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Issues on the Constitutive Formulation at Large Elastoplastic Deformations, Part 1: Kinematics

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With 1 Figure

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Summary

The kinematics at large elastoplastic deformations are analyzed within the framework of a general macroscopic constitutive theory with tensorial structure variables. The key concept is the distinction between the kinematics of the continuum and its underlying substructure. The proper definition of physically plausible corotational and corodeformational rates for the kinematical and state variables, shows the equivalence of the effect that the choice of an unstressed configuration has, on the transformation of these variables and their rates under superposed rigid body rotations. Along these lines, issues debated in the past are given definitive answers, and comparisons of different approaches are presented.

1. Introduction

The macroscopic constitutive formulation at large elastoplastic deformations has been a much debated subject in the mechanics community. To understand the fundamental reasons for this debate, one must first distinguish the two important aspects of the subject matter: the kinematics and the kinetics. The kinematics pertain to the geometrical aspects of the elastic and plastic mechanisms of deformation, and their effect on the values of the structure variables which characterize the material substructure. The kinetics address the question of the constitutive rate equations of evolution for both the kinematical and structure variables. Thus the principal reasons for the aforementioned debate, in our opinion, are two. First, the analysis of kinematics in most theories does not distinguish between the kinematics of the continuum and the kinematics of the underlying substructure. The absence of distinction is due to the influence of the elasticity theory, where the kinematics of the continuum and its substructure are determined by the same transformations. The second reason is the incomplete way the coupling between kinematics and kinetics has been accounted for, in order to reflect physical reality. Hence, the present work can naturally be divided in two parts, the kinematics in part 1 and the kinetics in part 2. However, this section must be considered as an introduction to both parts, because they are intimately interrelated.

The debate on the subject matter has been related in the past with a number of specific issues on which different researchers disagree or follow different approaches. The present work attempts to offer an answer to these issues. But the presentation does not focus on that aspect only; rather a general theory is developed systematically, mostly influenced by Mandel's work [1], [2], and along the lines of development answers to the different issues are provided. Consequently, a number of other aspects of the general theory which have not been discussed in the past are examined. Nevertheless, as an indication of the points of interest, the following are among the issues to be addressed, not necessarily in the order presented, in both parts of this work.

1. The effect of the choice of the unstressed configuration: is there any?

2. The effect of "full" or "partial" invariance requirements under superposed rigid body rotation at the current and unstressed configurations.

3. The additive decomposition of the elastic and plastic parts of the velocity gradient.

4. The effect of the rate choice for the stress and the structure variables.

5. Prager's preference for Jaumann rates, based on the stationarity of isotropic invariants.

6. Differences and similarities between Mandel's and Onat's approach.

7. The notion of director vectors in Mandel's work; are they necessary?

8. Difference between hardening and purely orientational structure variables.

9. Distinction between material isotropy and analytical isotropy of constitutive functions.

10. Residual stresses and their role on a physically acceptable definition of the unstressed configuration.

Although answers to some of these issues have been presented in the past, they were either incomplete or not viewed within the general and complete constitutive framework presented here. Furthermore, some issues such as number 10 for which Aris Phillips has so much contributed with his experimental work, require the introduction and investigation of the effect of novel concepts, such as the effectively unstressed configuration.

In reference to notation, tensors will be denoted usually by boldface characters in direct notation. Assuming the summation convention over repeated indices, the following symbolic operations are implied: $\mathbf{a}\boldsymbol{\sigma} \Rightarrow a_{ij}\sigma_{jk}$, $\mathbf{a}:\boldsymbol{\sigma} \Rightarrow a_{ij}\sigma_{ji}$, $\mathbf{a}\cdot\boldsymbol{\sigma} \Rightarrow a_{ij}\sigma_{ij}$, $\mathbf{a}\otimes\boldsymbol{\sigma} \Rightarrow a_{ij}\sigma_{kl}$, with proper extension to different order tensors. The prefix tr indicates the trace, a superscript T the transpose and a -1 the inverse, subscripts s and a the symmetric and antisymmetric parts, and a superposed dot the material time derivative or rate. Under an orthogonal transformation

$\boldsymbol{Q}$, the notation $\boldsymbol{Q}[\boldsymbol{b}]$ implies the corresponding transformation of the tensor $\boldsymbol{b}$, e.g. $\boldsymbol{Q}[\boldsymbol{b}] = \boldsymbol{Q}\boldsymbol{b}\boldsymbol{Q}^{\mathrm{T}}$ if $\boldsymbol{b}$ is of second order, and $\boldsymbol{Q}[\boldsymbol{b}] = \boldsymbol{Q}\boldsymbol{b}$ is $\boldsymbol{b}$ is of first order (vector).

2. General Concepts

The mapping of the neighborhood of a material point from a reference configuration κ_r to a current κ, is determined by the deformation gradient $\boldsymbol{F}$ according to $d\boldsymbol{x} = \boldsymbol{F}d\boldsymbol{X}$, where $d\boldsymbol{X}$ and $d\boldsymbol{x}$ are material line segments at κ_r and κ, respectively. While this conclusion is merely geometrical in nature, it requires further elaboration in order to become useful in a constitutive theory. In elastoplasticity the distinctly different kinematical mechanisms of the elastic and plastic deformation processes, can be best described by the multiplicative decomposition $\boldsymbol{F} = \boldsymbol{F}^{\mathrm{e}}\boldsymbol{F}^{\mathrm{p}}$, formally introduced in continuum mechanics by Lee and Liu [3] and Lee [4]. In reference always to an infinitesimal material neighborhood, it states that the κ_r is mapped first into an unstressed (relaxed or intermediate) configuration by the plastic part $\boldsymbol{F}^{\mathrm{p}}$, which reflects the kinematics associated with plastic deformation only; subsequently the unstressed configuration is carried into the current κ, by the elastic part $\boldsymbol{F}^{\mathrm{e}}$, embodying the kinematics associated with the elastic deformation. Hence, a geometrical decoupling between elastic and plastic kinematics is achieved, providing the proper framework for the development of realistic constitutive relations reflecting the different physical characteristics of elasticity and plasticity.

However, the situation is not as simple as it may appear at first. While the definition of κ_r and κ is unequivocal, the multiplicity of the possible unstressed configurations from the point of view of their orientation and the corresponding effect on invariance requirements, gave rise to lengthy discussions, arguments and counterarguments [5]–[8]. Mandel [1], [2] addressed this problem by introducing a triad of director vectors embedded in the material substructure, whose orientation in reference to a fixed cartesian coordinate system defines the orientation of the relaxed configuration, whichever it may be. The concept of director vectors has been criticized because while it may be well defined in cases such as a single crystal, it is not well defined for polycrystals or other complex material structures. Assuming that the material substructure can be macroscopically defined by a collection $\boldsymbol{s}$ of tensorial structure variables, Onat and co-workers [9], [10] have very simply stated that the orientation of $\boldsymbol{s}$ defines as well the orientation of the corresponding configuration, and material symmetries are direct consequence of the symmetries conveyed by the tensorial nature of $\boldsymbol{s}$. Note that Mandel [1], [2] has also introduced tensorial structure variables $\boldsymbol{s}$ in defining material symmetries, but preferred to use the director vectors concept as the primary orientation tool. In fact Mandel's and Onat's approaches of defining the orientation are not in conflict. Assuming the possibility to describe the material symmetries by $\boldsymbol{s}$ in reference to the director vectors, and using the representation theorems for isotropic functions [11], Dafalias [12], [13] has shown that the explicit presence of the director vectors in the constitutive functions of Mandel's theory disappears, and the Onat's definition of orientation is retrieved. While this may suggest that the director vectors are a redundancy, recall that the foregoing is valid only if it is possible to reasonably well describe the material substructure by means of $\boldsymbol{s}$, which is the case for most materials. If the material substructure is so defined that does not conform with specific symmetries describable by orthogonal transformations which leave certain $\boldsymbol{s}$ invariants, it is necessary to introduce a triad of director vectors in reference to which all constitutive functions can have a specific analytical description, not withstanding the elusiveness of the definition of such triad. The concept of director vectors will be reexamined in the sequel from a slightly different perspective than that of Mandel's.

3. Continuum Versus Substructural Kinematics

Mandel's great contribution in setting up a kinematical macroscopic framework in relation to the multiplicative decomposition of $\boldsymbol{F}$, is not so much the introduction of the director vectors (it has already been seen how one can in general dispense with the direct definition of them [12], [13]), but the underlying cause for such introduction. Motivated by single crystal plasticity, Mandel wanted to emphasize the difference of the kinematics of the continuum from the kinematics of its underlying substructure in elastoplasticity. It is in this sense that Mandel's work parts from the work of Onat and co-workers [9], [10], and in fact converges with corresponding suggestions made by Kratochvil [14], [15]. Havner [16] describes this concept of relative kinematics for metals as "the movement of the macroscopic material relative to the underlying crystalline structure", and the same theme underlines the work of Nemat-Nasser and Mehrabadi [17] in defining the fabric and related kinematics for granular media. The key novel quantity emerging from such a distinction of kinematics is the concept of plastic spin which has been the primary focus of recent papers by Dafalias [12], [13], [18]–[21] and Loret [22]. Simply put, the plastic spin is the difference of the rate of rotation (spin) of the substructure from the rate of rotation of the continuum (material spin). This is in direct analogy to the definition of the plastic rate of deformation as the difference of the substructural rate of deformation (elastic) from the rate of deformation of the continuum. The plastic spin is determined by proper constitutive relations, as does the plastic rate of deformation. During the elastic deformation, the kinematics of the continuum and its substructure are determined by the elastic part of $\boldsymbol{F}$, but are not necessarily identical to each other. The definition of the spin of the substructure as the difference between the material and plastic spins is very important because the law of evolution of the structure variables $\boldsymbol{s}$ (which in fact define the substructure) must, therefore, be expressed in terms of rates involving corotation with the substructural spin in reference to a fixed cartesian coordinate system. To be precise, the embedding of s with the elastic deformation introduces the so-called corodeformational rates [20], [21], used in their evolution equations, which include both the corotation with the substructural spin and the convection with the elastic deformation. These concepts have already been presented and will be further elaborated.

The question of the non-uniqueness of the unstressed configuration, however, has not been answered yet. The authors in [1], [2], [18]–[22] presented their development utilizing different such configurations, and it may appear as if they followed fundamentally different approaches. It is one of the major objectives of this section to show that the same final result is obtained, whichever choice of the relaxed configuration is made. To this extent, the following important concept must be developed. One can integrate the spin of the substructure (defined before as the difference between material and plastic spins) to obtain an orthogonal tensor. This tensor can be thought as representing the rotation of a director-vectors triad from the reference to the current unstressed configuration, whose spin reflects in a global sense the substructural spin. Notice the important conceptual difference from Mandel's original suggestion: here the director-vectors triad is not defined by being related to some specific substructural characteristics (although it may be in particular cases such as the single slip mechanism or orthotropic symmetries), but has been analytically derived. Henceforth, this definition of the director-vectors triad will be used only as an auxiliary tool in defining the corresponding spins at the different unstressed configurations.

At the risk of becoming redundant, it is thought important to comment once more on the following. One may question the necessity to define such a triad of director vectors since, after all, the orientation of the unstressed configuration can be defined by the orientation of s ([9], [10]). This line of thought, however, neglects the fact that is not just the orientation which is of importance, but even more are the relative spins of the continuum and its substructure as discussed earlier; the so-defined director vectors will help us to visualize and analytically describe these spins. Along the same lines one also can propose to consider the rotation of s from the reference to the unstressed configuration as a measure of the substructural rotation. This is true and coincides with the rotation of the director vectors as defined above, only if the s are purely orientational in nature, e.g. the normal to the slip plane or the principal axes of persisting orthotropy. In the general case, however, the s are evolving tensorial structure variables whose principal directions (eigenvectors) change not only due to the substructural spin (a rigid body spin), but also due to the appearance of "shear" rate components along the existing principal directions which alter their eigenvectors and eigenvalues (a "constitutive spin"). A typical example is the case of a backstress tensor in kinematic hardening, considered as a tensorial structure variable. Only the analytically "induced" by integration, rotation of the director vectors can globally account for a meaningful substructural rotation.

with the s being purely orientational or not ([13], [19], [21]). At this point the reader should not form the idea that the rotation of the director vectors is the primary focus; this is only a conceptual derivation, while the key to the development is the constitutive equations for the plastic spin. Once the plastic spin is so determined, it will automatically provide the substructural spin by subtraction from the total material spin, without even being necessary to define explicity the director vectors. The concept of relative continuum-substructure spin (i.e. the plastic spin) is relevant only for anisotropic materials with preferred disrections; for isotropic materials the plastic spin is zero, as it can be easily shown ([1], [2], [15], [18], [19]).

4. Analytical Description of Deformations and Rotations

The foregoing will be described analytically and illustrated by the schematic diagrams of Fig. 1 a, b. Figure 1 a shows the reference configuration κ_r, the current κ and three unstressed configurations κ_u, κ_o, and κ_i. On each one of theses configurations the director-vectors triad, obtained by the integration of the substructural spin, is schematically shown by two orthogonal axes with arrows. Notice that in all but the current configuration κ the axes are shown as straight line segments indicating their rotation, while in κ the axes are "bended" (but still may be orthogonal in a curvilinear sense) indicating their elastic embedding which takes place during the transformation from each one of the unstressed to the current configuration. The configuration κ_u is the most general having an arbitrary orientation; the elastic part $\boldsymbol{F}^e$ of the deformation gradient $\boldsymbol{F}$ includes both rotation and elastic stretching. The configuration κ_o (used primarily in [6], [7], [9], [10], [12], [13], [18]–[21]), is such that the elastic part $\boldsymbol{V}$ of $\boldsymbol{F}$ is symmetric, $\boldsymbol{V} = \boldsymbol{V}^T$, including only elastic stretching. The configuration κ_i (used primarily in [22]) is the so-called isoclinic configuration [1], [2] and is so defined that the director vectors at κ_i have a fixed orientation in reference to a global system, conveniently chosen if desired to be the same as the corresponding orientation of the director vectors in κ_r. Hence, the substructures at κ_r and κ_i may have the same orientation (although the s defining the substructure have in general evolved from κ_r to κ_i by constitutive laws). The concept of the isoclinic configuration has been reinvented in a recent paper by Dashner [23], called the reference cell. The elastic part $\boldsymbol{F}_i^e$ of $\boldsymbol{F}$ includes both rotation and stretching. The plastic part of $\boldsymbol{F}$ in reference to κ_u, κ_o and κ_i is represented by $\boldsymbol{F}^p$, $\boldsymbol{P}$ and $\boldsymbol{F}_i^p$, respectively. The principal stretch directions are shown as orthogonal line-segments (without arrows), discontinuous for the plastic part of $\boldsymbol{F}$ at κ_r, κ_u, κ_o and κ_i, and solid for the total $\boldsymbol{F}$ at κ and κ_r (notice their different orientation). None of the foregoing principal stretch directions are the ones associated with the elastic part of $\boldsymbol{F}$ (not shown for simplicity).

While Fig. 1 a illustrates the kinematics of the continuum, Fig. 1 b represents schematically the kinematics of the substructure via the transformation of the director vectors. From κ_r to κ_u, κ_o and κ_i, the substructural transformation is

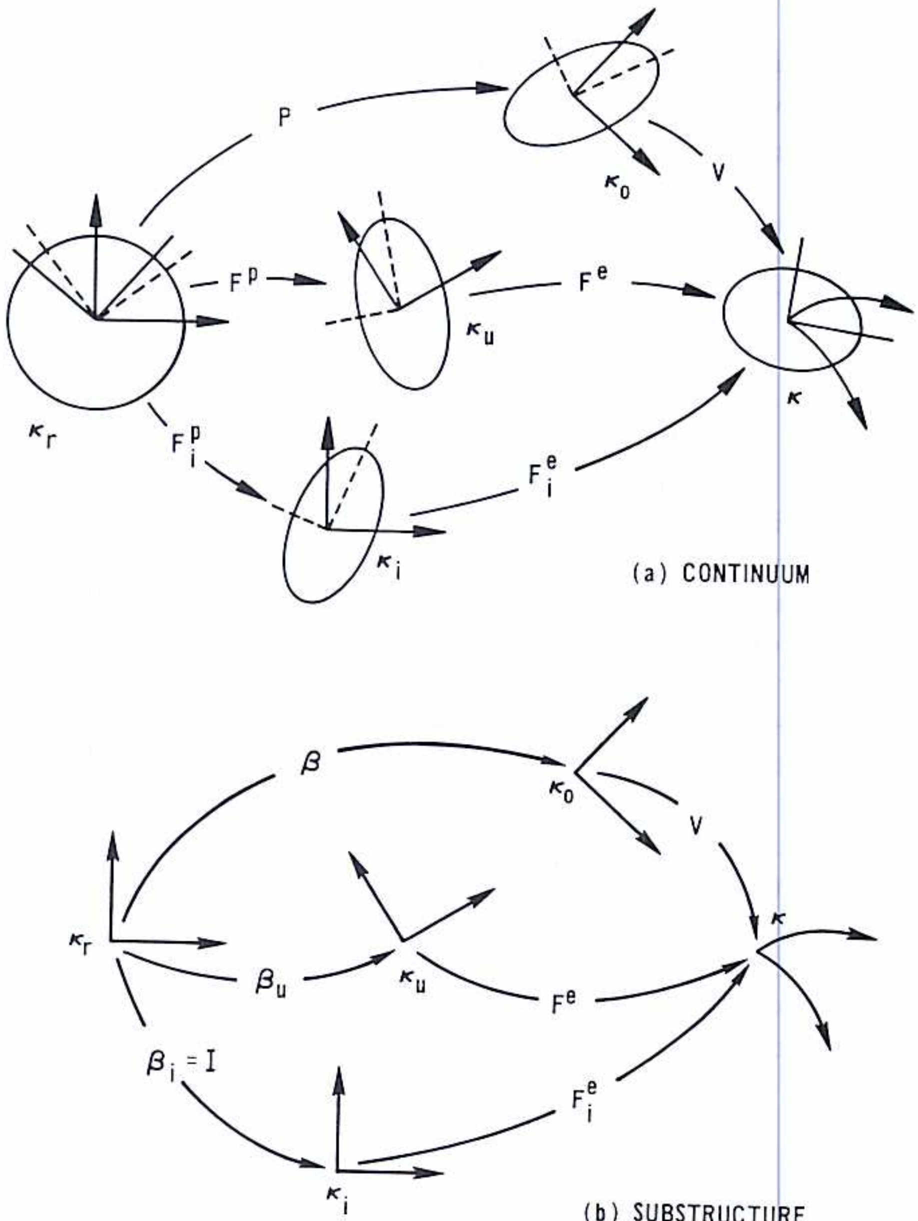

Fig. 1 a, b. Schematic diagram of the kinematics of the continuum and the substructure

represented by a rotation expressed by the orthogonal tensors $\boldsymbol{\beta}_u$, $\boldsymbol{\beta}$ and $\boldsymbol{\beta}_i$, respectively ($\boldsymbol{\beta}\boldsymbol{\beta}^{\mathrm{T}} = \boldsymbol{\beta}_u\boldsymbol{\beta}_u{}^{\mathrm{T}} = \boldsymbol{\beta}_i\boldsymbol{\beta}_i{}^{\mathrm{T}} = \boldsymbol{I}$(identity)); notice that $\boldsymbol{\beta}_i = \boldsymbol{I}$, consistent with the definition of the isoclinic configuration κ_i. From κ_u, κ_o and κ_i to κ, the substructural transformation is properly determined by the continuum elastic transformation given by $\boldsymbol{F}^e$, $\boldsymbol{V}$ and $\boldsymbol{F}_i^e$, respectively, but is not identical to it in the sense that the transformations of material and substructural line segments are, in general, differently related to the elastic part of $\boldsymbol{F}$. An example is the transformation of the slip and the normal to the slip-plane directions in single crystals [20].

Employing now the polar decomposition for the elastic and plastic parts of $\boldsymbol{F}$, with a self-evident notation, one has for the continuum (Fig. 1 a)

$$\boldsymbol{F} = \boldsymbol{F}^e \boldsymbol{F}^p = \boldsymbol{V}\boldsymbol{R}^e_u \boldsymbol{R}^p_u \boldsymbol{U} = \boldsymbol{F}^e_i \boldsymbol{F}^p_i = \boldsymbol{V}\boldsymbol{R}^e_i \boldsymbol{R}^p_i \boldsymbol{U} = \boldsymbol{V}\boldsymbol{R}^{ep} \boldsymbol{U} = \boldsymbol{V}\boldsymbol{P}. \tag{1}$$

The $\boldsymbol{V}$ and $\boldsymbol{U}$ are the elastic left and the plastic right stretch tensors, respectively. In a similar way, denoting by $\boldsymbol{F}_s$ the deformation gradient applied to the substructure, one can write (Fig. 1 b)

$$\boldsymbol{F}_s = \boldsymbol{F}^e \boldsymbol{\beta}_u = \boldsymbol{V}\boldsymbol{R}^e_u \boldsymbol{\beta}_u = \boldsymbol{F}^e_i = \boldsymbol{V}\boldsymbol{R}^e_i = \boldsymbol{V}\boldsymbol{\beta}. \tag{2}$$

Given κ and κ_r, and the specific definition of κ_o and κ_i, clearly the $\boldsymbol{V}$, $\boldsymbol{U}$, $\boldsymbol{R}^{ep} = \boldsymbol{R}^e_u \boldsymbol{R}^p_u = \boldsymbol{R}^e_i \boldsymbol{R}^p_i$, $\boldsymbol{R}^e_i$, $\boldsymbol{R}^p_i$ are unique, and consequently so are the $\boldsymbol{P} = \boldsymbol{R}^{ep}\boldsymbol{U}$, $\boldsymbol{F}^p_i = \boldsymbol{R}^p_i \boldsymbol{U}$ and $\boldsymbol{F}^e_i = \boldsymbol{V}\boldsymbol{R}^e_i$; but the $\boldsymbol{F}^e = \boldsymbol{V}\boldsymbol{R}^e_u$ and $\boldsymbol{F}^p = \boldsymbol{R}^p_u \boldsymbol{U}$ are not unique due to the arbitrariness of κ_u. It is of utmost importance now to realize the fact that in each configuration the substructural rotation is different from the continuum plastic rotation, the latter expressed analytically by the orthogonal part of the polar decomposition of the plastic part of $\boldsymbol{F}$. It means that $\boldsymbol{\beta}_u \neq \boldsymbol{R}^p_u$, $\boldsymbol{\beta} \neq \boldsymbol{R}^{ep}$ and $\boldsymbol{\beta}_i = \boldsymbol{I} \neq \boldsymbol{R}^p_i$ for κ_u, κ_o and κ_i, respectively. This observation may be considered the single most important one among the kinematical concepts developed so far; it is illustrated in Fig. 1 a where the pairs of solid segments with arrows (substructure) and discontinuous segments (principal plastic stretch directions of the continuum) have a different relative orientation at κ_r and any one of κ_u, κ_o, κ_i (where they have the same). A very simple example has been presented in [19]: considering a deck of cards lying on the table one can "shear" them inducing a large "plastic rotation" of the continuum aspect (the deck of cards), but the underlying substructure, expressed by the normal to the plane of the cards, has not rotated at all. In fact, it follows easily from Eqs. (1) and (2) that

$$\boldsymbol{V} = \boldsymbol{F}^e \boldsymbol{R}^{e^T}_u = \boldsymbol{F}^e_i \boldsymbol{R}^{e^T}_i; \quad \boldsymbol{\beta} = \boldsymbol{R}^e_i = \boldsymbol{R}^e_u \boldsymbol{\beta}_u = \boldsymbol{R}^{ep} \boldsymbol{R}^{p^T}_i. \tag{3}$$

Notice that in all the preceding elaboration it was never necessary to refer to the polar decomposition of the total deformation gradient $\boldsymbol{F}$.

5. Analytical Description of Rates of Deformations and Rotations (Spins)

In this subsection the definition and interrelation among the different rates of deformation and spins will be established. To this extent, the general definition of the corotational rate of a representative second order tensor $\boldsymbol{a}$ and vector $\boldsymbol{m}$ with respect to a spin Ω is given by

$$\frac{D\,a}{Dt} = \dot{a} - \Omega\, a + a\,\Omega; \quad \frac{D\,m}{Dt} = \dot{m} - \Omega\, m. \tag{4.1–2}$$

Based on the definition of $\boldsymbol{\beta}_u$, $\boldsymbol{\beta}$, and $\boldsymbol{\beta}_i$, the corresponding substructural spins (rates of rotation of the director-vector triads) in the configuration κ_u, κ_o and κ_i are given by

$$\boldsymbol{\omega}_u = \dot{\boldsymbol{\beta}}_u \boldsymbol{\beta}^T_u; \quad \boldsymbol{\omega} = \dot{\boldsymbol{\beta}}\boldsymbol{\beta}^T; \quad \boldsymbol{\omega}_i = \dot{\boldsymbol{\beta}}_i \boldsymbol{\beta}^T_i = \boldsymbol{0} \tag{5}$$

respectively, where clearly $\boldsymbol{\omega}_i = \boldsymbol{0}$ since by definition $\boldsymbol{\beta}_i = \boldsymbol{I}$ always. The rates of deformation and rotation and the rates of the structure variables used in the constitutive relations at the unstressed configurations, must be thought as taking place in reference to the existing substructure since the latter, and not the continuum, is the supporting element [21]. Hence, it is necessary to define the corresponding corotational with the substructure rates for these quantities, referred to any fixed cartesian coordinate system. Here, these rates for the kinematical quantities will be introduced. In order to avoid the general but cumbersome notation D/Dt, Eq. (4), the corotational rates with respect to $\boldsymbol{\omega}_u$ and $\boldsymbol{\omega}$ will be denoted by a superposed $\triangle$ and $\circ$, respectively; the corotational rates in reference to the director vectors in the isoclinic configuration are simply the usual material time derivatives because $\boldsymbol{\omega}_i = \boldsymbol{0}$. On the basis of the foregoing definitions, in particular the way of attachment of the different quantities with the substructure at the different configurations, it follows that

$$\overset{\triangle}{\boldsymbol{F}^e} = \dot{\boldsymbol{F}}^e + \boldsymbol{F}^e \boldsymbol{\omega}_u; \quad \overset{\triangle}{\boldsymbol{F}^p} = \dot{\boldsymbol{F}}^p - \boldsymbol{\omega}_u \boldsymbol{F}^p \tag{6.1–2}$$

$$\overset{\circ}{\boldsymbol{V}} = \dot{\boldsymbol{V}} - \boldsymbol{\omega}\, \boldsymbol{V} + \boldsymbol{V}\boldsymbol{\omega}; \quad \overset{\circ}{\boldsymbol{P}} = \dot{\boldsymbol{P}} - \boldsymbol{\omega}\, \boldsymbol{P} \tag{6.3–4}$$

$$\overset{\triangle}{\boldsymbol{R}^e_u} = \dot{\boldsymbol{R}}^e_u + \boldsymbol{R}^e_u \boldsymbol{\omega}_u; \quad \boldsymbol{\omega} = \overset{\triangle}{\boldsymbol{R}^e_u} \boldsymbol{R}^{e^T}_u = \dot{\boldsymbol{R}}^e_u \boldsymbol{R}^{e^T}_u + \boldsymbol{R}^e_u \boldsymbol{\omega}_u \boldsymbol{R}^{e^T}_u. \tag{6.5–6}$$

Notice that the form of corotational rates for the second order tensors entering Eqs. (6) do not conform exactly with the definition, Eq. (4.1). This is because these are two-point tensors with their two indices attached to different configurations. Hence, since the spin $\boldsymbol{\omega}_u$ or $\boldsymbol{\omega}$ applies to the director vectors in one configuration (κ_u or κ), it is only the corresponding index attached to the spinning triad which brings-in the corotational aspect (the + or − in Eqs. (6) depends on whether the relevant index is the first or second). Notable exception is the corotational rates for $\boldsymbol{V}$, Eq. (6.3), because when the director vectors at κ_u spin by $\boldsymbol{\omega}$, so do also at κ given the fact that by definition the transport from κ_u to κ involves only distortion (elastic embedding) without rotation; hence, both indices of $\boldsymbol{V}$ bring-in the corotation of $\boldsymbol{V}$ in reference to a fixed cartesian coordinate system. Using Eqs. (2) and (6), it can be shown that

$$\dot{\boldsymbol{F}}_s \boldsymbol{F}_s^{-1} = \overset{\triangle}{\boldsymbol{F}^e_i} \boldsymbol{F}^{e^{-1}}_i = \dot{\boldsymbol{F}}^e_i \dot{\boldsymbol{F}}^{e^{-1}}_i = \boldsymbol{\omega} + \overset{\circ}{\boldsymbol{V}} \boldsymbol{V}^{-1} \tag{7.1}$$

$$\boldsymbol{F}^e \overset{\triangle}{\boldsymbol{F}^p} \boldsymbol{F}^{p^{-1}} \boldsymbol{F}^{e^{-1}} = \boldsymbol{F}^e_i \dot{\boldsymbol{F}}^p_i \boldsymbol{F}^{p^{-1}}_i \boldsymbol{F}^{e^{-1}}_i = \boldsymbol{V} \overset{\circ}{\boldsymbol{P}} \boldsymbol{P}^{-1} \boldsymbol{V}^{-1}. \tag{7.2}$$

It is interesting to observe that Eq. (7.1) represents the velocity gradient of the substructure, $\dot{\boldsymbol{F}}_s \boldsymbol{F}_s^{-1}$, hence, no plastic rate of deformation or spin appears.

Based on Eqs. (1), (6), and (7) the total velocity gradient $\dot{\boldsymbol{F}} \boldsymbol{F}^{-1}$ can be decomposed as follows

$$\begin{aligned} \dot{\boldsymbol{F}} \boldsymbol{F}^{-1} &= \dot{\boldsymbol{F}}^e \boldsymbol{F}^{e^{-1}} + \boldsymbol{F}^e \dot{\boldsymbol{F}}^p \boldsymbol{F}^{p^{-1}} \boldsymbol{F}^{e^{-1}} = \overset{\triangle}{\boldsymbol{F}^e} \boldsymbol{F}^{e^{-1}} + \boldsymbol{F}^e \overset{\triangle}{\boldsymbol{F}^p} \boldsymbol{F}^{p^{-1}} \boldsymbol{F}^{e^{-1}} \\ &= \dot{\boldsymbol{F}}^e_i \boldsymbol{F}^{e^{-1}}_i + \boldsymbol{F}^e_i \dot{\boldsymbol{F}}^p_i \boldsymbol{F}^{p^{-1}}_i \boldsymbol{F}^{e^{-1}}_i = \boldsymbol{\omega} + \overset{\circ}{\boldsymbol{V}} \boldsymbol{V}^{-1} + \boldsymbol{V} \overset{\circ}{\boldsymbol{P}} \boldsymbol{P}^{-1} \boldsymbol{V}^{-1}. \end{aligned} \tag{8.1–4}$$

Observe that Eq. (8) can be obtained by adding the corresponding terms of Eqs. (7.1) and (7.2), i.e. adding the plastic velocity gradient (Eq. (7.2)), to the velocity gradient of the substructure (Eq. (7.1)). Taking the symmetric and antisymmetric parts of Eq. (8) one has

$$(\dot{\boldsymbol{F}}\boldsymbol{F}^{-1})_s = \boldsymbol{D} = \boldsymbol{D}^e + \boldsymbol{D}^p \tag{9.1}$$

$$(\dot{\boldsymbol{F}}\boldsymbol{F}^{-1})_a = \boldsymbol{W} = \boldsymbol{W}^* + \boldsymbol{W}^p = \boldsymbol{\omega} + \boldsymbol{W}^e + \boldsymbol{W}^p \tag{9.2}$$

with the definitions for the elastic and plastic rates of deformation and spins $\boldsymbol{D}^e$, $\boldsymbol{D}^p$, $\boldsymbol{W}^e$ and $\boldsymbol{W}^p$ at the current configuration κ given by

$$\boldsymbol{D}^e = (\overset{\triangle}{\boldsymbol{F}^e}\boldsymbol{F}^{e^{-1}})_s = (\dot{\boldsymbol{F}}_i^e\boldsymbol{F}_i^{e^{-1}})_s = (\overset{\circ}{\boldsymbol{V}}\boldsymbol{V}^{-1})_s \tag{10.1}$$

$$\boldsymbol{W}^* = (\overset{\triangle}{\boldsymbol{F}^e}\boldsymbol{F}^{e^{-1}})_a = (\dot{\boldsymbol{F}}_i^e\boldsymbol{F}_i^{e^{-1}})_a = \boldsymbol{\omega} + (\overset{\circ}{\boldsymbol{V}}\boldsymbol{V}^{-1})_a = \boldsymbol{\omega} + \boldsymbol{W}^e \tag{10.2}$$

$$\boldsymbol{D}^p = (\boldsymbol{F}^e\overset{\triangle}{\boldsymbol{F}^p}\boldsymbol{F}^{p^{-1}}\boldsymbol{F}^{e^{-1}})_s = (\boldsymbol{F}_i^e\dot{\boldsymbol{F}}_i^p\boldsymbol{F}_i^{p^{-1}}\boldsymbol{F}_i^{e^{-1}})_s = (\boldsymbol{V}\overset{\circ}{\boldsymbol{P}}\boldsymbol{P}^{-1}\boldsymbol{V}^{-1})_s \tag{10.3}$$

$$\boldsymbol{W}^p = (\boldsymbol{F}^e\overset{\triangle}{\boldsymbol{F}^p}\boldsymbol{F}^{p^{-1}}\boldsymbol{F}^{e^{-1}})_a = (\boldsymbol{F}_i^e\dot{\boldsymbol{F}}_i^p\boldsymbol{F}_i^{p^{-1}}\boldsymbol{F}_i^{e^{-1}})_a = (\boldsymbol{V}\overset{\circ}{\boldsymbol{P}}\boldsymbol{P}^{-1}\boldsymbol{V}^{-1})_a. \tag{10.4}$$

It is interesting to observe four points in relation to Eqs. (8)–(10). First, that the rates of deformation and spins are expressed in three equivalent ways using the elastic and plastic deformation gradients pertaining to the three intermediate unstressed configurations. Second, that these expressions involve the corresponding corotational rates $\overset{\triangle}{\boldsymbol{F}^e}$, $\overset{\triangle}{\boldsymbol{F}^p}$, $\overset{\circ}{\boldsymbol{V}}$, $\overset{\circ}{\boldsymbol{P}}$ in reference to κ_a and κ_a, and not the usual material time derivatives: this is based on the concept of the spinning substructure as discussed earlier, and it will be of fundamental importance in studying the constitutive invariance under superposed rigid body rotation. Notice that for $\boldsymbol{F}_i^e$ and $\boldsymbol{F}_i^p$ the material time derivatives and the corotational rates coincide since $\boldsymbol{\omega}_i = \boldsymbol{0}$. Despite the apparent similarity, the pair of $\dot{\boldsymbol{F}}_i^e\boldsymbol{F}_i^{e^{-1}}$ and $\dot{\boldsymbol{F}}_i^p\boldsymbol{F}_i^{p^{-1}}$ express totally different concepts from the pair of $\dot{\boldsymbol{F}}^e\boldsymbol{F}^{e^{-1}}$ and $\dot{\boldsymbol{F}}^p\boldsymbol{F}^{p^{-1}}$: the former, but not the latter, are properly invariant due to the definition of κ_i as it will be shown subsequently. A third interesting point, which expresses a personal preference of the author and has been pursued in [12]–[13], [19]–[21], is that the decomposition of the rates using the configuration κ_a offers greater clarity, especially in reference to the spin terms. From Eq. (9.2) it follows that the total material spin $\boldsymbol{W}$ in additively decomposed into the rigid-body substructural spin $\boldsymbol{\omega}$, the elastic spin $\boldsymbol{W}^e$ caused by the antisymmetric part $(\overset{\circ}{\boldsymbol{V}}\boldsymbol{V}^{-1})_a$ of the substructural elastic rate of distortion, and the plastic spin $\boldsymbol{W}^p$. The appearance of $\boldsymbol{\omega}$ as a part of $\boldsymbol{W}$ is explicit only in reference to κ_a, while it is implicit (hidden) in the second and third members of Eq. (10.2) if one uses the κ_a and κ_i. The $\boldsymbol{W}^* = \boldsymbol{\omega} + \boldsymbol{W}^e$ has been often called the elastic spin. In addition, the use of κ_a makes easier the study of small elastic deformations by setting $\boldsymbol{V} \simeq \boldsymbol{I}$ [13], [20]–[21], while by using κ_a or κ_i one must set $\boldsymbol{F}^e \simeq \boldsymbol{R}_a^e$ or $\boldsymbol{F}_i^e \simeq \boldsymbol{R}_i^e$.

The fourth point of interest is that the definition of the plastic rate of deformation $\boldsymbol{D}^p$ and plastic spin $\boldsymbol{W}^p$ at κ, involves the corresponding elastic deformation gradients $\boldsymbol{F}^e$, $\boldsymbol{F}^e_i$ and $\boldsymbol{V}$ in reference to κ_u, κ_i and κ_o. This has been a point of considerable concern in the literature [6], [7], [24]–[26], with arguments and counterarguments on the validity of the additive decomposition of $\boldsymbol{D}$, Eq. (9.1). However, the intervention of the elastic deformation gradients is expected since the plastic rate of deformation and spin occur first in the intermediate configuration (whichever it may be), and the elastic deformation gradients simply transport the plastic kinematical rates to the current configuration. This point of view has been assumed in Mandel's work [1], [2] and made explicit by Nemat-Nasser [26]. The "purity" of $\boldsymbol{D}^p$ (and $\boldsymbol{W}^p$) as only plastic, has been correctly debated by Lee et al. [6], [7], [24], but as long as the foregoing arguments on elastic transport are understood and incorporated in the constitutive relations, there is no other but semantic reason in continuing this debate. In fact, the "purely plastic" definitions of the plastic rates of deformation and plastic spins in reference to the three intermediate configurations κ_u, κ_o and κ_i are given by

$$\boldsymbol{D}^p_u = (\hat{\bar{\boldsymbol{F}}}^p \boldsymbol{F}^{p^{-1}})_s; \quad \boldsymbol{D}^p_o = (\mathring{\boldsymbol{P}}\boldsymbol{P}^{-1})_s; \quad \boldsymbol{D}^p_i = (\dot{\boldsymbol{F}}^p_i \boldsymbol{F}^{p^{-1}}_i)_s \tag{11.1}$$

$$\boldsymbol{W}^p_u = (\hat{\bar{\boldsymbol{F}}}^p \boldsymbol{F}^{p^{-1}})_a; \quad \boldsymbol{W}^p_o = (\mathring{\boldsymbol{P}}\boldsymbol{P}^{-1})_a; \quad \boldsymbol{W}^p_i = (\dot{\boldsymbol{F}}^p_i \boldsymbol{F}^{p^{-1}}_i)_a \tag{11.2}$$

where the used notation is self-evident. Based on the polar decomposition and Eqs. (6) and (7), one can interrelate the above quantities according to

$$\boldsymbol{D}^p_o = \boldsymbol{R}^e_u \boldsymbol{D}^p_u \boldsymbol{R}^{e^{\mathrm{T}}}_u = \boldsymbol{R}^e_i \boldsymbol{D}^p_i \boldsymbol{R}^{e^{\mathrm{T}}}_i; \quad \boldsymbol{W}^p_o = \boldsymbol{R}^e_u \boldsymbol{W}^p_u \boldsymbol{R}^{e^{\mathrm{T}}}_u = \boldsymbol{R}^e_i \boldsymbol{W}^p_i \boldsymbol{R}^{e^{\mathrm{T}}}_i. \tag{12}$$

It must be pointed out however from Eq. (10.3), that it is not the transport of $\boldsymbol{D}^p_o$ alone from κ_o to κ, but the symmetric part of the transport of $\boldsymbol{D}^p_o + \boldsymbol{W}^p_o$ which yields the $\boldsymbol{D}^p$. The fact that the $\boldsymbol{W}^p_o$ at κ_o contributes via its transport to the value of $\boldsymbol{D}^p$ at κ, must not be surprising: it occurs in the fundamental mechanism of single slip, as it will be seen in the second part of this work [27]. The foregoing point applies also for the $\boldsymbol{W}^p$, and in reference to any other intermediate configuration.

6. The Elastic Embedding

In what follows the procedure developed in [20], [21] is presented, but instead of restricting the presentation in reference to κ_o, all three intermediate configurations will be examined with emphasis on κ_u, as being the most general. It will be shown that identical results are obtained at the end, irrespective of the choice of the intermediate configuration.

The material state will be defined at the current configuration κ in terms of the Cauchy stress $\boldsymbol{\sigma}$ (temperature is omitted for simplicity) and a collection $\boldsymbol{s} = \{\boldsymbol{a}, \boldsymbol{m}, k\}$ of structure variables, which for the sake simplicity will be restricted to be second order tensors $\boldsymbol{a}$, vectors $\boldsymbol{m}$ and scalars k. The $\boldsymbol{\sigma}$ and $\boldsymbol{s}$ are the actual current values of the state variables. The $\boldsymbol{s}$ are transported by the elastic deformation gradient to the corresponding unstressed configuration in a number of different ways, depending on their physical meaning. This elastic transport is called elastic embedding [20], [21], and the transported structure variables will be symbolized by $\boldsymbol{S}=\{\boldsymbol{A},\boldsymbol{M},K\}$ for tensors, vectors and scalars, respectively. For example, in reference to κ_u one may have the following transports of $\boldsymbol{s}$:

Contravariant:	$\boldsymbol{A}=\lvert\boldsymbol{F}^e\rvert^w\boldsymbol{F}^{e^{-1}}\boldsymbol{a}\,\boldsymbol{F}^{e^{-T}}$;	$\boldsymbol{M}=\lvert\boldsymbol{F}^e\rvert^w\boldsymbol{F}^{e^{-1}}\boldsymbol{m}$	(13.1)
Covariant:	$\boldsymbol{A}=\lvert\boldsymbol{F}^e\rvert^w\boldsymbol{F}^{e^{T}}\boldsymbol{a}\,\boldsymbol{F}^{e}$;	$\boldsymbol{M}=\lvert\boldsymbol{F}^e\rvert^w\boldsymbol{m}\,\boldsymbol{F}^{e}$	(13.2)
Mixed:	$\boldsymbol{A}=\lvert\boldsymbol{F}^e\rvert^w\boldsymbol{F}^{e^{T}}\boldsymbol{a}\,\boldsymbol{F}^{e^{-T}}$;	$\boldsymbol{A}=\lvert\boldsymbol{F}^e\rvert^w\boldsymbol{F}^{e^{-1}}\boldsymbol{a}\,\boldsymbol{F}^{e}$	(13.3)
Scalar:	$K=\lvert\boldsymbol{F}^e\rvert^w k$		(13.4)

where w is the weight, if $\boldsymbol{s}$ is a relative tensor, and $\lvert\boldsymbol{F}^e\rvert$ denotes the determinant of $\boldsymbol{F}^e$. Similarly the transport of $\boldsymbol{\sigma}$ will be denoted by $\boldsymbol{\Pi}$, and can be obtained from Eqs. (13) by substituting $\boldsymbol{\sigma}$ and $\boldsymbol{\Pi}$ for $\boldsymbol{a}$ and $\boldsymbol{A}$, respectively. The most common stress transport used in relation to the elastic constitutive laws, is the 2nd Piola-Kirchhoff stress tensor, defined by $\boldsymbol{\Pi}=\lvert\boldsymbol{F}^e\rvert\boldsymbol{F}^{e^{-1}}\boldsymbol{\sigma}\boldsymbol{F}^{e^{-T}}$ in reference to κ_u. If the transport is in reference to κ_o and κ_i, it suffices to substitute $\boldsymbol{V}$ and $\boldsymbol{F}^e_i$, respectively, for $\boldsymbol{F}^e$ in Eqs. (13) and in the transport of $\boldsymbol{\sigma}$ (the use of $\boldsymbol{V}$ was followed in [20], [21]). Hence the transported $\boldsymbol{A}$, $\boldsymbol{M}$ and $\boldsymbol{\Pi}$ in the three unstressed configurations differ only by rotation and can be interrelated based on Eqs. (3), while the K is the same since $\lvert\boldsymbol{F}^e\rvert=\lvert\boldsymbol{V}\rvert=\lvert\boldsymbol{F}^e_i\rvert$.

There is, however, a difference of physical importance between the embedding (or transport) of a structure variable, such as $\boldsymbol{A}=\lvert\boldsymbol{F}^e\rvert^w\boldsymbol{F}^{e^{-1}}\boldsymbol{a}\boldsymbol{F}^{e^{-T}}$, and that of the stress tensor $\boldsymbol{\Pi}=\lvert\boldsymbol{F}^e\rvert\boldsymbol{F}^{e^{-1}}\boldsymbol{\sigma}\boldsymbol{F}^{e^{-T}}$. When the stress $\boldsymbol{\sigma}$ varies in the current configuration κ without causing any plastic loading (which would imply plastic constitutive changes), the $\boldsymbol{F}^e$ also varies due to the elastic deformation change following the $\boldsymbol{\sigma}$ variation. This simultaneous change of $\boldsymbol{\sigma}$ and $\boldsymbol{F}^e$ will cause a change of $\boldsymbol{\Pi}$, consistent with the elastic relations (to be presented in [27], part 2). However, due to its elastic embedding the $\boldsymbol{a}$ will vary in such a way, that in combination with the variation of $\boldsymbol{F}^e$ will keep the $\boldsymbol{A}$ unchanged (apart from possible rotations). This is in fact the full meaning of the elastic embedding which transports the $\boldsymbol{a}$ into $\boldsymbol{A}$, and will be reflected in their constitutive rate equations.

Anticipating the formulation of the rate constitutive relations for the structure variables, consider a typical example for the corotational with the substructure rates of a contravariantly elastically embedded second order tensor, in reference to all three unstressed configurations. In what follows, a superposed $*$ and $\triangledown$ denote corotational rates with respect to $\boldsymbol{W}^*$ and $\boldsymbol{W}$, respectively, a superposed $\vee$ denotes the convected derivative of relative tensors (for $w=1$ and contravariant embedding it yields the Truesdell derivative), and a superposed $\square$ denotes the so-called corodeformational rate [20], [21]. Then, based on Eqs.

(4), (6), (9), and (10), one has for

$$\kappa_u:\quad \boldsymbol{A} = |\boldsymbol{F}^{\rm e}|^{w}\,\boldsymbol{F}^{\rm e^{-1}}\,\boldsymbol{a}\,\boldsymbol{F}^{\rm e^{-T}};\qquad \overset{\triangle}{\boldsymbol{A}} = |\boldsymbol{F}^{\rm e}|^{w}\,\boldsymbol{F}^{\rm e^{-1}}\,\overset{\square}{\boldsymbol{a}}\,\boldsymbol{F}^{\rm e^{-T}} \tag{14.1}$$

$$\kappa_o:\quad \boldsymbol{A} = |\boldsymbol{V}|^{w}\,\boldsymbol{V}^{-1}\,\boldsymbol{a}\,\boldsymbol{V}^{-1};\qquad \overset{\circ}{\boldsymbol{A}} = |\boldsymbol{V}|^{w}\,\boldsymbol{V}^{-1}\,\overset{\square}{\boldsymbol{a}}\,\boldsymbol{V}^{-1} \tag{14.2}$$

$$\kappa_i:\quad \boldsymbol{A} = |\boldsymbol{F}_i^{\rm e}|^{w}\,\boldsymbol{F}_i^{\rm e^{-1}}\,\boldsymbol{a}\,\boldsymbol{F}_i^{\rm e^{-1}};\qquad \dot{\boldsymbol{A}} = |\boldsymbol{F}_i^{\rm e}|^{w}\,\boldsymbol{F}_i^{\rm e^{-1}}\,\overset{\square}{\boldsymbol{a}}\,\boldsymbol{F}_i^{\rm e^{-1}} \tag{14.3}$$

$$\begin{aligned}\kappa:\quad \overset{\square}{\boldsymbol{a}} &= \dot{\boldsymbol{a}} - \boldsymbol{a}\,\boldsymbol{F}^{\rm e^{-T}}\,\overset{\triangle}{\boldsymbol{F}}{}^{\rm eT} - \overset{\triangle}{\boldsymbol{F}}{}^{\rm e}\,\boldsymbol{F}^{\rm e^{-1}}\,\boldsymbol{a} + w\,\boldsymbol{a}\,{\rm tr}\,\boldsymbol{D}^{\rm e}\\ &= \overset{\circ}{\boldsymbol{a}} - \boldsymbol{a}\,\boldsymbol{V}^{-1}\,\overset{\circ}{\boldsymbol{V}} - \overset{\circ}{\boldsymbol{V}}\boldsymbol{V}^{-1}\,\boldsymbol{a} + w\,\boldsymbol{a}\,{\rm tr}\,\boldsymbol{D}^{\rm e}\\ &= \dot{\boldsymbol{a}} - \boldsymbol{a}\,\boldsymbol{F}_i^{\rm e^{-T}}\,\dot{\boldsymbol{F}}_i^{\rm eT} - \dot{\boldsymbol{F}}_i^{\rm e}\,\boldsymbol{F}_i^{\rm e^{-1}}\,\boldsymbol{a} + w\,\boldsymbol{a}\,{\rm tr}\,\boldsymbol{D}^{\rm e}\\ &= \overset{*}{\boldsymbol{a}} - \boldsymbol{a}\,\boldsymbol{D}^{\rm e} - \boldsymbol{D}^{\rm e}\,\boldsymbol{a} + w\,\boldsymbol{a}\,{\rm tr}\,\boldsymbol{D}^{\rm e}\end{aligned} \tag{15}$$

and with

$$\overset{\vee}{\boldsymbol{a}} = \dot{\boldsymbol{a}} - \boldsymbol{a}\,\boldsymbol{F}^{-T}\,\dot{\boldsymbol{F}}^{T} - \dot{\boldsymbol{F}}\boldsymbol{F}^{-1}\,\boldsymbol{a} + w\,\boldsymbol{a}\,{\rm tr}\,\boldsymbol{D} = \overset{\triangledown}{\boldsymbol{a}} - \boldsymbol{a}\,\boldsymbol{D} - \boldsymbol{D}\,\boldsymbol{a} + w\,\boldsymbol{a}\,{\rm tr}\,\boldsymbol{D} \tag{16}$$

the subtraction of Eq. (15) from Eq. (16) yields, using Eqs. (9) and (10),

$$\overset{\vee}{\boldsymbol{a}} - \overset{\square}{\boldsymbol{a}} = -\,(\boldsymbol{a}\,\boldsymbol{D}^{p} + \boldsymbol{D}^{p}\,\boldsymbol{a}) + (\boldsymbol{a}\,\boldsymbol{W}^{p} - \boldsymbol{W}^{p}\,\boldsymbol{a}) + w\,\boldsymbol{a}\,{\rm tr}\,\boldsymbol{D}^{p}. \tag{17}$$

Similar expressions can be obtained for the rates of the stress tensors, with $\boldsymbol{\sigma}$ and $\boldsymbol{\Pi}$ substituting for $\boldsymbol{a}$ and $\boldsymbol{A}$, respectively, and $w = 1$, in Eqs. (14)–(17). What is really important in the above equations can be seen in the second part of Eqs. (14): it shows that the corotational rates $\overset{\triangle}{\boldsymbol{A}}$, $\overset{\circ}{\boldsymbol{A}}$, $\dot{\boldsymbol{A}}$ in reference to the corresponding substructural spins ($\boldsymbol{\omega}_u$ for κ_u, $\boldsymbol{\omega}$ for κ_o, $\boldsymbol{\omega}_i = \boldsymbol{0}$ for κ_i) are related to the corodeformational rate $\overset{\square}{\boldsymbol{a}}$, in the same way as $\boldsymbol{A}$ is related to $\boldsymbol{a}$. Hence, when $\overset{\square}{\boldsymbol{a}} = \boldsymbol{0} \Rightarrow \overset{\triangle}{\boldsymbol{A}} = \overset{\circ}{\boldsymbol{A}} = \dot{\boldsymbol{A}} = \boldsymbol{0}$ and vice-versa, reflecting analytically the content of the discussion following Eqs. (13) on the meaning of the elastic embedding. The name corodeformational stems from the fact that any value of $\overset{\square}{\boldsymbol{a}}$ will represent a change of $\boldsymbol{a}$ apart from the changes due to the corotation with the substructure and the elastic embedding with it, i.e. a *true constitutive change*. This will be a key point in the subsequent formulation of constitutive relations. Observe from Eq. (15) that the $\overset{\square}{\boldsymbol{a}}$ can be expressed in a variety of equivalent ways, but it is unique. It is also interesting to observe in Eq. (15), that while the corotational rate $\overset{\circ}{\boldsymbol{a}}$ is used in conjunction with $\overset{\circ}{\boldsymbol{V}}$ at κ_o, it is the usual rate $\dot{\boldsymbol{a}}$ and not $\overset{\triangle}{\boldsymbol{a}}$ which must be used in conjunction with $\overset{\triangle}{\boldsymbol{F}}{}^{\rm e}$ at κ_u. The explanation of this seemingly inconsistent point must be sought in Eqs. (14.1) and (14.2). When $\overset{\square}{\boldsymbol{a}}$ is related to $\overset{\circ}{\boldsymbol{A}}$ at κ_o, Eq. (14.2), one must recall that the substructural spin $\boldsymbol{\omega}$ applied to $\boldsymbol{A}$ at κ_o must also apply to $\boldsymbol{a}$ at κ, due to the very definition of κ_o ($\boldsymbol{V} = \boldsymbol{V}^T$, i.e. the substructures at κ and κ_o spin by the same amount); hence the appearance of $\overset{\circ}{\boldsymbol{a}}$ in Eq. (15). But when $\overset{\square}{\boldsymbol{a}}$ is related to $\overset{\triangle}{\boldsymbol{A}}$ at κ_u, Eq. (14.1), the substructural spin $\boldsymbol{\omega}_u$ applied to $\boldsymbol{A}$ at κ_u *does not effect* $\boldsymbol{a}$ at κ, due to the independent spinning of the substructures in κ_u and κ; hence, the appearance of $\dot{\boldsymbol{a}}$. Of course in reference to κ_i all corotational rates are the usual rates since $\boldsymbol{\omega}_i = \boldsymbol{0}$. Based on the definition (16), which in fact can be derived from $\boldsymbol{A} = |\boldsymbol{F}|^w \boldsymbol{F}^{-1} \boldsymbol{a} \boldsymbol{F}^{-T} \Rightarrow \dot{\boldsymbol{A}} = |\boldsymbol{F}|^w \boldsymbol{F}^{-1} \overset{\vee}{\boldsymbol{a}} \boldsymbol{F}^{-T}$, Eq. (17) will allow to relate $\overset{\vee}{\boldsymbol{a}}$ and $\overset{\square}{\boldsymbol{a}}$ in the following. Similar relations to Eqs. (14)–(17) can be obtained for all other kinds of embedding of tensors, vectors and scalars but are not presented for brevity.

A point which is worth mentioning is that in the development no explicit appearance of a plastic strain measure arises as a primary variable of the theory. Clearly one can define Almansi plastic strain measures $\boldsymbol{A}^p_o$, $\boldsymbol{A}^p_u$, $\boldsymbol{A}^p_i$ in reference to κ_o, κ_u, κ_i, by $\boldsymbol{A}^p_o = \boldsymbol{R}^e_u \boldsymbol{A}^p_u \boldsymbol{R}^{e^T}_u = \boldsymbol{R}^e_i \boldsymbol{A}^p_i \boldsymbol{R}^{e^T}_i = (1/2)[\boldsymbol{I} - \boldsymbol{P}^{-T} \boldsymbol{P}^{-1}] = (1/2)\boldsymbol{R}^{ep}[\boldsymbol{I} - \boldsymbol{U}^{-2}]\boldsymbol{R}^{ep^T}$ (the equalities follow from Eq. (1)), and consider them as one of the $\boldsymbol{S}$. Subsequently, such a plastic strain measure can be transported at the current configuration κ as one of the $\boldsymbol{s}$. For example, considering the $\boldsymbol{A}^p_o$, one can define at κ the $\boldsymbol{a}^p = \boldsymbol{V}^{-1} \boldsymbol{A}^p_o \boldsymbol{V}^{-1} = (1/2)[\boldsymbol{V}^{-2} - \boldsymbol{F}^{-T} \boldsymbol{F}^{-1}]$. It is interesting now to observe that in a Lagrangian description the $\boldsymbol{s}$ will be transported from κ to κ_r. Along these lines the $\boldsymbol{a}^p$ can be transported at κ_r as $\boldsymbol{A}^p_r = \boldsymbol{F}^T \boldsymbol{a}^p \boldsymbol{F} = (1/2)[\boldsymbol{P}^T \boldsymbol{P} - \boldsymbol{I}] = (1/2)[\boldsymbol{F}^{p^T} \boldsymbol{F}^p - \boldsymbol{I}] = (1/2)[\boldsymbol{U}^2 - \boldsymbol{I}]$, which is nothing else but the Green plastic strain measure between κ_r and any one of the intermediate configurations. Notice that such Green measure was not introduced as a primary variable of the general theory, as done in other works, but simply followed the initial introduction of an Almansi measure (appropriate for an Eulerian description) and its subsequent transports to the current and the reference configurations. Whether or not, however, such a plastic strain measure in any one of the aforementioned forms can play an important role in defining the substructure, i.e. can be considered as one of the $\boldsymbol{S}$, is a debatable question. Nevertheless, if such a conclusion can be reached, the use of an Almansi plastic strain measure is recommended as one of the $\boldsymbol{S}$ [20], because it is referred to an intermediate configuration where also all other $\boldsymbol{S}$ are referred to (semi-Eulerian description). It can be shown that its corotational rate is given by $\overset{\circ}{\boldsymbol{A}}{}^p_o = \boldsymbol{D}^p_o - (\boldsymbol{A}^p_o \boldsymbol{D}^p_o + \boldsymbol{D}^p_o \boldsymbol{A}^p_o) - (\boldsymbol{A}^p_o \boldsymbol{W}^p_o - \boldsymbol{W}^p_o \boldsymbol{A}^p_o)$, if one chooses the κ_o configuration; hence, its evolution depends on the constitutive equations for $\boldsymbol{D}^p_o$ and $\boldsymbol{W}^p_o$ [20], [21].

7. Transformation Under Superposed Rigid Body Rotations

In proposing a constitutive relation it is necessary to impose the restrictions derived from invariance requirements under superposed rigid body rotation, henceforth abbreviated as s.r.b.r. Therefore, one must know how the different quantities and their rates transform under s.r.b.r., and this will be the objective of the present section. The question whether invariance requirements must be imposed at the current configuration only, or at both the current and intermediate, has been debated in the literature [5]–[8], [23]. It will be shown that such a question does not arise if one uses the concept of substructural orientation and spin. In fact the problem will be studied in each one of the three unstressed configurations, yielding the same results at the end.

Henceforth, assume that the current configuration κ is subjected to a s.r.b.r. described analytically by the orthogonal tensor $\boldsymbol{Q}(t)$. The arbitrariness of the orientation of the configuration κ_u allows us to assume that it is subjected to a s.r.b.r. described by a different orthogonal tensor $\boldsymbol{Q}_u(t)$ [5] (in the following the notation for the dependence of $\boldsymbol{Q}$ and $\boldsymbol{Q}_u$ on t will be suppressed). By its very definition the isoclinic configuration κ_i is not affected by a $\boldsymbol{Q}$ occurring at the current configuration κ, i.e. the corresponding rotation $\boldsymbol{Q}_i = \boldsymbol{I}$. Similarly, the definition of κ_o, linked by $\boldsymbol{V}$ with κ, necessitates that when κ is subjected to $\boldsymbol{Q}$ so does κ_o in order to maintain the symmetry relation $\boldsymbol{V} = \boldsymbol{V}^T$ [7]. Bearing these physical observations in mind and denoting by a superscript * a quantity transformed by s.r.b.r., the following relations can be stated

For the continuum at

$$\kappa_u: \quad \boldsymbol{F}^{e*} = \boldsymbol{Q}\boldsymbol{F}^e\boldsymbol{Q}_u^T; \quad \boldsymbol{F}^{p*} = \boldsymbol{Q}_u\boldsymbol{F}^p \tag{18.1}$$

$$\kappa_o: \quad \boldsymbol{V}^* = \boldsymbol{Q}\boldsymbol{V}\boldsymbol{Q}^T; \quad \boldsymbol{P}^* = \boldsymbol{Q}\boldsymbol{P} \tag{18.2}$$

$$\kappa_i: \quad \boldsymbol{F}_i^{e*} = \boldsymbol{Q}\boldsymbol{F}_i^e; \quad \boldsymbol{F}_i^{p*} = \boldsymbol{F}_i^p. \tag{18.3}$$

For the substructure at

$$\kappa_u: \quad \boldsymbol{\beta}_u^* = \boldsymbol{Q}_u\boldsymbol{\beta}_u \tag{19.1}$$

$$\kappa_o: \quad \boldsymbol{\beta}^* = \boldsymbol{Q}\boldsymbol{\beta} \tag{19.2}$$

$$\kappa_i: \quad \boldsymbol{\beta}_i^* = \boldsymbol{\beta}_i = \boldsymbol{I}. \tag{19.3}$$

For the substructural spins at

$$\kappa_u: \quad \boldsymbol{\omega}_u^* = \dot{\boldsymbol{\beta}}_u^*\boldsymbol{\beta}_u^{*T} = \boldsymbol{Q}_u\boldsymbol{\omega}_u\boldsymbol{Q}_u^T + \dot{\boldsymbol{Q}}_u\boldsymbol{Q}_u^T \tag{20.1}$$

$$\kappa_o: \quad \boldsymbol{\omega}^* = \dot{\boldsymbol{\beta}}^*\boldsymbol{\beta}^{*T} = \boldsymbol{Q}\boldsymbol{\omega}\boldsymbol{Q}^T + \dot{\boldsymbol{Q}}\boldsymbol{Q}^T \tag{20.2}$$

$$\kappa_i: \quad \boldsymbol{\omega}_i^* = \boldsymbol{\omega}_i = \boldsymbol{0}. \tag{20.3}$$

Next, based on Eqs. (6), (18) and (20) and the relation $\dot{\boldsymbol{Q}}\boldsymbol{Q}^T + \boldsymbol{Q}\dot{\boldsymbol{Q}}^T = \boldsymbol{0}$ for $\boldsymbol{Q}\boldsymbol{Q}^T = \boldsymbol{I}$, the transformation under s.r.b.r. of the elastic and plastic rates of deformation and spins will be derived in relation to the use of each one of the three intermediate configurations.

In relation to κ_u

$$\overset{\triangle}{F}{}^{e*} = \dot{F}^{e*} + F^{e*}\omega_u^* = Q\dot{F}^e Q_u^T + \dot{Q}F^e Q_u^T + QF^e \dot{Q}_u^T + QF^e Q_u^T(\dot{Q}_u Q_u^T + Q_u \omega_u Q_u^T)$$

$$= Q[\dot{F}^e + F^e \omega_u] Q_u^T + \dot{Q}F^e Q_u^T = Q\overset{\triangle}{F}{}^e Q_u^T + \dot{Q}F^e Q_u^T \tag{21.1}$$

$$\overset{\triangle}{F}{}^{p*} = \dot{F}^{p*} - \omega_u^* F^{p*} = Q^u \dot{F}^p + \dot{Q}_u F^p - (\dot{Q}_u Q_u^T + Q_u \omega_u Q_u^T) Q_u F^p$$

$$= Q_u(\dot{F}^p - \omega_u F^p) = Q_u \overset{\triangle}{F}{}^p \tag{21.2}$$

$$\overset{\triangle}{F}{}^{e*} F^{e*-1} = Q\overset{\triangle}{F}{}^e Q_u^T Q_u F^{e-1} Q^T + \dot{Q}F^e Q_u^T Q_u F^{e-1} Q^T \tag{21.3}$$
$$= Q\overset{\triangle}{F}{}^e F^{e-1} Q^T + \dot{Q}Q^T$$

$$\overset{\triangle}{F}{}^{p*} F^{p*-1} = Q_u \overset{\triangle}{F}{}^p F^{p-1} Q_u^T \tag{21.4}$$

$$F^{e*}\overset{\triangle}{F}{}^{p*} F^{p*-1} F^{e*-1} = QF^e Q_u^T Q_u \overset{\triangle}{F}{}^p F^{p-1} Q_u^T Q_u F^{e-1} Q^T \tag{21.5}$$
$$= QF^e \overset{\triangle}{F}{}^p F^{p-1} F^{e-1} Q^T.$$

In relation to κ_n

$$\overset{\circ}{V}{}^* = \dot{V}^* - \omega^* V^* + V^* \omega^*$$
$$= \dot{Q}VQ^T + \dot{Q}VQ^T + QV\dot{Q}^T - (\dot{Q}Q^T + Q\omega Q^T)QVQ^T + QVQ^T(\dot{Q}Q^T + Q\omega Q^T)$$

$$= Q[\dot{V} - \omega V + V\omega]Q^T = Q\overset{\circ}{V}Q^T \tag{22.1}$$

$$\overset{\circ}{P}{}^* = \dot{P}^* - \omega^* P^* = \dot{Q}P + Q\dot{P} - (\dot{Q}Q^T + Q\omega Q^T)QP$$
$$= Q(\dot{P} - \omega P) = Q\overset{\circ}{P} \tag{22.2}$$

$$\overset{\circ}{V}{}^* V^{*-1} = Q\overset{\circ}{V}Q^T QV^{-1}Q^T = Q\overset{\circ}{V}V^{-1}Q^T \tag{22.3}$$

$$\overset{\circ}{P}{}^* P^{*-1} = Q\overset{\circ}{P}P^{-1}Q^T \tag{22.4}$$

$$V^* \overset{\circ}{P}{}^* P^{*-1} V^{*-1} = QVQ^T Q\overset{\circ}{P}P^{-1}Q^T QV^{-1}Q^T = QV\overset{\circ}{P}P^{-1}V^{-1}Q^T. \tag{22.5}$$

In relation to κ_i

$$\dot{F}_i^{e*} = \dot{Q}F_i^e + Q\dot{F}_i^e; \quad \dot{F}_i^{p*} = \dot{F}_i^p \tag{23.1}$$

$$\dot{F}_i^{e*} F_i^{e*-1} = Q\dot{F}_i^e F_i^{e-1} Q^T + \dot{Q}Q^T; \quad \dot{F}_i^{p*} F_i^{p*-1} = \dot{F}_i^p F_i^{p-1} \tag{23.2}$$

$$F_i^{e*} \dot{F}_i^{p*} F_i^{p*-1} F_i^{e*-1} = QF_i^e \dot{F}_i^p F_i^{p-1} F_i^{e-1} Q^T. \tag{23.3}$$

It follows now from Eqs. (21)–(23) and the definitions given by Eqs. (10)

and (11), that for the unstressed configurations

$$\kappa_u: \quad \boldsymbol{D}^{p*}_u = \boldsymbol{Q}_u \boldsymbol{D}^p_u \boldsymbol{Q}^T_u; \quad \boldsymbol{W}^{p*}_u = \boldsymbol{Q}_u \boldsymbol{W}^p_u \boldsymbol{Q}^T_u \tag{24.1}$$

$$\kappa_o: \quad \boldsymbol{D}^{p*}_o = \boldsymbol{Q}\boldsymbol{D}^p_o \boldsymbol{Q}^T; \quad \boldsymbol{W}^{p*}_o = \boldsymbol{Q}\boldsymbol{W}^p_o \boldsymbol{Q}^T \tag{24.2}$$

$$\kappa_i: \quad \boldsymbol{D}^{p*}_i = \boldsymbol{D}^p_i; \quad \boldsymbol{W}^{p*}_i = \boldsymbol{W}^p_i \tag{24.3}$$

and for the current configuration

$$\kappa: \quad \boldsymbol{D}^{e*} = \boldsymbol{Q}\boldsymbol{D}^e \boldsymbol{Q}^T; \quad \boldsymbol{W}^{e*} = \boldsymbol{Q}\boldsymbol{W}^e \boldsymbol{Q}^T \tag{25.1}$$

$$\boldsymbol{D}^{p*} = \boldsymbol{Q}\boldsymbol{D}^p \boldsymbol{Q}^T; \quad \boldsymbol{W}^{p*} = \boldsymbol{Q}\boldsymbol{W}^p \boldsymbol{Q}^T. \tag{25.2}$$

Hence, we have reached the important conclusion represented by Eqs. (25), that whichever is the choice of the unstressed configuration the elastic and plastic rates of deformation and spins at the current coonfiguration are properly invariant, i.e. they transform only by the corresponding rotation $\boldsymbol{Q}$. This was based on the physical requirement for full invariance under s.r.b.r. [5] for the choice κ_u, or partial invariance for the choice κ_o [6], [7] or κ_i [1], [2]. The equivalence of the final result, Eqs. (25), should therefore put aside all the arguments and counterarguments presented in [5]–[8] because it shows that each one is correct if viewed from a proper perspective. Equally important is the conclusion represented by Eqs. (24), namely that the plastic rate of deformation and spin at the unstressed configurations are properly invariant in reference to the corresponding rotations, that is $\boldsymbol{Q}_u$ for κ_u, $\boldsymbol{Q}$ for κ_o and $\boldsymbol{I}$ for κ_i. These conclusions, and the ones represented by Eqs. (18) and (19), play a fundamental role in the formulation of the constitutive relations, as it will be seen subsequently.

The importance of the foregoing conclusions can be better appreciated if one attempts to define the elastic and plastic rates of deformation and spin without using the proper corotational with the substructure rates of the different deformation gradients, as erroneously has been often suggested in the literature. Indeed, referring to Eq. (8.2), it is straightforward to show using Eq. (18.1) (full invariance requirement [5]) that

$$\dot{\boldsymbol{F}}^{e*}\boldsymbol{F}^{e*-1} = \boldsymbol{Q}\dot{\boldsymbol{F}}^e \boldsymbol{F}^{e-1}\boldsymbol{Q}^T + \dot{\boldsymbol{Q}}\boldsymbol{Q}^T - \boldsymbol{Q}\boldsymbol{F}^e \boldsymbol{Q}^T_u \dot{\boldsymbol{Q}}_u \boldsymbol{F}^{e-1}\boldsymbol{Q}^T \tag{26.1}$$

$$\dot{\boldsymbol{F}}^{p*}\boldsymbol{F}^{p*-1} = \boldsymbol{Q}_u \dot{\boldsymbol{F}}^p \boldsymbol{F}^{p-1}\boldsymbol{Q}^T_u + \dot{\boldsymbol{Q}}_u \boldsymbol{Q}^T_u \tag{26.2}$$

$$\boldsymbol{F}^{e*}\dot{\boldsymbol{F}}^{p*}\boldsymbol{F}^{p*-1}\boldsymbol{F}^{e*-1} = \boldsymbol{Q}\boldsymbol{F}^e \dot{\boldsymbol{F}}^p \boldsymbol{F}^{p-1}\boldsymbol{F}^{e-1}\boldsymbol{Q}^T + \boldsymbol{Q}\boldsymbol{F}^e \boldsymbol{Q}^T_u \dot{\boldsymbol{Q}}_u \boldsymbol{F}^{e-1}\boldsymbol{Q}^T. \tag{26.3}$$

Identifying now the elastic and plastic rate of deformation and spin in reference to κ as the symmetric and antisymmetric parts of the velocity gradients of Eqs. (26.1) and (26.3), and the plastic rate of deformation and spin in reference to κ_u similarly from Eq. (26.2), it is seen that the proper invariance under s.r.b.r. is not satisfied (compare with Eqs. (24.1) and (25)). Similar conclusion negating proper invariance is reached using Eq. (18.2), if one works in reference to κ_o with $\dot{\boldsymbol{V}}\boldsymbol{V}^{-1}$, $\dot{\boldsymbol{P}}\boldsymbol{P}^{-1}$ and $\boldsymbol{V}\dot{\boldsymbol{P}}\boldsymbol{P}^{-1}\boldsymbol{V}^{-1}$ instead of $\overset{\circ}{\boldsymbol{V}}\boldsymbol{V}^{-1}$, $\overset{\circ}{\boldsymbol{P}}\boldsymbol{P}^{-1}$ and $\boldsymbol{V}\overset{\circ}{\boldsymbol{P}}\boldsymbol{P}^{-1}\boldsymbol{V}^{-1}$, Eqs. (10).

Next, the transformation of the structure variables and their rates under s.r.b.r. will be studied. The computations are straightforward and are based on Eqs. (18) and (20)–(25), and the fact the $\boldsymbol{A}$ and $\boldsymbol{a}$ rotate with the substructure under s.r.b.r.. Hence, omitting superfluous algebra one can show that in reference to

$$\kappa_u: \quad \boldsymbol{A}^* = \boldsymbol{Q}_u \boldsymbol{A} \boldsymbol{Q}_u^T; \quad \overset{\triangle}{\boldsymbol{A}}{}^* = \boldsymbol{Q}_u \overset{\triangle}{\boldsymbol{A}} \boldsymbol{Q}_u^T \tag{27.1}$$

$$\kappa_o: \quad \boldsymbol{A}^* = \boldsymbol{Q} \boldsymbol{A} \boldsymbol{Q}^T; \quad \overset{\circ}{\boldsymbol{A}}{}^* = \boldsymbol{Q} \overset{\circ}{\boldsymbol{A}} \boldsymbol{Q}^T \tag{27.2}$$

$$\kappa_i: \quad \boldsymbol{A}^* = \boldsymbol{A}; \quad \dot{\boldsymbol{A}}^* = \dot{\boldsymbol{A}} \tag{27.3}$$

$$\kappa: \quad \boldsymbol{a}^* = \boldsymbol{Q} \boldsymbol{a} \boldsymbol{Q}^T; \quad \overset{\square}{\boldsymbol{a}}{}^* = \boldsymbol{Q} \overset{\square}{\boldsymbol{a}} \boldsymbol{Q}^T; \quad \overset{\vee}{\boldsymbol{a}}{}^* = \boldsymbol{Q} \overset{\vee}{\boldsymbol{a}} \boldsymbol{Q}^T. \tag{27.4}$$

For example, the $\overset{\square}{\boldsymbol{a}}{}^* = \boldsymbol{Q} \overset{\square}{\boldsymbol{a}} \boldsymbol{Q}^T$ can be easily obtained by the last expression for $\overset{\square}{\boldsymbol{a}}$ in Eq. (15), and Eqs. (25.1). Similar transformation under s.r.b.r. are obtained for vector valued structure variables and their rates, e.g. in ref. to κ_o $\boldsymbol{M}^* = \boldsymbol{Q}\boldsymbol{M}$, $\overset{\circ}{\boldsymbol{M}}{}^* = \boldsymbol{Q}\overset{\circ}{\boldsymbol{M}}$, and in reference to κ $\boldsymbol{m}^* = \boldsymbol{Q}\boldsymbol{m}$, $\overset{\square}{\boldsymbol{m}}{}^* = \boldsymbol{Q}\overset{\square}{\boldsymbol{m}}$ and $\overset{\vee}{\boldsymbol{m}}{}^* = \boldsymbol{Q}\overset{\vee}{\boldsymbol{m}}$.

8. Conclusion

The first part of this work has addressed in detail the analysis of the kinematics of both the continuum and its substructure in large deformation elastoplasticity. Emphasis was placed on formulating the analytical description of the kinematics in reference to each one of three possible definitions of the unstressed configuration, and showing that a unique analytical formulation is obtained in reference to the current configuration, as physically expected. In particular the transformation of the different kinematical and structure variables and their rates under superposed rigid body rotation was studied in detail. The obtained results will be instrumental in obtaining restrictions on the form of the constitutive equations to be presented in the second part of this work, dealing with the kinetics of the problem. Although a number of the issues listed in the introduction were addressed, the complete answer to all of them will be given in the development of the second part. This is because many of these issues cannot be answered, or their effect cannot be fully understood, unless they are considered from both the kinematics and kinetics points of view simultaneously.

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