III. Decision Analysis

PROBLEM III.1: Developing a New Software for the Department of Defense

The Department of Defense (DoD) is considering two companies to develop a new software application to improve guidance and control on ballistic munitions.

DESCRIPTION

Two software manufacturers responded with proposals: we'll call them Company A and Company B. The DoD's desired software performance specs included cost overrun estimates within the proposal to allow adequate budgeting. Both companies bid approximately $3.0 million for DoD's initial software development expenditure. Because the proposal specifications from both companies were compatible, DoD will award the contract based on cost overrun estimates.

PART A: METHODOLOGY

Using the *fractile method,* find the solution and analyze the result.

DoD preferred Cost Overrun Ratios (in Thousands):

Best Case:	$0
Worst Case:	$250
Most Likely:	$75
50-50 Chance:	$25 +/- from Most Likely

SOLUTION

The cost overrun estimates from the software manufacturers were as follows:

Company A (in Thousands):

Best Case:	$0
Worst Case:	$350
Most Likely:	$150
50-50 Chance:	$75 +/- from Most Likely

Company B (in Thousands):

Best Case:	$0
Worst Case:	$500
Most Likely:	$75
50-50 Chance:	$50 +/- from Most Likely

Table III.1.1. Parameters for the Fractile Distribution Representing Cost Overruns

	Cost Overruns ($T)		
Fractile	DoD	A	B
0.00	$0	$0	$0
0.25	$50	$75	$25
0.50	$75	$150	$75
0.75	$100	$225	$125
1.00	$250	$350	$500

$E[X_{DoD}]$ = (0.25)($0 + ($50-$0)/2) + (0.25)($50 + ($75-$50)/2) + (0.25)($75 + ($100-$75)/2) + (0.25)($100 + ($250-$100)/2)
= (0.25)($25) + (0.25)($62.5) + (0.25)($87.5) + (0.25)($175)
= $87.5

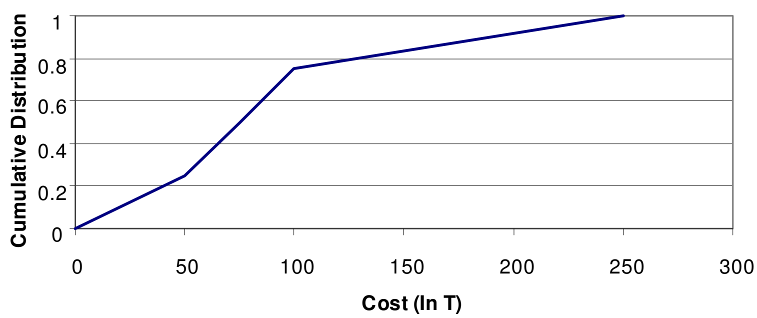

Figure III.1.1. DoD Preferred Cumulative Cost Distribution

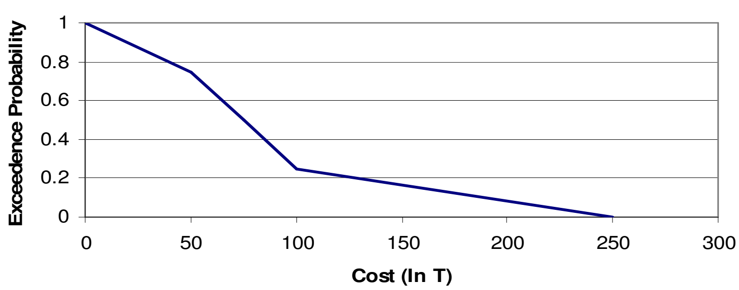

Figure III.1.2. DoD Preferred Cost Exceedance (Over Expense) Distribution

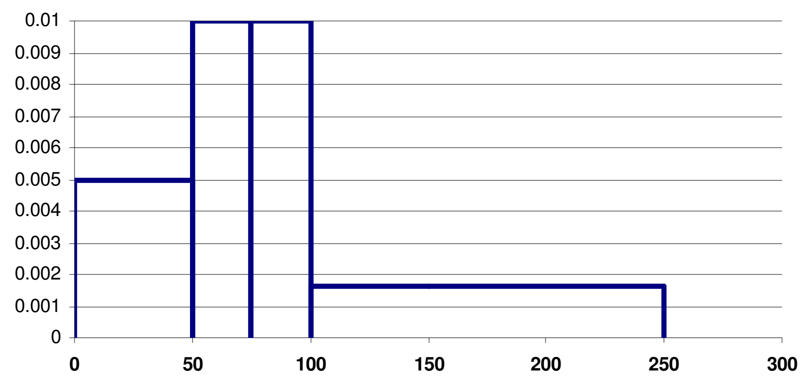

Figure III.1.3. Fractile Distribution for DoD's Preferred Software (Cost, in $)

$E[X_A]$ = (0.25)($0 + ($75-$0)/2) + (0.25)($75 + ($150-$75)/2) + (0.25)($150 + ($225-$150)/2) + (0.25)($225 + ($350-$225)/2)
= (0.25)($37.5) + (0.25)($112.5) + (0.25)($187.5) + (0.25)($287.5)
= $156.25

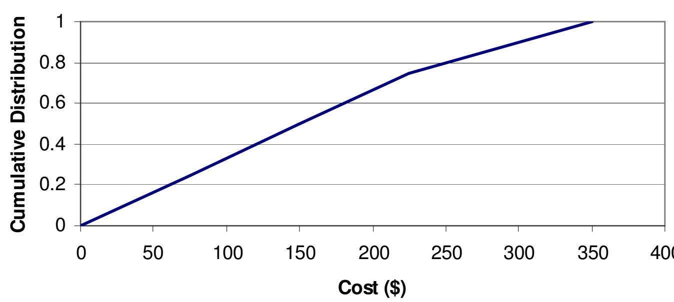

Figure III.1.4. Company A Cumulative Cost Distribution

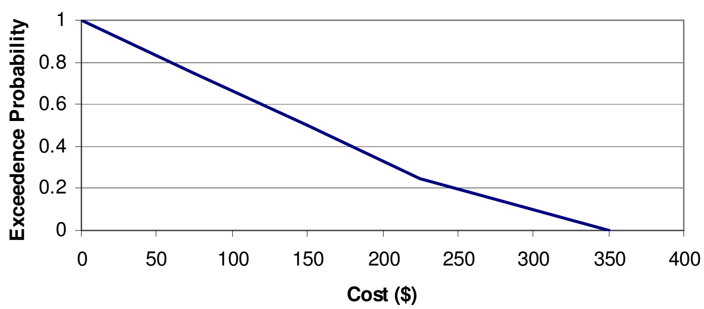

Figure III.1.5. Company A Cost Exceedance Distribution

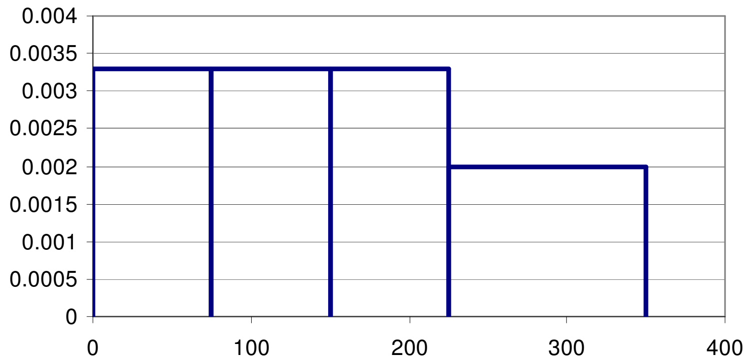

Figure III.1.6. Company A Cost PDF (Cost, in \$)

$E[X_B]$ = (0.25)(\$0 + (\$25-\$0)/2) + (0.25)(\$25 + (\$75-\$25)/2) + (0.25)(\$75 + (\$125-\$75)/2) + (0.25)(\$125 + (\$500-\$125)/2)
= (0.25)(\$12.5) + (0.25)(\$50) + (0.25)(\$100) + (0.25)(\$312.5)
= \$118.75

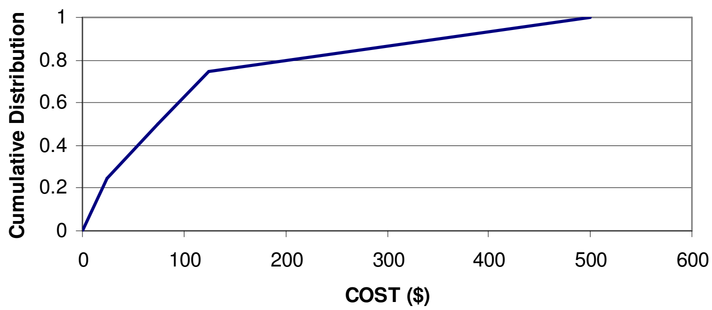

Figure III.1.7. Company B Cumulative Cost Distribution

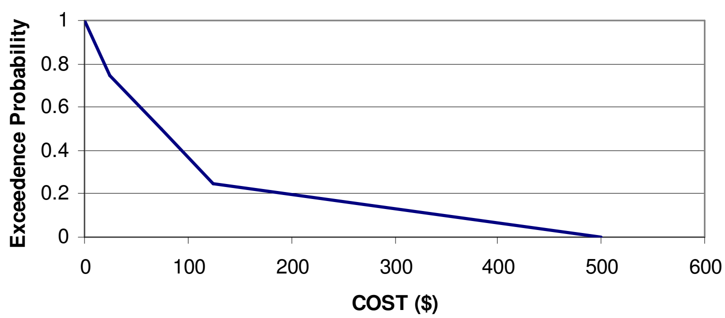

Figure III.1.8. Company B Cost Exceedance Distribution

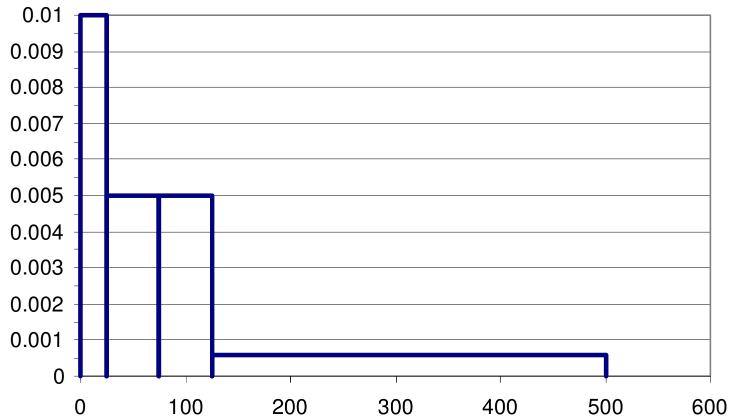

Figure III.1.9. Company B Cost PDF

ANALYSIS

From the calculations, the Department of Defense has a preferred cost overrun expected value of $87.5 thousand. Company A's proposal cost overrun estimation has an expected value of $156.25. Company B's proposal cost overrun estimation has an expected value of $118.75. Neither of the two software manufacturers is within the cost overrun range preferred by the Department of Defense. DoD can: 1) accept the lower estimated cost overrun of $118.75 from Company B; 2) relax some of its software requirements and lower the proposal costs and cost overruns; or 3) budget extra funds for the project to cover cost overruns, including the $31.25 ($118.75 – $87.5 = $31.25).

Furthermore, 50% of Company B's cost overruns are less than or equal to DoD's preferences. Company B exceeds DoD slightly at the 75% range. If the cost overruns do exceed 75% probability, then Company B's cost experiences a tremendous jump. Company A is not within DoD's range for any of the values, but experiences a much smaller jump if the probability exceeds 75%. Given that the decisionmakers have $\alpha = 0.6$ and are determined to move forward on awarding the contract to either Company A or B, they would choose Company B.

PART B: METHODOLOGY

Solve the same problem and analyze the result using the following *triangular distribution* for the construction of the probabilities.

Table III.1.2. Parameters for the Triangular Distribution Representing Cost Overruns

Cost Overruns ($T)			
Values	DoD	A	B
Lowest (a)	0	0	0
Highest (b)	250	350	500
Most Likely (c)	75	150	75

SOLUTION

Department of Defense (DoD):

Density triangle height:
$\underline{p}(c) = 2/(b - a) = 2/(250 - 0) = 0.008$

Density $[p(x)]$	=	$[2(x\text{-}0)]/[(250\text{-}0)(75\text{-}0)]$	If $0 \le x \le 75$
	=	$[2(250\text{-}x)]/[(250\text{-}0)(250\text{-}75)]$	If $75 < x \le 250$
	=	0	otherwise

Distribution $[P(x)]$	$= 0$	If $x < 0$
	$= [(x\text{-}0)^2]/[(250\text{-}0)(75\text{-}0)]$	If $0 \le x \le 75$
	$= 1 - [(250\text{-}x)^2]/[(250\text{-}0)(250\text{-}75)]$	If $75 < x \le 250$
	$= 1$	If $x > 250$

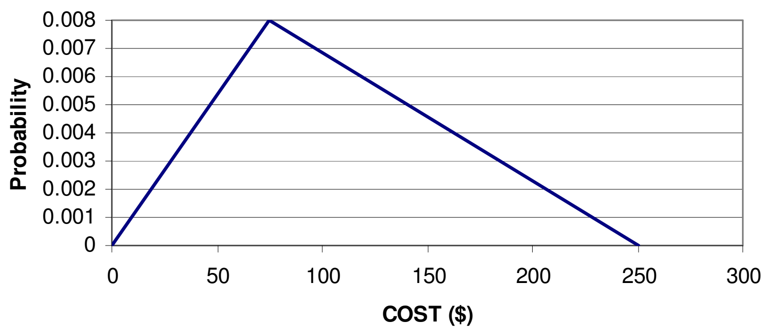

Figure III.1.10. Triangular Distribution for DoD's Preferred Software

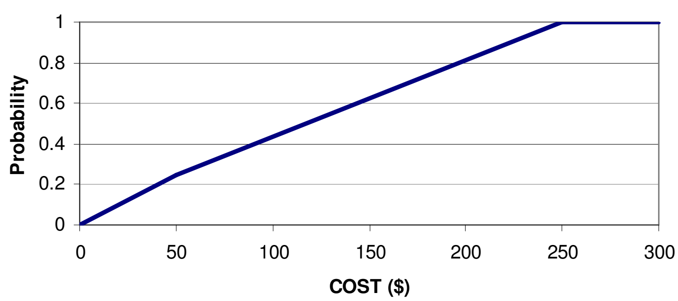

Figure III.1.11. Cumulative Distribution for DoD's Preferred Software

Mean = $E[X_{DoD}] = (a + b + c)/3 = (\$0+\$250+\$75)/3 = \$108.33$

Variance = $(a^2 + b^2 + c^2 - ab - ac - bc)/18$
$= (0^2 + 250^2 + 75^2 - (0)(250) - (0)(75) - (250)(75))/18$
$= (0 + 62500 + 5625 - 0 - 0 - 18750)/18$
$= (49375)/18 = 2743.06$ dollars^2

Standard deviation = $\sqrt{Variance}$ = $\sqrt{2743.60}$ = \$52.37

Company A:

Density triangle height:
$p(c) = 2/(b - a) = 2/(350 - 0) = 0.005714$

Density $[p(x)]$	=	$[2(x\text{-}0)]/[(350\text{-}0)(150\text{-}0)]$	If $0 \le x \le 150$
	=	$[2(350\text{-}x)]/[(350\text{-}0)(350\text{-}150)]$	If $150 < x \le 350$
	=	0	otherwise

Distribution $[P(x)]$	= 0	If $x < 0$
	$= [(x\text{-}0)^2]/[(350\text{-}0)(150\text{-}0)]$	If $0 \le x \le 150$
	$= 1 - [(350\text{-}x)^2]/[(350\text{-}0)(350\text{-}150)]$	If $150 < x \le 350$
	= 1	If $x > 350$

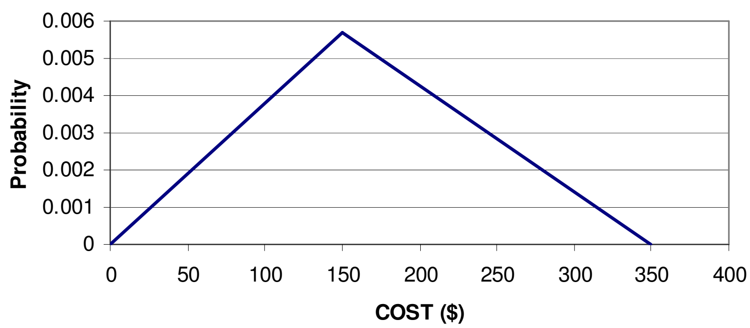

Figure III.1.12. Company A Cost Overrun PDF

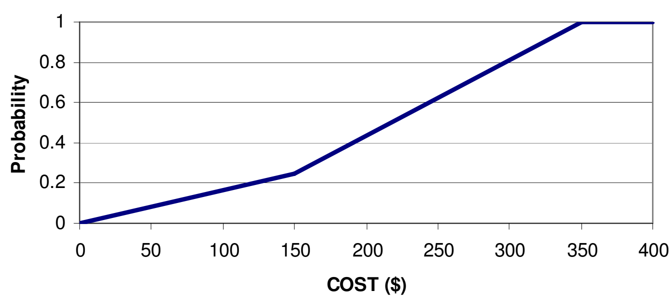

Figure III.1.13. Company A Cost Overrun Cumulative Distribution

Mean = $E[X_A] = (a + b + c)/3 = (0+350+150)/3 = \$166.67$

Variance = $(a^2 + b^2 + c^2 - ab - ac - bc)/18$
$= (0^2+350^2+150^2-(0)(350)-(0)(150)-(350)(150))/18$
$= (0 + 122500 + 22500 - 0 - 0 - 52500)/18$
$= (92500)/18 = 5138.89$ dollars^2

Standard deviation = $\sqrt{Variance} = \sqrt{5138.89} = \$71.69$

Company B:

Density triangle height:
$p(c) = 2/(b - a) = 2/(500 - 0) = 0.004$

Density [$p(x)$]	=	$[2(x\text{-}0)]/[(500\text{-}0)/(75\text{-}0)]$	If $0 \le x \le 75$
	=	$[2(500\text{-}x)]/[(500\text{-}0)(500\text{-}75)]$	If $75 < x \le 500$
	=	0	otherwise

Distribution [$P(x)$]	= 0	If $x < 0$
	$= [(x\text{-}0)^2]/[(500\text{-}0)(75\text{-}0)]$	If $0 \le x \le 75$
	$= 1 - [(500\text{-}x)^2]/[(500\text{-}0)(500\text{-}75)]$	If $75 < x \le 500$
	= 1	If $x > 500$

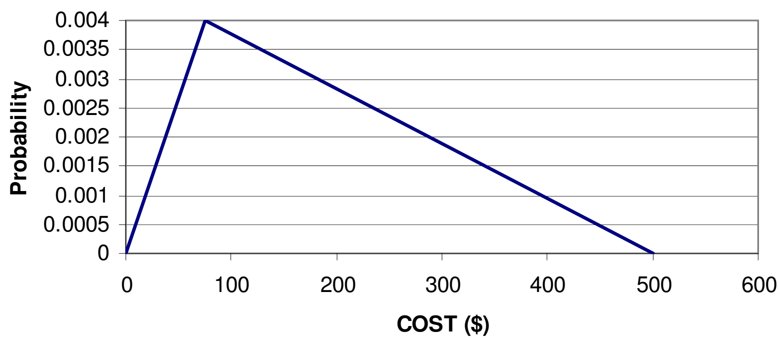

Figure III.1.14. Company B Overrun PDF

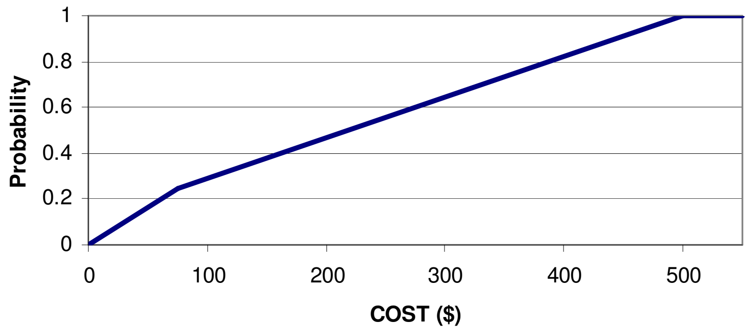

Figure III.1.15. Company B Overrun Cumulative Distribution

Mean = $E[X_B] = (a + b + c)/3 = (0+500+75)/3 = \$191.67$

Variance = $(a^2 + b^2 + c^2 - ab - ac - bc)/18$
$= (0^2 + 500^2 + 75^2 - (0)(500) - (0)(75) - (500)(75))/18$
$= (0 + 250000 + 5625 - 0 - 0 - 37500)/18$
$= (218125)/18 = 12118.06$ dollars^2

Standard deviation = $\sqrt{Variance} = \sqrt{12118.06} = \$110.08$

ANALYSIS

Using triangular distributions, the DoD prefers a cost overrun expected value of \$108.33, a variance of 2743.05 dollars^2, and a standard deviation of \$52.3741. Company A has a software development cost overrun expected value of \$166.66, a variance of 5138.88 dollars^2, and a standard deviation of \$71.6860. Company B has a software development cost overrun expected value of \$191.67, a variance of 12118.05 dollars^2, and a standard deviation of \$110.08. Neither of the two manufacturers is within the cost overrun desired by the DoD. Using triangular distribution, Company A is closer to the desired cost overrun expectation, but still requires a \$58,330 decrease to meet the DoD expectation mean value.

Summary of results from Part A (the *fractile method*) and Part B (the *triangular method*) is shown in the following table.

Table III.1.3. Comparison of Expected Overrun Values from Fractile and Triangular Distributions

Cost Overrun Expectations ($)

DoD		Company A		Company B	
Fractile	Triangular	Fractile	Triangular	Fractile	Triangular
\$87.5	\$108.33	\$156.25	\$166.66	\$118.75	\$191.66
Difference	23.81%	Difference	6.66%	Difference	61.40%

Comparing the two methods using this scenario, the triangular distribution gave higher expected values for all three organizations. Two of the three fractile method expected values are within one standard deviation of the triangular distribution expected values; the highest percent change is Company B at 61.40%. The expected value with the closest correlation using the two methods is Company A with a 6.66% difference. Due to the data point spread in this particular scenario, we can have more confidence in the results from the fractile method, since more data points are considered. Hence, from the fractile distribution, Company B is the logical choice.

The bottom line for both manufacturers is that neither one has a cost overrun estimation for their software development proposal that meets DoD's desired expectations, regardless of which method is used for data comparison.

PROBLEM III.2: Selection of a Car Service

A businessman who travels often must decide which airport car service to hire considering his preferences and risk aversion towards being late.

DESCRIPTION

Bill, a business traveler, is attempting to determine which car service he will use for his frequent trips to the airport. He elicits information from three different rental companies about price, on-time arrival information, and delay statistics. Bill figures that in addition to the cost of the car service, every minute he is late costs him $5 in the first 10 minutes. For every minute he is late after ten minutes, it costs him $20 a minute in stress, increased odds of missed flights, possible missed customer time, etc.

PART A: METHODOLOGY

Use the *fractile method* to derive the lowest expected cost for the car service.

SOLUTION

Car Service A – Cost to Airport, $40.00

Best-case arrival time = 0 minutes late (on time)
Worst-case arrival time = 30 minutes late
Median value arrival time = 7 minutes late
There is a 50-50 chance that the arrival time is ± 5 minutes of median arrival time.

Car Service B – Cost to Airport, $45.00

Best-case arrival time = 0 minutes late (on time)
Worst-case arrival time = 15 minutes late
Median value arrival time = 10 minutes late
There is a 50-50 chance that the arrival time is ± 2 minutes of median arrival time

Car Service C – Cost to Airport, $30.00

Best-case arrival time = 0 minutes late (on time)
Worst-case arrival time = 60 minutes late
Median value arrival time = 5 minutes late
25% chance arrival time = 2 minutes late
75% chance arrival time = 20 minutes late

Table III.2.1. Comparative Cumulative Distribution Functions (CDFs)

	Time Delay (minutes)		
Fractile	**Car Service A**	**Car Service B**	**Car Service C**
0.0	0	0	0
0.25	2	8	2
0.50	7	10	5
0.75	12	12	20
1.00	30	15	60

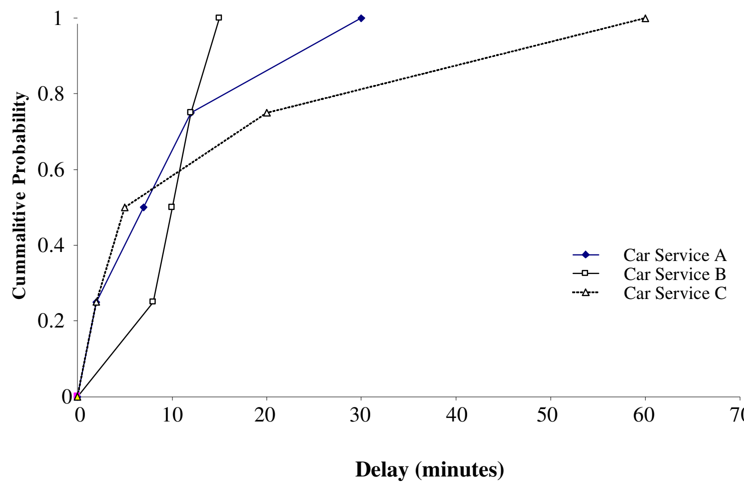

Figure III.2.1. Graph of Car Service Delay

Generate the expected value of risk of time delay for each choice.

Car Service A

Expected value delay = E[X] = 0.25 * [0 + (2-0) / 2] + 0.25 * [2 + (7-2) / 2] + 0.25 * [7 + (12-7) / 2] + 0.25 * [12 + (30-12) / 2]

E[X] = 0.25 * 1 + 0.25 * 4.5 + 0.25 * 9.5 + 0.25 * 21 = 9 min.

Total cost = cab cost + expected delay *(cost/delay)
Total cost = $40 + 9 min * ($5/min) = $85

Car Service B

Expected value delay = E[X] = 0.25 * [0 + (8-0) / 2] + 0.25 * [8 + (10-8) / 2] + 0.25 * [10 + (12-10) / 2] + 0.25 * [12 + (15-12) / 2]

E[X] = 0.25 * 4 + 0.25 * 9 + 0.25 * 11 + 0.25 * 13.5 = 9.375 min.

Total cost = cab cost + expected delay * (cost/delay)

Total cost = \$45 + 9.375 min * (\$5/min) = \$91.88

Car Service C

Expected value delay = E[X] = 0.25 * [0 + (2-0) / 2] + 0.25 * [2 + (5-2) / 2] + 0.25 * [5 + (20-5) / 2] + 0.25 * [20 + (60-20) / 2]

E[X] = 0.25 * 1 + 0.25 * 3.5 + 0.25 * 12.5 + 0.25 * 40 = 14.25 min.

Total cost = cab cost + expected delay * (cost/delay)
Total cost = \$30 + 10 min * (\$5/min) + 4.25 min * (\$20/min) = \$165

The cost vs. the expected value of risk is plotted in Figure III.2.2.

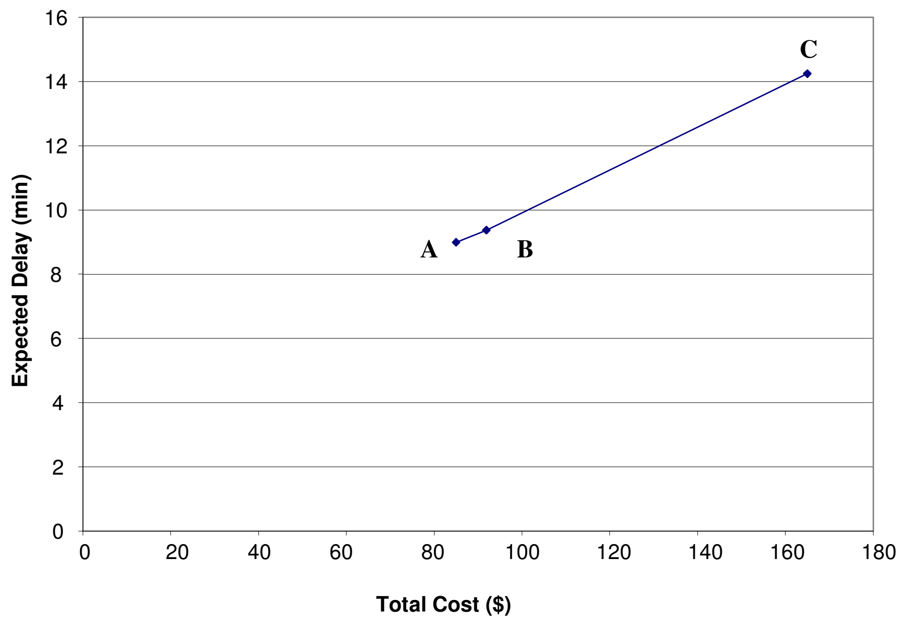

Figure III.2.2. Cost vs. the Expected Value of Risk, Fractile Method

ANALYSIS

Based on the results of the fractile method, *Car Service A* has the lowest total expected cost at \$85, even though it is the second most expensive car service. This is because it has the lowest expected delay at 9 minutes. *Car Service B* is close at a total cost of \$91.88. It is slightly more expensive due to a larger initial price and a slightly greater expected delay of 9.375 minutes. *Car Service C,* with a total expected cost of \$165, is considerably more expensive although it has the lowest price; this is due to the high expected delay.

PART B: METHODOLOGY

For the same problem, use the *triangular distribution* for the construction of the probabilities. Follow the same stages as in Part A.

SOLUTION

Car Service A – Cost to Airport, $40.00
Best-case arrival time (a) = 0 minutes late (on time)
Most-likely arrival time (c) = 7 minutes late
Worst-case arrival time (b) = 30 minutes late

Car Service B – Cost to Airport, $45.00
Best-case arrival time (a) = 0 minutes late (on time)
Most-likely arrival time (c) = 10 minutes late
Worst-case arrival time (b) = 15 minutes late

Car Service C – Cost to Airport, $30.00
Best-case arrival time (a) = 0 minutes late (on time)
Most-likely arrival time (c) = 5 minutes late
Worst-case arrival time (b) = 60 minutes late

Use the triangular distribution as follows:

Car Service A
Expected value delay = E[X] = (a + b + c) / 3

E[X] = (0 + 7 + 30) / 3 = 12.333 min

Total cost = cab cost + expected delay * cost/delay
Total cost = $40 + 10 min * ($5/min) + 2.333 min * ($20/min) = $136.67

Car Service B
Expected value delay = E[X] = (a + b + c) / 3

E[X] = (0 + 10 + 15) / 3 = 8.333 min

Total cost = cab cost + expected delay * cost/delay
Total cost = $45 + 8.333 min * ($5/min) = $86.67

Car Service C
Expected value delay = E[X] = (a + b + c) / 3

E[X] = (0 + 5 + 60) / 3 = 21.667 min

Total cost = cab cost + expected delay * cost/delay
Total cost = $30 + 10 min * ($5/min) + 11.66 min * ($20/min) = $313.33

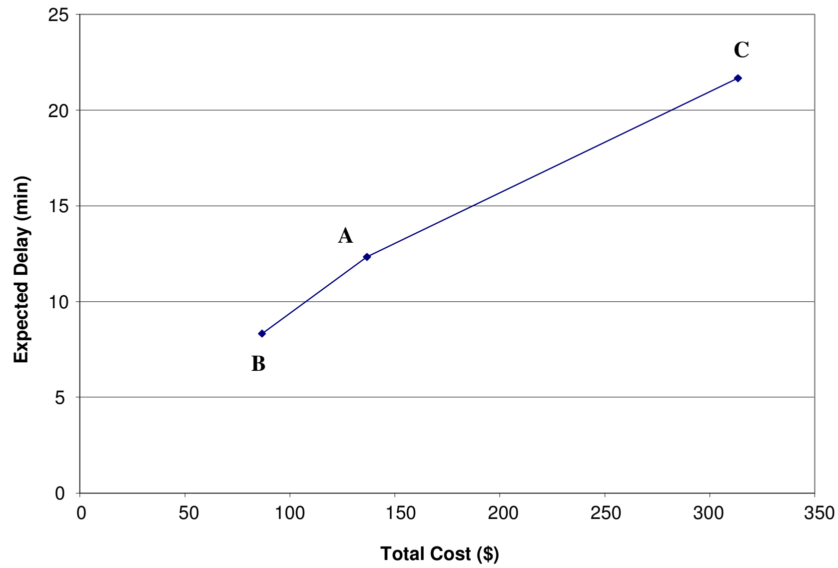

Figure III.2.3. Cost vs. Expected Value of Risk, Triangular Distribution Method

ANALYSIS

Based on the results of triangular distribution, *Car Service B* has the lowest total expected cost at $86.67, even though it is the most expensive option. This is because it has the lowest expected delay at 8.33 minutes. *Car Service A* is second with a total cost of $136.67, which is more expensive due to the larger expected delay of 12.33 min. *Car Service C,* with a total expected cost of $313.33, is considerably more expensive although it has the lowest price; this is due to the high expected delay of 21.67 minutes.

The solutions obtained using Fractile (Part A) and triangular (Part B) distributions vary because of the different methods used to analyze the car services. In both cases *Car Service C* is the least viable option. This is due to the fact that its worse-case scenario is considerably higher than for options A and B. This contributes to high expected delay times using both methods, which in turns leads to a high expected total cost. *Car Service A* has the lowest expected cost using the fractile method, and the second-lowest expected cost using the triangular distribution. *Car Service B* has the lowest expected cost using triangular distribution and the second lowest using the fractile method. The key differences between the two methods is that *Car Service A* has lower delay times for the lower 75% percentile than *Car Service B,* yet *A* has a considerably higher worst-case scenario (30 min. vs. 15 min.). This results in a lower expected value when all the fractiles are taken into account, yet a higher expected value when using the triangular distribution (due to the fact that the worse-case scenario has a higher weighting).

PROBLEM III.3: Cafeteria Entrée

A cafeteria is planning to add a new entrée to its menu.

DESCRIPTION

The cafeteria menu choices are *steak, chicken, and fish,* and their estimated profits are as shown in Table III.3.1.

Table III.3.1: Estimated Profits as a Function of Entrée and Market Appetite

	MARKET APPETITE		
ENTRÉE	Hungry (Excellent)	Moderate (Good)	Full (Poor)
Steak	$4000	$3000	-$2000
Chicken	$3000	$1500	-$1000
Fish	$3000	$750	$500

PART A: METHODOLOGY

Using the Hurwitz rule for decision analysis, solve the problem and analyze the result.

SOLUTION

Based on the estimated profits in Table III.3.1, the *payoff matrix* is shown in Table III.3.2.

Table III.3.2. Payoff Matrix in $Thousands

	$j=1$ (s_1)	$j=2$ (s_2)	$j=3$ (s_3)
$i=1$ (a_1)	4	3	-2
$i=2$ (a_2)	3	1.5	-1
$i=3$ (a_3)	3	.75	.5

The *opportunity loss matrix* in Table III.3.3 represents the potential profits we lose out on if we choose a particular entrée. For example: if we choose chicken and the market is hungry, the chart represents the potential profits lost.

Table III.3.3. Opportunity Loss Matrix

	MARKET APPETITE		
ENTRÉE	Hungry (Excellent)	Moderate (Good)	Full (Poor)
Steak	0	0	2.5
Chicken	1	1.5	1.5
Fish	1	2.25	0

1) *Pessimistic case:* Minimizing our losses (we want to lose the least):

For a_1 : min (4, 3, -2) = -2
For a_2 : min (3, 1.5, -1) = -1
For a_3 : min (3, .75, .5) = .5

We can take the maximum of the a values to minimize our losses:

Max (a_1, a_2, a_3) $\Rightarrow$ Max (-2, -1, .5) = .5 $\Rightarrow a_3$

2) *Optimistic case:* Maximizing potential profit:

For a_1 : max (4, 3, -2) = 4
For a_2 : max (3, 1.5, -1) = 3
For a_3 : max (3, .75, .5) = 3

We can take the maximum of the a values to maximize our potential profit:

Max (a_1, a_2, a_3) $\Rightarrow$ Max (4, 3, 3) = 4 $\Rightarrow a_1$

3) *Apply the Hurwitz rule:* We want to find a compromise between the pessimistic and optimistic rules used in (1) and (2) using the index α :

$$\underset{\substack{1 \le i \le 3 \\ (a_1, a_2, a_3)}}{\text{Max}} \{ \mu_i(\alpha) = \alpha \underset{1 \le j \le 3}{\min} \mu_{ij} + (1-\alpha) \underset{1 \le j \le 3}{\max} \mu_{ij} \},\ 0 \le \alpha \le 1$$

For α = 1; Pessimistic & α = 0; Optimistic

$$\begin{array}{lcl} \text{At } a_1: \ \mu_1(\alpha) = -2\alpha + 4(1-\alpha) & & a_1: \ \mu_1(\alpha) = 4 - 6\alpha \\ \text{At } a_2: \ \mu_2(\alpha) = -1\alpha + 3(1-\alpha) & \Rightarrow & a_2: \ \mu_2(\alpha) = 3 - 4\alpha \\ \text{At } a_3: \ \mu_3(\alpha) = .5\alpha + 3(1-\alpha) & & a_3: \ \mu_3(\alpha) = 3 - 2.5\alpha \end{array}$$

4) The *graph* below is the result of plotting these equations as a function of alpha.

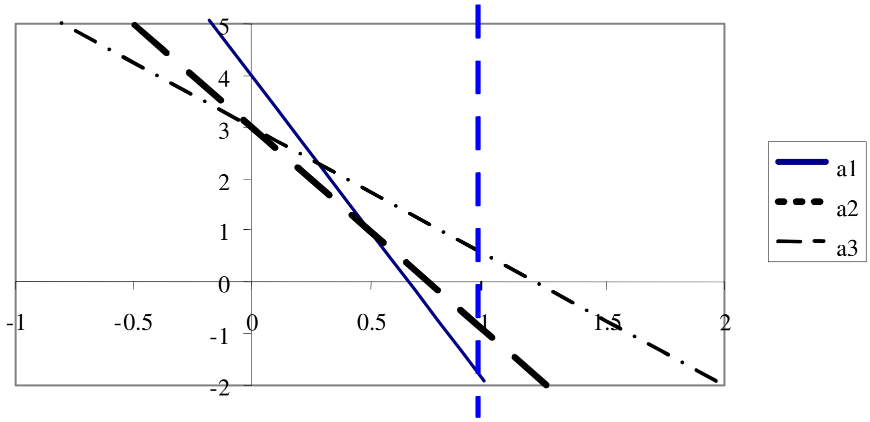

Figure III.3.1. Hurwitz Rule for Cafeteria Entrée Selection

$0 \le \alpha \le 1$; therefore the x-axis is the lower bound of 0 and the dashed blue line is the upper bound of 1.

ANALYSIS

Since $a \geq 0$; and a_3 dominates a_2 above 0, we can rule out chicken as an entrée. In order to have the potential for the best profits, management should never choose chicken (a_2). For $0 \le \alpha \le \frac{2}{7}$, the best entrée is steak (a_1); and for $\frac{2}{7} \le \alpha \le 1$, the best entrée is fish (a_3)

PART B: METHODOLOGY

Modify the above problem by adding your knowledge of the probabilities of payoff. To decide which entrée to offer, use the *triangular method to create a decision tree* to minimize the opportunity loss. Analyze your results.

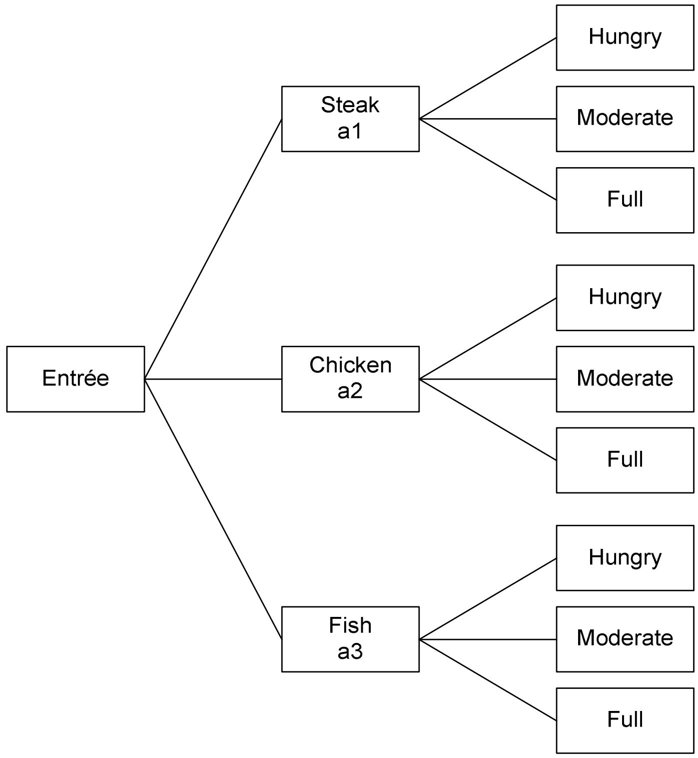

Figure III.3.2. Decision Tree for Cafeteria Entrée Choices

SOLUTION

Referring to Table III.3.1 (Market Appetite), we assign the following probabilities:

Hungry (Excellent) = .5;
Moderate (Good) = .4;
Full (Poor) = .1

Multiplying the probabilities with the figures from Table III.3.1, we get:

Steak:

.5*$4,000+
.4*$3,000+
.1*(-$2,000) = $3,000

Chicken:

.5*$3,000+
.4*$1,500+
.1*(-$1,000) = $2,000

Fish:

.5*$3,000+
.4*$750+
.1*$500 = $1,850

ANALYSIS

Using these probabilities, we would choose the steak because it will provide the highest expected profits. The Hurwitz Rule approach provides the cafeteria a flexible entrée selection by varying the level of optimism as shown in Figure III.3.1.

PROBLEM III.4: Replacing Seat Belts on School Buses

Seat belts are wearing out on a county's school buses.

DESCRIPTION

The School Board wants to find out the tradeoffs between potentially preventing students' injuries and the cost of replacing seatbelts on all school buses, some buses, or not replacing any at all.

PART A: METHODOLOGY

Solve the problem and analyze the results using the *fractile method.*

The values in Table III.4.1 represent the percent of possible student injuries on school buses under the three policies being considered.

Table III.4.1. Rate of Potential School Bus Injuries

Fractile	*Policy 1:* Replace Seatbelts on All Buses	*Policy 2:* Replace Seatbelts on Some Buses	*Policy 3:* Do Not Replace Any Seatbelts
0.00	0%	0%	0%
0.25	5%	10%	20%
0.50	10%	15%	40%
0.75	15%	25%	60%
1.00	20%	30%	80%

SOLUTION

The cumulative distribution functions (CDF's) of the possible injuries under each policy are represented in the following graphs.

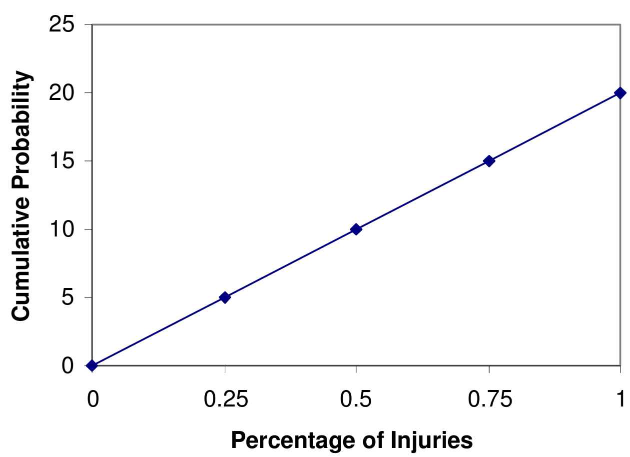

Figure III.4.1. Policy 1 CDF: All Buses have Seat Belts

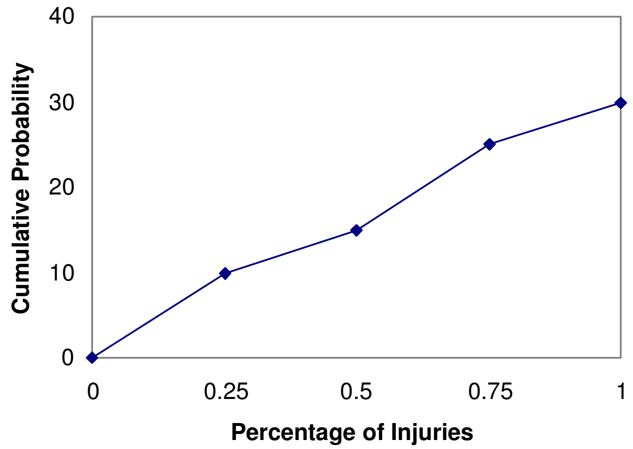

Figure III.4.2. Policy 2 CDF: Some Buses have Seat Belts

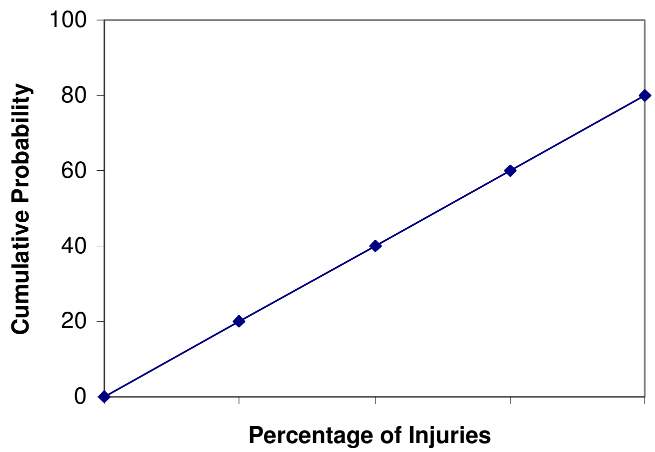

Figure III.4.3. Policy 3 CDF: No Buses have Seat Belts

The following graphs show the probability density functions (PDF's)

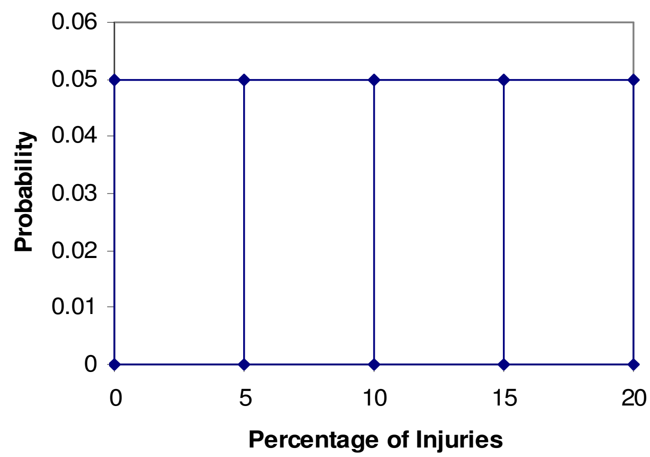

Figure III.4.4. Policy 1 PDF: All Buses have Seat Belts

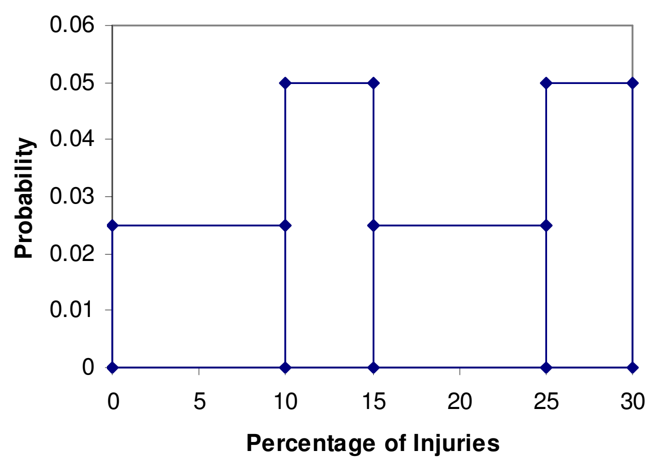

Figure III.4.5. Policy 2 PDF: Some Buses have Seat Belts

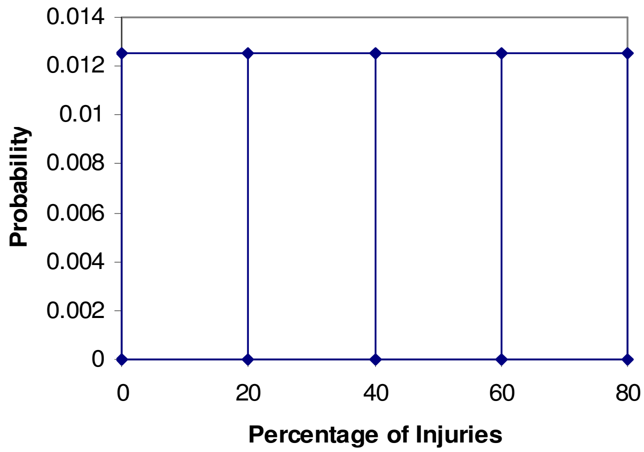

Figure III.4.6. Policy 3 PDF: No Buses have Seat Belts

***Policy 1*: ** $E[X] = \sum_{i=1}^{4} p_i x_i =$

$$.25\left[0 + \frac{5-0}{2}\right] + .25\left[5 + \frac{10-5}{2}\right] + .25\left[10 + \frac{15-10}{2}\right] + .25\left[15 + \frac{20-15}{2}\right]$$

= 0.625 + 1.875 + 3.125 + 4.375
= 10

***Policy 2*: ** $E[X] = \sum_{i=1}^{4} p_i x_i =$

$$.25\left[0 + \frac{10-0}{2}\right] + .25\left[10 + \frac{15-10}{2}\right] + .25\left[15 + \frac{25-15}{2}\right] + .25\left[25 + \frac{30-25}{2}\right]$$

= 1.25 + 3.125 + 5 + 6.875
= 16.25

***Policy 3*: ** $E[X] = \sum_{i=1}^{4} p_i x_i =$

$$.25\left[0 + \frac{20-0}{2}\right] + .25\left[20 + \frac{40-20}{2}\right] + .25\left[40 + \frac{60-40}{2}\right] + .25\left[60 + \frac{80-60}{2}\right]$$

= 2.5 + 7.5 + 12.5 + 17.5
= 40

Next, we assign costs to each policy, then graph the costs vs. the expected value of risk.

Policy 1 = \$1 M
Policy 2 = \$750 K
Policy 3 = \$250 K

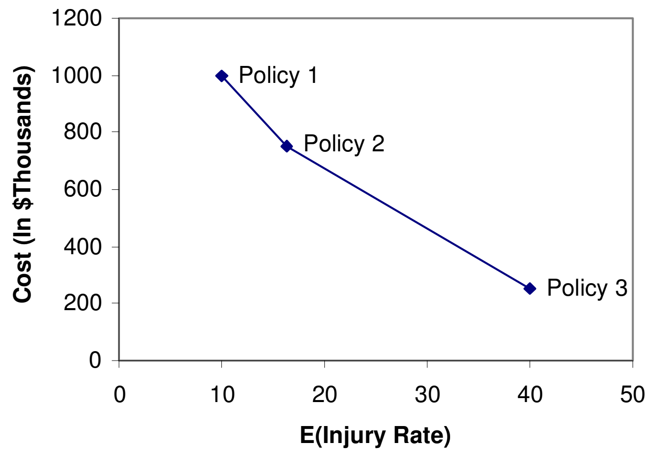

Figure III.4.7. Expected Injury Rate vs. Cost

ANALYSIS

In this case, the number of injuries drops dramatically as spending increases. Spending \$750,000 on Policy 2 will save more lives per dollar than spending the \$1M it would take to achieve an expected value of risk of 10% injuries with Policy 1.

PART B: METHODOLOGY

For the same problem, use the *triangular distribution* to construct the probabilities. Then compare the results to those in Part A.

SOLUTION

In this solution, most of the values are based on the results we obtained from the fractile method.

$$\text{Mean} = E[X] = \frac{a+b+c}{3}$$

where a = minimum, c = mode, and b = maximum parameters of a triangular distribution.

Solving for c:

$$3E[X] = a + b + c$$
$$c = 3E[X] - a - b$$

Now we must find the value of c for each policy:

Policy 1: $c_1 = 3 * 10 - 0 - 20 = 10$
Policy 2: $c_2 = 3 * 16.25 - 0 - 30 = 18.75$
Policy 3: $c_1 = 3 * 40 - 0 - 80 = 40$

Using the values we found for c we can find the height of each triangle $p(c)$,
Using $p(c) = \dfrac{2}{b - a}$

Policy 1: $p(c_1) = \dfrac{2}{20 - 0} = \dfrac{1}{10}$

Policy 2: $p(c_2) = \dfrac{2}{30 - 0} = \dfrac{1}{15}$

Policy 3: $p(c_1) = \dfrac{2}{80 - 0} = \dfrac{1}{40}$

The following graphs summarize the distributions above.

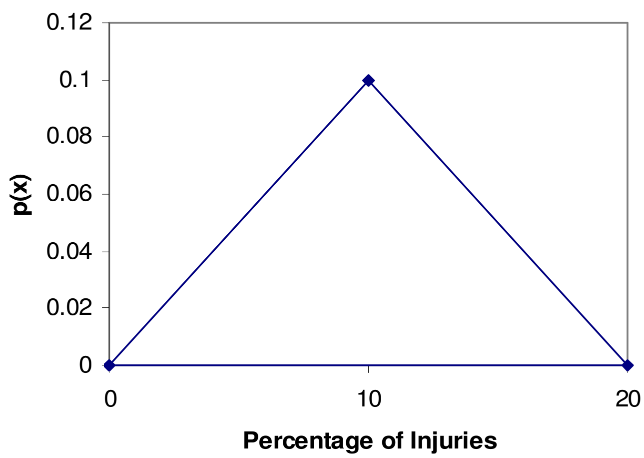

Figure III.4.8. Policy 1 Triangular Distribution

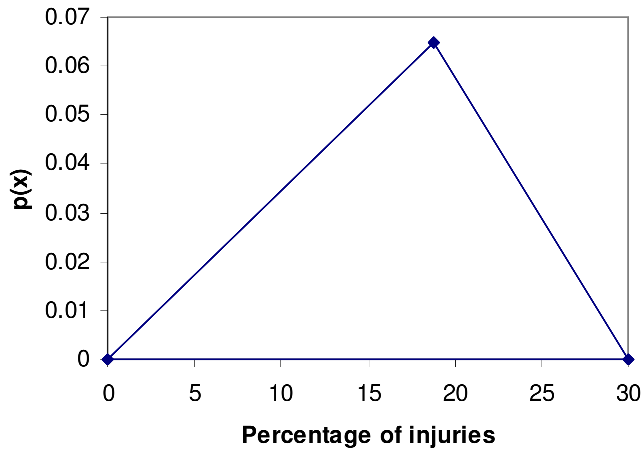

Figure III.4.9. Policy 2 Triangular Distribution

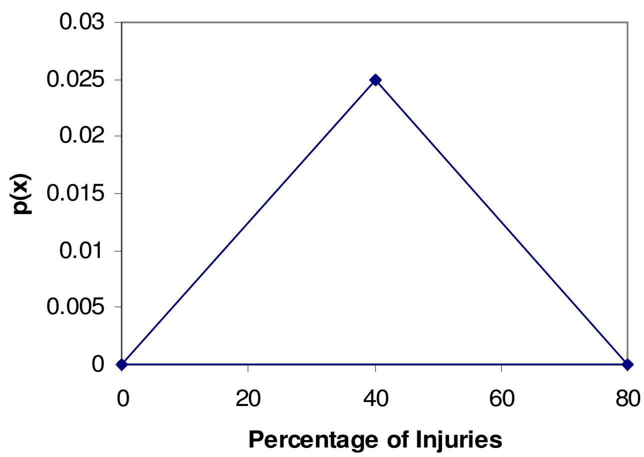

Figure III.4.10. Policy 3 Triangular Distribution

ANALYSIS

Recall that the expected value of a triangular distribution is as follows:

$$E[X] = \frac{a+b+c}{3}$$

Calculating the expected value of the three policies will give the following values (in %):

For Policy 1: $E[X] = \dfrac{0 + 20 + 10}{3} = 10$

For Policy 2: $E[X] = \dfrac{0 + 30 + 18.75}{3} = 16.25$

For Policy 3: $E[X] = \dfrac{0 + 80 + 40}{3} = 40$

Assuming that the most effective policy is based on the lowest value of the expected value of percentage on injuries, then Policy 1 (all buses have seatbelts) is recommended.

PROBLEM III.5: Testing Aircraft Parts before Installation
The objective of this problem is to analyze how to reduce the probability of a faulty part of an aircraft engine and to minimize costs associated with testing and repair.

DESCRIPTION
A part of an aircraft engine can be given a test before installation. The test has only a 75% chance of either revealing or passing a possibly defective part. Whether or not the part has been tested, it may undergo an expensive reworking which is certain to produce a part free from defects. If a defective part is installed in the engine, the property loss is \$1,000,000. If the reworking is done, the cost is \$200,000. Initially, one out of every eight of the parts is defective. Calculate how much you should pay for the test and determine all the optimum decisions in order to minimize the expected property loss (including cost).

METHODOLOGY
Use decision tree analysis to solve the aircraft problem using the following specifications.

States	**Actions**	**Priors**
θ_1 = Defective	A_1 = Install	$P(\text{Defective}) = 1/8 = 0.125$
θ_2 = Not Defective	A_2 = Rework	$P(\text{Not Defective}) = 7/8 = 0.875$

Test Results	**Conditional Probabilities**
X_1 = Reveals Part as Defective	$P(X_1 \mid \theta_1) = .75$ $P(X_2 \mid \theta_1) = .25$
X_2 = Reveals Part as Not Defective	$P(X_1 \mid \theta_2) = .25$ $P(X_2 \mid \theta_2) = .75$

SOLUTION

Find the marginal and posterior probabilities:

Marginal probabilities:

$$P(X_1) = P(X_1 \mid \theta_1)P(\theta_1) + P(X_1 \mid \theta_2)P(\theta_2)$$
$$= (0.75)(0.125) + (0.25)(0.875) = 0.3125$$
$$P(X_2) = (0.25)(0.125) + (0.75)(0.875) = 0.6875$$

Posterior probabilities (by Bayesian Theorem):

$$P(\theta_i \mid X_j) = P(X_j \mid \theta_i)P(\theta_i)/P(X_j)$$
$$P(\theta_1 \mid X_1) = (0.75)(0.125)/(0.3125) = 0.3$$
$$P(\theta_2 \mid X_1) = (0.25)(0.875)/(0.3125) = 0.7$$
$$P(\theta_1 \mid X_2) = (0.25)(0.125)/(0.6875) = 0.04545$$
$$P(\theta_2 \mid X_2) = (0.75)(0.875)/(0.6875) = 0.9545$$

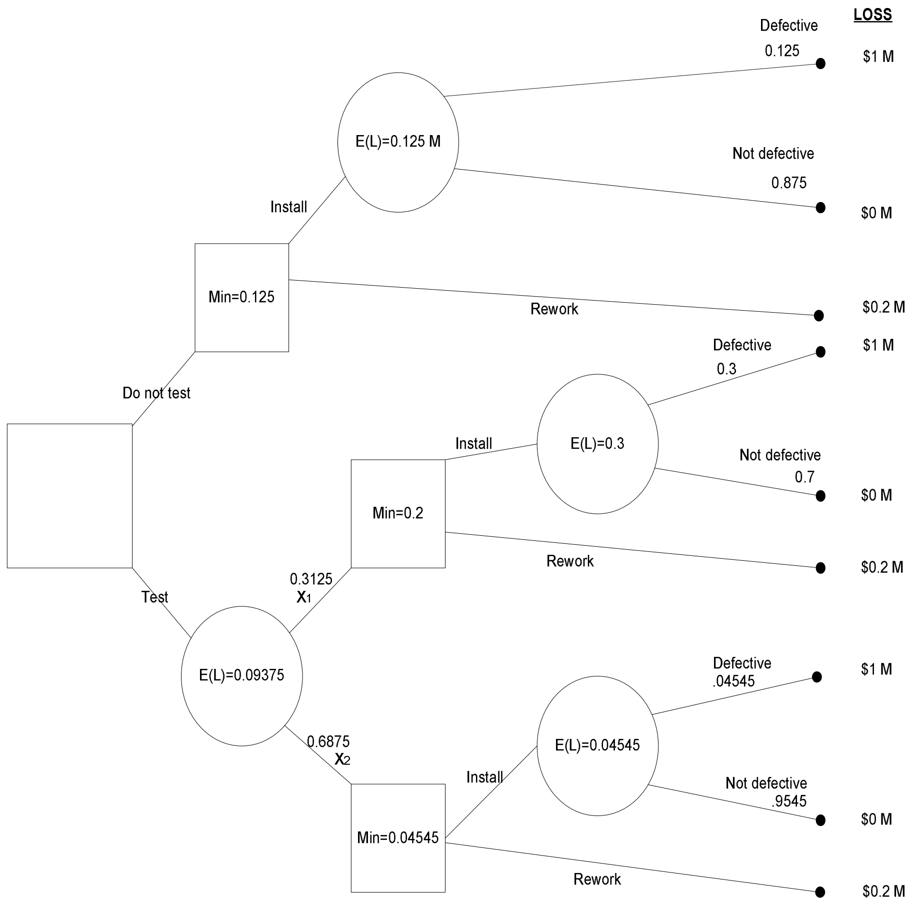

Figure III.5.1. Decision Tree

ANALYSIS

Based on the decision tree in Figure III.5.1:

- The expected loss without testing amounts to $0.125 million.
- The expected loss with testing amounts to $0.09375 million.
- Therefore, the cost of the test to break even would be $(0.125 - 0.09375) * 10^6 = \$31{,}250$.
- If the test reveals a part as defective $\rightarrow$ Rework part.
- If the test reveals part as not defective $\rightarrow$ Install part.
- If no test is performed $\rightarrow$ Install part.

PROBLEM III.6: Magic Beanstalk

Jack, a savvy businessman, climbs up a magic beanstalk and finds himself in a giant's kingdom. He sees two huge bags of gold coins, a gold-feathered duck that lays golden eggs, and a gold harp that plays really sweet music. Then a friendly giant appears and says that Jack may take one item—any one he chooses. Jack realizes that he can sell any of those items on the market and make a fortune. As he can take only one item, which should it be?

DESCRIPTION

Without knowing the prevailing market conditions, should Jack choose the gold coins (A), gold duck (B), or the gold harp (C)? The market for these items fluctuates between excellent, good, and poor, affecting the price he may receive for each of them.

Table III.6.1. Profit as a Function of Market Condition and Item Choice

	Sales Potential ($)		
Item	Excellent	Good	Poor
a1	900,000	600,000	150,000
a2	850,000	700,000	50,000
a3	400,000	300,000	200,000

PART A: METHODOLOGY

If Jack had the time, he could first solve this problem using decision analysis (DA) and the Hurwitz Rule.

SOLUTION

Table III.6.2. Payoff Matrix ($1000s)

	j=1	j=2	j=3
	s_1	s_2	s_3
i=1(a_1)	900	600	150
i=2(a_2)	850	700	50
i=3(a_3)	400	300	200

Table III.6.3. Opportunity Loss Matrix w/ Pessimistic and Optimistic Rules

	M_1-mu_{11} j=1	M_2-mu_{22} j=2	M_3-mu_{33} J=3	Pessimistic	Optimistic
i=1(a_1)	0	100	50	100	0
i=2(a_2)	50	0	150	150	0
i=3(a_3)	500	400	0	500	0

Summarizing the pessimistic and optimistic outcomes for each decision:

For alpha = 1: pessimistic
For alpha = 0: optimistic

mu1(alpha) = 100,000 * alpha + 0 * (1 - alpha) = 100,000 * alpha
mu2(alpha) = 150,000 * alpha + 0 * (1 - alpha) = 150,000 * alpha
mu3(alpha) = 500,000 * alpha + 0 * (1 - alpha) = 500,000 * alpha

Table III.6.4. Applying the Hurwitz Rule

alpha	mu1	mu2	mu3
0	0	0	0
0.1	10000	15000	50000
0.2	20000	30000	100000
0.3	30000	45000	150000
0.4	40000	60000	200000
0.5	50000	75000	250000
0.6	60000	90000	300000
0.7	70000	105000	350000
0.8	80000	120000	400000
0.9	90000	135000	450000
1	100000	150000	500000

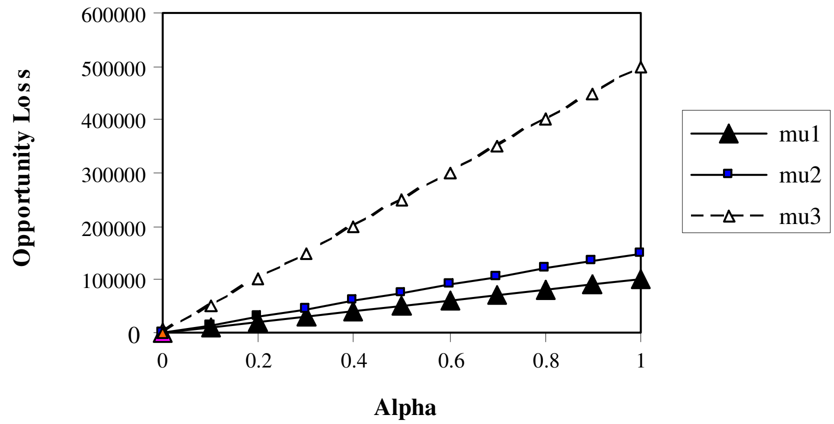

Figure III.6.1. Hurwitz Rule Results

ANALYSIS

The line denoting mu1 in Figure III.6.1 always dominates the other options. Hurwitz's rule tells us that Option a1 (gold coins) should be chosen. This case indicates that all optimistic views would choose not to have any opportunity loss, resulting in an amount of 0. Jack picks the gold coins due to their $900,000 value, the best option. Because there are no probabilities, Hurwitz tells us that if one option has a high maximum above others, then the opportunity loss can lead to a biased decision.

PART B: METHODOLOGY

Jack can also apply the Decision Tree method to solve the problem.

SOLUTION

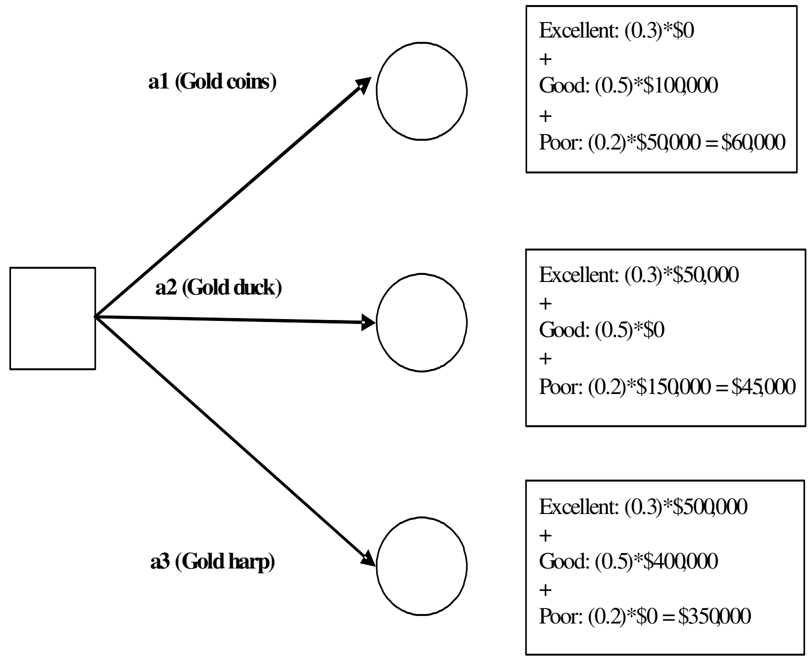

Figure III.6.2. Decision Tree with EOL measure

ANALYSIS

The solution obtained using the decision tree methodology is consistent with the results from the Hurwitz Rule methodology. The tree shows the golden duck as the choice with the lowest expected value for opportunity loss. One reason is that in the gold market, the duck has an opportunity loss of 0 (which has the highest probability). The expected values of opportunity loss for the duck and the gold coins are fairly close and may vary with different probabilities of market conditions, while the gold harp has the highest opportunity loss by far.

PROBLEM III.7: Issuing a Credit Card

A credit card ("bankcard") company must decide whether to extend a line of credit to an individual who has applied for one. This is a common situation, and one in which the decision made by the institution has ramifications on its future ability to make similar decisions.

This is a simple decision process under uncertainty. The primary uncertainty arises from the behavior of the individual once the line of credit has been issued. We consider three possible behavior patterns:

- *"Good" Behavior:* This is the situation where the individual granted the line of credit pays at least the minimum amount due for the billing cycle every time. The credit card company makes money on the interest of the balances carried between cycles, but the customer adheres to the credit agreement.
- *"Late" Behavior:* An individual who is granted credit misses some percentage of payments, but still pays with sufficient regularity for the credit card company to leave the line of credit open. We assume that these individuals will ultimately fulfill their credit obligation and not default on the debt. This is the most profitable (therefore most desirable) situation for the company because money is made on interest (on balances carried from period to period) and from late fees.
- *Charge-Off:* An individual is granted a line of credit, but defaults on the debt. The credit card company either pursues the individual through some type of internal asset recovery division or sells the debt to another company. This situation is expensive for the company and little money is made; there is also the potential for loss.

This is a single-stage decision under uncertainty. The problem can be solved by formulating a decision tree incorporating the following descriptions:

The action under consideration, **a**, is whether to grant a line of credit to a certain individual:

$\mathbf{a} \in \{a_1, a_2\}$, where:

a_1 = grant application for line of credit
a_2 = reject application for line of credit

We define the random variable **b** to describe the customer behavior:

$\mathbf{b} \in \{b_1, b_2, b_3\}$, where:

b_1 = "good" customer behavior
b_2 = "late" customer behavior
b_3 = charge-off behavior

On the other hand, only one outcome can stem from the action a_2 (reject application for line of credit), denoted by b_4 (no customer).

The probability of the outcome of b_i is given by p_i, which is assumed to be constant and independent of the action **a**.

Build the corresponding decision tree. In addition, complete your analysis by supplementing reasonable and relevant decision rules on the decision tree you build.

PROBLEM III.8: Stocking a Specialty Ice Cream Parlor

A popular ice cream parlor in an upscale neighborhood wants to offer a new flavor to its patrons. Which one of three types of flavors should it add to its stock?

The parlor has the following payoff matrix for different flavors of ice cream.

Table III.8.1. Payoff Matrix

		Daily Profits (Dollars)		
		Good	Most Likely	Poor
	a_1	200	175	-25
Flavor	a_2	150	125	-50
	a_3	125	100	25

Use the Hurwitz rule to decide how to minimize the opportunity loss by following this procedure: (1) Based on the payoff matrix, create the opportunity loss matrix, (2) Applying the pessimistic rule, minimize the maximum loss, (3) Applying the optimistic rule, minimize the minimum loss, (4) Apply the Hurwitz rule, which compromises between two extremes through the use of the index α. (5) Show your results graphically, and (6) Analyze your results.

In addition, construct a decision tree to the ice cream selection problem. For your analysis, assume that the probabilities of three states are assigned as follows:

- Pr(Good) = 0.3
- Pr(Most Likely) = 0.5
- Pr(Poor) = 0.2

PROBLEM III.9: Choosing Quality of Wine to Produce

A Vineyard is trying to determine which quality of wine it should produce given it has 100 acres of spare land that can be used for planting.

After soil tests come in, a vineyard's owner realizes that 100 more acres than expected are ready for planting. The vineyard's owner needs to decide which of his three wines it would like to produce from the extra acreage—high, medium, or low quality wine. Depending on the weather during the upcoming year, the revenue the three wines will bring in is displayed in Table III.9.1.

Table III.9.1. Revenues of different types of wine given weather conditions

		Weather	
Quality of Wine	**Favorable**	**Fair**	**Poor**
High	$500,000	$250,000	-$200,000
Medium	$300,000	$200,000	-$10,000
Low	$100,000	$80,000	$25,000

Use the Hurwitz rule to decide how to minimize the opportunity loss by following this procedure: (1) Based on the payoff matrix, create the opportunity loss matrix, (2) Applying the pessimistic rule, minimize the maximum loss, (3) Applying the optimistic rule, minimize the minimum loss, (4) Apply the Hurwitz rule, which determines the sensitivity of wine qualities through the use of the index α. (5) Show your results graphically, and (6) Analyze your results.

Use a decision tree to derive the Expected Opportunity Loss based on the weather. Assume that probabilities of three states are assigned as follows:

- Pr(Favorable) = 0.3
- Pr(Fair) = 0.55
- Pr(Poor) = 0.15

PROBLEM III.10: Expanding Aircraft Fleet

An airline is interested in expanding its fleet of aircrafts and must decide what type of aircraft to purchase as an addition to the fleet.

An airline is interested in growth and development of its business. In order to do so, the airline decides to purchase a new aircraft to add to the airline's existing fleet. Through the use of decision analysis and several other techniques, the airline will decide in which new aircraft invest.

PART A

A regional airline is interested in expanding its current fleet of aircrafts. It would like to determine which size plane to purchase based on profits (gross ticket revenue), number of passengers per type of plane, and amount of cargo space. For simplicity, this problem uses the metric "passenger capacity" to denote both number of passengers and associated cargo space (Table III.10.1).

Table III.10.1. Profit as a Function of Ticket Sales and Plane Size

		Ticket Sales Revenue		
	Passenger Capacity	**Full**	**Half-full**	**Mostly Empty**
Plane Size	**Small**	$200,000	$50,000	($80,000)
	Medium	$300,000	$175,000	($40,000)
	Large	$150,000	$100,000	$40,000

To make optimal decisions, utilize Pessimistic, Optimistic and Hurwitz rules in your analysis.

PART B

Modify the above problem by adding your knowledge of the payoff probabilities. Create a decision tree to decide how to minimize the opportunity loss. Analyze your results.

Assume that probabilities of three states are assigned as follows:

- Pr(Full) = 0.3
- Pr(Half-full) = 0.5
- Pr(Mostly Empty) = 0.2

PROBLEM III.11: Dealing with Inefficient Machines

A manufacturing company must decide how to deal with equipment that is not producing at its full capacity.

A manufacturing company has realized that one of its machines is producing at 50% of its usual capacity. After performing some diagnostic tests, the company realizes that it has three options, as follows:

1) Do nothing and continue manufacturing
   Cost = $0
2) Fix the Machine
   Cost = $40
3) Replace the machine
   Cost = $100

Derive Probability Density Functions (PDFs), Cumulative Density Functions (CDF) using Table III.11.1 with the Fractile method. Analyze your result by adding Expected Productivity Loss.

Table III.11.1. Fractile Method % Productivity Loss

	Best **0**	**25th** **0.25**	**Median** **0.5**	**75th** **0.75**	**Worst** **1**
Do Nothing	50	60	75	85	100
Fix	20	30	40	50	60
Replace	0	10	15	20	25

In addition, do the same analysis using the triangular distribution.

Table III.11.2. Triangular Distribution

	Best	**Most Likely**	**Worst**
Do Nothing	50	75	100
Fix	20	40	60
Replace	0	15	25

PROBLEM III.12: Minimizing Opportunity Loss

A cardboard box manufacturer is trying to decide what size boxes to produce for the upcoming Christmas Season.

For this problem, the decisionmaker must estimate profits as a function of sales potential and box size, as shown in Table III.12.1.

Table III.12.1. Sales Potential (in thousands of dollars)

Box Size	Excellent	Good	Fair
Small	1500	1200	1000
Medium	1700	1400	1200
Large	1600	1500	1400

Perform decision analysis to solve the above problem by following the steps below:

- Create the Payoff Matrix.
- Based on the Payoff Matrix, create the Opportunity Loss Matrix.
- Apply the Pessimistic Rule (maximize the minimum gain).
- Apply the Optimistic Rule (maximize the maximum gain).
- Apply the Hurwitz Rule that compromises between the two extremes through the use of index α.
- Show your results graphically.
- Analyze your results.

Extend your results with Decision Tree method by assuming that probabilities of three states are as follows:

- Pr(Excellent) = 0.1
- Pr(Good) = 0.4
- Pr(Fair) = 0.5

PROBLEM III.13: Snow for a Ski Resort

The owner of the *Sliding By* Ski Slopes in the southern Pennsylvania mountains is trying to decide whether or not to rent, and perhaps later buy, snow-making equipment for the coming years. He has lived in the area and operated the ski resort for only the past four winters.

There are three rental equipment options: 1) rent none; 2) rent enough to provide snow for about 30% of the trails; and 3) rent enough to provide snow for about 60% of the trails. He has projected profits for each of these alternatives under three conditions: very little snow, average snow, and heavy snow. The data is included in the following table.

Table III.13.1. Projected Profits

Alternative Actions	**State**		
	Little or no snow	Average snow	Heavy snow
No Rental	$-200,000	$200,000	$600,000
30% trails open	$-100,000	$200,000	$500,000
60% trails open	$100,000	$200,000	$400,000

The owner has decided to use decision analysis to help him make the decision. Previously, he was leaning toward renting the minimal amount of equipment. However, the past two or three years have had decent amounts of snowfall and this owner intends to draw on recent past experience for examples to help him make up his mind. Also, being a cautious man, if he does decide to rent the equipment, he will use the results of the rental season(s) to help him decide whether or not to buy the equipment.

Analyze the decisions of the owner by using Hurwitz Rule and Decision Tree. Assume the following probability values:

- Pr(Little or no snow) = 0.25
- Pr(Average snow) = 0.5
- Pr(Heavy snow) = 0.25