxidation number of $+1$ (this is not a metal hydride) and since there are three hydrogen atoms in the molecule, then the sum of the oxidation numbers of these three hydrogen atoms is $+3$.

   Putting this together gives:

   $$x + (+3) = 0$$
   $$x = -3$$

   So the oxidation number of nitrogen is -3.

   The oxidation number of hydrogen in the products is 0 and in the reactants it is $+1$ The oxidation number has increased (become more positive).

   The oxidation number of nitrogen in the products is 0 and in the reactants it is -3 The oxidation number has decreased (become more negative).

1. Consider the following chemical equations:

$$\text{Fe (s)} \rightarrow \text{Fe}^{2+}\text{(aq)} + 2e^-$$

$$4\text{H}^+\text{(aq)} + \text{O}_2\text{(g)} + 4e^- \rightarrow 2\text{H}_2\text{O (l)}$$
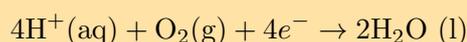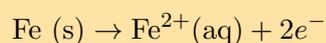
Which one of the following statements is correct?

   a) Fe is oxidised and H^+ is reduced
   b) Fe is reduced and O_2 is oxidised
   c) Fe is oxidised and O_2 is reduced
   d) Fe is reduced and H^+ is oxidised

(DoE Grade 11 Paper 2, 2007)

Solution:

Fe is oxidised and O_2 is reduced

2. Which one of the following reactions is a redox reaction?

   a) $\text{HCl (aq)} + \text{NaOH (aq)} \rightarrow \text{NaCl (aq)} + \text{H}_2\text{O (l)}$
   b) $\text{AgNO}_3\text{(aq)} + \text{NaI (aq)} \rightarrow \text{AgI (s)} + \text{NaNO}_3\text{(aq)}$
   c) $2\text{FeCl}_3\text{(aq)} + 2\text{H}_2\text{O (l)} + \text{SO}_2\text{(aq)} \rightarrow \text{H}_2\text{SO}_4\text{(aq)} + 2\text{HCl (aq)} + 2\text{FeCl}_2\text{(aq)}$
   d) $\text{BaCl}_2\text{(aq)} + \text{MgSO}_4\text{(aq)} \rightarrow \text{MgCl}_2\text{(aq)} + \text{BaSO}_4\text{(s)}$

Solution:

$$\text{FeCl}_3\text{(aq)} + 2\text{H}_2\text{O (l)} + \text{SO}_2\text{(aq)} \rightarrow \text{H}_2\text{SO}_4\text{(aq)} + 2\text{HCl (aq)} + 2\text{FeCl}_2\text{(aq)}$$
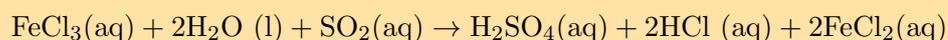
3. Balance the following redox reactions:

   a) $\text{Zn (s)} + \text{Ag}^+\text{(aq)} \rightarrow \text{Zn}^{2+}\text{(aq)} + \text{Ag (s)}$
   b) $\text{Cu}^{2+}\text{(aq)} + \text{Cl}^-\text{(aq)} \rightarrow \text{Cu (s)} + \text{Cl}_2\text{(g)}$
   c) $\text{Pb}^{2+}\text{(aq)} + \text{Br}^-\text{(aq)} \rightarrow \text{Pb (s)} + \text{Br}_2\text{(aq)}$
   d) $\text{HCl (aq)} + \text{MnO}_2\text{(s)} \rightarrow \text{Cl}_2\text{(g)} + \text{Mn}^{2+}\text{(aq)}$
      This reaction takes place in an acidic medium.

Solution:

a) Write a reaction for each compound:

$$Zn \rightarrow Zn^{2+}$$

$$Ag^+ \rightarrow Ag$$

The atoms are balanced.

Add electrons to each reaction so that the charges balance. We add the electrons to the side with the greater positive charge.

$$Zn \rightarrow Zn^{2+} + 2e^-$$

$$Ag^+ + e^- \rightarrow Ag$$
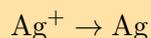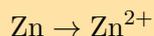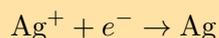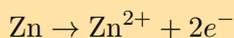
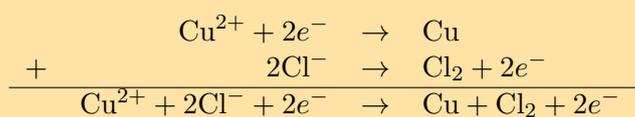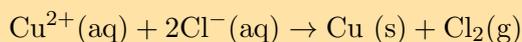We now make sure that the number of electrons in both reactions is the same.

The reaction for zinc has two electrons, while the reaction for silver has one electron. So we must multiply the reaction for silver by 2 to ensure that the charges balance.

$$Zn \rightarrow Zn^{2+} + 2e^-$$

$$2Ag^+ + 2e^- \rightarrow 2Ag$$

We combine the two half-reactions:

$$
\begin{array}{rcl}
Zn & \rightarrow & Zn^{2+} + 2e^- \\
+ \quad 2Ag^+ + 2e^- & \rightarrow & 2Ag \\
\hline
Zn + Ag^+ + 2e^- & \rightarrow & Zn^{2+} + 2Ag + 2e^-
\end{array}
$$

Cancelling out the electrons gives:

$$Zn \ (s) + 2Ag^+(aq) \rightarrow Zn^{2+}(aq) + 2Ag \ (s)$$

b) Write a reaction for each compound:

$$Cu^{2+} \rightarrow Cu$$

$$Cl^- \rightarrow Cl_2$$

Balance the atoms:

$$Cu^{2+} \rightarrow Cu$$

$$2Cl^- \rightarrow Cl_2$$

Add electrons to each reaction so that the charges balance. We add the electrons to the side with the greater positive charge.

$$Cu^{2+} + 2e^- \rightarrow Cu$$

$$2Cl^- \rightarrow Cl_2 + 2e^-$$

We now make sure that the number of electrons in both reactions is the same.

The reaction for copper has two electrons and the reaction for chlorine also has two electrons. So the charges balance.

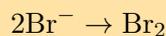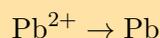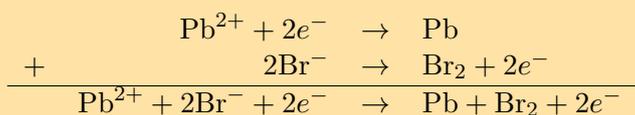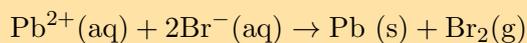We combine the two half-reactions:

$$\begin{array}{rcl} Cu^{2+} + 2e^- & \rightarrow & Cu \\ + \qquad\qquad 2Cl^- & \rightarrow & Cl_2 + 2e^- \\ \hline Cu^{2+} + 2Cl^- + 2e^- & \rightarrow & Cu + Cl_2 + 2e^- \end{array}$$

Cancelling out the electrons gives:

$$Cu^{2+}(aq) + 2Cl^-(aq) \rightarrow Cu\ (s) + Cl_2(g)$$

c) Write a reaction for each compound:

$$Pb^{2+} \rightarrow Pb$$

$$Br^- \rightarrow Br_2$$

Balance the atoms:

$$Pb^{2+} \rightarrow Pb$$

$$2Br^- \rightarrow Br_2$$

Add electrons to each reaction so that the charges balance. We add the electrons to the side with the greater positive charge.

$$Pb^{2+} + 2e^- \rightarrow Pb$$

$$2Br^- \rightarrow Br_2 + 2e^-$$

We now make sure that the number of electrons in both reactions is the same.

The reaction for lead has two electrons and the reaction for bromine also has two electrons. So the charges balance.

We combine the two half-reactions:

$$\begin{array}{rcl} Pb^{2+} + 2e^- & \rightarrow & Pb \\ + \qquad\qquad 2Br^- & \rightarrow & Br_2 + 2e^- \\ \hline Pb^{2+} + 2Br^- + 2e^- & \rightarrow & Pb + Br_2 + 2e^- \end{array}$$

Cancelling out the electrons gives:

$$Pb^{2+}(aq) + 2Br^-(aq) \rightarrow Pb\ (s) + Br_2(g)$$

d) Write a reaction for each compound:

$$HCl \rightarrow Cl_2$$

$$MnO_2 \rightarrow Mn^{2+}$$
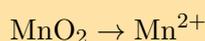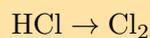
Balance the atoms.

In the first reaction we have 1 chlorine atom on the left hand side and 2 chlorine atoms on the right hand side. So we multiply the left hand side by 2:

$$2HCl \rightarrow Cl_2$$
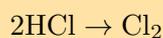
Now we note that there are 2 hydrogen atoms on the left hand side and no hydrogen atoms on the right hand so we add 2 hydrogen ions to the right hand side:

$$2HCl \rightarrow Cl_2 + 2H^+$$

The equation is now balanced.

For the second equation we need to add two water molecules to the right hand side:

$$MnO_2 \rightarrow Mn^{2+} + 2H_2O$$
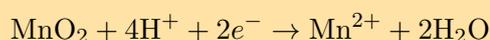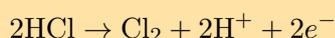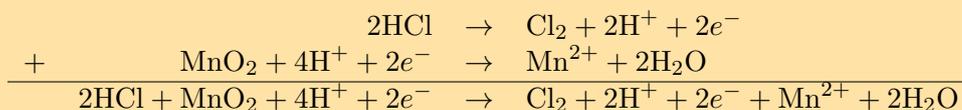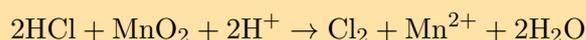
We now need to add four hydrogen ions to the left hand side to balance the hydrogens:

$$MnO_2 + 4H^+ \rightarrow Mn^{2+} + 2H_2O$$

This equation is now balanced.

Add electrons to each reaction so that the charges balance. We add the electrons to the side with the greater positive charge.

$$2HCl \rightarrow Cl_2 + 2H^+ + 2e^-$$

$$MnO_2 + 4H^+ + 2e^- \rightarrow Mn^{2+} + 2H_2O$$

We now make sure that the number of electrons in both reactions is the same.

The reaction for chlorine has two electrons and the reaction for manganese also has two electrons. So the charges balance.

We combine the two half-reactions:

$$
\begin{array}{rcl}
2HCl & \rightarrow & Cl_2 + 2H^+ + 2e^- \\
+ \quad MnO_2 + 4H^+ + 2e^- & \rightarrow & Mn^{2+} + 2H_2O \\
\hline
2HCl + MnO_2 + 4H^+ + 2e^- & \rightarrow & Cl_2 + 2H^+ + 2e^- + Mn^{2+} + 2H_2O
\end{array}
$$

Cancelling out the electrons and hydrogen ions gives:

$$2HCl + MnO_2 + 2H^+ \rightarrow Cl_2 + Mn^{2+} + 2H_2O$$

# 13.4   Chapter summary

Exercise 13 – 9:

1. Give one word/term for each of the following descriptions:

    a) A chemical reaction during which electrons are transferred.

    b) A substance that takes up protons and is said to be a proton acceptor.

    c) The loss of electrons by a molecule, atom or ion.

    d) A substance that can act as either an acid or as a base.

    Solution:

    a) Redox reaction

b) Bronsted-Lowry base

c) Oxidation

d) Amphoteric substance

2. Given the following reaction:

$$HNO_3(aq) + NH_3(aq) \rightarrow NH_4^+(aq) + NO_3^-(aq)$$
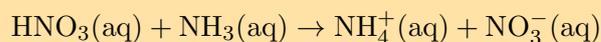
Which of the following statements is true?

a) HNO_3 accepts protons and is a Bronsted-Lowry base

b) HNO_3 accepts protons and is a Bronsted-Lowry acid

c) NH_3 donates protons and is a Bronsted-Lowry acid

d) HNO_3 donates protons and is a Bronsted-Lowry acid

Solution:

HNO_3 donates protons and is a Bronsted-Lowry acid

3. When chlorine water (Cl_2 dissolved in water) is added to a solution of potassium bromide, bromine is produced. Which one of the following statements concerning this reaction is correct?

a) Br is oxidised

b) Cl_2 is oxidised

c) Br^- is the oxidising agent

d) Cl^- is the oxidising agent

(IEB Paper 2, 2005)

Solution:

Br^- is the oxidising agent (chlorine is reduced in this reaction)

4. Given the following reaction:

$$H_2SO_3(aq) + 2KOH\ (aq) \rightarrow K_2SO_3(aq) + 2H_2O\ (l)$$
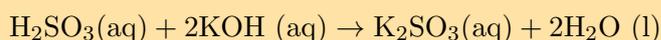
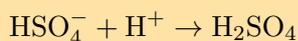Which substance acts as the acid and which substance acts as the base?

Solution:

H_2SO_3 is the Bronsted-Lowry acid and KOH is the Bronsted-Lowry base.

5. Use balanced chemical equations to explain why HSO_4^- is amphoteric.

Solution:

HSO_4^- accepts a proton to form sulfuric acid (in other words it acts as a Bronsted-Lowry base):

$$HSO_4^- + H^+ \rightarrow H_2SO_4$$

HSO_4^- also donates a proton to form the sulfate ion (in other words it acts as a Bronsted-Lowry acid):

$$HSO_4^- \rightarrow SO_4^{2-} + H^+$$

Since HSO_4^- can act as either an acid or as a base it is said to be amphoteric.

6. Milk of magnesia is an example of an antacid and is only slightly soluble in water. Milk of magnesia has the chemical formula $Mg(OH)_2$ and is taken as a powder dissolved in water. Write a balanced chemical equation to show how the antacid reacts with hydrochloric acid (the main acid found in the stomach).
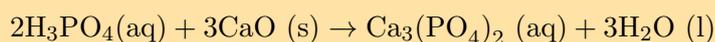
Solution:

$$2HCl\ (aq) + Mg(OH)_2(aq) \rightarrow MgCl_2(aq) + 2H_2O\ (l)$$
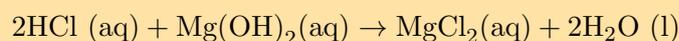
7. In an experiment sodium carbonate was used to neutralise a solution of hydrochloric acid. Write a balanced chemical equation for this reaction.

Solution:

$$2HCl\ (aq) + Na_2CO_3(aq) \rightarrow 2NaCl\ (aq) + H_2O\ (l) + CO_2(g)$$

8. Write a balanced chemical equation for phosphoric acid (H_3PO_4) reacting with calcium oxide (CaO).

Solution:

$$2H_3PO_4(aq) + 3CaO\ (s) \rightarrow Ca_3(PO_4)_2\ (aq) + 3H_2O\ (l)$$

9. Label the acid-base conjugate pairs in the following equation:

$$HCO_3^-\ (aq) + H_2O \rightarrow CO_3^{2-}\ (aq) + H_3O^+(aq)$$

Solution:

conjugate pair

$$HCO_3^- + H_2O \longrightarrow H_3O^+ + CO_3^{2-}$$

acid 1   base 2   acid 2   base 1

conjugate pair
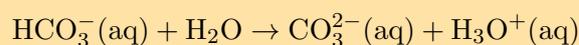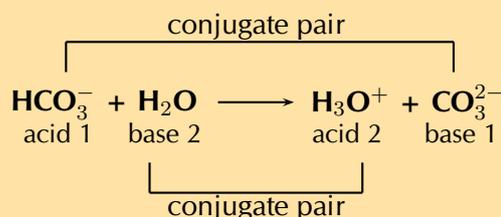
10. Look at the following reaction:

$$2H_2O_2(l) \rightarrow 2H_2O\ (l) + O_2(g)$$
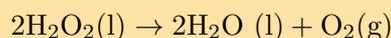
a) What is the oxidation number of the oxygen atom in each of the following compounds?

    i. H_2O_2

    ii. H_2O

    iii. O_2

b) Does the hydrogen peroxide (H_2O_2) act as an oxidising agent or a reducing agent or both, in the above reaction? Give a reason for your answer.

Solution:

a)   i. In the compound H_2O_2, the sum of the oxidation numbers must be 0 (rule 4).

Let the oxidation number of oxygen be x (this is a peroxide and so oxygen does not have an oxidation number of -2). We know that hydrogen has an oxidation number of $+1$ (this is not a metal hydride) and since there are two hydrogen atoms in the molecule, the sum of the oxidation numbers of these two hydrogen atoms is $+2$.

Putting this together gives:

$$2x + (+2) = 0$$
$$x = -1$$

So the oxidation number of oxygen is -1 in H_2O_2. (Note that this confirms what has been stated about peroxides (see rule 5).)

  ii. In the compound H_2O, the sum of the oxidation numbers must be 0 (rule 3).

This compound is not a metal hydride, so the oxidation number of hydrogen is $+1$ (rule 6). This compound is also not a peroxide, so the oxidation number of oxygen is -2 (rule 5).

We confirm that this gives us a sum of 0: $2(+1) + (-2) = 0$. So the oxidation number of oxygen is -2 in H_2O.

  iii. In the compound O_2, the oxidation number of oxygen is 0.

b) Hydrogen peroxide (H_2O_2) acts as both an oxidising agent and a reducing agent in the above reaction. In this example there is only one reactant and this reactant decomposes into two parts.

11. Challenge question: Zinc reacts with aqueous copper sulfate ($CuSO_4(aq)$) to form zinc sulfate ($ZnSO_4(aq)$) and copper ions. What is the oxidation number for each element in the reaction?
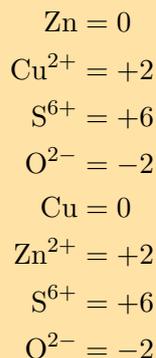
Solution:

Start by writing the balanced equation:

$$\text{Zn (s)} + \text{CuSO}_4\text{(aq)} \rightarrow \text{ZnSO}_4\text{aq} + \text{Cu (s)}$$
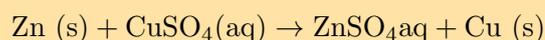
Next we determine the oxidation number of each element in the reactants. Zinc is an element and so has oxidation number of 0 (rule 1). The copper sulfate consists of Cu^{2+} ions and SO_4 ions. The copper ions will have an oxidation number of $+2$ (rule 3). Oxygen usually has an oxidation number of -2 (rule 6). In the polyatomic SO_4^{2-} ion, the sum of the oxidation numbers must be -2 (rule 4). Since there are four oxygen atoms in the ion, the total charge of the oxygen is -8. If the overall charge of the ion is -2, then the oxidation number of sulfur must be $+6$. ($x + 4(-2) = -2$ therefore $x - 8 = -2$).

Next we determine the oxidation number of each element in the products. Copper is an element and so has oxidation number of 0 (rule 1). The zinc sulfate consists of Zn^{2+} ions and SO_4 ions. The zinc ions will have an oxidation number of $+2$ (rule 3). Oxygen usually has an oxidation number of -2 (rule 6). In the polyatomic SO_4^{2-} ion, the sum of the oxidation numbers must be -2 (rule 4). Since there are four oxygen atoms in the ion, the total charge of the oxygen is -8. If the overall charge of the ion is -2, then the oxidation number of sulfur must be $+6$. ($x + 4(-2) = -2$ therefore $x - 8 = -2$).
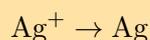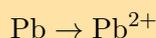
Write the final answer:

$$Zn = 0$$
$$Cu^{2+} = +2$$
$$S^{6+} = +6$$
$$O^{2-} = -2$$
$$Cu = 0$$
$$Zn^{2+} = +2$$
$$S^{6+} = +6$$
$$O^{2-} = -2$$

12. Balance the following redox reactions:

   a) $Pb\ (s) + Ag^{+}(aq) \rightarrow Pb^{2+}(aq) + Ag\ (s)$

   b) $Zn^{2+}(aq) + I^{-}(aq) \rightarrow Zn\ (s) + I_2(g)$

   c) $Fe^{3+}\ (aq) + NO_2(aq) \rightarrow Fe\ (s) + NO_3^{-}(aq)$

   This reaction takes place in an acidic medium.

   Solution:

   a) Write a reaction for each compound:

   $$Pb \rightarrow Pb^{2+}$$

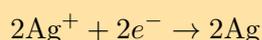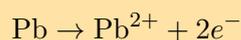 $$Ag^{+} \rightarrow Ag$$

   The atoms are balanced.

   Add electrons to each reaction so that the charges balance. We add the electrons to the side with the greater positive charge.

   $$Pb \rightarrow Pb^{2+} + 2e^{-}$$

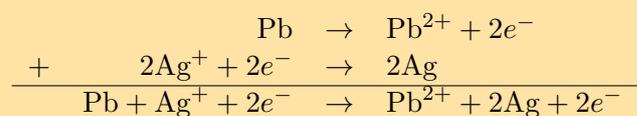 $$Ag^{+} + e^{-} \rightarrow Ag$$

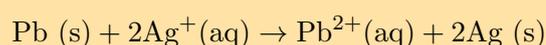   We now make sure that the number of electrons in both reactions is the same.

   The reaction for lead has two electrons, while the reaction for silver has one electron. So we must multiply the reaction for silver by 2 to ensure that the charges balance.
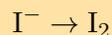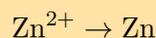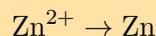
   $$Pb \rightarrow Pb^{2+} + 2e^{-}$$

   $$2Ag^{+} + 2e^{-} \rightarrow 2Ag$$

   We combine the two half-reactions:

   $$\begin{array}{rrcl} & Pb & \rightarrow & Pb^{2+} + 2e^{-} \\ + & 2Ag^{+} + 2e^{-} & \rightarrow & 2Ag \\ \hline & Pb + Ag^{+} + 2e^{-} & \rightarrow & Pb^{2+} + 2Ag + 2e^{-} \end{array}$$

   Cancelling out the electrons gives:

   $$Pb\ (s) + 2Ag^{+}(aq) \rightarrow Pb^{2+}(aq) + 2Ag\ (s)$$

b) Write a reaction for each compound:

$$Zn^{2+} \rightarrow Zn$$

$$I^- \rightarrow I_2$$

Balance the atoms:

$$Zn^{2+} \rightarrow Zn$$

$$2I^- \rightarrow I_2$$

Add electrons to each reaction so that the charges balance. We add the electrons to the side with the greater positive charge.

$$Zn^{2+} + 2e^- \rightarrow Zn$$
$$2I^- \rightarrow I_2 + 2e^-$$

We now make sure that the number of electrons in both reactions is the same.

The reaction for copper has two electrons and the reaction for chlorine also has two electrons. So the charges balance.

We combine the two half-reactions:

$$
\begin{array}{rcl}
Zn^{2+} + 2e^- & \rightarrow & Zn \\
+ \quad\quad 2I^- & \rightarrow & I_2 + 2e^- \\
\hline
Zn^{2+} + 2I^- + 2e^- & \rightarrow & Zn + I_2 + 2e^-
\end{array}
$$

Cancelling out the electrons gives:

$$Zn^{2+}(aq) + 2I^-(aq) \rightarrow Zn\ (s) + I_2(g)$$

c) Write a reaction for each compound:

$$Fe^{3+} \rightarrow Fe$$

$$NO_2 \rightarrow NO_3^-$$

Balance the atoms.

The first equation is balanced.

For the second equation we need to add one water molecule to the left hand side:

$$H_2O + NO_2 \rightarrow NO_3^-$$

We now need to add two hydrogen ions to the right hand side to balance the hydrogens:

$$H_2O + NO_2 \rightarrow NO_3^- + H^+$$

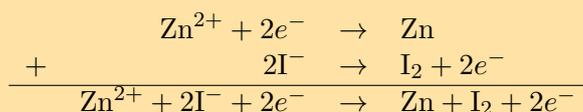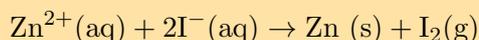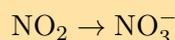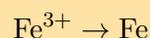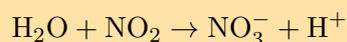This equation is now balanced.

Add electrons to each reaction so that the charges balance. We add the electrons to the side with the greater positive charge.
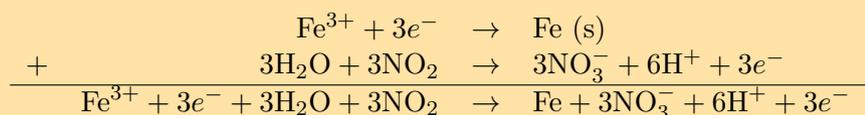
$$Fe^{3+} + 3e^- \rightarrow Fe\ (s)$$

$$H_2O + NO_2 \rightarrow NO_3^- + 2H^+ + e^-$$
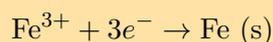
We now make sure that the number of electrons in both reactions is the same.

The reaction for iron has two electrons and but the reaction for NO_2 only has one. So we multiply the second equation by 3:
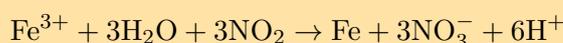
$$3H_2O + 3NO_2 \rightarrow 3NO_3^- + 6H^+ + 3e^-$$
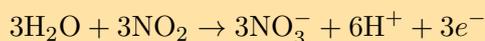
The number of electrons is now the same.

We combine the two half-reactions:

$$
\begin{array}{rcl}
Fe^{3+} + 3e^- & \rightarrow & Fe\ (s) \\
+ \quad\quad 3H_2O + 3NO_2 & \rightarrow & 3NO_3^- + 6H^+ + 3e^- \\
\hline
Fe^{3+} + 3e^- + 3H_2O + 3NO_2 & \rightarrow & Fe + 3NO_3^- + 6H^+ + 3e^-
\end{array}
$$

Cancelling out the electrons gives:

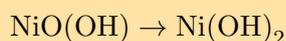$$Fe^{3+} + 3H_2O + 3NO_2 \rightarrow Fe + 3NO_3^- + 6H^+$$

13. A nickel-cadmium battery is used in various portable devices such as calculators. This battery uses a redox reaction to work. The equation for the reaction is:

$$Cd\ (s) + NiO(OH)\ (s) \rightarrow Cd(OH)_2(s) + Ni(OH)_2(s)$$
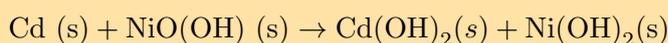
This reaction takes place in a basic medium. Balance the equation.

Solution:

Write a reaction for each compound:

$$Cd \rightarrow Cd(OH)_2$$

$$NiO(OH) \rightarrow Ni(OH)_2$$

Balance the atoms.

In the first reaction we have 0 oxygen atoms and 0 hydrogen atoms on the left hand side. On the right hand side we have 2 oxygen atoms and 0 hydrogen atoms (or two hydroxide ions). So we add two hydroxide ions to the left hand side (we don't add water as this would cause the number of hydrogen ions to be unbalanced):

$$CdOH^- \rightarrow Cd(OH)_2$$
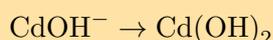
The equation is now balanced.

For the second equation we we have 2 oxygen atoms and 1 hydrogen atom on the left hand side (or one hydroxide ion and one oxygen atom). On the right hand side we have 2 oxygen atoms and 0 hydrogen atoms (or two hydroxide ions). So we add one hydroxide ion to the right hand side:

$$NiO(OH) \rightarrow Ni(OH)_2 + OH^-$$
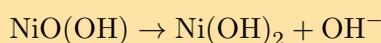
We now need to add one water molecule to the left hand side to balance the hydrogens and oxygens:
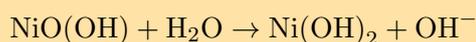
$$NiO(OH) + H_2O \rightarrow Ni(OH)_2 + OH^-$$

This equation is now balanced.

Add electrons to each reaction so that the charges balance. We add the electrons to the side with the greater positive charge.

$$CdOH^- \rightarrow Cd(OH)_2 + 2e^-$$

$$NiO(OH) + H_2O + e^- \rightarrow Ni(OH)_2 + OH^-$$

We now make sure that the number of electrons in both reactions is the same.

The reaction for cadmium has two electrons, but the reaction for nickel only has one electron. So we multiply the reaction for nickel by 2:

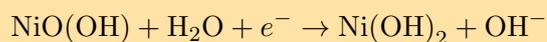$$2NiO(OH) + 2H_2O + 2e^- \rightarrow 2Ni(OH)_2 + 2OH^-$$
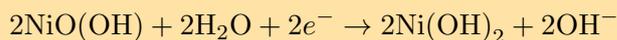
The charges are now balanced.

We combine the two half-reactions:

$$
\begin{array}{rcl}
CdOH^- & \rightarrow & Cd(OH)_2 + 2e^- \\
+ \quad 2NiO(OH) + 2H_2O + 2e^- & \rightarrow & 2Ni(OH)_2 + 2OH^- \\
\hline
CdOH^- + 2NiO(OH) + 2H_2O + 2e^- & \rightarrow & Cd(OH)_2 + 2e^- + 2Ni(OH)_2 + 2OH^-
\end{array}
$$

Cancelling out the electrons gives:

$$Cd\ (s) + 2NiO(OH)\ (s) + 2H_2O\ (l) \rightarrow Cd(OH)_2(s) + 2Ni(OH)_2(s) + 2OH^-(aq)$$
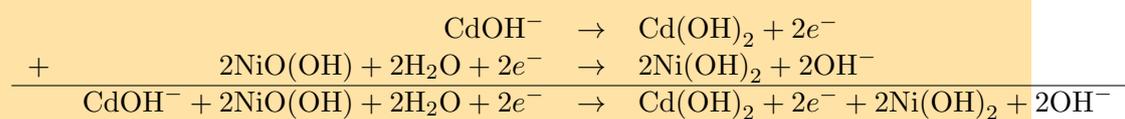
CHAPTER 14

The lithosphere

14.1   Introduction

This chapter looks at the lithosphere and explores mining in more detail. Mining is very important in South Africa as a large part of the economy depends on gold, diamond and coal mining. This chapter looks at the history of mankind, what the lithosphere is, what is in the lithosphere and then goes on to look at mining. The general techniques used across all types of mining are looked at and then gold mining is explored in greater detail. Learners can do the mining section as a project (either in groups or individually) that they then present to the class. If learners do a project ensure that each mineral listed in CAPs (iron, phosphate, coal, diamond, copper, platinum, zinc, chrome, asbestos, manganese and gold) is covered by at least one learner in the class.
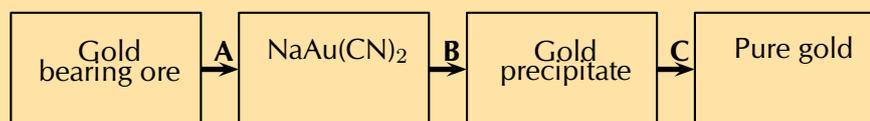
This topic provides a great opportunity to look at social justice and economic concerns. The various class discussions and debates provided give learners the chance to think about some critical issues in South Africa.

14.3   Mining and mineral processing

Characteristics and uses of gold

Exercise 14 – 1: Gold mining

1. In Mapungubwe (in the Limpopo Province) there is evidence of gold mining in South Africa as early as 1200. Today, South Africa is a world leader in the technology of gold mining. The following flow diagram illustrates some of the most important steps in the recovery of gold from its ore.

```
┌──────────────┐  A  ┌──────────────┐  B  ┌──────────────┐  C  ┌──────────────┐
│     Gold     │ ──> │  NaAu(CN)₂   │ ──> │     Gold     │ ──> │  Pure gold   │
│  bearing ore │     │              │     │  precipitate │     │              │
└──────────────┘     └──────────────┘     └──────────────┘     └──────────────┘
```

$Gold\ bearing\ ore \xrightarrow{A} NaAu(CN)_2 \xrightarrow{B} Gold\ precipitate \xrightarrow{C} Pure\ gold$

a) Name the process indicated by A.

b) During process A, gold is removed from the ore. Is gold oxidised or reduced during this process?

c) Use oxidation numbers to explain your answer to the question above.

d) Name the chemical substance that is used in process B.

e) During smelting (illustrated by C in the diagram), gold is sent to a large oven called a furnace. Why do you think this process is needed, and explain what happens to the gold during this process.

Solution:

a) Oxidation of gold by sodium cyanide solution.

b) Gold is oxidised.

c) The equation for the reaction is:

$$4Au + 8NaCN + O_2 + 2H_2O \rightarrow 4NaAu(CN)_2 + 4NaOH$$

Refer to chapter 13, types of reactions to calculate oxidation numbers. In the reactants gold has an oxidation number of 0. In the products gold occurs as $NaAu(CN)_2$. The cyanide ion (CN^-) has an oxidation number of -1. The sodium ion (Na^+) has an oxidation number of $+1$. The overall compound must have a total oxidation number of 0. Let gold's oxidation number be x. Then:

$$+1 + x + 2(-1) = 0$$
$$x = +1$$

So gold has an oxidation number of $+1$ in the products. Since this is more positive (greater than) the oxidation number of gold in the reactants gold has lost electrons and so being oxidised.

d) Zinc

e) The high temperature used during smelting causes the gold to become a liquid. This can then be removed from the molten mixture.

14.5   Summary

Exercise 14 – 2:

1. Give one word/term for each of the following descriptions:

   a) The part of the Earth that includes the crust and in which all minerals are found.

   b) The process in which minerals are extracted from the ores.

   c) An age in which humans discovered the use of fire to improve the properties of stone.

   Solution:

   a) Lithosphere

   b) Minerals processing or extraction

   c) Middle Stone Age

2. Read the following extract and answer the questions that follow.

   More profits, more poisons

   Since the last three decades gold miners have made use of cyanidation to recover gold from the ore. Over 99% of gold from ore can be extracted in this way. It

allows miners to obtain gold flakes – too small for the eye to see. Gold can also be extracted from the waste of old operations which sometimes left as much as a third of the gold behind.

The left-over cyanide can be re-used, but is more often stored in a pond behind a dam or even dumped directly into a local river. A teaspoonful of two-percent solution of cyanide can kill a human adult.

Mining companies insist that cyanide breaks down when exposed to sunlight and oxygen which render it harmless. They also point to scientific studies that show that cyanide swallowed by fish will not 'bio-accumulate', which means it does not pose a risk to anyone who eats the fish. In practise, cyanide solution that seeps into the ground will not break down because of the absence of sunlight. If the cyanide solution is very acidic, it could turn into cyanide gas which is toxic to fish. On the other hand, if the solution is alkaline, the cyanide does not break down.

There are no reported cases of human death from cyanide spills. If you don't see corpses, everything is okay.

a) What is cyanidation?

b) What type of chemical reaction takes place during this process: precipitation; acid-base; redox?

c) Is the solution after cyanidation acidic, basic or neutral?

d) How is the solid gold recovered from the solution?

e) Refer to cyanidation and discuss the meaning of the heading of the extract: More profits, more poisons.

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Solution:

a) Cyanidation is the addition of cyanide to the gold ore.

b) Redox reaction

c) acidic

d) By addition of zinc metal and then heating in a furnace.

e) Cyanidation extracts over 99% of gold from the ore, this leads to more (i.e. process is very effective) profits. However, left over cyanide can end up in rivers and aquifers that can be (i.e. negative effect of process on environment) harmful to life.

GRADE 11
PHYSICAL SCIENCES