VII. Partitioned Multiobjective Risk Method

PROBLEM VII.1: Taiwan High Speed Rail Selection

The Taiwan High Speed Rail is currently that country's most important means of transportation. It will be a key component of the economic future of Taiwan and the first public infrastructure project based on the Build, Operate, and Transfer (BOT) method in Taiwan. General features of this project include:

- Two tracks (northbound and southbound); multiple tracks at stations.
- A 350 km/hr design speed and maximum operating speed of 300 km/hr.
- Approximately 345 km from Taipei in the north to Kaohsiung in the south.
- Stations will be located along the High Speed Rail system; there are at most 12 counties on this line.
- Beside the stations, this system also needs some main depots and stabling yards.

The Taiwan High Speed Rail Corporation (THSRC), the winner of this project, not only builds High Speed Rail but also develops station areas. Income from both tickets and stations are the main profits of THSRC.

More stations will bring in more money but reduce the speed of rail transportation, which is not permitted by the government. Therefore, THSRC will assess expected cost overrun from three plans in order to verify whether a plan meet its needs as well as the government's initiatives.

DESCRIPTION

Table VII.1.1 depicts three station development plans for this project:

Table VII.1.1. The Station Development Plan

Plan	Budget (millions)	Number of Stations	Number of Main Depots	Number of Stabling Yards
A	95	7	1	1
B	105	9	1	2
C	120	11	1	3

The CEO of THSRC needs a more detailed analysis of the overrun risk behind these plans. He wants to know the expected overrun cost of a better probability of 15% (the best condition is no overrun) and the worst, 10%.

METHODOLOGY

We use the PMRM (Partitioned Multiobjective Risk Method) to do this analysis. The cost increase percentages of each plan are shown in Table VII.1.2, and we plot the Cumulative Distribution Function (CDF) curve shown in Figure VII.1.1 for each plan. Based on Figure VII.1.1, Table VII.1.2, and the intention of the CEO, traditional and conditional expected value will be calculated.

Table VII.1.2. Cost Increase Percentages

	Project Cost Increase (%)		
Fractile	**Plan A**	**Plan B**	**Plan C**
0	0	0	0
0.2	3	8	5
0.4	8	14	13
0.6	16	20	23
0.8	26	28	29
1	40	50	55

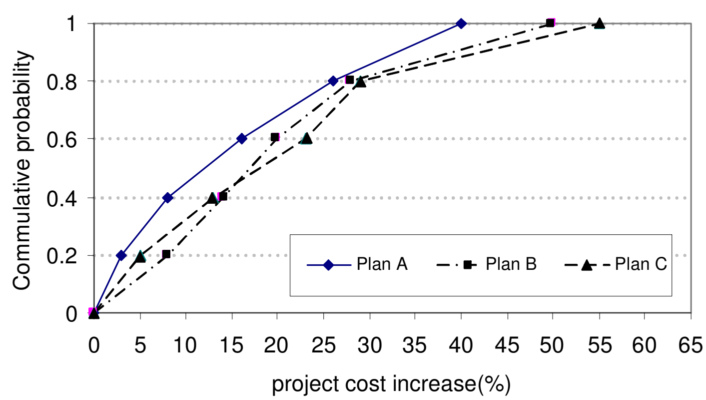


Figure VII.1.1. CDF for Project Cost Increase for Each Plan

SOLUTION

CDF Curve for Plan A:

1. Using the CDF, we calculate the Probability Density Function (PDF) as follows, and summarize the PDF data in Table VII.1.3.

Let p_i be the probabilities of each event.

$0.2 = p_1 * (3-0) \Rightarrow p_1 = 0.067$
$0.2 = p_2 * (8-3) \Rightarrow p_2 = 0.040$
$0.2 = p_3 * (16-8) \Rightarrow p_3 = 0.025$
$0.2 = p_4 * (26-16) \Rightarrow p_4 = 0.020$
$0.2 = p_5 * (40-26) \Rightarrow p_5 = 0.014$

2. We plot the PDF curve as shown in Figure VII.1.2.

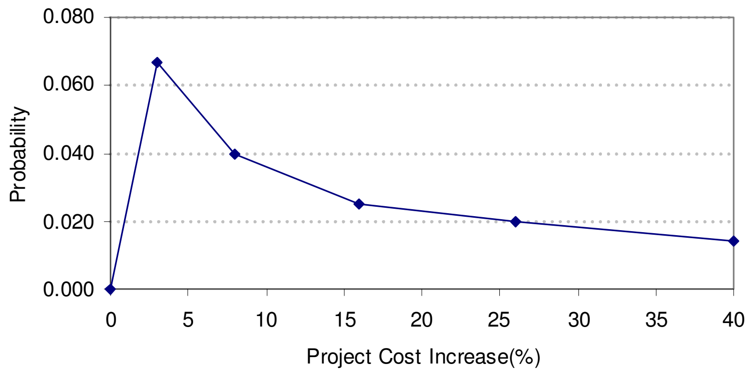


Figure VII.1.2. PDF Curve for Project Cost Increase (%) for Plan A

The PDF and CDF results are summarized in Table VII.1.3.

Table VII.1.3. CDF and PDF Summary of Plan A

Project Cost Increase (%)		
CDF	**Value**	**PDF**
0.00	0	0.000
0.20	3	0.067
0.40	8	0.040
0.60	16	0.025
0.80	26	0.020
1.00	40	0.014

The unconditional expected value of cost overrun, $f_5(\cdot)$, was calculated as follows:

$$f_5(\cdot) = \int_0^3 xp_1 dx + \int_3^8 xp_2 dx + \int_8^{16} xp_3 dx + \int_{16}^{26} xp_4 dx + \int_{26}^{40} xp_5 dx$$

$$\approx \int_0^3 0.067 x dx + \int_3^8 0.040 x dx + \int_8^{16} 0.025 x dx + \int_{16}^{26} 0.02 x dx + \int_{26}^{40} 0.014 x dx$$

$$= 0.067 \frac{x^2}{2}\bigg|_0^3 + 0.04 \frac{x^2}{2}\bigg|_3^8 + 0.025 \frac{x^2}{2}\bigg|_8^{16} + 0.02 \frac{x^2}{2}\bigg|_{16}^{26} + 0.014 \frac{x^2}{2}\bigg|_{26}^{40}$$

$$= 0.067 * (4.5) + 0.04 * (27.5) + 0.025 * (96) + 0.02 * (210) + 0.014 * (462)$$

$$= 0.3015 + 1.1 + 2.4 + 4.2 + 6.468$$

$$= 14.47\%$$

So, we can say that the expected value of cost overrun is $95* 14.47%=$13.75 million. Thus, the total cost will be $95+13.75=$108.75 million.

3. Using the information in Figure VII.1.1 we can get the exceedance probability as in Figure VII.1.3.

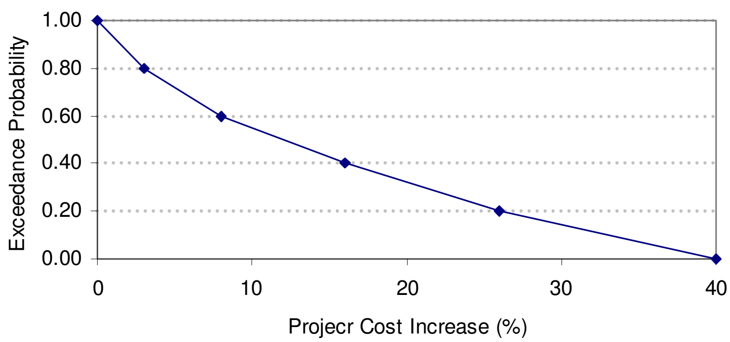


Figure VII.1.3 Exceedance Probability for project cost increase (%) for plan A

- In the worst 10% scenario, from Figure VII.1.3 we find there is a one-to-one relationship given that the overrun occurs with a probability of 0.1 or low. We need to calculate the conditional expected value when $1-\alpha = 0.1$ or $\alpha = 0.9$.

Since there is linear relationship between the exceedance probability and overrun cost, we can use Figure VII.1.3 to get the overrun cost for $1-\alpha = 0.1$. This will be $26 + \frac{(40-26)}{2} = 26 + 7 = 33\%$ for $\alpha = 0.9$

The conditional expected value of cost overrun under the scenario of an 0.1 probability of exceeding the original cost estimate can be computed as follows:

$$f_4(\cdot)=\frac{\int_{33}^{40} xp(x)dx}{\int_{33}^{40} p(x)dx}=\frac{\int_{33}^{40} xkdx}{\int_{33}^{40} kdx}=\frac{\left.\frac{x^2}{2}\right|_{33}^{40}}{\left.x\right|_{33}^{40}}=\frac{255.5}{7}=36.5\%$$

From the value of f_4, 36.5% or 34.7 million, we can understand that even the unconditional expected value of the project cost increase is \$13.75 million, but there is a 10% chance that the overrun cost will exceed 33% of the planned cost. By the way, in this condition there is an expected increase of 36.5%, or a \$34.68 million cost.

- In the better 15% scenario, using the same method as in the above case, we can let $\alpha = 0.15$. Then $1-\alpha = 0.85$, and there is also a one-to-one relationship between 8% and 14%. So the cost will be:

$$\frac{1.0-0.85}{1.0-0.8}=\frac{0.15}{0.20}=\frac{x-0}{3-0} \Rightarrow x = 2.25\%$$

So we plot the conditional expected overrun cost in this scenario as:

$$f_2(\cdot)=\frac{\int_{0}^{2.25} xp(x)dx}{\int_{0}^{2.25} p(x)dx}=\frac{\int_{0}^{2.25} xkdx}{\int_{0}^{2.25} kdx}=\frac{\left.\frac{x^2}{2}\right|_{0}^{2.25}}{\left.x\right|_{0}^{2.25}}=\frac{2.53}{2.25}=1.125\%$$

and we understand that there is a 15% chance that the overrun cost will be below 2.25%, or $95 * 2.25\% = \$2.1$ million. The conditional expected overrun cost will be $95 * 1.125\% = \$1.1$ million.

CDF Curve for Plan B:

1. Using the same method as in Plan A, we summarize the PDF data in Table VII.1.4 and plot them in Figure VII.1.4.

Table VII.1.4. CDF and PDF of Plan B

Project Cost Increase (%)		
CDF	**Value**	**PDF**
0.00	0	0.000
0.20	8	0.025
0.40	14	0.033
0.60	20	0.033
0.80	28	0.025
1.00	50	0.009

Figure VII.1.4 shows the PDF curve:

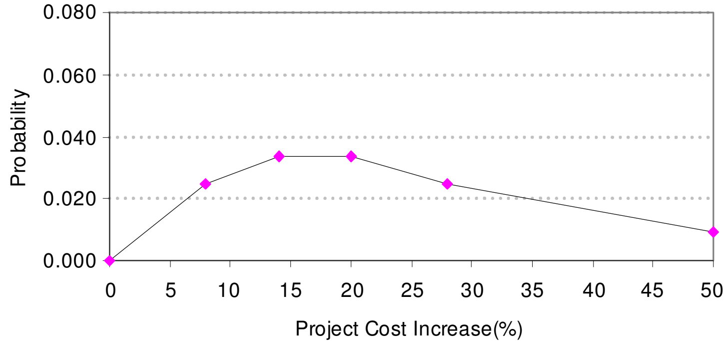


Figure VII.1.4. PDF for Project Cost Increase(%) for Plan B

The unconditional expected value of cost overrun, $f_5(\cdot)$, was calculated as follows:

$$f_5(\cdot) = \int_0^8 xp_1 dx + \int_8^{14} xp_2 dx + \int_{14}^{20} xp_3 dx + \int_{20}^{30} xp_4 dx + \int_{30}^{50} xp_5 dx$$

$$\approx \int_0^8 0.025x dx + \int_8^{14} 0.033x dx + \int_{14}^{20} 0.033x dx + \int_{20}^{30} 0.025x dx + \int_{30}^{50} 0.009x dx$$

$$= 0.025\frac{x^2}{2}\Big|_0^8 + 0.033\frac{x^2}{2}\Big|_8^{14} + 0.033\frac{x^2}{2}\Big|_{14}^{20} + 0.025\frac{x^2}{2}\Big|_{20}^{28} + 0.009\frac{x^2}{2}\Big|_{28}^{50}$$

$$= 0.025*(32) + 0.033*(66) + 0.033*(102) + 0.025*(192) + 0.009*(858)$$

$$= 0.8 + 2.178 + 3.366 + 4.8 + 7.722$$

$$= 18.866\%$$

So, we can say that the expected value of the cost overrun is \$105*18.866%=\$19.81 million. This means that the total cost will be \$105+19.81=\$124.81 million.

2. Again, using the information in Figure VII.1.1 we can get the exceedance probability as shown in Figure VII.1.5.

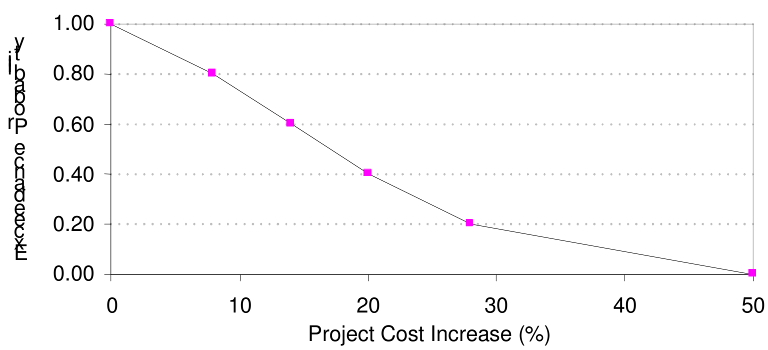


Figure VII.1.5. Exceedance Probability

- In the worst 10% scenario, from Figure VII.1.5 we find that there is a one-to-one relationship, given that the overrun occurs with a probability of 0.1 or low. We need to calculate the conditional expected value when $1-\alpha=0.1$ or $\alpha=0.9$.

Since there is a linear relationship between the exceedance probability and the overrun cost, we can use Figure VII.1.5 to get the overrun cost for $1-\alpha=0.1$ will be $28+\frac{(50-28)}{2}=28+11=39\%$ for $\alpha=0.9$.

The conditional expected value of cost overrun under the scenario of a 0.1 probability of exceeding the original cost estimate can be computed as follows

$$f_4(\cdot)=\frac{\int_{39}^{50} xp(x)dx}{\int_{38}^{50} p(x)dx}=\frac{\int_{39}^{50} xkdx}{\int_{39}^{50} kdx}=\frac{\left.\frac{x^2}{2}\right|_{39}^{50}}{\left.x\right|_{39}^{50}}=\frac{489.5}{11}=44.5\%$$

From the value of f_4, 44.5% or \$46.73 million, we can understand that even the unconditional expected value of the project cost increase is \$19.81 million, but there is a 10% chance that the overrun cost will exceed 39% of the planned cost. By the way, in this condition the cost is expected to increase by 44.5% or \$46.73 million.

- In the better 15% scenario, using the same method as in the above case, we can let $\alpha=0.15$. Then $1-\alpha=0.85$, and there is also a one-to-one relationship between 8% and 14%. So the cost will be:

$$\frac{1.0-0.85}{1.0-0.8}=\frac{0.15}{0.2}=\frac{x-0}{8-0} => x=6.0\%$$

The conditional expected overrun cost in this scenario is:

$$f_2(\cdot)=\frac{\int_{0}^{6} xp(x)dx}{\int_{0}^{6} p(x)dx}=\frac{\int_{0}^{6} xkdx}{\int_{0}^{6} kdx}=\frac{\left.\frac{x^2}{2}\right|_{0}^{6}}{\left.x\right|_{0}^{6}}=\frac{6}{2}=3.0\%$$

and we understand that there is a 15% chance that the overrun cost will be below 6.0%, or $\$105 * 6.0\% = \$6.3$ million. The conditional expected overrun cost will be $\$105 * 3.0\% = \$3.15$ million.

CDF Curve for Plan C

1. Using the CDF, we calculate the PDF as follows and plot the curve as in Figure VII.1.6.

Let p_i be the probability of each event.

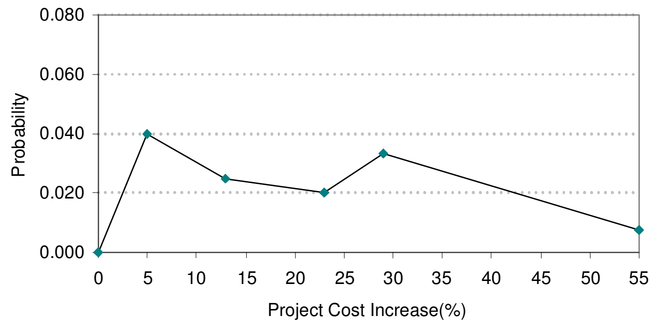


Figure VII.1.6. PDF for Project Cost Increase(%) for Plan C

Table VII.1.5 displays the summary.

Table VII.1.5. PDF Summary for Plan C

CDF	value	PDF
0.00	0	0.000
0.20	5	0.040
0.40	13	0.025
0.60	23	0.020
0.80	29	0.033
1.00	55	0.008

The unconditional expected value of cost overrun, $f_5(\cdot)$, was calculated as follows:

$$
\begin{aligned}
f_5(\cdot) &= \int_0^5 xp_1dx + \int_5^{13} xp_2dx + \int_{13}^{23} xp_3dx + \int_{23}^{29} xp_4dx + \int_{29}^{55} xp_5dx \\
&\approx \int_0^5 0.04xdx + \int_5^{13} 0.025xdx + \int_{13}^{23} 0.02xdx + \int_{23}^{29} 0.033xdx + \int_{29}^{55} 0.008xdx \\
&= 0.04\frac{x^2}{2}\Big|_0^5 + 0.025\frac{x^2}{2}\Big|_5^{13} + 0.02\frac{x^2}{2}\Big|_{13}^{23} + 0.033\frac{x^2}{2}\Big|_{23}^{29} + 0.008\frac{x^2}{2}\Big|_{29}^{55} \\
&= 0.04*(12.5) + 0.025*(72) + 0.02*(180) + 0.033*(156) + 0.008*(1092) \\
&= 0.5 + 1.8 + 3.6 + 5.15 + 8.74 \\
&= 19.79\%
\end{aligned}
$$

So, we can say that the expected value of the cost overrun is \$120*19.79%= \$23.75million Thus, the total cost will be \$120+23.75=\$143.75 million.

2. Once more using the information in Figure VII.1.1, we get the exceedance probability as shown in Figure VII.1.7.

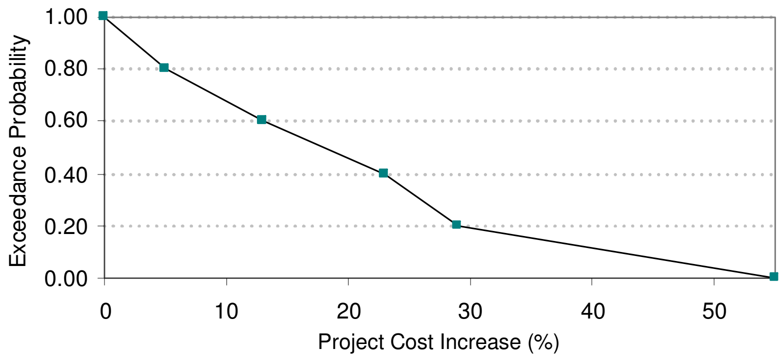


Figure VII.1.7. Exceedance Probability for Project Cost Increase (%) for Plan C

- In the worst 10% scenario, from Figure VII.1.7 we find that there is a one-to-one relationship given that the overrun occurs with a probability of 0.1 or low. We need to calculate the conditional expected value when $1-\alpha = 0.1$ or $\alpha = 0.9$.

Since there is a linear relationship between the exceedance probability and the overrun cost, we can use Figure VII.1.3 to get the overrun cost for $1-\alpha = 0.1$ will be $29 + \frac{(55-29)}{2} = 29 + 13 = 42\%$ for $\alpha = 0.9$

The conditional expected value of the cost overrun under the scenario of a 0.1 probability of exceeding the original cost estimate is computed as follows:

$$f_4(\cdot) = \frac{\int_{42}^{55} xp(x)dx}{\int_{42}^{55} p(x)dx} = \frac{\int_{42}^{55} xkdx}{\int_{42}^{55} kdx} = \frac{\left.\frac{x^2}{2}\right|_{42}^{55}}{\left.x\right|_{42}^{55}} = \frac{630.5}{13} = 48.5\%$$

From the value of f_4, 48.5% or \$58.2 million, we can understand that even the unconditional expected value of the project cost increase is 23.75%, but there is a 10% chance that the overrun cost will exceed 42% of the planned cost. By the way, in this condition there is expected to be a cost increase of 48.5% or \$58.2 million.

- In the better 15% scenario, using the same method as in the above case, we can let $\alpha = 0.15$. Then $1-\alpha = 0.85$, and there is also a one-to-one relationship between 8% and 14%. So the cost will be:

$$\frac{1.0-0.85}{1.0-0.8} = \frac{0.15}{0.20} = \frac{x-0}{5-0} \Rightarrow x = 3.75\%$$

Thus, the conditional expected overrun cost in this scenario is:

$$f_2(\cdot)=\frac{\int_0^{3.75} xp(x)dx}{\int_0^{3.75} p(x)dx}=\frac{\int_0^{3.75} xkdx}{\int_0^{3.75} kdx}=\frac{\left.\frac{x^2}{2}\right|_0^{3.75}}{\left.x\right|_0^{3.75}}=\frac{3.75}{2}=1.88\%$$

We also understand that there is a 15% chance that the overrun cost will be below 3.75%, or $\$120*3.75\% = \$4.5$ million. The conditional expected overrun cost will be $\$120*1.88\% \;\; = \$2.26$ million

ANALYSIS

From above analysis, we can summarize the results as follows:

Table VII.1.6. Summary of Results

	Cost $M	Unconditional expected value (f_5)		Worst scenario 10% (f_4) Threshold		Worst scenario 10% (f_4) Expected value		Better scenario 15% (f_2) Threshold		Better scenario 15% (f_2) Expected value	
Plan A	95	14.47%	13.75	33.00%	31.35	36.50%	34.68	2.25%	2.14	1.13%	1.07
Plan B	105	18.87%	19.81	39.00%	40.95	44.50%	46.73	6.00%	6.30	3.00%	3.15
Plan C	120	19.79%	23.75	42.00%	50.40	48.50%	58.20	3.75%	4.50	1.88%	2.26

- We can plot the values of costs f_2, f_4, and f_5 for each plan in the same diagram, as shown in Figure VII.1.8.

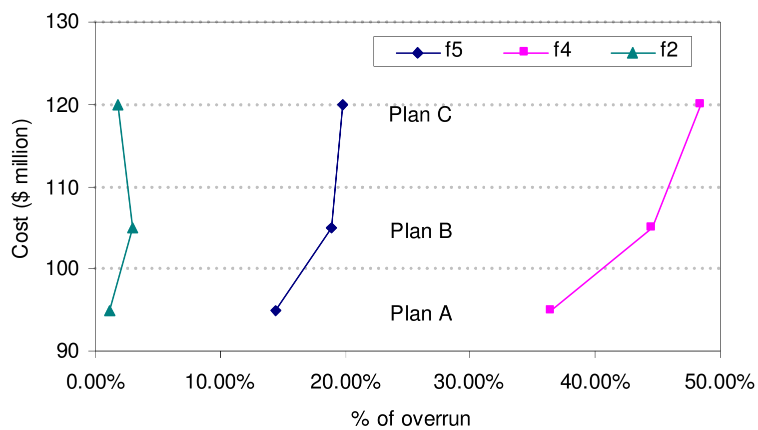


Figure VII.1.8. Comparison of the Conditional and Traditional Expected Values

- The minimum unconditional expected overrun cost is Plan A
- In the worst scenario Plan C has the most overrun cost, but Plan B is higher than the others in the better scenario.

- When comparing the differences between the expected values of f_5 and f_4 for each plan, we can understand that Plan C has the biggest one**,** 28.71% (48.5%-19.79%) and Plan A has the smallest one**,** 22.03% (36.50%-14.47%).
- Plan C has more risk in both the worst and normal scenarios, but it also has the most stations—that is, more profit. If the THSRC has a good civil subcontractor and project management, Plan C may be considered as the best choice.

PROBLEM VII.2: Supplier Selection

A company has recently adopted a policy that specifies dealing only with a single supplier of its product. It is evaluating its two contractor companies, A and B. How can it evaluate contractors' performances?

DESCRIPTION

Performance data on the two candidates was obtained from their transaction records. These contain the cost overruns (defined as % increase in cost over the normal cost) resulting from failure of the contractor to deliver on time, deliver the required units of product, deliver the required quality of products, and other sources of increased costs (*see Appendix*). The choice is difficult since the conventional expected value measure did not yield a significant difference between the cost overruns of the contractors.

Table VII.2.1. Average Cost Overrun by Subcontractor

Subcontractor	Average Cost Overrun (%)
A	50.3
B	53.3

The management feels that investigating the contractor's reliability can't be truly represented by the expected value alone. They cite instances of very costly transactions with A in the past, and would like to look into that aspect as well.

Referring to the data shown below and in the Appendix, the following cumulative distribution functions of Subcontractors A and B are superimposed to make preliminary deductions as to which of the two is more reliable.

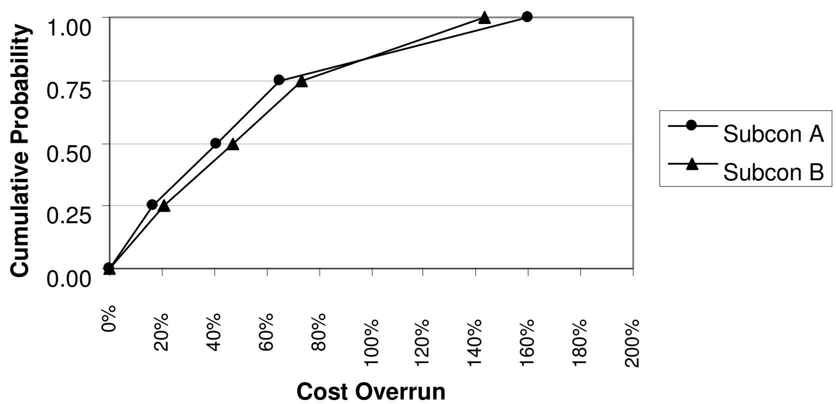


Figure VII.2.1. CDF of Subcons' Cost Coverruns

Although Subcon A shows superiority in terms of the 25th, 50th, and 75th quartiles, this does not necessarily guarantee that it is the better option. It should be noted that historical performance suggests that Subcon B has a lower maximum cost overrun

(143%) than Subcon A (160%). Furthermore, although the mean of Subcon B is slightly higher than A, it is evident from their probability distribution functions that Subcon B has a shorter tail, which implies that their behavior significantly differs at extreme values of cost overruns.

Choosing the better subcontractor can be assessed not only by using the "business as usual" definition of expected value. This problem exemplifies the case where the PMRM (Partitioned Multiobjective Risk Method) will be very handy and meaningful.

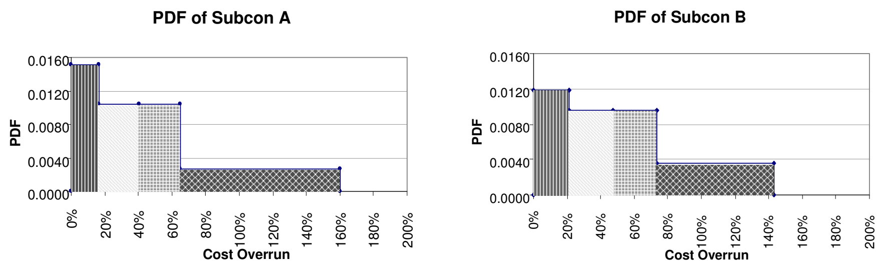


Figure VII.2.2. PDFs by Subcontractor

The preceding probability distribution functions were constructed by taking off from the CDF, using the definition that:

CDF $= \sum_x p(x)$ and consequently, it follows that $p(x) = \frac{\Delta y}{\Delta x}$;

where y is the Cumulative Probability and x is the Cost Overrun.

This means that to know the height of the PDF, say between 0 and the 25th quartile, we need to get the slope of the CDF at the specified interval.

It is also worthwhile to show the Exceedance Graph, which is just 1-CDF. This will help visualize the subsequent analysis using the Partitioned Multiobjective Risk Method (PMRM).

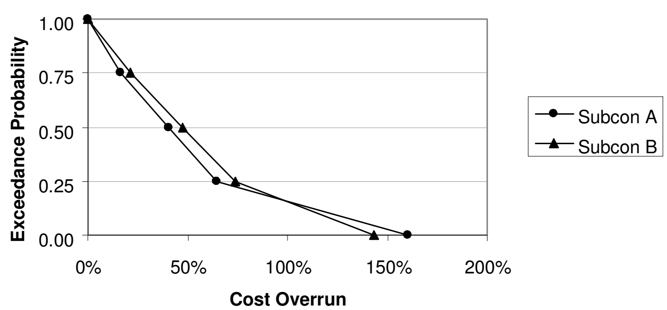


Figure VII.2.3. Exceedance Probabilities of Subcons' Cost Overrun

METHODOLOGY

For this exercise, the subcontractors are evaluated according to cost of overrun using the PMRM. It is necessary to compute the values of conditional expected-value risk functions f_4 and unconditional expected-value risk function f_5 using the following general expression:

$$f_i(s_j) = \frac{\int_{\beta_{ii-2,j}}^{\beta_{i-1,j}} x p_x(x; s_j)dx}{\int_{\beta_{ii-2,j}}^{\beta_{i-1,j}} p_x(x; s_j)dx}, \qquad i = 4 \quad j = A, B$$

where,

s_j : subcontractor j, j = A,B
x : cost overrun associated to s_j
$p_x(x,s_j)$: denotes the pdf of the cost overruns
$f_4(\cdot)$: is of low exceedance probability and high severity
β_I : unique cost overrun point corresponding to the exceedance probability $(1-\alpha_i)$, where α_i is the range of severity relevant to the analysis

A cost overrun with an exceedance probability of 0.1 represents the point at which the extreme consequence begins. These cost overrun values for Subcons A and B are calculated as follows (*refer to the previously-shown PDFs*):

Since the upper quartile of the PDF represents 0.25 probability, dividing the range by 2.5 will yield a range in the upper quartile that represents 0.10 probability. Therefore,

$$\text{Cost overrun with a probability of 0.1} = \frac{\text{Range of Upper Quartile}}{2.5}$$

Cost overrun for Subcon A = (160-64.78) / 2.5 = 38.088
Cost overrun for Subcon B = (143-73.66) / 2.5 = 27.736

SOLUTION

The conditional expected values for the high-consequence, low-probability regions of the subcontractors are:

Subcontractor A:

$$f_4(\cdot) = \frac{\int_{121.9}^{160} xf(x)dx}{\int_{121.9}^{160} f(x)dx} = \frac{\frac{160^2 - 121.9^2}{2}(0.0026)}{0.0026(160 - 121.9)} = 140.956 \text{ \% } \textit{Cost Overruns}$$

Subcontractor B:

$$f_4(\cdot) = \frac{\int_{115.264}^{143} xf(x)dx}{\int_{115.264}^{143} f(x)dx} = \frac{\frac{143^2 - 115.3^2}{2}(0.0036)}{0.0036(143 - 115.3)} = 129.132\ \%\ \textit{Cost Overruns}$$

And the expected values (f_5) of the cost overruns of subcontractors A and B are computed as:

Subcontractor A:

$$\begin{aligned} f_5(\cdot) = \int_0^{\infty} xp(x)dx &= \frac{16.57^2 - 0}{2}(0.0151) + \frac{40.67^2 - 16.57^2}{2}(0.0104) \\ &+ \frac{64.78^2 - 40.67^2}{2}(0.0104) + \frac{160^2 - 64.78^2}{2}(0.0026) \\ &= 50.29138 \end{aligned}$$

Subcontractor B:

$$\begin{aligned} f_5(\cdot) = \int_0^{\infty} xp(x)dx &= \frac{21.07^2 - 0}{2}(0.0119) + \frac{47.37^2 - 21.07^2}{2}(0.0095) \\ &+ \frac{73.66^2 - 47.37^2}{2}(0.0095) + \frac{143^2 - 73.66^2}{2}(0.0036) \\ &= 53.34703 \end{aligned}$$

ANALYSIS

The conditional expected value contributes significantly to the analysis of the two subcontractors' expected performance. At the extreme 10% cost overrun, Subcontractor B has a significantly lower conditional expected cost overrun than Subcontractor A. The following graph shows the conditional and unconditional expected values of the two cost overruns. It is worth noting that when considering only the extreme cases, the conditional expected overrun is much greater than the unconditional overrun -- more than two times in magnitude.

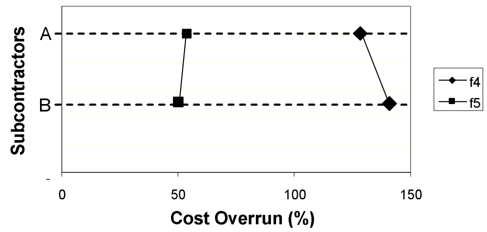
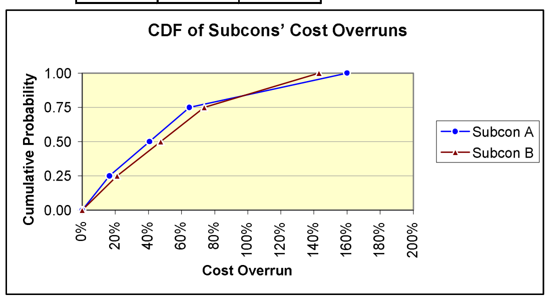


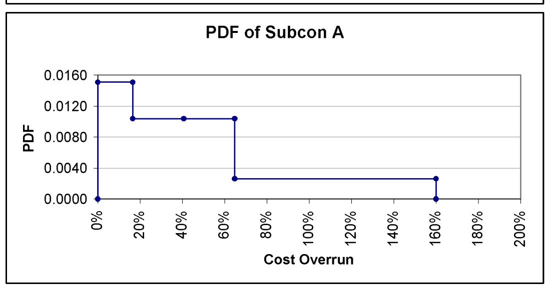
Figure VII.2.4. Plot of f4 and f5

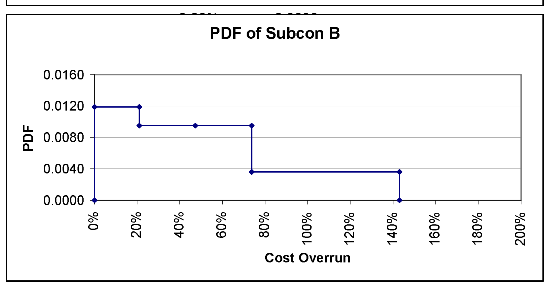
APPENDIX. Cost Overrun Data of Subcons' A and B

	Overrun A	Overrun B
	14.00%	17.00%
	71.00%	85.00%
	38.00%	46.00%
	55.00%	66.00%
	0.00%	1.00%
	68.00%	82.00%
	50.00%	60.00%
	5.00%	6.00%
	78.00%	94.00%
	1.00%	1.00%
	78.00%	93.00%
	26.00%	31.00%
	4.00%	4.00%
	14.00%	17.00%
	57.00%	69.00%
	23.00%	28.00%
	160.00%	115.00%
	84.00%	101.00%
	26.00%	31.00%
	47.00%	56.00%
	106.00%	127.00%
	17.00%	21.00%
	1.00%	2.00%
	30.00%	36.00%
	59.00%	71.00%
	29.00%	35.00%
	10.00%	12.00%
	26.00%	31.00%
	95.00%	114.00%
	1.00%	1.00%
	4.00%	5.00%
	6.00%	7.00%
	19.00%	23.00%
	21.00%	25.00%
	85.00%	102.00%
	8.00%	10.00%
	119.00%	143.00%
	31.00%	37.00%
	21.00%	26.00%
	93.00%	112.00%
	33.00%	39.00%
	79.00%	95.00%
	22.00%	26.00%
	29.00%	34.00%
	8.00%	10.00%
	6.00%	7.00%
	36.00%	44.00%
	14.00%	16.00%
	24.00%	29.00%
	52.00%	62.00%
	46.00%	55.00%
	86.00%	103.00%
AVERAGE	**40.67%**	**47.37%**
STDEV	**24.10%**	**26.30%**

Subcon A	Subcon B	Fractiles
0.00%	0.00%	0.00
16.57%	21.07%	0.25
40.67%	47.37%	0.50
64.78%	73.66%	0.75
160.00%	143.00%	1.00







PROBLEM VII.3: Job Hunting

Sally was recently laid off. Though she has some savings, she is desperate to find a job. Her field is accounting, but she knows that if she can't find an accounting job, she may need to consider retail or waitressing. Depending on the industry, Sally is prepared to dip into her savings. She would like to find out what percentage of her current salary she might lose in her next job; this will help her determine whether she will be able to pay her mortgage. If she cannot, she may need to sell her house or find a roommate. To keep her house, she must make at least 75% of her original salary or make 50% of her original salary and take in a roommate. Her current options are listed below. For each, Sally projects a certain cost to her savings pool.

1. Find work in the accounting industry.
2. Find work in the retail industry.
3. Find work in the restaurant industry.

DESCRIPTION

From market data, the following table and graph were constructed to represent the probabilities of Sally's yearly salary loss for each of the three industries.

Table VII.3.1. Annual Salary Loss by Industry

Option	Best (0)	25th	Median 50th	75th	Worst (100)
Accounting	0%	10%	20%	30%	40%
Retail	25%	40%	50%	55%	60%
Restaurant	40%	50%	60%	65%	70%

Figure VII.3.1. CDF for Sally's Potential Salary

METHODOLOGY

For this problem, it is necessary that each job should be evaluated according annual salary loss using the PMRM. In order to do this analysis, computing the values of conditional expected-value f_4 with finding partitioning points and unconditional expected-value f_5 is prerequisite.

SOLUTION

The expected value of salary loss (see below) for each of the industries was computed using fractile method equations, based on the PDF.

Table VII.3.2. Expected Value of Salary Loss

	1st	2nd	3rd	4th	Total EV
Accounting	1.25	3.75	6.25	8.75	20
Retail	8.13	11.25	13.13	14.38	47
Restaurant	11.25	13.75	15.63	16.88	58

The following table includes the computed expected values as well as the estimated cost to Sally's savings for each.

Table VII.3.3. Summary of Expected Value of Salary Loss & Cost

	Estimated Cost to Savings ($K yr)	$f_5(.)$ %
Accounting	0.00	20.00
Retail	15.00	46.88
Restaurant	20.00	57.50

As the expected value does not account for extreme values, we now calculate the conditional expected values using the exceedance probability. First, we must determine the integration points to use to calculate the conditional expected values. The graph below is used as a visual check for the following numerical method.

Accounting

$$\frac{x-30}{40-30}=\frac{0.25-(1-\alpha)}{0.25}$$
$$x-30=\frac{0.15*10}{0.25}$$
$$x=\frac{1.5}{0.25}+30$$
$$x=36\%$$

Retail

$$\frac{x-55}{60-55}=\frac{0.25-(0.1)}{0.25}$$
$$\frac{x-55}{5}=\frac{0.15}{0.25}$$
$$x=\frac{0.15*5}{0.25}+55$$
$$x=58\%$$

Restaurant

$$\frac{x-65}{70-65}=\frac{0.25-(0.1)}{0.25}$$
$$\frac{x-65}{5}=\frac{0.15}{0.25}$$
$$x=\frac{0.15*5}{0.25}+65$$
$$x=68\%$$

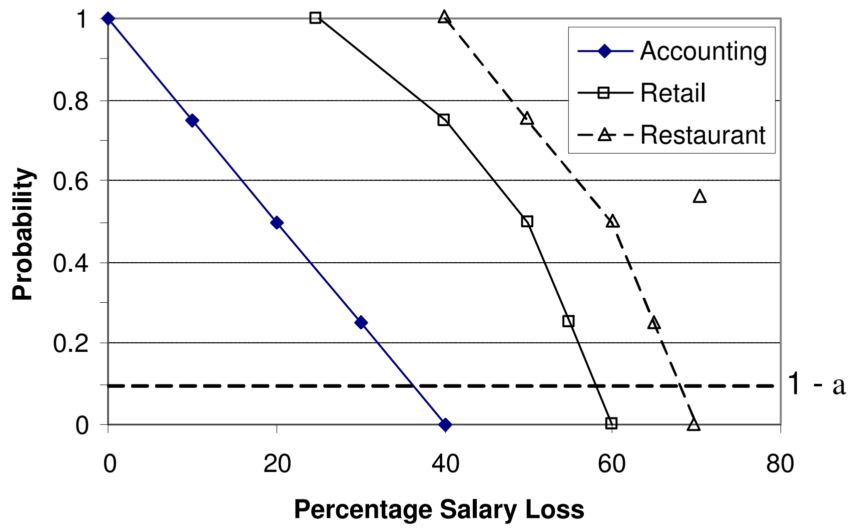


Figure VII.3.2. Exceedance Probability by Industry

With the 10% partition points known, the integration can be done to determine the conditional expected values.

Accounting

$$f_4(.) = \frac{\int_{36}^{40} xKdx}{\int_{36}^{40} Kdx} = \frac{K\int_{36}^{40} Kdx}{K\int_{36}^{40} dx} = \frac{\left.\frac{x^2}{2}\right|_{36}^{40}}{\left.x\right|_{36}^{40}} = \frac{1600-1296}{2(40-36)} = 38\%$$

Restaurant

$$f_4(.) = \frac{\int_{58}^{60} xKdx}{\int_{58}^{60} Kdx} = \frac{K\int_{58}^{60} Kdx}{K\int_{58}^{60} dx} = \frac{\left.\frac{x^2}{2}\right|_{58}^{60}}{\left.x\right|_{58}^{60}} = \frac{3600-3364}{2(60-58)} = 59\%$$

Retail

$$f_4(.) = \frac{\int_{68}^{70} xKdx}{\int_{68}^{70} Kdx} = \frac{K\int_{68}^{70} Kdx}{K\int_{68}^{70} dx} = \frac{\left.\frac{x^2}{2}\right|_{68}^{70}}{\left.x\right|_{68}^{70}} = \frac{4900-4624}{2(70-68)} = 69\%$$

Table VII.3.4. Summary of f_4, f_5 and Cost

	Estimated Cost to Savings ($K yr)	$f_5(\cdot)$ %	$f4(\cdot)$ %
Accounting	0.00	20	38
Retail	15.00	47	59
Restaurant	20.00	58	69

ANALYSIS

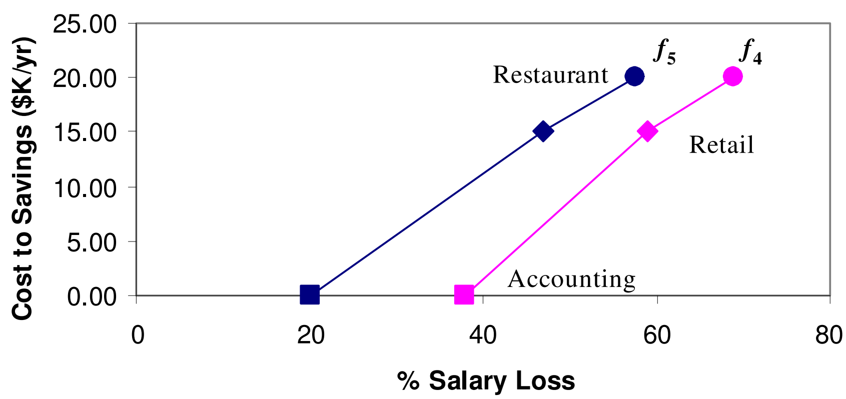


Figure VII.3.3. Comparison of f_4 and f_5

As is shown by both the tabular and graphic results, the expected values of Sally's salary loss are significantly different when using the extreme value method. As long as she is able to find a job in accounting, it is fair to expect that she will not have anything to worry about, in terms of her savings. Based on her original calculations, Sally feels confident that if she gets a job in accounting, she can still comfortably pay her mortgage without taking on a roommate or dipping into her savings. If she works in retail, she will surely need to find a roommate as well as deplete her savings quite a bit. If she takes a job as a waitress, neither her savings nor a roommate will allow her to afford her house. Based on this analysis, she can see that she must focus her efforts on jobs in accounting and retail.

PROBLEM VII.4: Church facility development

There are three options to be evaluated for a new church facility. It should have 844 seats for worship, 10 offices for the staff, 16 classrooms for Bible study and Sunday School, and 211 parking spaces with the ability to add 80 more spaces within the design. Each element of the design should also accommodate a minimum of 20% growth through additions and modifications.

The church has three options that will satisfy their facility requirements/needs. They have evaluated their funding and initially anticipate the costs of all options to be $13,686,000. While any of the three options satisfies the requirements, the church understands that cost overruns are possible.

Which option would be the best option and how could it be assessed when considering evens within the 15% worse case?

DESCRIPTION

They would like to have a better understanding of such cost overruns. The differences in each option are identified as follows:

Option A:
They can develop a new facility on the current church site, where the improvements have already passed the county zoning board and the project will provide for the needs of the church within 13 months.

Option B:
They can acquire land in a new location and have a building designed specifically for that site. The county would have to review any design and approve it for the specific site. Meanwhile, church operations would be carried on at the current location, and worship services would continue to be held in local high schools.

Option C:
They can purchase an existing office building or warehouse and modify it to meet the church needs. The county would have to review and approve the final design prior to starting any building or land modification activities. The church would have to sell their existing facility to afford the purchase.

Table VII.4.1. Percentage of Cost Increase for Each Option and Initial Budget

Probability	Building Options			Initial Budget
	Existing A	New B	Update C	$13,686,000
	% Cost Increase			Exceedance
0.00	0	0	0	1.00
0.25	10	20	15	0.75
0.50	15	40	20	0.50
0.75	20	50	30	0.25
1.00	25	75	50	0.00

METHODOLOGY

For this problem, each option is evaluated according to cost overrun using the PMRM. It is necessary to compute the values of conditional expected-value f_4 and unconditional expected-value f_5 complemented by the property of the *fractile* method.

SOLUTION

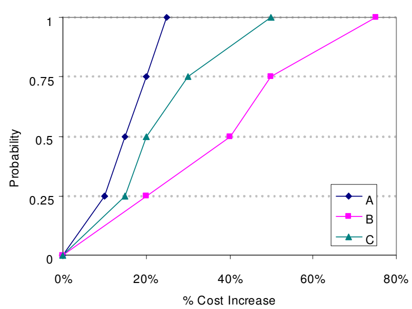
The church's budget, or anticipated cost, for all three options is $13,686,000, but they recognize that potential cost overruns for the three project options would vary greatly. The church would like to utilize this analysis in order to have a more realistic budget expectation when they move forward with one of the building plans. This can be accomplished through comparing traditional expected values with conditional expected one.

The following tables and figures reveal the expected values and the associated probabilities of a cost overrun for each option. They depict the risk in the form of percentages of possible cost increases and overruns and show the actual cost increases over the target value.

Table VII.4.2. Expected Value (f_5) of the Percentage of Project Cost Increase

Option		Fractile Range				Total		Final Cost
		0-0.25	0.25-0.50	0.50-0.75	0.75-1.0	% over	$ over	
A	f_5	1.1250	3.125	4.375	5.625	14.375		
		$171,075	$427,688	$598,763	$769,838		$1,967,363	$15,653,363
B	f_5	2.500	7.500	11.250	15.626	36.875		
		$342,150	$1,026,450	$1,539,675	$2,138,438		$5,046,713	$18,732,713
C	f_5	1.875	4.375	6.250	10.000	22.500		
		$256,613	$598,763	$855,375	$1,368,600		$3,079,350	$16,765,350

The cumulative distribution function for the fractile approach is shown below.



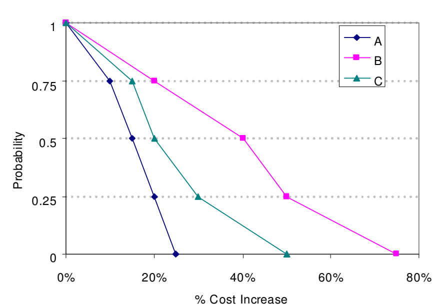


Figure VII.4.1. CDF and Exceedance Probability by Option

To calculate f_4, the first step is to figure out partitioning points for each option and apply the property of the fractile method to compute them:

Option A

$$\frac{x-20}{25-20}=\frac{0.25-(1-0.85)}{0.25};\ x=22\%$$

Option B

$$\frac{x-50}{75-50}=\frac{0.25-(1-0.85)}{0.25};\ x=60\%$$

Option C

$$\frac{x-30}{50-30}=\frac{0.25-(1-0.85)}{0.25};\ x=38\%$$

According to the partitioning points, conditional expected values are computed as follows;

Option A

$$f_4(\cdot)=\frac{20+22}{2};\ x=21\%$$

Option B

$$f_4(\cdot)=\frac{60+75}{2};\ x=67.5\%$$

Option C

$$f_4(\cdot)=\frac{38+50}{2};\ x=44\%$$

We can summarize unconditional and conditional expected values by policy in Table VII.4.3.

Table VII.4.3. Summaries of f_5, and f_4 (Conditional Expected Value)

	f_5	f_4
Option A	14.375%	21%
Option B	36.875%	67.5%
Option C	22.5%	44%

ANALYSIS

By analyzing the results of f_5, Option A has the least opportunity for cost overrun due to the level of prior planning, understanding of the site specifics, prior approval by the county, and previous architectural and geological surveys. Each of the other options will have more inherent risks due to the unknowns for those options. Of these two options, the update to an existing facility will have less risk than the new building and site.

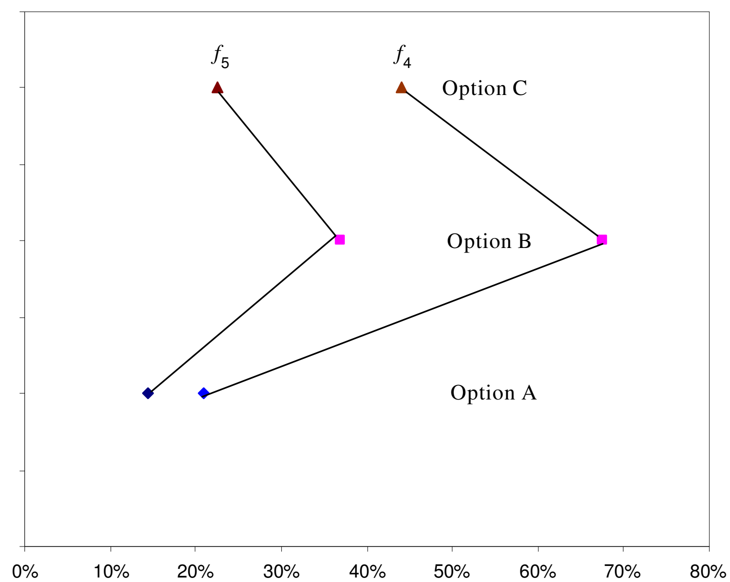


Figure VII.4.2. Comparison of f_4 vs. f_5 by Option

Moreover, even though the 15% worst case occurs, Option A shows the least variation from its unconditional expected value. Therefore, Option A can be recommended by consulting with both f_4 and f_5 but Option B, New Land Acquisition and Construction should not be selected in any case since it shows the worst cost overrun in either case.

PROBLEM VII.5: Investment on the construction of a meteorological observatory

The more precise the weather forecast, the smaller the loss from severe weather events such as heavy rainfall or snow. How can a state government improve the accuracy of weather forecast so minimize cascading losses from incorrect forecast?

DESCRIPTION

A state government considers constructing a meteorological observatory in order to forecast weather more precisely.

The tradeoff between the cost of construction and the loss resulting from severe weather is described in Table VII.5.1. Assume that independent loss functions are normally distributed.

Table VII.5.1. Tradeoff for Policies with Standard Deviation (in million dollars)

Policy	Construction Cost	Estimated Loss	Standard Deviation of Loss
1	10	50	9
2	20	45	7
3	30	40	5
4	40	35	3
5	50	30	1

METHODOLOGY

In order to use the PMRM, traditional and conditional expected values need to be calculated. These values are based on the probability distributions as follows:

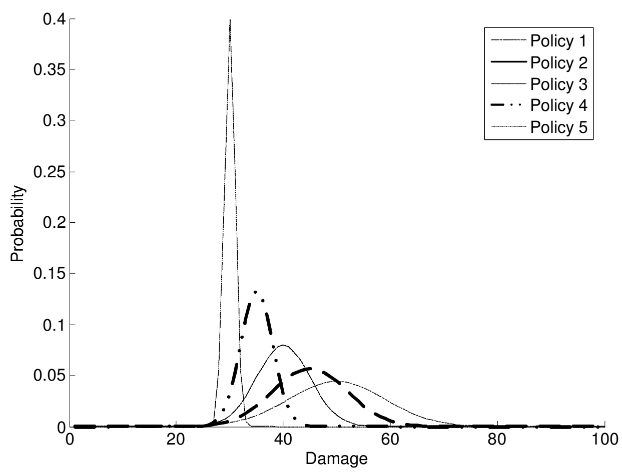


Figure VII.5.1. Probability Density Functions for Five Policy Options

Based on the PDFs, CDFs and Exceedance probability functions can be shown:

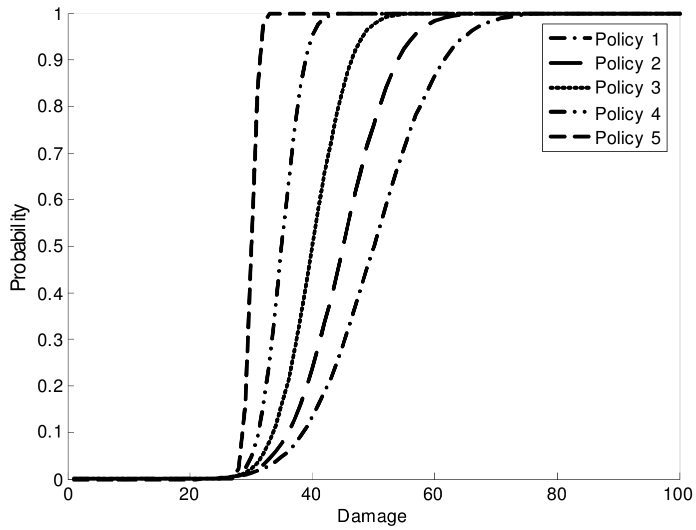


Figure VII.5.2. Cumulative Distribution Functions for Five Policy Options

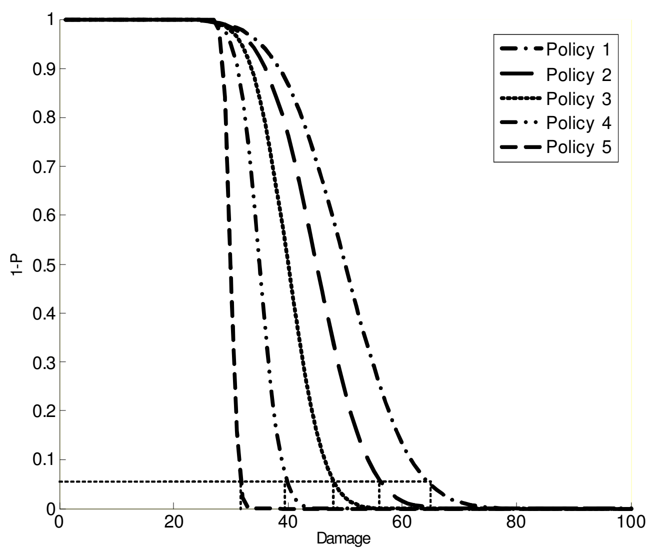


Figure VII.5.3. Partitioning the Exceedance Probability Axis onto the Damage Axis

The plot of the exceedance probability axis partitioned onto the damage axis. The red line represents Policy Option 5, which dominates the four other policies. That is, for forecasting extreme weather events, at a glance a decisionmaker can prefer Policy Option 5 to other options even though the cost is much higher.

SOLUTION

Given Table VII.5.1, the PMRM table and Pareto-optimal frontier incorporating every policy can be calculated and formulated.

Table VII.5.2. PMRM Summary

	Policy 1	Policy 2	Policy 3	Policy 4	Policy 5
μ	50	45	40	35	30
σ	9	7	5	3	1
α	0.95	0.95	0.95	0.95	0.95
β	64.80	56.51	48.22	39.93	31.64
$f(\beta)$	0.0115	0.0147	0.0206	0.0344	0.1031
$1- F(\beta)$	0.0500	0.0500	0.0500	0.0500	0.0500
f_4	68.5644	59.4390	50.3136	41.1881	32.0627
f_5	50	45	40	35	30

ANALYSIS

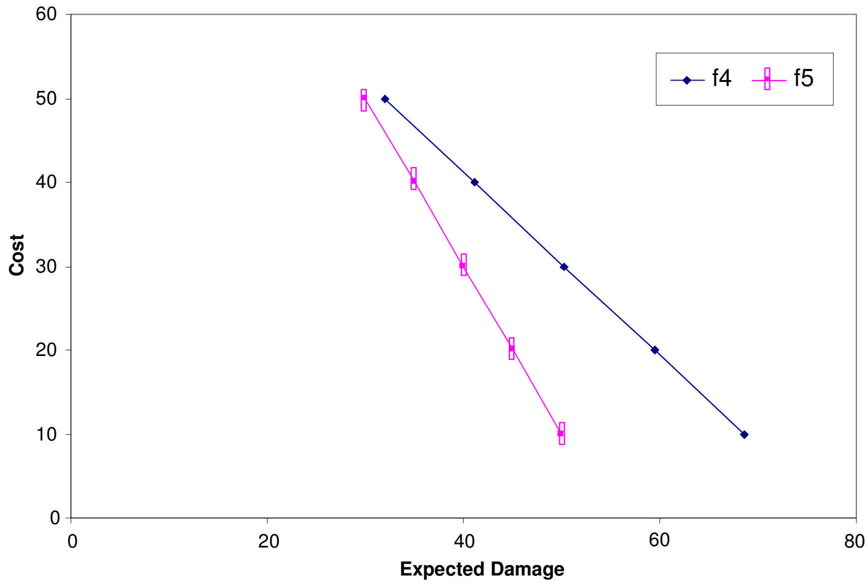


Figure VII.5.4. Pareto-optimal Frontier

In Figure VII.5.4, two Pareto-optimal frontiers can be observed with respect to unconditional and conditional expected damage versus the estimated cost of constructing the observatory. For instance, opting for Policy 1 indicates a $50 million loss in unconditional expected damage but approximately a $69 million loss in conditional expected damage. Policy Option 1 shows the largest variance estimation for damage. The difference between the two expected values is also the largest compared to the other policy options. Therefore, this variance difference can be taken into consideration in deciding the amount of investment and can support the decisionmaker's decision whether it is right or not.

PROBLEM VII.6: Contractor Selection

SM Construction Consultants (SMCC) is reviewing proposals for the construction of a four-story building scheduled for completion in September 2009. SMCC's task is to estimate the total cost of erecting the building and to choose the contractor according to the estimates.

DESCRIPTION

Five area contractors submitted cost estimates after reviewing preliminary plans. SMCC must evaluate each bid and determine a projected cost for the project. Details from each contractor are listed in Table VII.6.1.

Table VII.6.1. Project Cost by Contractor

Contractor	Project Cost
Contractor "A"	$1,125,000
Contractor "B"	$1,375,000
Contractor "C"	$1,050,000
Contractor "D"	$1,250,000
Contractor "E"	$1,075,000

To more accurately estimate the cost of the project, SMCC has captured data and statistics from prior projects of each of the contractors. Included are estimates of the percentage of cost overrun from the original projected cost. From this data, which is summarized in Table VII.6.2, a more precise estimate of the final project cost can be determined.

Table VII.6.2. Percentage Cost Overrun for Selected Contractors

	Best (0)	25^{th}	Median 50^{th}	75^{th}	Worst (100)
Contractor "A"	2	15	22	42	50
Contractor "B"	0	10	15	25	40
Contractor "C"	4	25	40	50	60
Contractor "D"	3	12	25	35	70
Contractor "E"	2	20	35	50	80

METHODOLOGY

In the selection of a new contractor, it is necessary that SMCC should estimate the projected cost overrun using the PMRM and for this analysis, unconditional and conditional expected values (f_5 and f_4) are calculated with the properties of the *fractile* method and this general equation:

$$f_4(x) = \frac{\int_{\beta_{p,j}}^{\beta_{u,j}} xp(x)dx}{\int_{\beta_{p,j}}^{\beta_{u,j}} p(x)dx}, \qquad j = 1, 2$$

where β_{uj} : Upper bound for jth contractor

β_{pj} : Partitioning point for jth contractor

SOLUTION

First, the percentage cost overrun for each contractor is detailed in both a cumulative probability distribution function and a probability distribution function and is analyzed utilizing the fractile method (and checked by integration). The appropriate graphs are detailed below for each contractor.

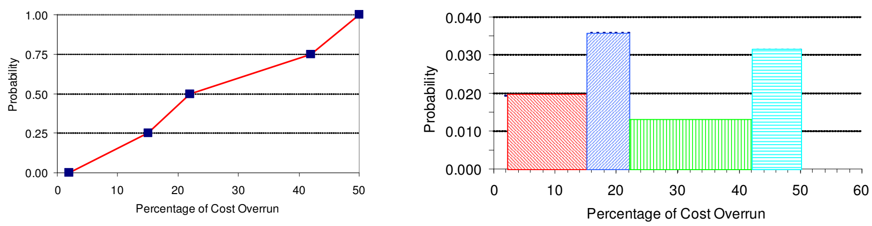


Figure VII.6.1. Cumulative Density Function(CDF) and Probability Distribution Funtion(PDF) for Contractor A

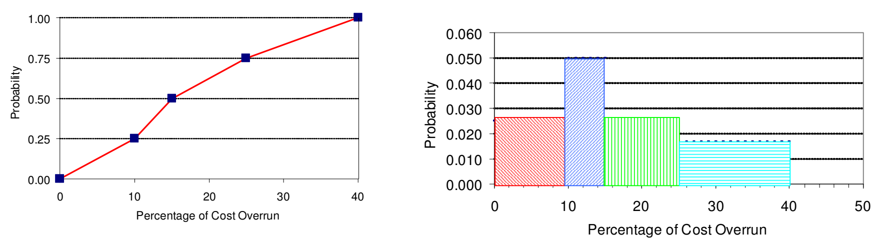


Figure VII.6.2. CDF and PDF for Contractor B

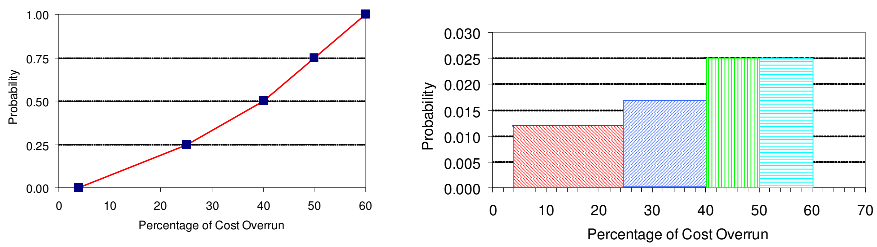


Figure VII.6.3. CDF and PDF for Contractor C

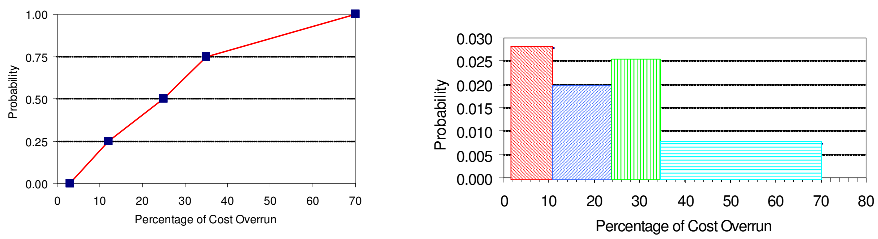


Figure VII.6.4. CDF and PDF for Contractor D

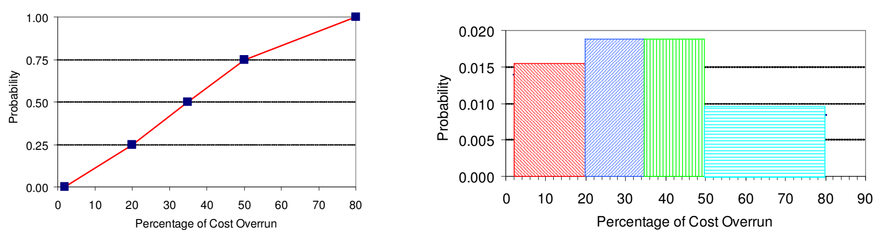


Figure VII.6.5. CDF and PDF for Contractor E

Next, the expected value $f_5(x)$ of the percentage of cost overrun is determined (and checked with integrals).

(i) Contractor A

Fractile Method

$$f_5(x) = (0.25)(2+(15-2)/2) + (0.25)(15+(22-15)/2) + (0.25)(22+(42-22)/2) + (0.25)(42+(50-42)/2) = 2.125 + 4.625 + 8.000 + 11.500$$

$$f_5(x) = 26.250\%$$

Integral Method

$$f_5(x) = \int_2^{15} xf(x)dx + \int_{15}^{22} xf(x)dx + \int_{22}^{42} xf(x)dx + \int_{42}^{50} xf(x)dx$$
$$= \int_2^{15} 0.0192xdx + \int_{15}^{22} 0.0357xdx + \int_{22}^{42} 0.0125xdx + \int_{42}^{50} 0.0313xdx$$
$$= (2.160 - .0.384) + (8.639 - 4.016) + (11.025 - 3.025) + (39.125 - 27.607)$$
$$f_5(x) = 25.917\%$$

(ii) Contractor B

Fractile Method

$$f_5(x) = (0.25)(10 - 0)/2 + (0.25)(10 + (15 - 10)/2) + (0.25)(15 + (25 - 15)/2)$$
$$+ (0.25)(25 + (40 - 25)/2) = 1.250 + 3.125 + 5.000 + 8.125$$
$$f_5(x) = 17.500\%$$

Integral Method

$$f_5(x) = \int_0^{10} xf(x)dx + \int_{10}^{15} xf(x)dx + \int_{15}^{25} xf(x)dx + \int_{25}^{40} xf(x)dx$$
$$= \int_0^{10} 0.0250xdx + \int_{10}^{15} 0.0500xdx + \int_{15}^{25} 0.0250xdx + \int_{25}^{40} 0.0167xdx$$
$$= 1.250 + (5.625 - 2.500) + (7.8125 - 2.8125) + (13.360 - 5.218)$$
$$f_5(x) = 17.516\%$$

(iii) Contractor C

Fractile Method

$$f_5(x) = (0.25)(4 + (25 - 4)/2) + (0.25)(25 + (40 - 25)/2) + (0.25)(40 + (50 - 40)/2)$$
$$+ (0.25)(50 + (60 - 50)/2) = 3.620 + 8.125 + 11.250 + 13.750$$
$$f_5(x) = 36.745\%$$

Integral Method

$$f_5(x) = \int_4^{25} xf(x)dx + \int_{25}^{40} xf(x)dx + \int_{40}^{50} xf(x)dx + \int_{50}^{60} xf(x)dx$$

$$= \int_4^{25} 0.0119xdx + \int_{25}^{40} 0.0167xdx + \int_{40}^{50} 0.0250xdx + \int_{50}^{60} 0.0250xdx$$

$$= (3.7188 - 0.1592) + (13.360 - 5.2188) + (31.250 - 20.000) + (45.000 - 31.250)$$

$$f_5(x) = 36.700\%$$

(iv) Contractor D

Fractile Method

$$f_5(x) = (0.25)(3 + (12 - 3)/2) + (0.25)(12 + (25 - 12)/2) + (0.25)(25 + (35 - 25)/2)$$
$$+ (0.25)(35 + (70 - 35)/2) = 1.875 + 4.625 + 7.500 + 13.125$$

$$f_5(x) = 27.125\%$$

Integral Method

$$f_5(x) = \int_3^{12} xf(x)dx + \int_{12}^{25} xf(x)dx + \int_{25}^{35} xf(x)dx + \int_{35}^{50} xf(x)dx$$

$$= \int_3^{12} 0.0278xdx + \int_{12}^{25} 0.0192xdx + \int_{25}^{35} 0.0250xdx + \int_{35}^{50} 0.0071xdx$$

$$= (2.0016 - 0.1251) + (6.000 - 1.3824) + (15.3125 - 7.8125) + (17.395 - 4.3488)$$

$$f_5(x) = 27.0403\%$$

(v) Contractor E

Fractile Method

$$f_5(x) = (0.25)(2 + (20 - 2)/2) + (0.25)(20 + (35 - 20)/2) + (0.25)(35 + (50 - 35)/2)$$
$$+ (0.25)(50 + (80 - 50)/2) = 2.750 + 6.875 + 10.625 + 16.250$$

$$f_5(x) = 36.500\%$$

Integral Method

$$f_5(x) = \int_{2}^{20} xf(x)dx + \int_{20}^{35} xf(x)dx + \int_{35}^{50} xf(x)dx + \int_{50}^{80} xf(x)dx$$

$$= \int_{2}^{20} 0.0139xdx + \int_{20}^{35} 0.0167xdx + \int_{35}^{50} 0.0167xdx + \int_{50}^{80} 0.0083xdx$$

$$= (2.780 - 0.0278) + (10.2288 - 3.340) + (20.875 - 10.2288) + (26.560 - 10.375)$$

$$f_5(x) = 36.472\%$$

A representation of the total expected cost of the project from each contractor is shown in Table VII.6.3.

Table VII.6.3. Total Expected Cost by Contractor

	Bid Price	Cost Overrun $f_5(x)$	Total Expected. Cost
Contractor "A"	$1,125,000	26.250%	$1,420,312
Contractor "B"	$1,375,000	17.500%	$1,615,625
Contractor "C"	$1,050,000	36.745%	$1,435,823
Contractor "D"	$1,250,000	27.040%	$1,588,000
Contractor "E"	$1,075,000	36.500%	$1,467,375

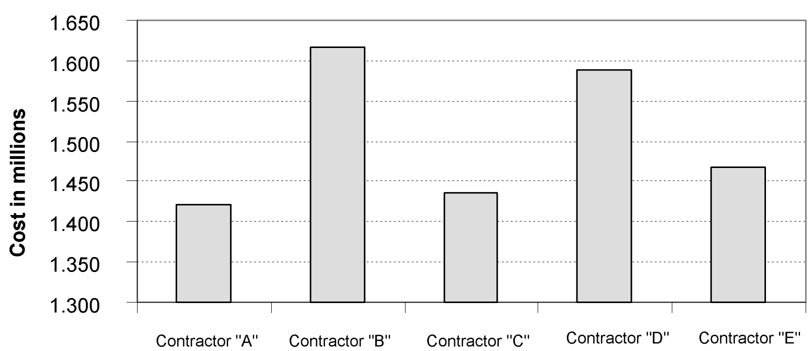


Figure VII.6.6. Comparison of Expected Cost of Project

Generate Conditional Expected Value E[X] = $f_4(x)$ using both the fractile method and the integration method

SMCC is now interested in the worst 10% scenario, or the conditional expected value of percentage of cost overrun, given that the cost overrun occurs with a probability of 0.10 or lower. The partition point on the damage axis corresponding to (1-α) = 0.1 is computed. Therefore, the damage axis must be partitioned at α =

0.9. Using simple geometry, the percentage of cost overrun associated with a probability of exceedance of 0.1 is computed as follows:

(i) ***Contractor "A"***

$$\frac{x-42}{50-42}=\frac{0.25-(1-\alpha)}{0.25}$$
$$\frac{x-42}{8}=\left(\frac{0.25-0.1}{0.25}\right)$$
$$x=46.8\%$$

(ii) ***Contractor "B"***

$$\frac{x-25}{40-25}=\frac{0.25-(1-\alpha)}{0.25}$$
$$\frac{x-25}{15}=\left(\frac{0.25-0.1}{0.25}\right)$$
$$x=34.0\%$$

(iii) ***Contractor "C"***

$$\frac{x-50}{60-50}=\frac{0.25-(1-\alpha)}{0.25}$$
$$\frac{x-50}{10}=\left(\frac{0.25-0.1}{0.25}\right)$$
$$x=56.0\%$$

(iv) ***Contractor "D"***

$$\frac{x-35}{70-35}=\frac{0.25-(1-\alpha)}{0.25}$$
$$\frac{x-35}{35}=\left(\frac{0.25-0.1}{0.25}\right)$$
$$x=56.0\%$$

(v) ***Contractor "E"***

$$\frac{x-50}{80-50}=\frac{0.25-(1-\alpha)}{0.25}$$
$$\frac{x-50}{30}=\left(\frac{0.25-0.1}{0.25}\right)$$
$$x=68.0\%$$

The conditional expected values $f_4(x)$ are now computed with the above partition points. Since the CDF is a straight line between the above partition points, the conditional expected value is the average between the lowest and highest values.

The conditional expected value has also been computed using integration, as follows:

(i) Contractor "A"

$$f_4(x) = \frac{46.8+50}{2} = 48.4\%$$

<u>*Integral Method*</u>

$$f_4(x) = \frac{\int_{46.8}^{50} xp(x)dx}{\int_{46.8}^{50} p(x)dx} = \frac{\int_{46.8}^{50} xKdx}{\int_{46.8}^{50} Kdx} = \frac{\left.\frac{x^2}{2}\right|_{46.8}^{50}}{\left.x\right|_{46.8}^{50}} = \frac{(2500-2190.24)}{2(50-46.8)}$$

$$f_4(x) = 48.4\%$$

(ii) Contractor "B"

$$f_4(x) = \frac{34.0+40}{2} = 37.0\%$$

<u>Integral Method</u>

$$f_4(x) = \frac{\int_{34}^{40} xp(x)dx}{\int_{34}^{40} p(x)dx} = \frac{\int_{34}^{40} xKdx}{\int_{34}^{40} Kdx} = \frac{\left.\frac{x^2}{2}\right|_{34}^{40}}{\left.x\right|_{34}^{40}} = \frac{(1600-1156)}{2(40-34)}$$

$$f_4(x) = 37.0\%$$

(iii) Contractor "C"

$$f_4(x) = \frac{56.0+60}{2} = 58.0\%$$

<u>Integral Method</u>

$$f_4(x) = \frac{\int_{56}^{60} xp(x)dx}{\int_{56}^{60} p(x)dx} = \frac{\int_{56}^{60} xKdx}{\int_{56}^{60} Kdx} = \frac{\left.\frac{x^2}{2}\right|_{56}^{60}}{\left.x\right|_{56}^{60}} = \frac{(3600-3136)}{2(60-56)}$$

$$f_4(x) = 58.0\%$$

Contractor "D"

$$f_4(x) = \frac{56.0+70}{2} = 63.0\%$$

<u>Integral Method</u>

$$f_4(x) = \frac{\int_{56}^{70} xp(x)dx}{\int_{56}^{70} p(x)dx} = \frac{\int_{56}^{70} xKdx}{\int_{56}^{70} Kdx} = \frac{\left.\frac{x^2}{2}\right|_{56}^{70}}{\left.x\right|_{56}^{70}} = \frac{(4900-3136)}{2(70-56)}$$

$$f_4(x) = 63.0\%$$

(iv) Contractor "E"

$$f_4(x) = \frac{68.0+80}{2} = 74.0\%$$

<u>Integral Method</u>

$$f_4(x) = \frac{\int_{68}^{80} xp(x)dx}{\int_{68}^{80} p(x)dx} = \frac{\int_{68}^{80} xKdx}{\int_{68}^{80} Kdx} = \frac{\left.\frac{x^2}{2}\right|_{68}^{80}}{\left.x\right|_{68}^{80}} = \frac{(6400-4624)}{2(80-68)}$$

$$f_4(x) = 74.0\%$$

Table VII.6.4 and Figure VII.6.7 summarize the results:

Table VII.6.4. Total Conditional Expected Cost by Contractor

	Bid Price	Cost Overrun f_4(x)	Total Conditional Expected. Cost
Contractor "A"	$1,125,000	48.4%	$1,669,500
Contractor "B"	$1,375,000	37.0%	$1,883,750
Contractor "C"	$1,050,000	58.0%	$1,659,000
Contractor "D"	$1,250,000	63.0%	$2,037,500
Contractor "E"	$1,075,000	74.0%	$1,870,500

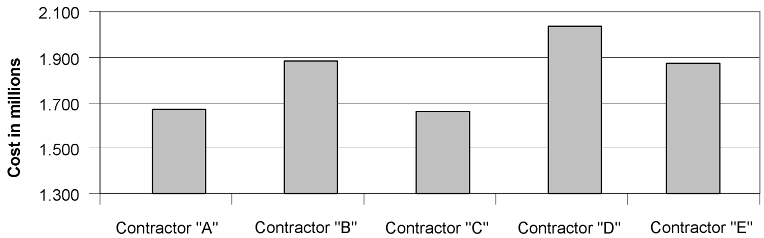


Figure VII.6.7. Comparison of Conditional Expected Cost of Project

From the bid price, expected project cost, and the conditional project cost values, a more realistic determination of the project cost can be determined. These figures are shown below.

Table VII.6.5. Summary of f_4 and f_5

	Bid Price	Cost overrun f_5(x)	Total Expected Cost	Cost overrun f_4(x)	Total Conditional Expected Cost
Contractor “A”	$1,125,000	26.250%	$1,420,312	48.4%	$1,669,500
Contractor “B”	$1,375,000	17.500%	$1,615,625	37.0%	$1,883,750
Contractor “C”	$1,050,000	36.745%	$1,435,823	58.0%	$1,659,000
Contractor “D”	$1,250,000	27.040%	$1,588,000	63.0%	$2,037,500
Contractor “E”	$1,075,000	36.500%	$1,467,375	74.0%	$1,870,500

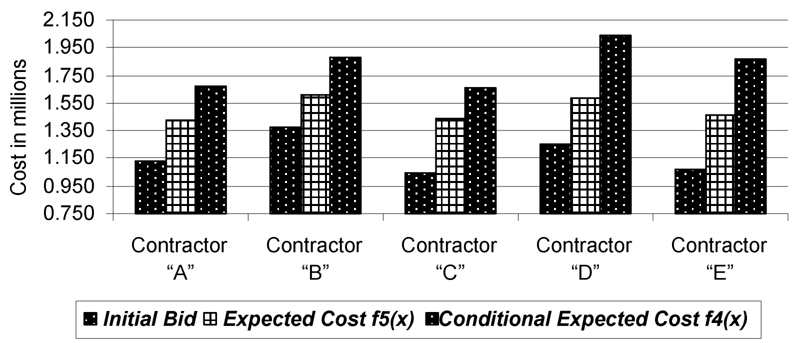


Figure VII.6.8. Summary of Projected Cost Estimates

ANALYSIS

SMCC can utilize the above results to make a more accurate prediction of the construction costs of the building. From the table and graph above, it is clear that there is a clear disparity between each contractor's bid price and the expected actual price of the building. For instance, Contractor "D" submitted a bid of $1,075,000. However, in the worst 10% case, their estimate would be $1,870,500, resulting in a 74% increase in their original bid. Utilizing these disparities, SMCC can formulate a separate cost estimate combining all aspects of the above bids and expected values to achieve the most accurate cost estimate for their customer.

PROBLEM VII.7: Water Supply Treatment Selection

In order to secure the safe level of chloride concentration, Metropolitan Manila is considering where to build a new facility.

DESCRIPTION

Metropolitan Manila is the capital of the Philippines and among the world's thirty most populous metropolitan areas. It contains the city of Manila, as well as sixteen surrounding cities and municipalities. In some of these surrounding cities and municipalities, water is not yet supplied by bulk water treatment plants. The people obtain water from either deep wells or ambulant water suppliers (i.e., trucks selling water). To address current and future water needs, the recently privatized Manila Water Company is studying the feasibility of abstracting water from Laguna Lake, the Philippines' largest lake.

Two sites are being considered for the proposed 40,000 cubic meter Bulk Water Supply Treatment Plant—Muntinlupa City and Paranaque City. Because of the difference in location along the lake's bay, degree of tidal water (seawater) intrusion, and level of industrial and aquaculture activities in these two cities, the quality of raw water is different. Among the water quality parameters, chloride concentration is one of the most important because this cannot be removed by physical processes or removed economically even by chemical processes. Often, chloride is addressed by desalination which is expensive with regard to both capital and operating expenditures. Assuming that all other parameters and other factors are equal (an oversimplification of the problem), the designers/analysts are presented the following table of chloride concentrations on which they will make their recommendations.

Table VII.7.1. Prediction of Chloride Concentration by 2010 (Projected Project Completion Year)

	Muntinlupa	Paranaque
Best case chloride concentration, mg/L	200	180
Worst case chloride concentration, mg/L	1000	1200
Most likely chloride concentration, mg/L	250	230

METHODOLOGY

For this PMRM exercise, chloride concentrations are assessed using a triangular distribution. Let *a, b,* and *c* denote the best, worst, and most likely respectively. Also, let the subscripts *M* and *P* denote Muntinlupa and Paranaque, respectively.

	M	P
a	200	180
b	1000	1200
c	250	230

The expected value of the triangular distribution is given by $f_5(\cdot)$:

$$f_5(\cdot) = \frac{a+b+c}{3}$$

$$f_{5M}(\cdot) = \frac{200+1000+250}{3} = 483.33$$

$$f_{5P}(\cdot) = \frac{180+1200+230}{3} = 536.67$$

The computation for the height of the triangular distribution is straightforward:

$$h = \frac{2}{b-a}$$

$$h_M = \frac{2}{1000-200} = 0.0025$$

$$h_P = \frac{2}{1200-180} = 0.001961$$

Figure VII.7.1 graphically depicts the chloride concentrations in Muntinlupa and Paranaque.

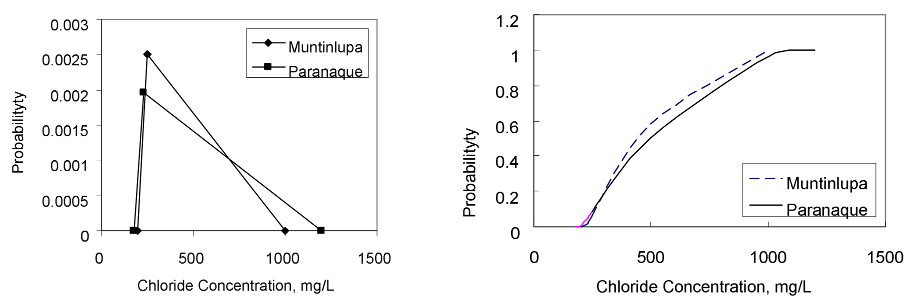


Figure VII.7.1. Chloride Concentrations (PDF & CDF)

SOLUTION

Since the Bulk Water Supply Treatment Plant is a public utility, we consider an event above 95 percent likelihood to be extreme.

$$\alpha = 0.95$$

The value of *x* (partition point on the damage axis) can be computed as follows,

$$x = b - \sqrt{\frac{2(1-\alpha)(b-c)}{h}}$$

$$x_M = 1000 - \sqrt{\frac{2(1-0.95)(1000-250)}{0.0025}} = 826.79$$
$$x_P = 1200 - \sqrt{\frac{2(1-0.95)(1200-230)}{0.001961}} = 977.58$$

Since the extreme region forms a right-angled triangle, we can compute the value of $f_4(\cdot)$ as its mean,

$$f_4(\cdot) = \frac{2x+b}{3}$$
$$f_{4M}(\cdot) = \frac{2(826.76)+1000}{3} = 884.53$$
$$f_{4P}(\cdot) = \frac{2(977.58)+1200}{3} = 1051.72$$

ANALYSIS

Suppose that the construction of treatment plants would require investment costs of \$100 million for Muntinlupa and \$30 million for Paranaque. The Pareto-optimal frontiers are shown in Figure VII.7.2.

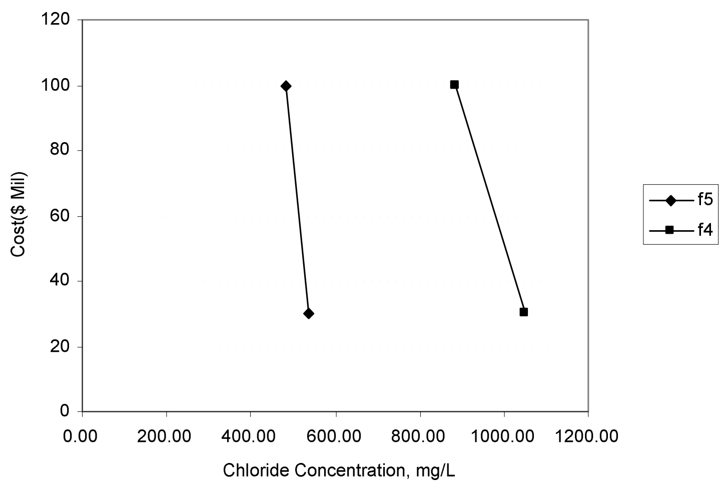


Figure VII.7.2. Pareto-optimal Frontiers of f1 versus f4 and f5

From this plot we can see that the Muntinlupa site is superior in terms of its overall chloride level and its expected extreme chloride level. Therefore, we can recommend that the board of Manila select Muntinlupa as the candidate place for the treatment plant, although this site would require approximately three times more budget than the Paranaque option.

There are several aspects of this problem that highlight the need for applying the Partitioned Multiobjective Risk Method (PMRM). First of all, the probability distribution is highly skewed over a large range. In situations like this, the expected value over the entire distribution gives a very poor picture of what actually can be expected. Also, in this example, expert opinion could be solicited regarding the concentration of chloride that should be considered dangerous. Rather than partitioning on probability, we could alternatively partition based on this factor. The extreme-event analysis would then cover the situation in which the concentration levels were dangerous.

PROBLEM VII.8: Architectural Style Selection

A local entrepreneur is building a new restaurant and is considering two different, interesting architectural styles. The construction company can build either type of building and provides cost estimates for each.

Given the complex designs and the uncertainty of labor and material costs, the construction company estimates the following fractiles. Costs for Designs A and B are in thousands of US dollars:

Table VII.8.1. Cost by Design

Fractile	Design A	Design B
0	700	600
0.25	750	650
0.5	770	750
0.75	800	850
1	850	1000

Figure VII.8.1 graphs the probability density functions.

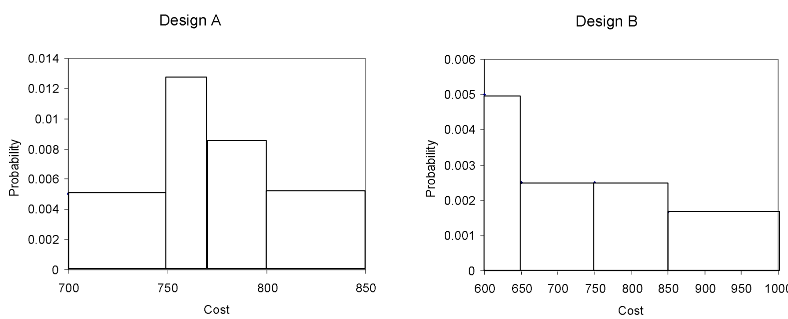


Figure VII.8.1. Probability Density Function by Design

Compare and evaluate the two styles using the PMRM with respect to cost and analyze your results. Use a probability partition (α) of 0.9 in calculating the conditional expected values (f_4).

PROBLEM VII.9: Selection of contractors for a highway project

Two contractors are being considered for a new highway construction project. We use the Partitioned Multiobjective Risk Method (PMRM) to help make the decision.

The contractors provided the following probabilistic estimates of their projected completion times for the project:

Contractor A

Triangular distribution with parameters:
Lowest estimate: 1 year
Most likely: 1.25 years
Highest estimate: 2 years

Contractor B

Fractile distribution with parameters:
Lowest estimate: 0.5 year
25^{th} fractile: 1.2 year
50^{th} fractile: 1.4 year
75^{th} fractile: 1.6 years
Highest estimate: 2 years

In the selection of a new contractor, evaluate the projected completion time using the PMRM and add your explanation for the results. We are interested in both the average and the conditional expected value representing the worst 10% scenario (i.e., $\alpha = 0.9$).

PROBLEM VII.10: Shipping company selection

A new online retailer is considering several shipping companies for distributing its wares to its customers. The company was able to narrow the choices down to three candidates: Company A, Company B, and Company C. The online retailer obtained cost estimates and past-year performance statistics for these shipping companies from an independent consulting firm. This firm provided information regarding shipping timeliness in the form of percentages of late deliveries.

The probabilities were derived using the fractile method. The expected value of risk for each shipping company was calculated and plotted against the estimated delivery cost. The consulting company provided statistics on the best, the worst, and the most likely percentages of late deliveries.

For the purposes of the fractile method, the most-likely percentage of late deliveries was considered the median. The worst case was placed at 1.00 fractile. The 0.25 and 0.75 fractiles were calculated based on the median +/- 5%.

The following table shows the fractiles for all three shipping companies.

Table VII.10.1. Delivery Options for a New DotCom

Fractile	Percentage of Late Delivery for Each Option		
	Company A	Company B	Company C
0	0	0	0
0.25	10	15	5
0.5	15	20	10
0.75	20	25	15
1	40	35	50

The PDF and CDF plots for these companies are shown below, using the data in the table above.

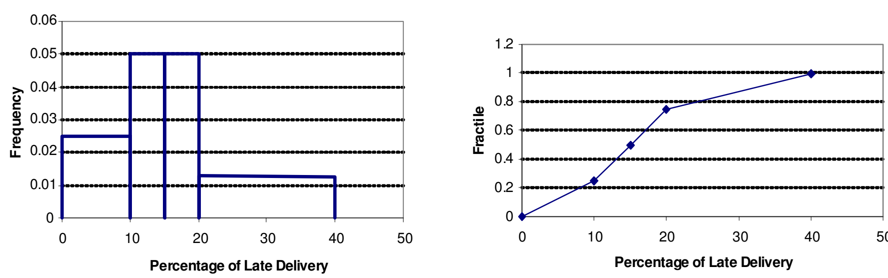


Figure VI.10.1. PDF and CDF of Company A

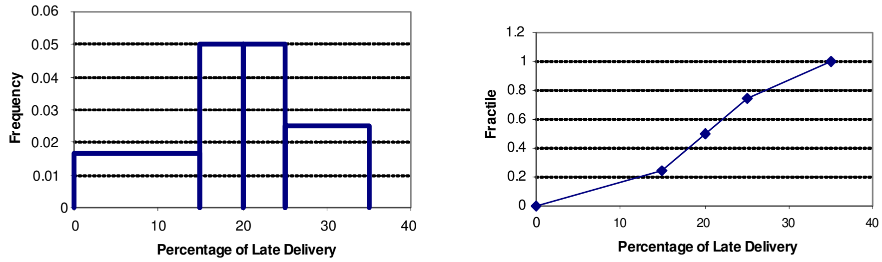


Figure VII.10.2. PDF and CDF of Company B

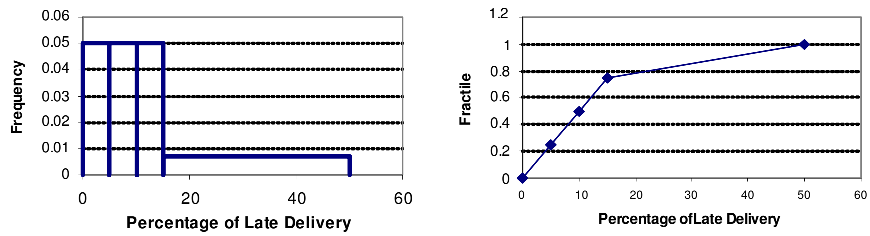


Figure VII.10.3. PDF and CDF of Company C

For this problem, each option is evaluated according to the Percentage of Late Delivery using the PMRM. Calculate the expected-value f_5 and conditional expected-value f_4 with a probability partition of $\alpha = 0.9$. Analyze the results.

PROBLEM VII.11: Reliability of Shuttle "O"-Rings

The failure density function of elastomeric "O"-rings can be described using a Weibull distribution[1], as follows:

Weibull Probability Density Function: $f(x) = \dfrac{\lambda}{\eta^{\lambda}} x^{\lambda-1} \exp\left[-\left(\dfrac{x}{\eta}\right)^{\lambda}\right]$

where x is the failure time in hours,
λ is the Weibull shape factor, and
η is the characteristic time parameter in hours.

Based on the failure density function, analyze reliability of Shuttle "O"-Rings.

DESCRIPTION

Consider the following "O"-ring alternatives, all with shape parameters of $\lambda = 1.0$. For a shape parameter of 1.0, the Weibull distribution reduces to an exponential distribution.

Table VII.11.1. Summary of η, Cost and Expected Failure Time of each alternative

Alternative	λ	η	Cost ($)	$f_5(\cdot)$	$f_4(\cdot)$ at α=0.95
"O"-ring *A*	1.0	10,000	10.00	10,000	39,957
"O"-ring *B*	1.0	15,000	20.00	15,000	59,936
"O"-ring *C*	1.0	20,000	15.00	20,000	79,915
"O"-ring *D*	1.0	25,000	50.00	25,000	99,893
"O"-ring *E*	1.0	30,000	100.00	30,000	119,872

Note: α is the upper-tail probability partition

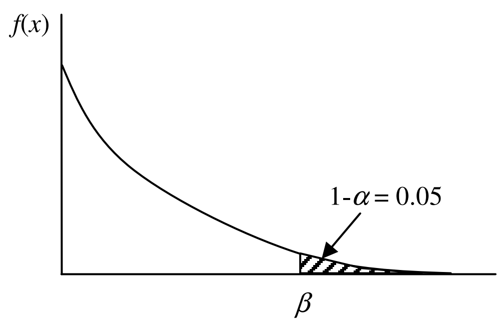


Figure VII.11.1. Exponential Probability Distribution Curve

[1] See Bloch, Heinz P. and Fred K. Geitner, 1994. *Practical Machinery Management for Process Plants, Volume 2: Machinery Failure Analysis and Troubleshooting, 2nd Edition.* Houston, TX: Gulf Publishing Company.

Since the values of $f_5(\cdot)$ and $f_4(\cdot)$ in the above table represent failure times that are maximization-type objectives, define the following measures of risk to be the reciprocal of failure time:

Expected value of risk (hour^{-1}): $\hat{f}_5(\cdot) = \left[f_5(\cdot)\right]^{-1}$

Conditional expected risk (hour^{-1}): $\hat{f}_4(\cdot) = \left[f_4(\cdot)\right]^{-1}$

With the reciprocal values of failure time, evaluate the reliability of each O-ring option using the PMRM. Use a probability partition of α = 0.95, as depicted in Figure VII.11.1.

PROBLEM VII.12: Budget allocation for counterterrorism

The Department of Homeland Security (DHS) has been asked to submit to the Executive Office a budget for counterterrorism measures. The overall budget is very tight, and while combating terrorism is a significant goal, DHS must be sure to spend its money wisely and choose an effective strategy.

Five potential strategies and their possible outcomes are given below.

Table VII.12.1. Summary of Fractile Distributions by Option

	Best	25th	Median	75th	Worst	Cost ($billion)
No Action	5	15	20	30	45	0
Increase Security	3	8	15	20	30	60
Increase Intelligence Budget	3	9	17	22	32	30
Increase Technology Budget	8	14	17	22	35	15
Preemptive War	4	11	14	15	17	300

The cumulative density function and exceedance probabilities for the strategies are shown below:

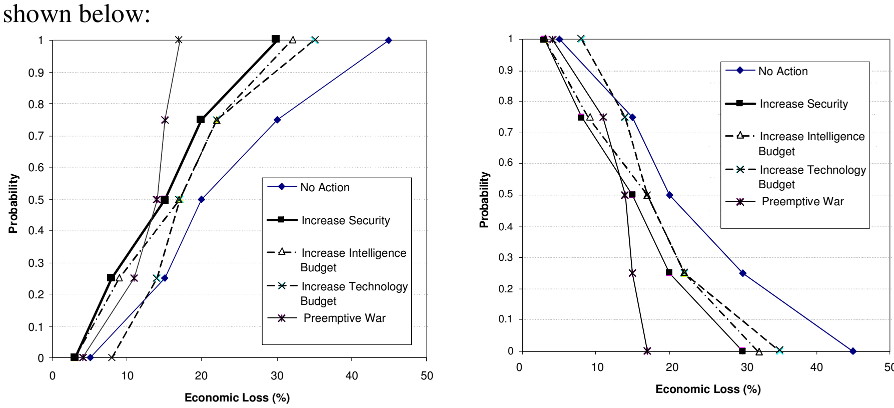


Figure VII.12.1. CDF and Exceedance Probability of Each Option

For this problem, each option is evaluated according economic loss using the PMRM. Calculate the expected-value f_5 and conditional expected-value f_4 with a probability partition of $\alpha = 0.85$. Analyze your results.

PROBLEM VII.13: Improving Road Safety

A recent spate of accidents on a dangerous mountain road has the nearby residents asking the town council for help.

During a specially held town meeting, five options were proposed to deal with the problem. These were: 1) Take no action, 2) Add signs, 3) Add speed bumps, 4) Widen the road, and 5) Build an alternate route.

The council employed the Partitioned Multiobjective Risk Method (PMRM) to help decide which course of action to take. First, they estimated the number of accidents in the best- and worst-case scenarios for each option. These figures are given in Table VII.13.1, and the costs for each option are given in Table VII.13.2.

Table VII.13.1. Number of Accidents per Year for Each Option

	Best (0)	25th	Median 50^{th}	75^{th}	Worst (100)
No Action, a_1	10	15	20	40	100
Signs, a_2	8	12	18	34	85
Speed Bumps, a_3	4	8	15	30	75
Widen Road, a_4	6	10	14	25	60
Alternate Rte., a_5	0	2	8	14	20

Table VII.13.2. Cost for Each Option

Alternative	Cost ($)
a_1	0
a_2	5,000
a_3	15,000
a_4	50,000
a_5	250,000

The town council then determined the number of accidents that would be considered “extreme.” They decided that a high-damage outcome, or the β value, should be set at 40.

For an alternative view of the data above, the council decided to partition the data on the probability axis as well. They wanted to see what would be likely to happen greater than one in ten years, i.e., an α value of 0.9.

Given the two specified approaches for partitioning (i.e., with respect to the damage axis, β, and probability axis, α), use the PMRM and analyze your results.

PROBLEM VII.14: Investor's Dilemma

A market theory asserts that investment returns, denoted by X, are normally distributed. For this problem, we interpret investment returns X as "opportunity losses." Therefore, the upper-tail region in a distribution of investment returns X corresponds to events that have high opportunity losses, although with low likelihoods of occurrence.

Suppose an investor who has faith in this market theory asked us to conduct in-depth analysis for the following four long-term bond investment alternatives.

For a given investment i, the notation $X_i \sim N(\mu,\sigma)$ is used to refer to a normal distribution with parameters μ and σ, which are the mean and standard deviation, respectively, of the underlying random variable X_i. These parameters were estimated from historical *annual* data.

(i) *Investment 1*: $X_1 \sim N_1(0.047, 0.010)$; Unit Cost = \$10
(ii) *Investment 2*: $X_2 \sim N_2(0.048, 0.015)$; Unit Cost = \$8
(iii) *Investment 3*: $X_3 \sim N_3(0.049, 0.020)$; Unit Cost = \$5
(iv) *Investment 4*: $X_4 \sim N_4(0.050, 0.025)$; Unit Cost = \$4

Evaluate opportunity losses using the PMRM so need to derive the expected values and conditional expected values in terms of each investment's parameter(μ and σ) and a specified upper-tail partitioning points, β_i (i = 1, 2, 3, and 4) respectively

PDF for the normal distribution is characterized with mean and standard variation:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2} \right]$$

Conditional expected value can be calculated as follows:

$$f_4(x) = \frac{\int_{\beta}^{\infty} x \cdot f(x)dx}{\int_{\beta}^{\infty} f(x)dx} = \frac{\int_{\beta}^{\infty} \frac{x}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2} \right] dx}{1-\alpha}$$

where $p(x \le \beta) = \alpha$

Assume that α is 0.95 for each investment case.

PROBLEM VII.15: Recommendation for Welding Processes

A consulting company is contracted to analyze potential welding processes for a new Sports Utility Vehicle (SUV).

The firm will analyze three options and the options are as follows:

1. Robotic welding
2. Semi-Automatic welding
3. Manual welding

To generate probability distributions, the fractile method is used. They have been determined by manufacturing experts for the number of defective units produced per 100, as shown:

Table VII.15.1. Fractile Distribution of Each Option

	Best	25th	Median	75th	Worst
Robotic	0	5	15	20	30
Semi-Automatic	5	20	25	30	40
Manual	10	20	30	40	60

For example, in the robotic welding option, the best-case outcome produces 0 defective units per 100 while the worst-case outcome produces 30. The cumulative distribution for each policy is graphed below:

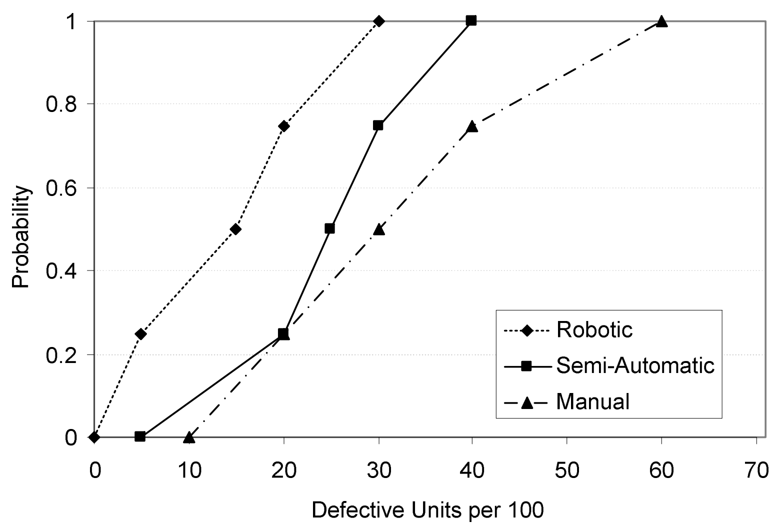


Figure VII.15.1. CDF of Each Option

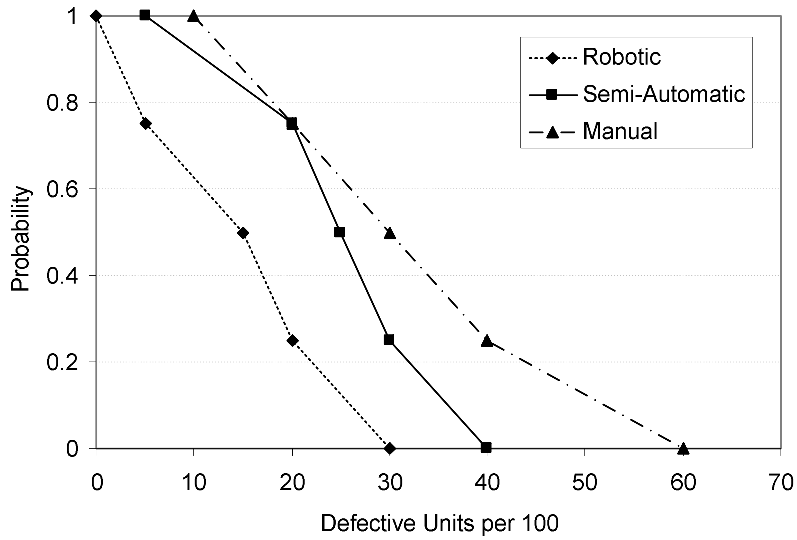


Figure VII.15.2. Exceedance Probability of Each Option

Conduct Portioned Multiobjective Risk Method (PMRM) of candidate welding processes for the two cases below:

Case I: Partition on the probability axis as follows: $\alpha = 0.9$
Case II: Partition on the damage axis as follows: the firm chooses $x \geq 35$ defective items.

PROBLEM VII.16: Automobile Company's options for building a safe car

A car-manufacturing company wants to incorporate into its vehicles components to reduce the number of serious injuries that result from high-velocity vehicle crashes. Such injuries are defined as those requiring more than three days of hospitalization.

The company is considering five approaches, adding: (1) safety features, (2) crumple zones, (3) a collapsible steering column, (4) fuel pump shutoff devices, and (5) a reinforced side door structure. To help the company arrive at a decision, Solve the problem using the Partitioned Multiobjective Risk Method (PMRM), applying triangular distribution

In detail the scenarios are:

(1) *Safety features*. Enhance all vehicles with existing safety features such as side air bags, anti-lock brakes, daytime running lights, and safety restraints. This option would be moderately inexpensive since these features are already popular options among consumers. There would be no need for research and development; the cost would be solely for making these options standard features on its vehicles.

(2) *Crumple zones.* Incorporate areas that will absorb the energy of an impact when the car hits something. This option would be very expensive due to research and development, as well as vehicle redesign. Preliminary research shows that this could potentially reduce the number of serious injuries.

(3) *Collapsible steering column.* This option would be moderately inexpensive. The main cost would be introducing it into the production process. Depending on the type of collision, this option may not be as effective as some of the others.

(4) *Fuel pump shutoff devices.* These would turn off gas flow in the event of a collision to prevent gasoline fires. This would be a minor modification to the current production process, making this option very inexpensive. However, as with Option 3, its effectiveness is limited in scope.

(5) *Reinforced side door structure.* Costs of additional materials to reinforce side doors would be moderately expensive. Since side-door impacts frequently occur, this option would be effective.

Because the company has not widely introduced any of these passive safety features, there is no historical data to perform statistical analysis. Therefore, it is assumed that the random variable X_j, which represents the rate of (number of) serious injuries per 1000 crashes for Scenario *j*, follows a triangular distribution. In addition, expert evidence was used to generate the lower bound, upper bound, and most-likely serious injury rate for each Scenario *j*.

Table VII.16.1. Design Data

Scenario	Cost ($Millions)	Lower Bound	Upper Bound	Most Likely
1	$30	30	120	70
2	$165	15	45	30
3	$50	80	260	185
4	$22	60	300	215
5	$100	20	80	45

Use the PMRM to evaluate the design scenarios (see Table VII.16.1) according to the number of serious injuries. Calculate the expected-value f_5 and conditional expected-value f_4 with a probability partition of $\alpha = 0.9$. Analyze your results.

PROBLEM VII.17: Energy Cost Estimation

A state government must determine the amount of budgetary dollars to allocate to energy costs for the next fiscal year. In order to obtain an estimate of these costs, the state has requested an energy cost analysis from two energy institutes with expertise in this area. One institute is conservative and one is liberal, selected in an attempt to satisfy concerns over skewing the estimate towards one end of the political spectrum. An internal state team will also perform the energy cost analysis. Based on the results from three sources, the state government will make a decision to allocate its limited resources.

All teams were required to provide data and estimates as follows:

Table VII.17.1. Estimate of Energy Cost Increase by Team

Evidence-based information	**State Team**	**Conservative Energy Institute**	**Liberal Energy Institute**
Best-case energy cost increase	0%	0%	10%
Worst-case energy cost increase	50%	30%	80%
Median value of energy cost increase	25%	10%	50%

Note: Current fiscal year energy costs = $100 M.

For each team, compute the expected-value f_5 and the conditional expected-value f_4 for the 10% worst case scenarios and analyze the results.