VIII. Multiobjective Decision Tree

PROBLEM VIII.1: Marikina River Flooding

The Marikina River in the Philippines endangers the residents of the city of Marikina when it overflows. However, during heavy rains, it is difficult to decide whether to warn or evacuate the population.

DESCRIPTION

The Marikina River is important to the economic and social activities of the surrounding community. During heavy rains, the river is continuously monitored for its current level and overflow probability. This data is used to give warnings and evacuation orders to the residents along the banks of the river and other affected areas. The decision not to evacuate too early (without indication of possible flooding) is due to the high cost incurred in the evacuation process. However, a late evacuation entails a high cost as well in terms of the higher risk to the residents and the use of more sophisticated operations such as helicopter rescues.

METHODOLOGY

Multiobjective Decision Tree (MODT) analysis can help Marikina city officials decide when it is necessary to evacuate during the rainy season.

The following conflicting objectives have been identified:

Minimize f_1:	Effective policy implementation cost, expressed in terms of $1\text{x}10^8$ PhP
Minimize f_2:	Risk to residents and rescuers

The river's water flow during heavy rains is represented by two equally likely *a priori* distributions, given as:

LN_1 ~ Lognormal (ln 150, 1)
LN_2 ~ Lognormal (ln 80, 1)

Chance Nodes:

There are two possibilities (i.e., chances) that can occur for the initial period: *Flood* or *No Flood.*

Flood stage is reached at water flow (W) = 60,000 cfs.

For the second period, the following events can occur:

> Water flow is high (30,000 $\leq W \leq$ 60,000 cfs)
> Water flow is moderate high (20,000 $\leq W \leq$ 30,000 cfs)
> Water flow is low (5,000 $\leq W \leq$ 20,000 cfs)

Decision Nodes:

The problem can be seen as a two-period decisionmaking process. The first period decision involves issuing either an evacuation order or a warning. If that decision is to issue only a warning, a second period decision is made after city officials receive additional information on the river's water flow.

Requirement:

Constrain and solve the problem using the Multiobjective Decision Tree (MODT) method.

SOLUTION

The decision tree depicting different combinations of the decisions and chances for the two periods is given in Figure VIII.1.1. The first column on the right shows the cost of effective policy implementation (in $1\text{x}10^8$ PhP) while the second column shows the risk (normalized between 0 and 1). The values of the objectives are assessed during the end of the second period (based on historical data). These are shown in Tables VIII.1.1-VIII.1.5 and Figure VIII.1.2 below.

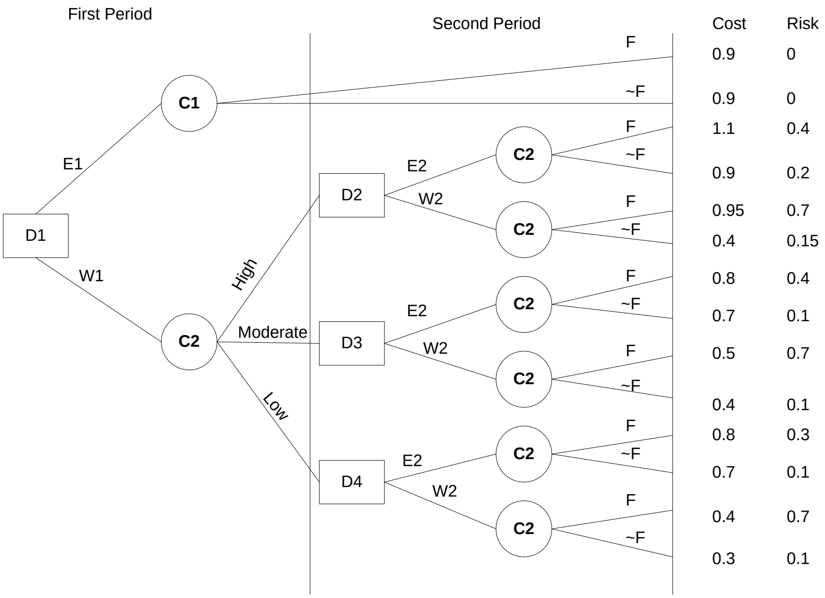

Figure VIII.1.1. Decision tree

Table VIII.1.1. First-Period Probabilities

P(Flood)	0.7167
P(No Flood)	0.2833

Table VIII.1.2. Second-Period Probabilities

P(High)	0.1747
P(Moderate)	0.0562
P(Low)	0.0508
P(Flood | High)	0.6877
P(No Flood | High)	0.3121
P(Flood | Moderate)	0.6718
P(No Flood | Moderate)	0.3282
P(Flood | Low)	0.6572
P(No Flood | Low)	0.3428

Table VIII.1.3. Expected Value of Vector of Objectives at Second Period

Decision	Arc	Chance	Cost	Risk	E[Cost]	E[Risk]
D2	E2	F | High	0.757	0.275	1.037	0.338
	E2	~F | High	0.281	0.062		
	W2	F | High	0.653	0.481	0.778	0.528
	W2	~F | High	0.125	0.047		
D3	E2	F | Mod	0.537	0.267	0.767	0.302
	E2	~F | Mod	0.230	0.033		
	W2	F | Mod	0.336	0.470	0.467	0.503
	W2	~F | Mod	0.131	0.033		
D4	E2	F | Low	0.526	0.177	0.766	0.231
	E2	~F | Low	0.240	0.034		
	W2	F | Low	0.263	0.460	0.366	0.494
	W2	~F | Low	0.103	0.034		

Table VIII.1.4. Non-Inferior Decisions for Second-Period Decision Nodes

Node	Non-Inferior Decision
D2	E2, W2
D3	E2, W2
D4	E2, W2

Table VIII.1.5. Decision for the First-Period Node

First-Period Decisions	Second-Period Decisions High	Moderate	Low	Objective Vector Cost	Risk
E1				0.645	0
				0.170	0
W1	E2	E2	E2	0.187	0.073
	E2	E2	W2	0.176	0.086
	E2	W2	E2	0.178	0.084
	E2	W2	W2	0.164	0.078
*Inferior	W2	E2	E2	0.171	0.107
	W2	E2	W2	0.158	0.122
	W2	W2	E2	0.160	0.120
	W2	W2	W2	0.146	0.134

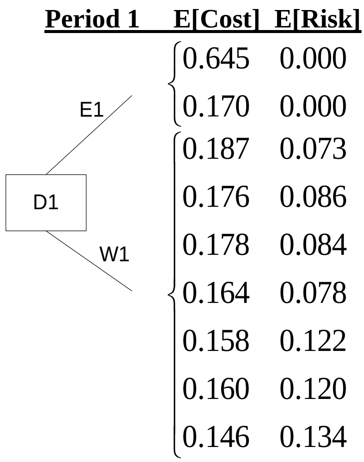

Figure VIII.1.2. Decision tree for the first stage

ANALYSIS

Plotting the vector solutions will yield the Pareto frontier as shown in Figure VIII.1.3.

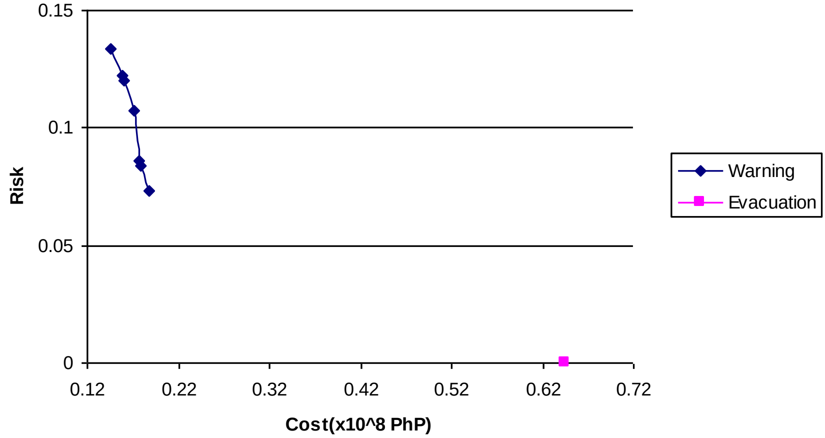

Figure VIII.1.3. Pareto Frontier

A significant gap can be seen between the set of optimal solutions for the *warning* decision and the optimal solution for the *evacuation* solution. This signals the decisionmakers to:

- generate some alternative X that will cost somewhere in-between the existing two sets of solutions with a lower risk level than that involved in the *warning* decision;
- make the emergency evacuation more cost-effective.

PROBLEM VIII.2: Highway Bridge Maintenance

The objective of this problem is to select a maintenance policy for a highway bridge using MODT based on the cost of the policy and mean time before failure (MTBF) of the bridge.

DESCRIPTION

A consulting firm was commissioned to model and analyze the maintenance policy for a bridge on an interstate highway. The policy options are: replace the bridge, repair it, or do nothing. The firm's management asked its risk analysis group to conduct the study. In preparing the work plan, it was decided that the problem be modeled using the Multiobjective Decision Tree (MODT). The two objectives considered are the cost of the policy option and the mean time before failure (MTBF) of the bridge.

METHODOLOGY

We solve the problem in the following way

i) Construct a complete decision tree and indicate the values of both objectives on each terminal node.
ii) Determine the set of Pareto-optimal decisions for the branch of the tree corresponding to a decision node.

Assumptions

The following are the assumptions made for this problem:

1. Cost of a new bridge is $1 million
2. The condition of the bridge can be judged by a parameter **s** which represents a declining factor of the age of the bridge.
3. The cost of repair depends upon the parameter **s** and is given by
$$C_{REPAIR} = 200,000 + 4,000,000(\mathbf{s}-0.05)$$
4. This parameter s is uncertain in nature and can take the following values
$$\mathbf{s} = s_1 = 0.050$$
$$\mathbf{s} = s_2 = 0.075$$
$$\mathbf{s} = s_3 = 0.100$$
5. The prior probability distribution of s is
$$p(s_1) = 0.25$$
$$p(s_2) = 0.50$$
$$p(s_3) = 0.25$$
6. A test to reduce the uncertainty in **s** can be performed at a cost of $50,000.
7. The test to reduce the uncertainty in **s** can have three possible outcomes.
8. The conditional probabilities of the test results are as follows:

$p(\text{lower}\mid s_1) = 0.50$	$p(\text{same}\mid s_1) = 0.25$	$p(\text{higher}\mid s_1) = 0.25$
$p(\text{lower}\mid s_2) = 0.25$	$p(\text{same}\mid s_2) = 0.50$	$p(\text{higher}\mid s_2) = 0.25$
$p(\text{lower}\mid s_3) = 0.25$	$p(\text{same}\mid s_3) = 0.25$	$p(\text{higher}\mid s_3) = 0.50$

9. The value of λ for the exponential distribution of failure of a new bridge is 0.1.
10. The value of λ for the exponential distribution of failure of a repaired bridge is 0.15.

Notes

a) Bridge failure is defined as any event that causes the closure of the bridge.
b) Mean Time Before Failure (MTBF) is defined as the amount of time that can be expected to pass before a bridge failure occurs.
c) For the exponential distribution with mean λ the mean time before failure is MTBF = $1/\lambda$
d) The probability density function of failure of a new bridge is given by an exponential distribution with mean λ.
e) The probability density function of failure of an old bridge is given by an exponential distribution with mean $(\lambda + s)$, where $\lambda = 0.1$.
f) If repair is done immediately, the probability density function of failure is given by an exponential distribution with mean $(\lambda + 0.05)$, where $\lambda = 0.1$.

SOLUTION

Constructing the Multi-Objective Decision Tree (MODT)

The decision tree for the problem is given in Figure VIII.2.1. The two objective functions are:

Maximize MTBF, and
Minimize cost

For an exponential distribution the MTBF is given by $1/\lambda$, where λ is the mean of the exponential distribution.

For a *new bridge,* we are given $\lambda = 0.1 \Rightarrow$ MTBF|Replace = 1/0.1 = 10 years

For a *repaired bridge,* we are given $\lambda = 0.15 \Rightarrow$ MTBF|Repair = 1/0.15 = 6.6667 years

For the *do nothing* option, the MTBF is a function of the value of s:

for $s = s_1$, $\lambda = 0.1+0.05 \Rightarrow \text{MTBF}|s_1 = 1/0.15 = 6.6667$ years
for $s = s_2$, $\lambda = 0.1+0.075 \Rightarrow \text{MTBF}|s_2 = 1/0.175 = 5.7143$ years
for $s = s_3$, $\lambda = 0.1+0.1 \Rightarrow \text{MTBF}|s_3 = 1/0.2 = 5$ years

For a *new bridge,* we are given: Cost = \$1 million =>Cost|Replace = \$1 million

For the *repair* option, the cost is a function of the value of s:

$$\text{Cost}_{repair}|s_1 = 200000+4000000(s_1-0.05) = 200000+4000000(0.05-0.05) = \$0.2 \text{ million (for } s = s_1)$$

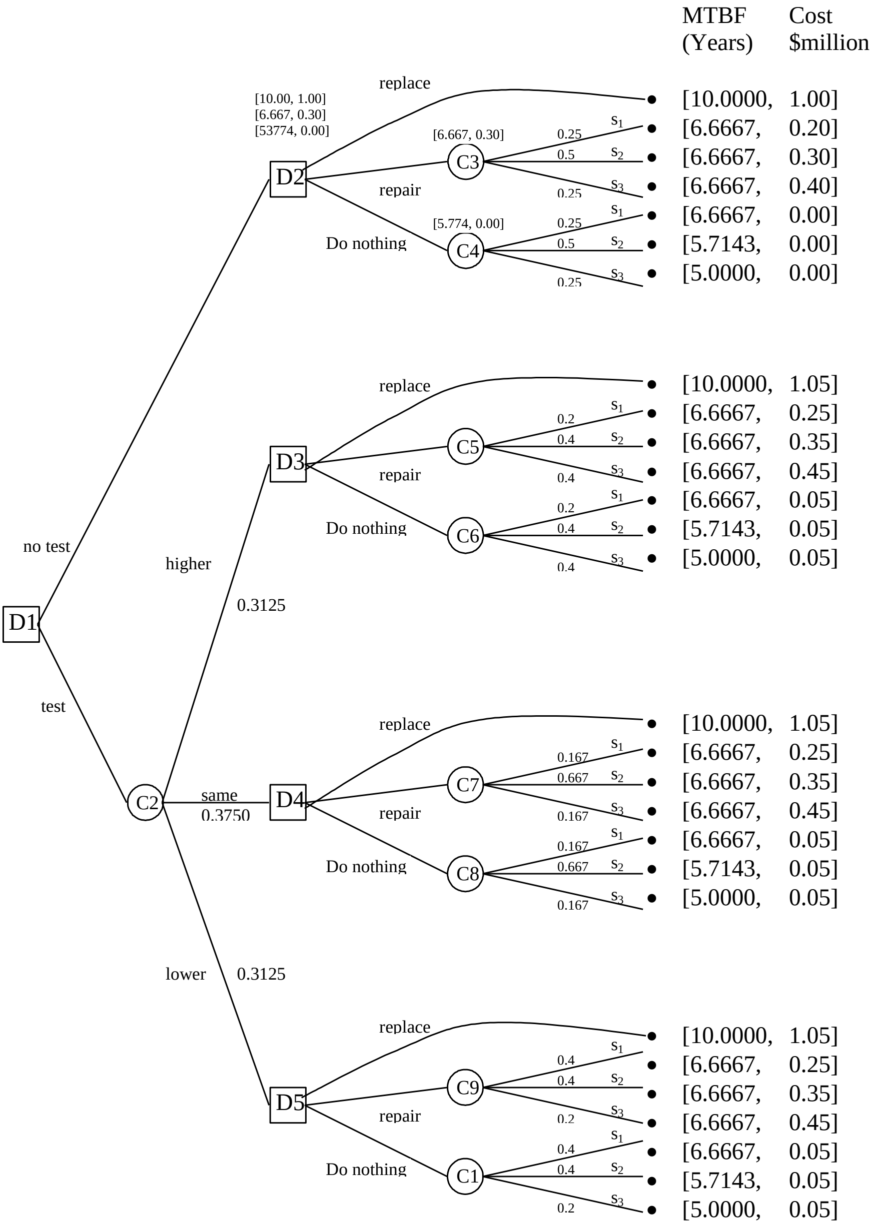

Figure VIII.2.1. Decision Tree for the Bridge Maintenance Problem

Similarly,

$Cost_{Repair}|s_2$ = \$0.3 million
$Cost_{Repair}|s_3$ = \$0.4 million

For the *test* option, the cost of testing, \$0.05 million, will be added to the costs at each terminal node. All the costs are shown at the terminal nodes in Figure VIII.2.1.

The computation of the Pareto-optimal set is shown in Figure VIII.2.1 for the Decision node D2.

To obtain the costs for each of the three arcs, we must average out the Chance nodes C3 and C4. Averaging out at the Chance node C3, we obtain:

$$\begin{bmatrix} 6.6667*0.25+0.6667*0.5+6.6667*0.25 \\ 0.20*0.25+0.30*0.5+0.40*0.25 \end{bmatrix} = \begin{bmatrix} 6.6667 \\ 0.30 \end{bmatrix}$$

which is the solution for the Chance node C3.

Similarly, for Chance node C4, we obtain $=\begin{bmatrix} 5.774 \\ 0.00 \end{bmatrix}$ as the required solution.

For the arc *replace,* we have $=\begin{bmatrix} 10.00 \\ 1.00 \end{bmatrix}$ as the required solution.

Neither of these three solutions is dominated by any other solution. Therefore, the required Pareto-optimal solutions for Decision node D2 are:

$$\begin{bmatrix} 10.00,1.00 \\ 6.667,0.30 \\ 5.774,0.00 \end{bmatrix}$$

The solutions for the other decision nodes can be similarly obtained by making use of the posterior probabilities.

For instance, we can calculate Pr(higher) and Pr(s1|higher) as follows:

Pr(higher) = Pr(higher|s1)·Pr(s1) + Pr(higher|s2) ·Pr(s2) + Pr(higher:s3) ·Pr(s3)
= 0.25 · 0.25 + 0.25 · 0.5 + 0.50 · 0.25
= 0.3125 (By Total Probability rule)

Pr(s1|higher) = Pr(higher|s1) · Pr(s1) / Pr(higher) = 0.25 · 0.25 / 0.3125 = 0.2
(By Bayes' Theorem)

ANALYSIS

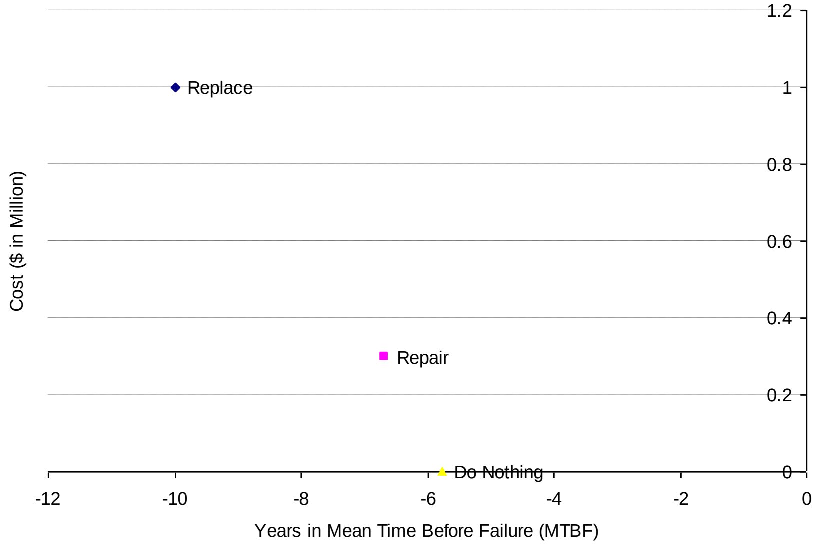

Figure VIII.2.2. Plot for Pareto Optimal Points

The above figure shows the Pareto Optimal points. Note that values of the MTBF (see *x*-axis in Figure VIII.2.2) have been negated to convert the objectives into a standard multiobjective minimization problem. From Figure VIII.2.2, we can see that the policy option "Do Nothing" is a good option, especially compare to the policy option "Repair". At the expense of $300,000, repairing the bridge just improved the MTBF by less than a year. But if we replace the bridge by using $1 million, we can improve MTBF by more than 4 years. Hence, at this point, either "Do Nothing" or "Replace the Bridge" would be good policy choices. If there is enough funding available, "Replace the Bridge" would be recommended, but if the financial constraint is strictly tight, "Do Nothing" policy would be a logical choice to implement.

PROBLEM VIII.3: Hiring a consultant for maximizing profit

The purpose of this problem is to determine the effectiveness of hiring a consultant in order to maximize the market share for a manufacturing company.

DESCRIPTION

A manufacturing company wants to maximize their market share. The demand for a product in the next period can be either increased by 50% or decrease by 5%. According to the demand change in the next period, the company needs to decide whether to continue same operation, increase employee overtime or investing in additional machines.

Since there is no reliable estimate available for the next period demand changes, the company is considering hiring a consultant who can provide a good estimate for a next period demand change. Does the company need to hire a consultant?

METHODOLOGY

Consider the MODT presented in Figure VIII.3.1 with the following specifications in terms of states of nature, actions, and objective functions[1]:

States of Nature:

$\theta_1 \rightarrow$ Demand for a product will increase by 20%.

$\theta_2 \rightarrow$ Demand for a product will decrease by 5%.

Actions:

a_1 = continue same operation
a_2 = put some employees on overtime
a_3 = buy additional machines

First objective function → maximize $

Payoff Matrix for Demand (million $)

		States θ_1	θ_2
Actions	a_1	1.5	1.4
	a_2	2.0	1.4
	a_3	2.1	1.0

Second objective function → maximize market share [0→100%]

[1] The MODT considered in this exercise is an extension of the single objective decision tree problem found in Vira Chankong and Yacov Y. Haimes. *Multiobjective Decision Making: Theory and Methodology.*. North Holland Series in System Science and Engineering, (Hardcover), 1983.

Payoff Matrix (Market Share % after 1 Year)

		States		
		θ_1	θ_2	
	a_1	45	40	← present level
Actions	a_2	55	35	
	a_3	60	30	

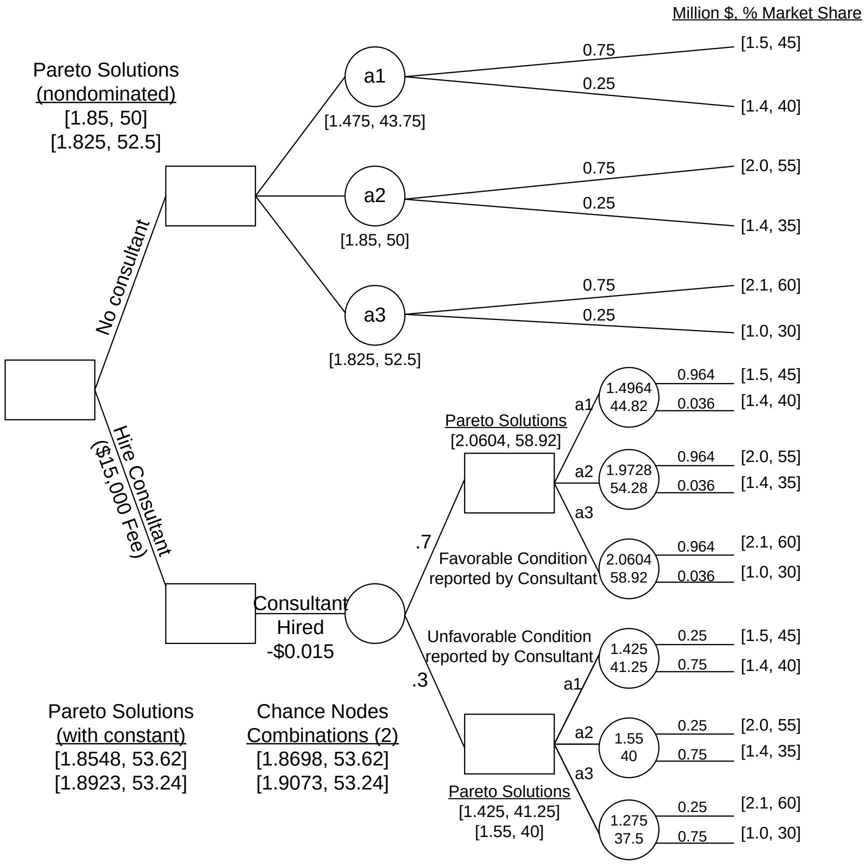

Figure VIII.3.1. Multiobjective Decision Tree

SOLUTION

Overall Pareto Solutions (both using a consultant):

	[f_1 (cost in million $), f_2 (market share in %)]
1)	[1.8548, 53.62]
2)	[1.8923, 53.24]

ANALYSIS

Based on the Pareto Optimal Solutions, it appears that the company will benefit from hiring a consultant to improve its market share. Also, from the above solution, the company must decide if Δf_2 is "worth" Δf_1. In this case, the company must ask himself: is a 0.38% market share increase worth spending $37,500? If so, the first solution would be the sole Pareto Optimal Solution. Likewise, if the 0.38% market share increase is not worth enough spending $37,500 then the second solution would be the sole Pareto Optimal Solution.

PROBLEM VIII.4: Business Decision Problem

The management committee of a consumer product company is considering several options when the peak season for its product is approaching. Note that this problem builds on and extends the previous problem with the addition of a new objective function (Mean Time to Failure). For completeness, the calculations from the previous problem are repeated here.

DESCRIPTION

The firm can increase its profits by either maximizing its market share or minimizing its Mean Time to Failure (MTTF). Given the allowable budget, it is contemplating the following options:

Do nothing (follow the same operation)
Purchase additional machines
Utilize an overtime workforce

They also consider hiring external sources, such as consultants to analyze market trends and suggest short-term strategies.

METHODOLOGY

Multiobjective Decision Tree (MODT) analysis is used to evaluate the trade-offs among the noncommensurate objectives.

SOLUTION

Part A. Trade-off Analysis between Profit and Market Share

Initially, a two-objective problem was specified dealing mainly with the objectives of maximizing profit and maximizing market share. The decision matrices corresponding to these two objectives are described below:

States of Nature:

$\theta_1 \rightarrow$ Demand for a product will increase by 20%.

$\theta_2 \rightarrow$ Demand for a product will decrease by 5%.

Actions:

a_1 = Do nothing (continue same operation)

a_2 = Put some employees on overtime

a_3 = Purchase additional machines

First objective function → maximize profit:

Payoff Matrix for Demand (Million $)

States

Actions		θ_1	θ_2
	a_1	1.5	1.4
	a_2	2	1.4
	a_3	2.1	1

Note that the solution to this problem will evaluate the trade-offs between profit and market share. A supplementary trade-off analysis will include another objective, Mean Time to Failure (MTTF).

Second objective function → maximize market share [0→100%]:

Payoff Matrix (Market Share % after 1 Year)

States

Actions		θ_1	θ_2	
	a_1	45	40	← present level
	a_2	55	35	
	a_3	60	30	

The probabilities for each state by actions are given as follows: For every action, θ_1 will occur at the probability of 75%, so θ_2 will be encountered at the probability of 25%. However, after hiring consultants, the probabilities will be changed from θ_1 and θ_2 to 96.4% and 3.6%, respectively. Normally, consultants suggest favorable reports at 70% of their practices.

Figure VIII.4.1 shows the Multiple Objective Decision Tree with market share.

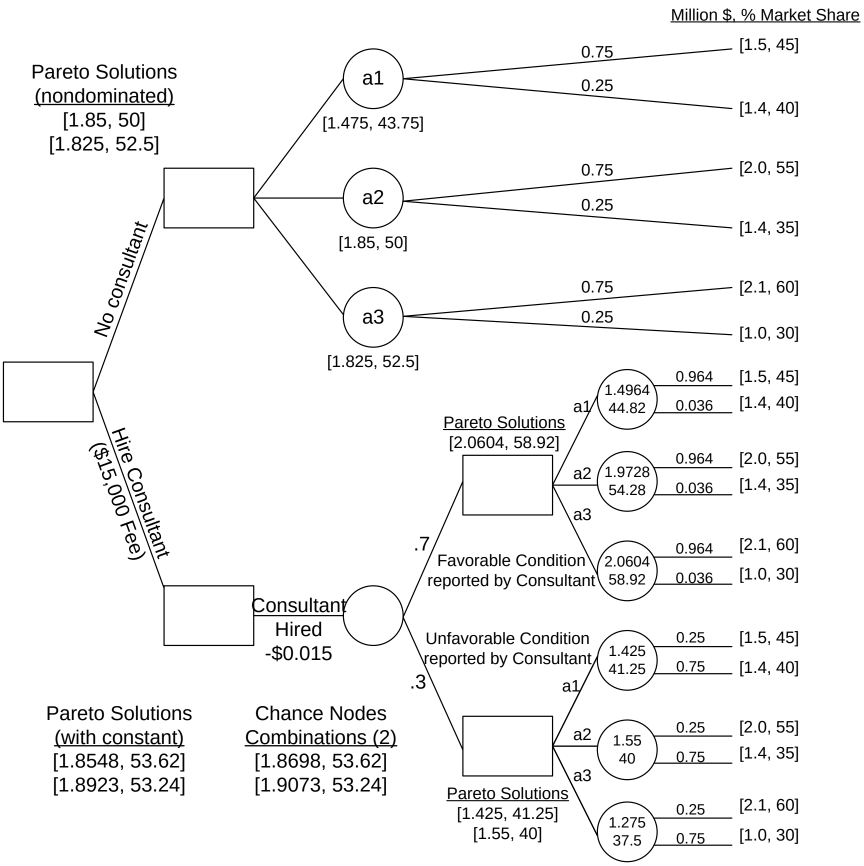

Figure VIII.4.1. MODT with market share

ANALYSIS

In Figure VIII.4.1, we can find overall Pareto solutions (both using consultant cases) as follows:

	[Million $,	% Market Share]
1)	[1.8548,	53.62]
2)	[1.8923,	53.24]

Thus, the decisionmaker must decide if Δf_2 is "worth" Δf_1. In this case, the decisionmaker must ask: Is spending $37,500 worth a 0.38% market share increase?

Part B. Trade-off Analysis between Profit and Mean Time to Failure (MTTF)

Along with profit, the Mean Time to Failure (MTTF) of the product is also considered. Note that the analysis from this point forward only comprises of profit and MTTF (i.e., market share is excluded from the trade-off analysis).

It is assumed that the pace of business is inversely proportional to the MTTF, and also that overtime and a new machine adversely affect MTTF.

Figure VIII.4.2 shows the decision tree taking into account Mean Time to Failure, and Figure VIII.4.3 graphically illustrates the noninferior solutions.

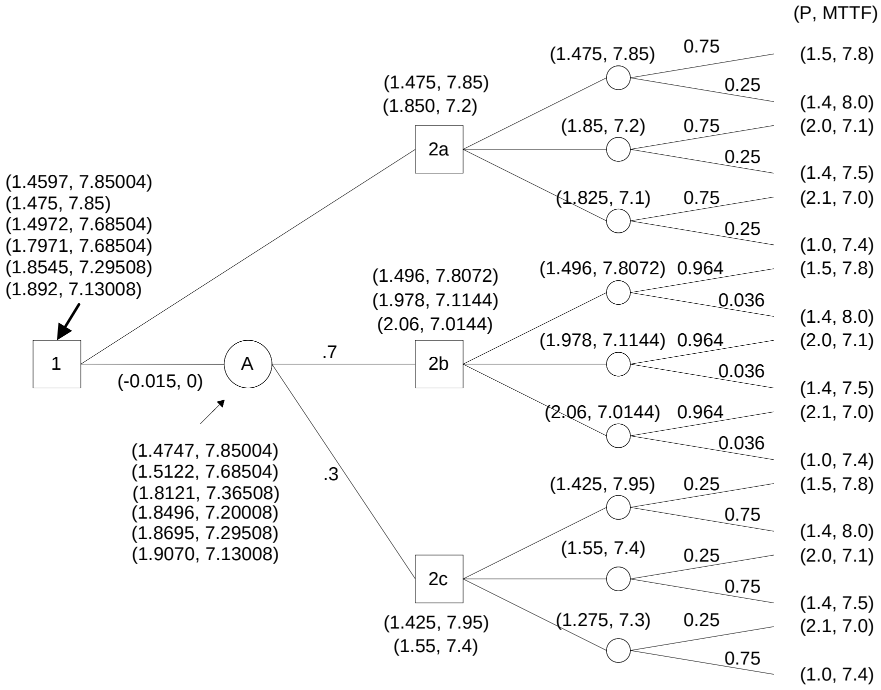

Figure VIII.4.2. Multiple objective decision tree (MODT) with MTTF

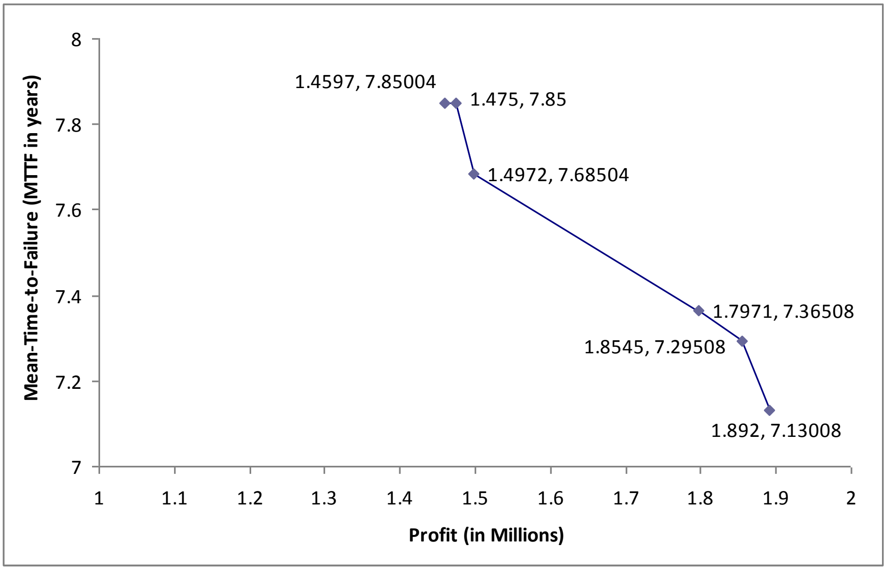

Figure VIII.4.3. Pareto-optimal frontier

ANALYSIS

After folding back to the initial decision nodes and considering only profit and MTTF, Figure VIII.4.3 reveals the noninferior solutions to be: [1.4597, 7.85004], [1.475, 7.85], [1.4972, 7.68504], [1.7971, 7.68504], [1.8545, 7.29508], [1.892, 7.13008].

PROBLEM VIII.5: Maintaining E-mail Service

The objective of this problem is to decide whether to hire an outside agency to manage a risk of an e-mail service interruption.

An e-mail service program is used as an important organizational tool. When the service is unavailable, the organization loses efficiency and possibly direct funds. The cost to the agency is estimated at $100,000 per incident if mail service is interrupted for more than 1 hour at a time. There is no cost associated with the server or the network being down for less than 1 hour. The probability of the server being down for more than 1 hour in the first stage is 10%. The probability of the server being down for more than 1 hour in the second stage is 20%. There is a client satisfaction rating which is 0 for satisfied clients and 1 for dissatisfied clients.

Management has two options. The first is to hire an outside agency that will automatically switch the system over if e-mail cannot be forwarded. The second option is to do nothing. This costs nothing but holds the risk of service being down.

Assess the cost of both options using a two-stage Multiobjective Decision Tree (MODT).

The following assumptions are made:

1. The cost estimate generated is the same for any time period over one hour. Anything less than one hour downtime is not counted.

2. There are two possible actions for the first time period (less than one hour):
   a. Purchase a service contract with an outside agency at a cost of $60,000 [OMS1].
   b. Do nothing at no cost [DN1].

3. There are two possible actions for the second time period (more than one hour):
   a. Purchase a service contract with an outside agency at a cost of $70,000 [OMS2].
   b. Do nothing at no cost [DN2].

PROBLEM VIII.6: Call Center

Call center staffing depends on uncertain volumes of customer calls. Staffing decisions are sequential processes wherein future decisions on how many operators to subcontract can be based on past and current customer call volumes.

With the importance of telephones in the modern commercial world, most business transactions are accomplished by the installation of call centers. Sales, marketing and many other corporate functions are handled efficiently using trunk lines (1-800 numbers) that are attended to by a number of company operators. The number of operators to be employed is very critical since there exists a tradeoff between:

(a) the financial considerations in employing a large number of operators; and
(b) the calling clients that forego the company's service due to long queue time

In an ideal call processing center, no business is lost since all customers' calls are answered immediately. Figure VIII.6.1 presents the following ideal schematic:

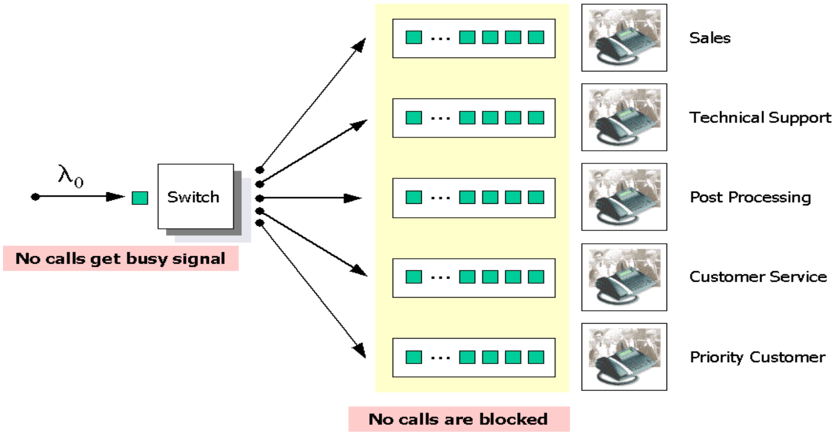

Figure VIII.6.1. Ideal Call Processing Center

However, a typical call center faces the reality of having to put a significant number of customers on hold (in a queue). This phenomenon usually happens during peak times, which consequently leads to complaints or worse, lost businesses. Figure VIII.6.2 diagrams this scenario

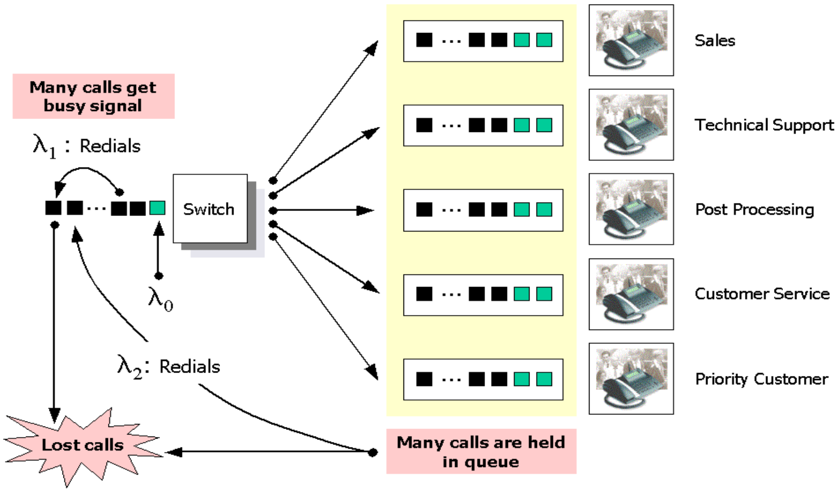

Figure VIII.6.2. Typical Call Center Service

Using a Multi-Objective Decision Tree (MODT) model, the following conflicting objectives have been identified:

(1) Minimize f1 → Proportion of time an operator is idle
(2) Minimize f2 → Proportion of time a customer will be in a queue

The call arrival process has two equally likely *a priori* distributions, given as:

LN1 ~ Lognormal (Ln 50, 1)
LN2 ~ Lognormal (Ln 10, 1)

CHANCE NODES

There are two possible types of calling periods, which are classified as slack- and peak-time chances to call. The *slack period (S)* occurs when the number of callers per minute is less than 20. The *peak period (P)* occurs when the number of callers for a given minute exceeds 20.

There are two 4-hour (240-minute) time periods in a given day. The first is from 8:00 AM – 12 noon, while the second ranges from 1:00 PM to 5:00 PM. For simplicity in this example, it shall be assumed that the entire 4-hour time period is either a slack or a peak calling period (i.e., if it's slack at 8:00 AM, then it shall be slack throughout the entire period or until 12 noon).

DECISION NODES

The company must determine the number of operators it needs, and it is trying to decide whether to assign 10 or 20 operators to a given period.

By using the Multiobjective Decision Trees (MODT) shown below, evaluate the Pareto-optimal solutions by folding the outcomes back to the initial decision node. Analyze your results.

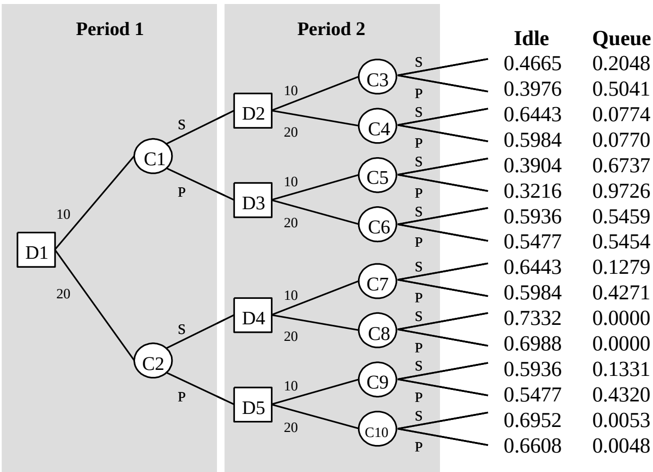

Figure VIII.6.3. Example of MODT

PROBLEM VIII.7: Determining when to Book Plans for Vacation

A student has a chance to go to the Cayman Islands for a week in the middle of the semester. Should he go, and when should he book the reservations?

The payoff of the vacation is an improved state of mind. The cost is the expense of airfare and hotel. The problem is that he will miss some work in school.

Because the objectives are conflicting, a Multiobjective Decision Tree (MODT) analysis can help to decide on the best strategy.

The *objectives* are:

- Minimize the level of stress. This objective is a function of the amount of work the student has to do and whether or not he takes the vacation.
- Minimize the cost of the plane fare and the hotel stay. This is measured on a straightforward monetary scale.

The *alternatives* in the month before the trip is planned are:

- Pay for a plane ticket and hotel (at a discount).
- Make plane and hotel reservations but do not pay yet.
- Wait until later to do anything.

The three possible *states* of nature that may occur in the two weeks before the trip are:

- The student finds he has *little* work to do
- He finds he has homework due that week (*some* work)
- He has two tests and a paper due that week (*much* work)

The student will not know for sure when the tests and paper are due until two weeks before the trip is planned. But he may have some idea of his future work by the amount of work he has now. From past experience, he knows that the work loads of now and later are negatively correlated. That is, if he has lots of work now, he will not have much later, but if he has only a little now, chances are he will have a lot later.

Let us define:

LWN = Little work to do now
SWN = Some work to do now
MWN = Much work to do now
LWL = Little work to do over the time of the trip
SWL = Some work to do over the time of the trip
MWL = Much work to do over the time of the trip

Data

By reviewing his calendar for the last four semesters, the student determines the probabilities for each work load.

The prior and conditional probabilities are as follows:

P(LWL) = .25
P(SWL) = .35
P(MWL) = .4

P(SWN/LWL) = .2
P(LWN/LWL) = .1
P(MWN/LWL) = .7
P(SWN/SWL) = .4
P(LWN/SWL) = .3
P(MWN/SWL) = .3
P(SWN/MWL) = .3
P(LWN/MWL) = .6
P(MWN/MWL) = .1

From the travel agent, the student knows that if he pays for airfare and a hotel now, it will cost $900. If he makes reservations now and pays later, it will cost $1300, but if he waits till later to make reservations and pay, it will cost $1650. If he makes reservations without paying and cancels, it costs nothing. If he pays in the first stage and then cancels later, it will cost 10% of the original price.

The Model

The first step is to define the state variables—those variables that define the system at any given point.

The *state variables* are:
- S1 – the benefit of the time spent on vacation
- S2 – the amount of work the student has to do or will miss
- S3 – the cost of the trip to the islands

The *decision variable* is:
- If and when to make reservations and pay for the airfare and hotel

The *exogenous variables* or parameters are:
- The cost of plane fare
- The cost of hotel accommodations

Quantifying the Objectives:

The objectives are as follows:
- f_1 = Minimize the level of stress
- f_2 = Minimize the cost of the trip

1) *Level of Stress* – f_1(S1,S2)

The level of stress is reduced by going on the trip, and is raised by having more work to do. The easiest way to measure these levels of stress in an ordinal scale is through subjective assessment. The different stress levels are accessed as a function of whether or not the student goes on vacation and how much work he has to do. The graph of Stress Level is show in Figure VIII.7.1:

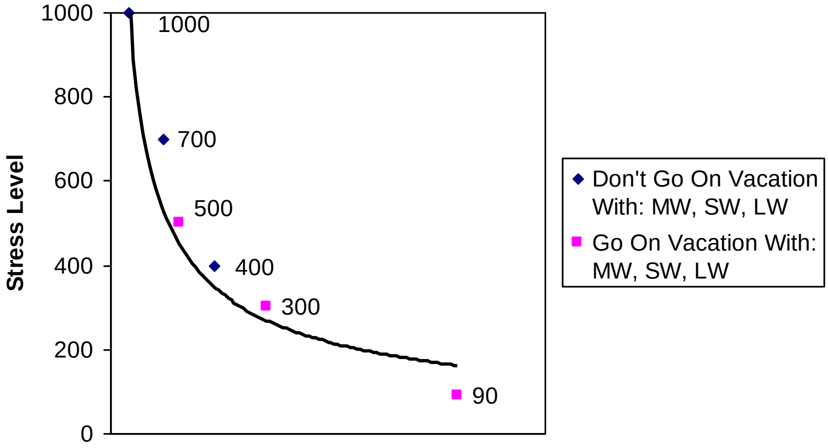

Figure VIII.7.1. Stress level according to vacation and work load

2) *Cost of Trip* – f_2(S3)

The cost is $f_2 = C_{ij}$ where

i = first-stage decision

j = second-stage decision

From the travel agent you know that:

C(Pay, Cancel) = \$90
C(Pay, Go) = \$900
C(Reserve, Go) = \$1300
C(Reserve, Cancel) = \$0
C(Do Nothing, Go) = \$1650
C(Do Nothing, Stay Home) = \$0

The Multiobjective Decision Tree (MODT)

The comprehensive decision tree for this problem is shown in Figure VIII.7.2 below:

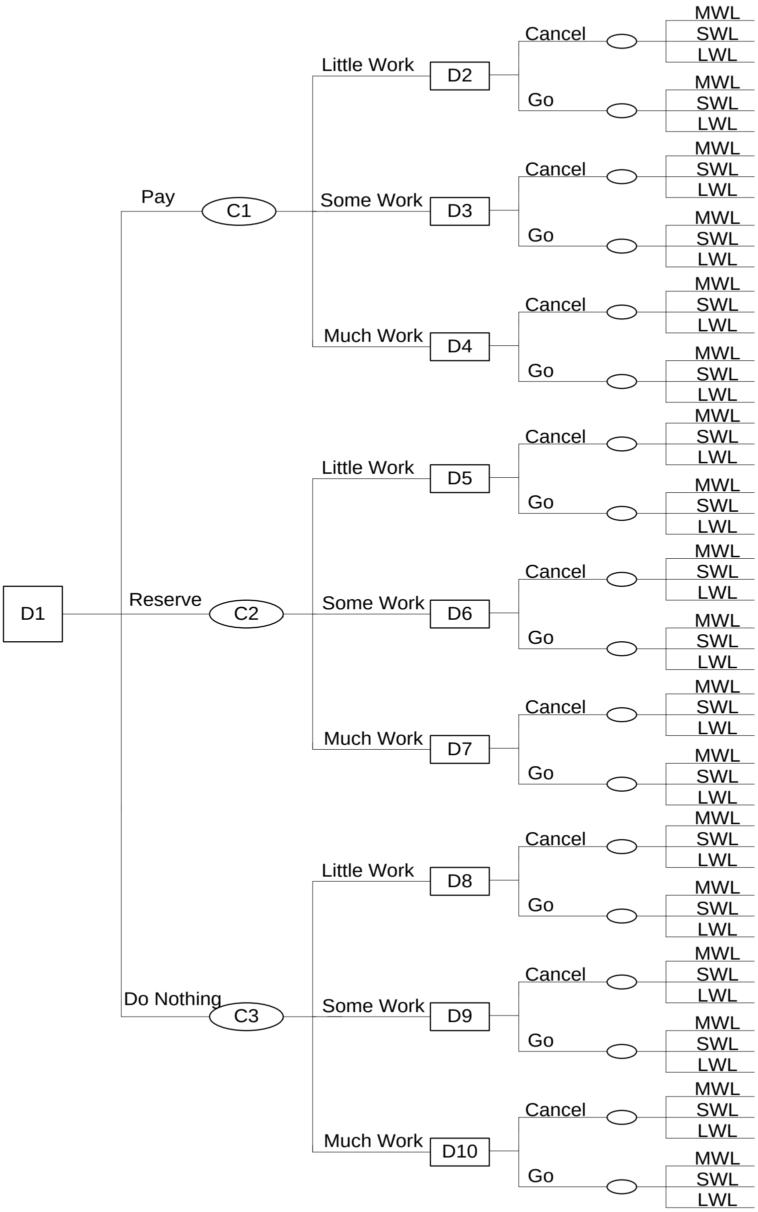

Figure VIII.7.2. Multiobjective decision tree for vacation decision

By using the Multiobjective Decision Trees (MODT) shown above, evaluate the Pareto-optimal solutions by folding the outcomes back to the initial decision node. Analyze your results.

PROBLEM VIII.8: Analysis of Alternate Routing System

The objective of this problem is to alleviate highway traffic congestion by considering an alternate routing system.

There are two possible actions: (a) alternate routing (AR) and (b) doing nothing (DN). The decision tree covers two time periods and the associated cost is a function of the period in which the action is taken.

The complete decision tree is shown in Figure VIII.8.1.

Assumptions

1. There are two possible actions associated with costs for the first period:
    a. Coming up with *alternate routing* for the traffic at a cost of $200,000 (AR1).
    b. *Doing nothing* at zero cost (DN1).
2. For the second period the actions and corresponding costs are:
    a. Alternate routing at a cost of $100,000 (AR2).
    b. Doing nothing at zero cost (DN2).
3. Travel time, T, is measured in hours. A *stall*, or gridlock, occurs when travel time (T) between two points, A and B, is greater than or equal to 4 hours.
4. There are two underlying probability distributions for the flow of traffic:
    a. T ~ lognormal (1.2527, 1), represented as LN1.
    b. T ~ lognormal (0.7419, 1), represented as LN2.

    The mean values of the lognormal distributions were arrived at by taking the log of the midpoint between time limits (T) for higher and lower traffic levels. For example, log(3.5) = 1.2527.

    The *a priori* probability that any of these probability density functions (pdfs) is the actual pdf is equal.
5. There are three possible events at the end of the first period:
    a. A stall or gridlock (T > = 4 hr.)
    b. Higher traffic (3 < = T < = 4 hr.)
    c. Same or lower traffic (3 < = T)
6. L and C are, respectively, the maximum possible *loss of lives* due to fatal accidents and *money lost* due to legal action ensuing from the accident, given no alternate routing.

Update the multiobjective decision tree with probabilities and expected values associated with each option. Analyze your results.

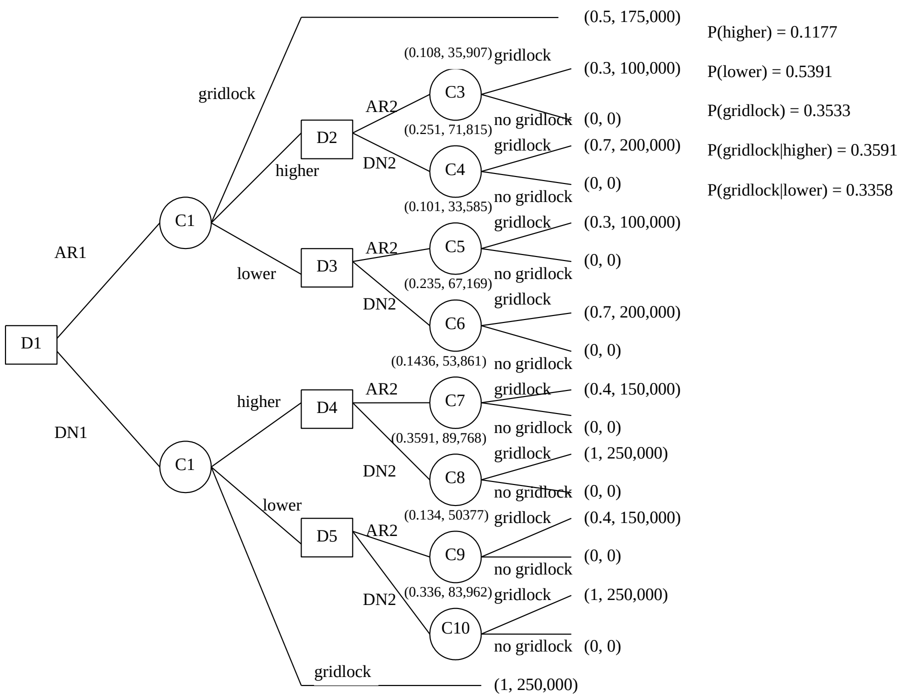

Figure VIII.8.1. Multiobjective Decision Tree (MODT)

PROBLEM VIII.9: Bobsled Training Strategy

A Central American four-man bobsled team entered in the Winter Olympics must train for many months in Sweden. The cost of travel, facility rental, and room and board for living in Sweden during this extended time period is expensive. Thus, the country can only afford two $45,000 bobsleds for its team and has a limited sled repair budget.

The condition of the ice (how slippery it is) on a bobsled track varies during the day. Lateral bobsled slippage on the track depends primarily on speed as well as on the condition of the ice, which is beyond the control of the bobsled team. High lateral slip can cause a bobsled to lose its grip on the ice in a turn and slide sideways into the wall, resulting in a crash. The bobsled can also crash into a wall without incurring lateral slippage, usually because of excess speed and loss of control. Damage to a bobsled in a crash is greater at higher speeds and if lateral slip occurs.

The Swedish training track has two tight turns on differing slope levels which are the main source of sled crashes. The team knows that accidents will happen during the training and the repair budget will be used. They cannot risk losing both bobsleds in accidents and being unable to repair at least one of them in time for the Olympics.

The team's objectives are to minimize damage to the bobsleds and to train effectively at competition speeds (maximize speed). They need to decide how much brake to apply in the two dangerous turns in order to safely negotiate the course and keep their bobsleds minimally harmed, while still receiving beneficial training. The team analyzes and averages Swedish training data and builds a Multiobjective Decision Tree (MODT) to show the bobsled's relationship to braking and lateral slip in the turns.

1. There are two possible actions with the associated bobsled speed for the first turn of the course:
   a. Brake hard and reduce speed 25 MPH
   b. Brake soft and reduce speed 10 MPH

2. For the second turn of the course, there are two possible actions with the associated bobsled speed:

   a. Brake hard and reduce speed 15 MPH
   b. Brake soft and reduce speed 5 MPH

3. There are two underlying probability density functions (pdfs) for lateral bobsled slippage (L) in a turn associated with the condition of the ice:

   a. L ~ normal $(0.6, (0.05)^2)$, represented as N_1
   b. L ~ normal $(0.5, (0.075)^2)$, represented as N_2
   The prior possibilities that any of these two pdfs is the actual pdf are equal.

4. There are two possible events at the end of the first turn of the course:
   a. Bobsled slips (L > 0.7 g), represented as C_0
   b. Bobsled grips (L <= 0.7 g), represented as C_1

5. D= $3,750 and S = 65 MPH are the maximum average bobsled damage and maximum average speed, respectively, resulting from a bobsled run.

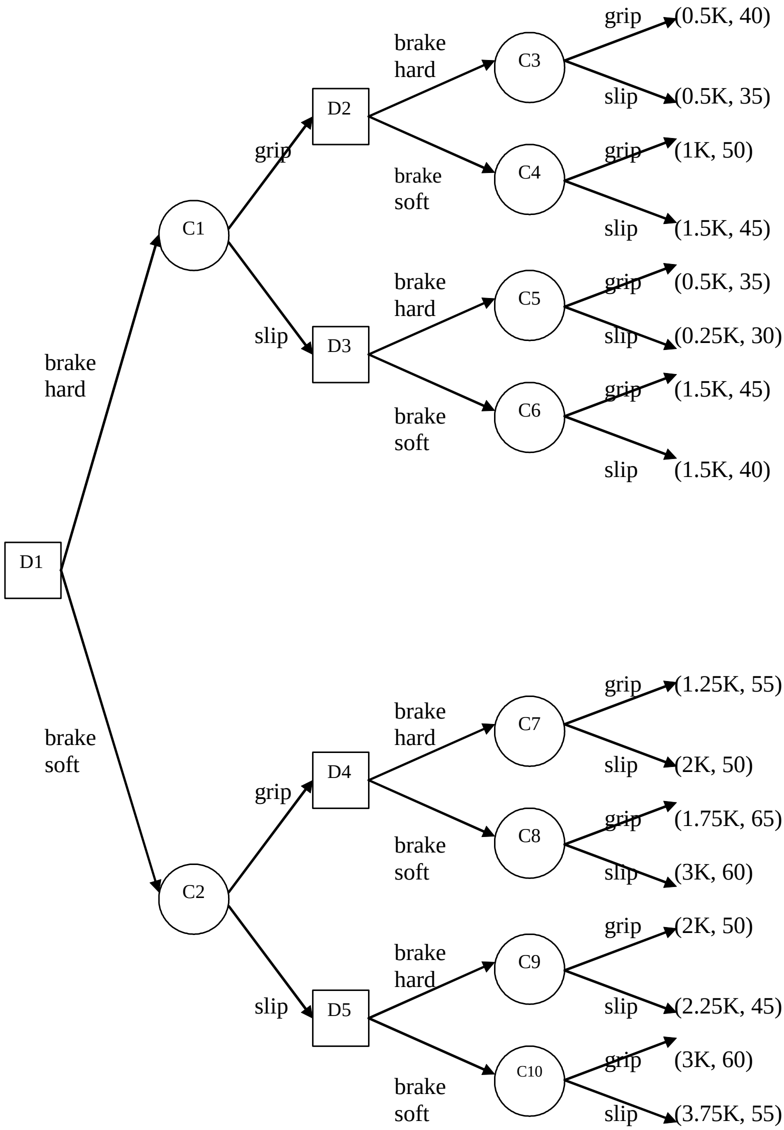

Figure VIII.9.1. Multiobjective Decision Tree

Perform MODT to analyze the possible bobsled training strategies.