Appendix 4-B

Insights from one-dimensional linearized Pierce theory about wideband traveling wave tubes with high space charge

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INTRODUCTION

There are several reasons why an examination of basic 1D analytic linear TWT theory is pertinent to current research and technology development. For example, ultrawideband TWT development for ECM applications requires careful attention to what happens at the band edges [1]. Teasing out additional performance at these band edges is a very high priority for some recent applications. Also, recent research shows that the level of harmonic distortion and harmonic injection for linearization depend on the linear growth rate of the frequency corresponding to the harmonic [2]. Therefore it is useful to have a good understanding of TWT growth rate behavior, even at the edges of positive gain. As the discussion in this paper illustrates, accurate theoretical predictions of gain near band edges requires a highly precise knowledge of the TWT circuit's phase velocities and space charge characteristics. In a related issue, accurate evaluation of the space charge reduction factor, R_{SC}, used in 1D TWT modeling [3-5] requires an *a priori* choice of using either the "cold circuit" wavenumber,

$$\beta_{cc} = \frac{\omega}{v_{cc}(\omega)} \tag{1}$$

or the "electronic" wavenumber

$$\beta_e = \frac{\omega}{v_0} \tag{2}$$

as an approximate substitute for the "hot" wavenumber, $\mathrm{Re}(\beta_z)$ of the growing wave on the TWT circuit. Above, $v_{cc}(\omega)$ is the phase velocity of a wave at frequency ω, in the absence of the electron beam. v_0 is the initial velocity of the electron beam. The results discussed in this paper provide guidance for the best choice of approximation for $\mathrm{Re}(\beta_z)$. A third point relates to appreciating the tradeoffs between the approximated but analytically accessible versus more exact but computationally solved solutions for small signal gain.

Background Discussion

Virtually all 1D TWT models share a common conceptual framework. This includes a beam-current-sourced wave equation for the slow-wave-circuit guided wave and a force law equation for the effect of the wave's longitudinal electric field component on the electrons' velocities [6]. A complete review of the previously published 1D TWT models is beyond the scope of this paper. However, we briefly review a subset of typical model equations, variable definitions, and conceptual assumptions in order to facilitate subsequent discussions.

First, we will assume a 1D electron force law of the form

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial z} = -\frac{e}{m}E_w - R_{sc}^2\frac{e}{m}E_{sc}\,. \qquad (3)$$

E_w and E_{sc} are the axial components of the wave's electric field and the beam space charge fields, respectively. $R_{sc} = \omega_q/\omega_p$ is the 1D "plasma frequency reduction" factor. It accounts for reductions in the magnitude of the axial component of space charge electric field due to either finite beam radius or close proximity to surrounding conducting walls [3-5].

Next, as discussed in many references, a traveling wave interaction requires the electron and wave velocities to remain approximately equal, or *synchronous*, for net energy transfer. For amplification of the wave, the electrons need to be traveling slightly faster than the wave, so that as they decelerate and the wave grows they don't immediately fall back into the accelerating phase and start taking energy back out of the wave. The difference between the electrons' (initial) velocity and the wave velocity is referred to as *velocity detuning*. One useful parameter to characterize the relationship between wave growth and velocity detuning is the *cold circuit velocity detuning parameter*

$$b_{cc} = \frac{v_0 - v_{cc}(\omega)}{C(\omega)v_{cc}(\omega)}, \qquad (4)$$

where C is the Pierce gain parameter (see below).

It is convenient to *a priori* assume that the wave will have a unique (complex) wavenumber β, i.e., pure harmonic in space $E_w \sim e^{j\beta z}$, and to express β in terms of the amount that it differs from the electronic wavenumber, i.e.,

$$\beta = \beta_e + \delta\beta = \beta_e + \beta_e C\xi\,. \qquad (5)$$

The final piece of mathematical apparatus needed is to describe the electron beam charge density in terms of the normalized beam plasma frequency

$$\Omega_p^2 = \frac{R_{sc}^2 \left(\omega_p^2 / \omega^2 \right)}{C^2} \quad , \tag{6}$$

where R_{SC} is the space charge reduction factor discussed above, ω_p is the usual plasma frequency, $\omega_p^2 = e\rho / m\varepsilon_0$, ρ is the volume charge density of the beam, and C is the Pierce gain parameter

$$C^3 = \frac{I_{b0} \overline{K}}{4V_{b0}} . \tag{7}$$

V_{b0} is the *dc* beam voltage, numerically given by $V_{b0} = mv_0^2 / 2e$, and $\overline{K}$ is the beam-averaged "interaction impedance" [4],

$$\overline{K} = \frac{1}{S_b} \iint_{S_b} \frac{|E_w|^2}{2\beta^2 P_w} ds \quad , \tag{8}$$

where S_b is the beam cross-sectional area and P_w is the power in the wave. To compare with traditional Pierce parameters, $\Omega_p^2 = 4QC$, called a "space charge parameter". This equivalence is demonstrated in numerous texts [7,8]. We will use both notations interchangeably, since the Ω_p^2 is more indicative of the physical meaning, while the $4QC$ representation has become universally adopted throughout the TWT literature and within the minds of most TWT designers and researchers.

One-Dimensional TWT LInearized Growth Rate

With the above parameter definitions, one can derive a determinantal equation for the assumed wavenumber β [7]. Substituting with variable definitions such as Eqs. (1), (2), (4) and (5), that equation takes the form

$$\frac{(1+C\xi)^2 (1+Cb_{cc})}{(1+C\xi)^2 - (1+Cb_{cc})^2} 2C - \Omega_p^2 + \xi^2 = 0 . \tag{9}$$

Equation (9) is a quartic equation for ξ,from which one can obtain four distinct solutions (roots) for the assumed wavenumber β through the use of Eq. (5). As discussed in many texts, it turns out that the general solution for the TWT wave propagating along the circuit at frequency ω is *not* pure harmonic in space. However, it can be described using a superposition of four harmonic functions using all four pure harmonic roots [3,6,7], e.g., of the form:

$$E_w(z) \approx \sum_{m=1}^{4} A_m e^{j\beta_m z} \; . \tag{10}$$

In practice, one is usually not interested in the full general solution, and so certain simplifications are acceptable. First, one of the four roots represents a backwards-traveling disturbance, while the other three represent forward-traveling disturbances. It is therefore common to assume an idealized circuit on which the backwards-traveling disturbance is never excited, and consequently neglect it. The second assumption is to focus attention on the region after the beam has been bunched and the wave is growing (i.e., the "exponential small signal growth" region). In this region, the disturbance *is* well-approximated by a spatially pure harmonic solution, and the wave is described in terms of a single complex root, β_z, of Eq. (9), i.e., the root associated with spatial growth. There are two other assumptions that are commonly employed for mathematical convenience. Specifically, one assumes that $|\beta-\beta_e| << \beta_e$, or, using Eq. (5), $|C\xi| << 1$, and $|v_0 - v_{cc}| << v_{cc}$, or, using Eq. (4) $|Cb| << 1$. These two assumptions reduce the quartic Eq. (9) to a more easily solved cubic equation

$$\xi^2 + \frac{1}{\xi - b_{cc}} - 4QC = 0 \; . \tag{11}$$

The exponentially growing wave is thus described as

$$E_w \sim e^{\mathrm{Im}(\beta_z)z} e^{j\omega t - j\,\mathrm{Re}(\beta_z)z} = e^{\beta_e C x z} e^{j\omega t - j\beta_e(1+Cy)z} \tag{12}$$

where $\xi = \mathrm{Re}(\xi) + j\,\mathrm{Im}(\xi) = y + jx$. The reason that assuming $|\beta-\beta_e| << \beta_e$ coincides with reducing the quartic equation to a cubic one is that this assumption implicitly excludes the backwards-traveling root from consideration.

The three roots of Eq. (11) can be expressed analytically [9]:

$$\xi_1 = \left(\sqrt[3]{R+\sqrt{D}} + \sqrt[3]{R-\sqrt{D}}\right) + \frac{b}{3} \tag{13a}$$

$$\xi_2 = \left[-\frac{1}{2}\left(\sqrt[3]{R+\sqrt{D}} + \sqrt[3]{R-\sqrt{D}}\right) + \frac{b}{3}\right] + j\frac{\sqrt{3}}{2}\left[\left(\sqrt[3]{R+\sqrt{D}} - \sqrt[3]{R-\sqrt{D}}\right)\right] \tag{13b}$$

$$\xi_3 = \left[-\frac{1}{2}\left(\sqrt[3]{R+\sqrt{D}} + \sqrt[3]{R-\sqrt{D}}\right) + \frac{b}{3}\right] - j\frac{\sqrt{3}}{2}\left[\left(\sqrt[3]{R+\sqrt{D}} - \sqrt[3]{R-\sqrt{D}}\right)\right] \tag{13c}$$

where

$$D = Q^3 + R^2, \tag{13d}$$

$$Q = -\frac{b^2 + 3\Omega_p^2}{9}, \text{ and} \tag{13e}$$

$$R = \frac{2b^3 - 18b\Omega_p^2 - 27}{54}. \tag{13f}$$

It can be shown that positive growth only occurs when $D > 0$, and that the exponentially growing mode root is ξ_2 when $D > 0$.

Figure 1 compares the normalized growth rate, x, for the growing root solution calculated using both Eqs. (9) and (11) versus the cold circuit velocity detuning parameter, b_{cc}. For these results, it has been assumed that C = 0.l and a range of values of the space charge parameter $\Omega_p^2 = 4QC$ have been examined spanning negligible to very large space charge regimes.

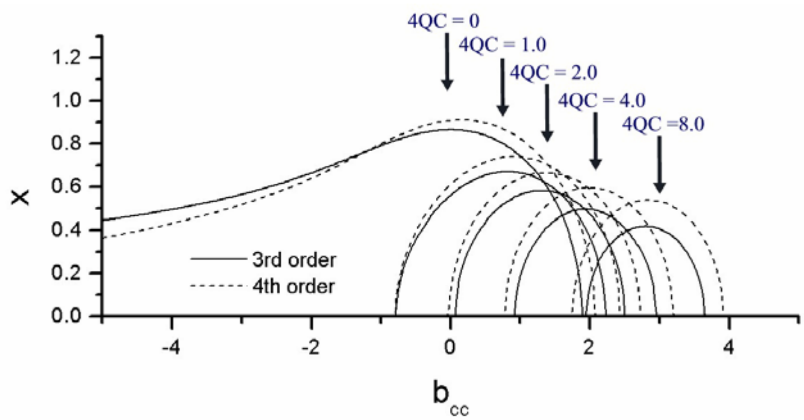


Fig.1. Normalized growth rate versus cold circuit velocity detuning parameter. In this example C = 0.1. Notice that the quartic root solution is generally always slightly greater than the cubic root solution, except for the unrealistic case of zero space charge and $b_{cc} < -1$.

There are several things to be learned from this figure. First, the simplified cubic solution always underestimates the growth rate of the growing wave, except in the unrealistic idealized case of $\Omega_p^2 = 0$, and even there, only for negative values of b_{cc} < -1. A second observation is that the magnitude of this underestimate can be quite significant for situations where total small signal gain of the device is of interest. This is because small signal gain scales as $\sim e^{\beta_e CxL}$, where L is the TWT circuit length. To provide a quantitative context, the underestimate in maximum growth rate varies from a minimum of ~ 5% for the $\Omega_p^2 = 0$ case to as much as 17% for $\Omega_p^2 = 4QC = 4$. Therefore, for a TWT with nominal gain of 20 dB and $4QC = 4$, the cubic root underestimates the overall gain by as much as 3.5 dB. This is not an insignificant error from the point of view of final stage design specifications. On the other hand, this reveals that even for large space charge TWTs, the error of the cubic solution are within bounds acceptable for draft design efforts. This is interesting because the cubic solution is available in analytic closed form. Therefore it offers advantages for incorporation into closed-form, derived expressions for optimized design points and for appreciation of scaling relationships between device parameters and growth rates.. Finally, the cubic and quartic solutions are in quite good overall qualititative agreement, especially with regard to the value of velocity detuning b_{cc} for which one obtains maximum gain. In this regard, the agreement improves with higher space charge.

The reason that the cubic solution underestimates the growth rate compared to the quartic derives from the errors implicit in assuming $\beta \approx \beta_e$ and $v_0 \approx v_{cc}$. As will be seen in later discussion, the assumption $\beta \approx \beta_e$ is only a good approximation for large and negative values of $b_{cc} \precsim -1$. Thus, the cubic approximate solution always underestimates the growth rate for $b_{cc} > 0$ where the assumption $\beta \approx \beta_e$ is not a good approximation. Meanwhile, for the example shown with C = 0.1, the assumption $v_0 \approx v_{cc}$ requires $|b_{cc}| \precsim 1$. In fact, for the case of C = 0.1, the two assumptions $\beta \approx \beta_e$ and $v_0 \approx v_{cc}$ are simultaneously very accurate in the vicinity of $b_{cc} \approx -1$. This is supported by the observation in Fig. 1 that the cubic and quartic solutions are equal for $b_{cc} \approx -1$ for both $4QC = 0$ and $4QC = 1.0$. For $b_{cc} < -2$ and $4QC = 0$, the $\beta \approx \beta_e$ assumption is very accurate, but $v_0 \approx v_{cc}$ is not a good assumption. Consequently, the cubic solution overestimates the growth rate in comparison to the more accurate quartic solution in this region.

The next insight one can glean from Fig. 1 is that accurate prediction of gain for some frequencies will be considerably more challenging than for other frequencies. The "inverted-U" shape of the growth rate versus velocity detuning means that the prediction of growth rate (and thus device gain) for frequencies with velocity detuning close to maximum growth rate (top of the "inverted-U") will be relatively insensitive to errors in either velocity or space charge parameters. In contrast, accurate prediction of gain for frequencies with velocity detuning corresponding to the edges of the "inverted-U" will be extremely sensitive to accurate and precise knowledge of the cold circuit phase velocity and the space charge parameter $4QC$. This issue is depicted in Fig. 2. Since all modern TWTs have non-negligible space charge ($4QC$ ranges from ~ 0.4 for space TWTs to as high as ~ 4 for TWTs used in electronic counter measures), this issue is a general challenge.

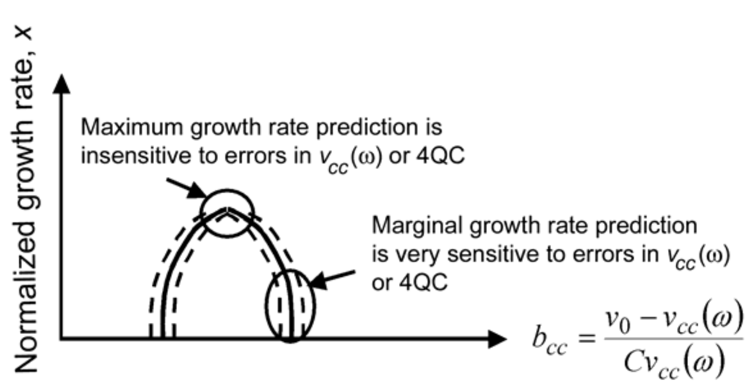


Fig. 2. Illustration of how the sensitivity of growth rate predictions and gain to errors in space

It is useful to examine which frequencies within the positive growth rate regime of a typical TWT correspond to maximum versus marginal growth rates. For this purpose, the XWING TWT [10] serves as a useful illustration, as it corresponds to a fairly high space charge device with $4QC \sim 4$. The measured small signal gain of the XWING TWT is plotted in Fig. 3. Figure 4 shows the (quartic) growth rate solution versus b_{cc} corresponding to the XWING's characteristics at 1, 3, and 5 GHz. Also shown by "lines" are the typical operating values of b_{cc} for 1, 3, and 5 GHz, respectively. As the frequency increases, the interaction increasingly becomes one of marginal growth. Hence, Fig. 4 illustrates the general principle that excessive velocity detuning determines the upper frequency limit to the gain bandwidth in a typical TWT. Extensive research experience confirms that accurate predictions of gain at 4-6 GHz in the XWING device are extremely sensitive to the values of $4QC$ and v_{cc}.

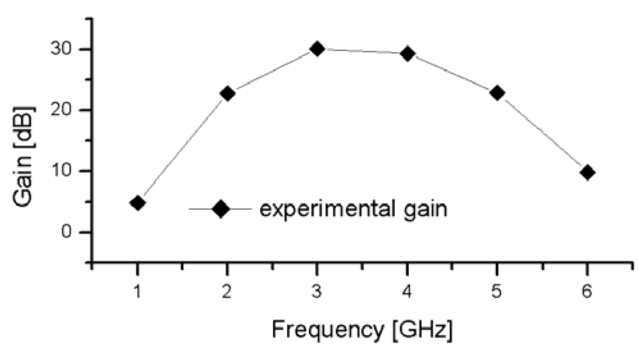


Fig. 3. Measured small signal gain versus frequency for the XWING TWT.

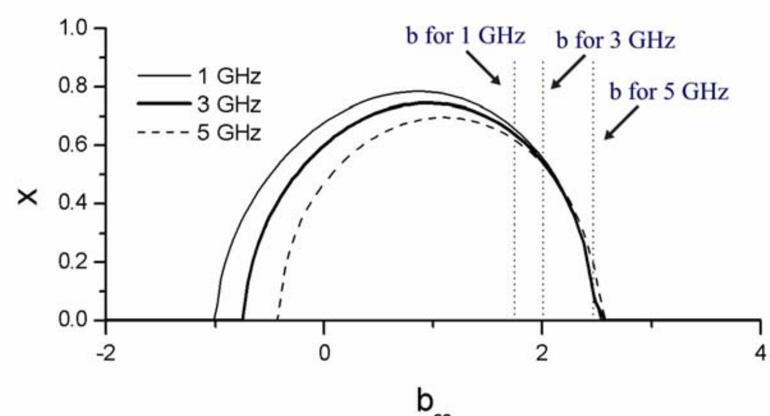


Figure 4: Normalized wave growth rate, x, versus cold circuit velocity detuning parameter b_{cc} for 1,3, and 5 GHz in XWING.

There are several situations that can frustrate the ability of a 1D TWT model to accurately predict the gain at these sensitive, upper frequency bandwidth edges. First, conventional experience indicates that a precise and accurate knowledge of the cold circuit phase velocity is not a trivial matter. Batch variations in helix rod permittivities or helix distortions arising during circuit assembly can result in cold circuit velocities that are not accurately predicted by three-dimensional electromagnetic simulations using original design parameter values. An alternative is to measure the phase velocities of every circuit prior to TWT assembly, but this introduces considerable labor and cost. Meanwhile, accurate knowledge of the space charge parameter Ω_p^2 requires accurate knowledge of the space charge reduction factor, R_{SC}, which requires accurate knowledge of the beam radius. It is impractical to incorporate beam radius measurements of each TWT during manufacture, so that 3D electron optics simulations must be relied upon. The accuracy of these simulations relies on assuming that the electron gun and magnetic circuit of each TWT performs according to original design specifications. Not only is the beam radius not typically measured, but only inferred through optics simulations, but the beam is assumed to be of constant density for simplicity in most interaction models. This condition is only approximately true, even for a 'good' optics design, and any increased current at the beam edge might be expected to play a disproportionate role in the actual TWT gain. Cold phase velocities are easily characterized for benchmarking purposes, but the beam size (including ripple and effects due to PPM stack errors) are not typically well known. Hence, prediction of small signal gain at the upper edge of the positive gain band in high space charge TWTs is more challenging for 1D models than predicting the gain near the center of the operating bandwidth.

While discussing the physics of gain at the edge of the TWT bandwidth, it is interesting to ask what determines the lower frequency limit of gain in a typical TWT. From Fig. 4, it is apparent that velocity detuning does not determine the low frequency limit of the bandwidth in the same way that it determines the high frequency limit. For example, in Fig. 3, 1 GHz clearly corresponds to the low frequency limit of the gain bandwidth yet in Fig. 4, the 1 GHz operating point is close to the maximum growth rate. The explanation to this apparent contradiction is that at low frequencies (< 1 GHz in XWING), the wave's wavelength becomes too long for significant bunching within the TWT's length. For example, at 1 GHz, the helix circuit in the XWING TWT is only approximately 5 wavelengths long. This does not allow for significant bunching and therefore negligible gain is observed at frequencies below 1 GHz

Velocity Detuning and Gain

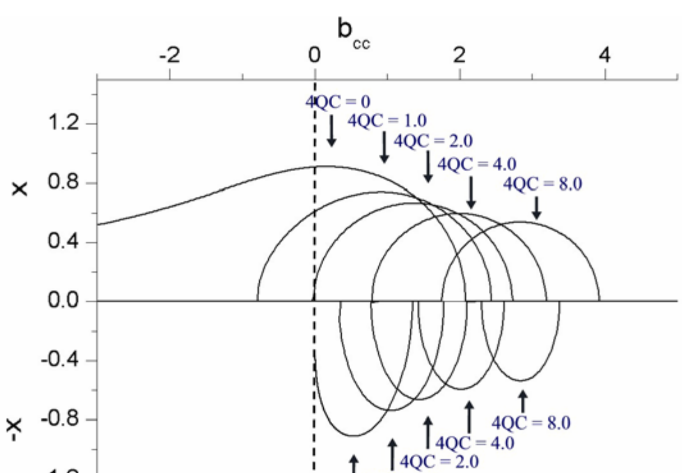
Examination of Fig. 1 inspires a fundamental question. If the condition of wave growth requires the beam velocity to be greater than the wave velocity, how is it that low space charge TWTs with small values of Ω_p^2 or $4QC$ have positive gain ($x > 0$) for negative values of velocity detuning, $b_{cc} < 0$? The answer is that b_{cc} only characterizes the velocity detuning between the beam velocity and the *cold circuit* wave phase velocity at frequency ω. The velocity detuning between the beam and the actual, beam-loaded, "hot" wave phase velocity v_h, is generally very different from b_{cc}. To illustrate this point, Eq. (9) was solved for ξ (and thus for β_z) as a function of b_{cc} . b_{cc} was allowed to vary between –5 and 5 and the five values of 0.0, 1, 2, 4, and 8 were considered for $4QC$. Then the beam-loaded hot phase velocity was calculated as

$$v_h = \frac{\omega}{\mathrm{Re}(\beta_z)} \tag{14}$$

from which a "hot" velocity detuning parameter was calculated

$$b_h = \frac{v_0 - v_h}{C v_h} \quad . \tag{15}$$

The results for normalized growth rate x have been plotted against both b_h and b_{cc} in Fig. 5 for comparison. It is immediately evident that positive gain always corresponds to positive velocity detuning, $b_h > 0$, i.e., a beam velocity that is greater than the wave phase velocity, v_h. Although this point is intuitively reasonable, it has never (to the authors' knowledge) been previously demonstrated in a publication.



Approximating β_{hot} for space charge reduction parameter calculations

There are additional, timely, insights that can be extracted from this re-examination of 1D Pierce theory for TWTs. Discussion above shows that an accurate 1D model prediction of TWT gain, especially at the upper frequency edge of the gain bandwidth, requires an accurate estimate of Ω_p^2 and thus R_{SC}. On the other hand, derivations of space charge reduction coefficient expressions all result in equations which require *a priori* knowledge of the hot wavenumber for the circuit wave, β_{hot} = Re(β_z). Since a determination of β_z requires an *a priori* knowledge of R_{SC}, while determination of R_{SC} requires *a priori* knowledge of Re(β_z), one is faced with a paradox. The conventional resolution is to calculate an estimate for R_{SC} by substituting an approximation for Re(β_z), either β_e, or β_{cc}. To determine which of these approximations are more accurate, Eq. (9) has been solved for C = 0.01 and 0.1, and for the same choices of 4QC as in previous calculations. In Fig. 6 both the normalized growth rate, x and the normalized wavenumber, 1+Cy, have been plotted versus b_{cc} for C = 0.1. In addition, curves have been plotted for normalized β_e and β_{cc}. Careful study of this figure shows that for all values of space charge, in the region of positive gain the hot wavenumber of the growing wave is better approximated by the cold circuit wavenumber

$$\mathrm{Re}(\beta_z) \approx \beta_{cc},$$

for frequencies inside the positive growth rate band, while the circuit wave's beam-loaded wavenumber is better approximated by β_e for frequencies outside the positive growth rate band,

$$\mathrm{Re}(\beta_z) \approx \beta_e .$$

The exception to this latter statement is that for unusually high space charge beams $\Omega_p^2 = 4QC > 4$,

$$\mathrm{Re}(\beta_z) \approx \beta_{cc},$$

for frequencies outside the positive growth rate band. However, such cases would be extremely rare in practical TWTs, because they would imply the need for a very strong focusing magnetic field.

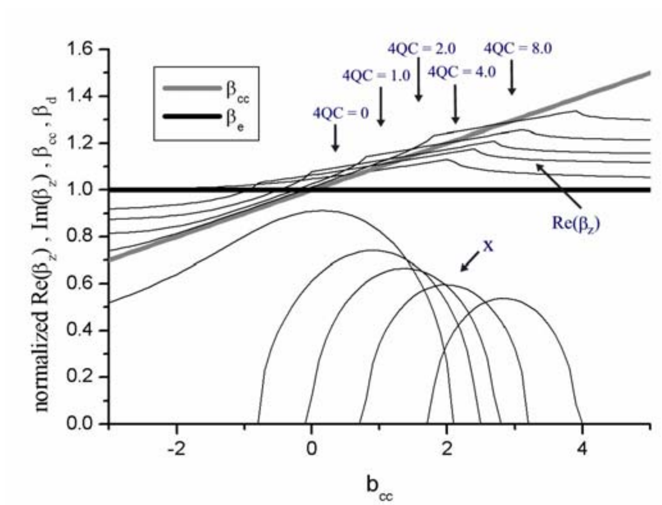
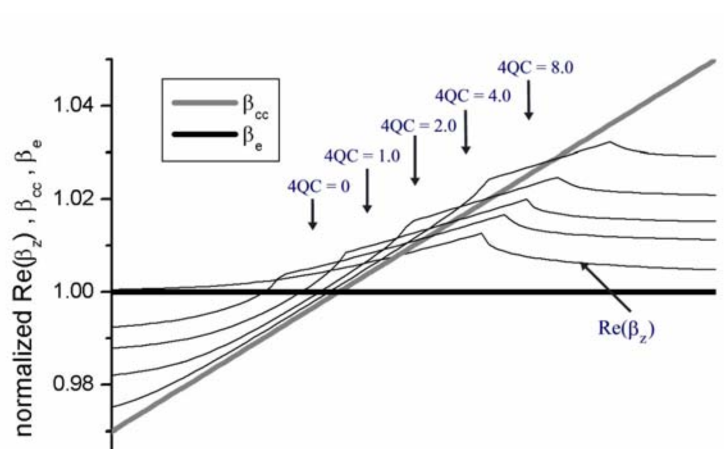


Fig.6. Normalized β_{cc}, β_e, growth rate Im(β_z), and hot wavenumber Re(β_z) for C = 0.1. Normalizations are Im(β_z/Cβ_e) =Im(ξ) = x , hot wavenumber Re(β_z/β_e) = 1+CRe(ξ) = 1+ Cy, cold circuit wavenumber β_{cc}/β_e, and electronic wavenumber β_e/β_e.

The results for C = 0.01 are shown in Fig. 7, and show that similar conclusions can be drawn for both large and small C values.



Maximum gain and the slow space charge wave

Finally, it is instructive to re-examine a conventional wisdom regarding the condition for maximum gain in a TWT. Figure 8 plots the normalized hot wavenumber of the growing mode (calculated from the quartic equation) versus b_{cc} for 4 values of $\Omega_p^2 = 4QC =$ 0.0, 0.4, 2.0, and 4.0. On each of the four curves, a filled shape (circle, square, diamond, pentagon) has been placed, showing the point of intersection of the slow space charge wave's wavenumber (normalized) and the growing mode's wavenumber. The slow space charge wave's wavenumber is defined to be

$$\beta_{sscw} = \beta_e + \beta_p = \beta_e\left(1 + C\sqrt{4QC}\right) = \beta_e\left(1 + C\Omega_p\right) \qquad (16)$$

where $\beta_p = \omega_q / v_o$ and ω_q is the reduced plasma frequency [6]. Hence, these four circles represent the points at which the hot phase velocity of the growing wave and the phase velocity of the slow space charge wave are equal. At the top of the figure are four labeled arrows, indicating the positions of peak gain (maximum x) for each of the four values considered for $\Omega_p^2 = 4QC$. These particular results were computed for C = 0.01, but the conclusion proves to be generally true for any realistic value of C. Specifically, it is often stated as a "rule-of-thumb" that the peak gain corresponds to the condition where the hot phase velocity of the growing wave equals the phase velocity of the slow space charge wave. What Fig. 8 clarifies, however, is that this statement is only valid for values of $4QC > 0.4$. From the results calculated here, in fact, the rule-of-thumb is reasonably accurate for $\Omega_p^2 = 4QC > \sim 1$ (or $QC > \sim 0.25$). This is also consistent with the results in Fig. 10-14 of [7]. Obviously, for $4QC = 0$, $\beta_{sscw} = \beta_e$ (i.e., there are no space charge waves), so it is clearly not appropriate to apply the rule-of-thumb in that case. However, for values of $4QC < 1$, such as might be encountered in many space TWTs, this rule-of-thumb is not a good approximation for intuitive reasoning or simplified initial design steps.

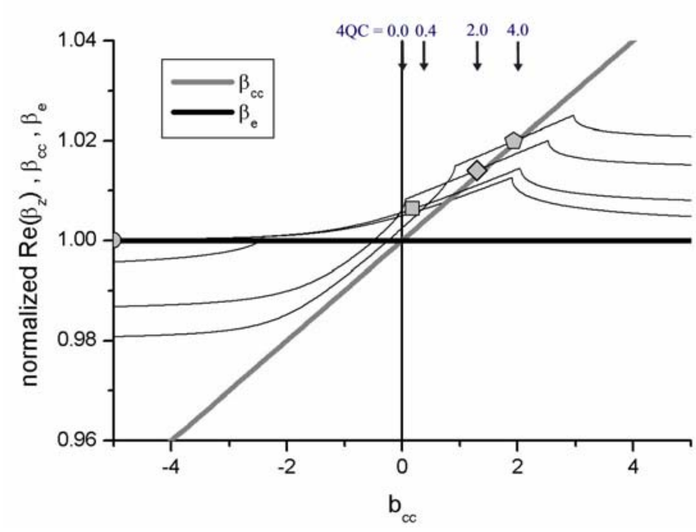


Fig. 8. Normalized β_{cc}, β_e, and Re(β_z) for C = 0.01. Normalizations are cold circuit wavenumber β_{cc}/β_e, electronic wavenumber β_e/β_e and hot wavenumber $\mathrm{Re}(\beta_z/\beta_e) = 1 + Cy$. The arrows at the top indicate the values of b_{cc} corresponding to maximum gain (x) for the indicated values of QC. The four filled shapes indicate the locations where the slow space charge wave velocity equals the hot phase velocity of the growing waves corresponding to the four values of $4QC$. Specifically, circle for 4QC = 0.0, square for 4QC = 0.4, diamond for 4QC = 1.0, and pentagon for 4QC = 2.0

Using the approximate cubic solutions, Eqs.(13), one can confirm and explain the observation illustrated by Fig. 8. First, it can be shown from Eqs. (13) that the normalized growth rate,

$$x = \mathrm{Im}(\xi_2) = \frac{\sqrt{3}}{2}\left(\sqrt[3]{R+\sqrt{D}} - \sqrt[3]{R-\sqrt{D}}\right), \tag{17}$$

is always maximized under the condition that $b^2 \approx \Omega_p^2$, for all values of Ω_p^2 . This is also revealed in graphical form in Fig. 10-14 of Ref. [7]. Further algebra then reveals for the maximum growth rate condition $b^2 \approx \Omega_p^2$, that $D = Q^3 + R^2 \approx R^2$, and hence,

$$y = \mathrm{Re}(\xi_2) \approx \frac{1}{2}\left[2\left(\frac{2\Omega_p}{3}\right)^3 + 1\right]^{1/3} + \frac{\Omega_p}{3}. \tag{18}$$

In the limit that $(2\Omega_p/3)^3 >> 1/2$, then $y \approx \Omega_p$, and $\mathrm{Re}(\beta) \approx \beta_e(1 + C\Omega_p)$, i.e., indeed the slow space charge wave. In the limit that $(2\Omega_p/3)^3 << 1/2$, then $y \approx 1/2$, and $\mathrm{Re}(\beta) \approx \beta_e(1 + C/2)$, in agreement with the classic Pierce solution for space charge free and synchronous (b = 0) conditions [6,7]. The demarcation between these two extremes occurs approximately when $(2\Omega_p/3)^3 \approx 1/2$, or $\Omega_p^2 \approx 1.4 \approx 1$. This is in good agreement with the main point of Fig. 8, especially considering the approximation of neglecting the contribution of Q^3 to D, and the fact that the maximum growth rate point does not precisely correspond to $b^2 = \Omega_p^2$. Implicit in this result is the point that when $\Omega_p^2 = 4QC < 1$, the space charge wave model of TWT interaction begins to lose meaning as the fast and slow modes of the coupled beam-circuit disturbance are no longer closely associated with the fast and slow beam space charge waves.

Conclusions

In summary, this re-examination of 1D linearized TWT theory leads to the following conclusions:

1) The assumptions $|\beta - \beta_e| << \beta_e$ and $|v_0 - v_{cc}| << v_{cc}$ produce qualitatively accurate predictions of the growth rate, but can result in quantitative errors in gain of ~ 3 dB for high space charge TWTs with 4QC ~ 4 and nominal gain ~ 20 dB. The error increases for higher gain devices and decreases for lower space charge. On the other hand, the errors are sufficiently bounded to justify the use of the analytic cubic solution in deriving closed-form expressions for optimized design starting points.
2) Prediction of the gain at the high frequency end of the gain bandwidth by 1D models is very sensitive to precise and accurate knowledge of the wave's cold circuit phase velocity and the normalized plasma frequency or effective (reduced) space charge parameter, $\Omega_p^2 = \left(R_{SC}\omega_p / C\omega\right)^2$ = 4QC. This sensitivity is greatest for high space charge beams with 4QC > 2.
3) Regardless of the value of 4QC, positive gain occurs when the *dc* beam velocity is faster than the "hot" circuit phase velocity of the wave, $v_0 > v_h$,
4) For 4QC > 1, maximum gain occurs when the hot phase velocity equals the slow space charge wave velocity, $v_h = v_{sscw}$. For 4QC < 1, the space charge wave interaction model begins to lose meaning as the fast and slow modes are no longer easily identified with the beam space charge waves.
5) For purposes of evaluating the 1D model's space charge reduction coefficient, it is recommended to use the approximation that $\mathrm{Re}(\beta_z) \approx \beta_{cc}$, for frequencies inside the gain bandwidth, and $\mathrm{Re}(\beta_z) \approx \beta_e$, for frequencies outside the gain bandwidth. The exception to this latter statement is that for unusually high space charge beams 4QC > 4, it is recommended to use $\mathrm{Re}(\beta_z) \approx \beta_{cc}$, for frequencies both inside and outside the gain bandwidth.

ACKNOWLEDGMENT

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