XII. Multiobjective Statistical Method

PROBLEM XII.1: Designing Water Treatment Plant Capacity and Efficiency

Following the recommendations from past financial and technical feasibility studies, a private water company was hired to construct a city bulk water treatment plant. Although the project site has been chosen, the treatment plant's capacity and removal efficiency must still be decided.

DESCRIPTION

For simplicity, the treatment plant capacity is limited to five natural numbers (in mega-gallons per day). Likewise, the treatment efficiency options are limited to three to represent the most common treatment levels for advanced secondary or tertiary treatment, and also because equipment and technology providers usually make available only the rounded efficiency of their systems. The removal efficiency is closely related to the choice of appropriate technologies.

METHODOLOGY

We apply the Multiobjective Statistical Method (MSM) to design a water treatment plant that will have the required capacity and most reliable removal capabilities for the city's needs, while minimizing the cost.

SOLUTION

Decision Variables

x_1 = Capacity of the treatment plant [mega-gallons per day, MGD]
where $x_1 \in \{2, \quad 4, \quad 6, \quad 8, \quad 10\}$

x_2 = Removal efficiency of the treatment plant,
where $x_2 \in \{0.85, \quad 0.90, \quad 0.95\}$

Due to uncertainties in population growth, migration, residential and industrial development, development of alternative water sources, efficiency of water distribution networks, and changes in per capita water consumption, the actual daily water demand when the plant is already operational should be considered as a random variable. Moreover, the quality of lake water is also probabilistic and is dependent on the implementation of environmental regulations, public awareness and participation, and other factors.

Random Variables

r_1 = Actual daily water demand [MGD]

r_2 = Actual average raw water (water abstracted from the lake) quality when the plant is already operational, measured in terms of Carbonaceous Biochemical Oxygen Demand (CBOD) concentration [mgO_2/L]

The probability distributions of r_1 and r_2 are given in Tables XII.1.1 and XII.1.2, respectively [Santos-Borja, 2004]. Figures XII.1.1 and XII.1.2 show the cumulative distribution of r_1 and r_2, respectively.

Table XII.1.1. Density and Cumulative Density Functions of Actual Daily Water Demand

Daily Demand MGD	Probability %	Cum. Probability %	Daily Demand MGD	Probability %	Cum. Probability %
0	0	0			
0.25	0	0	5.25	5	43
0.5	1	1	5.5	7	50
0.75	0	1	5.75	5	55
1	1	2	6	6	61
1.25	1	3	6.25	8	69
1.5	0	3	6.5	8	77
1.75	0	3	6.75	4	81
2	2	5	7	5	86
2.25	1	6	7.25	3	89
2.5	1	7	7.5	1	90
2.75	0	7	7.75	1	91
3	2	9	8	3	94
3.25	3	12	8.25	0	94
3.5	1	13	8.5	1	95
3.75	4	17	8.75	1	96
4	3	20	9	1	97
4.25	4	24	9.25	2	99
4.5	6	30	9.5	1	100
4.75	5	35	9.75	0	100
5	3	38	10	0	100

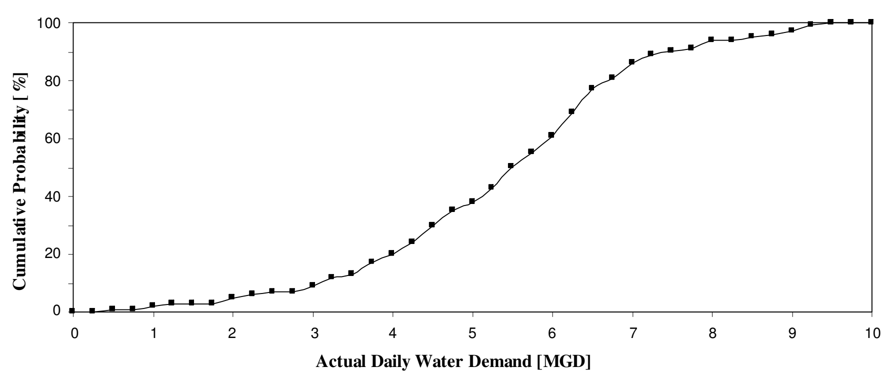

Figure XII.1.1. Cumulative Density Function of Actual Daily Water Demand (r_1)

Table XII.1.2. Density and Cumulative Density Functions of Actual Raw Water Quality

CBOD mgO_2/L	Probability %	Cum. Probability %
0	0	0
10	5	5
20	9	14
30	14	28
40	22	50
50	12	62
60	7	69
70	7	76
80	6	82
90	3	85
100	1	86
110	0	86
120	3	89
130	2	91
140	0	91
150	1	92
160	0	92
170	0	92
180	2	94
190	3	97
200	1	98
210	0	98
220	1	99
230	0	99
240	1	100
250	0	100

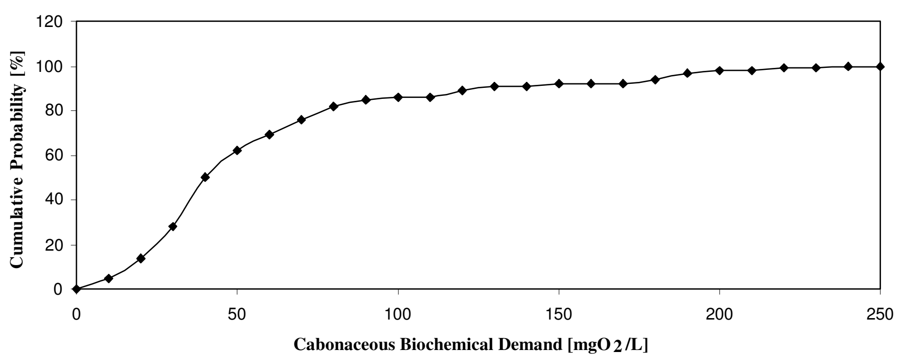

Figure XII.1.2. Cumulative Density Function of Actual CBOD (r_2)

The state of the system can be represented by the quantity and quality of water treated daily. Note that the plant capacity used as a decision variable is different from (although proportional to) the rate of treated water production. The quantity and quality of treated water are functions of the quality of raw water and the treatment efficiency.

State Variables

$s_1 =$ Daily water production

$s_2 =$ Quality of treated water

$$s_1(\cdot) = \begin{cases} r_1 & \text{if } r_1 \le x_1 \\ x_1 & \text{if } r_1 > x_1 \end{cases}$$

$$s_2(\cdot) = (1 - x_2)r_2$$

For this problem, the objectives are to maximize the reliability of the treatment plant as a whole and to minimize the cost.

Objective Functions

$f_1(\cdot) =$ Cost [million \$]

$f_2(\cdot) =$ Reliability

The first objective function, cost, refers to the sum of the capital expenditure and 3 years of operating expenditure. The capital expenditure is a function of the plant capacity, x_1. The operating expenditure is a function of the actual water production, s_1.

$$\min f_1(\cdot) = \begin{cases} 30.0 + 1.42 \ln x_1 + 0.78 s_1 & \text{if } x_2 = 0.85 \\ 33.3 + 1.05 \ln x_1 + 0.29 s_1 & \text{if } x_2 = 0.90 \\ 25 - 0.22 \ln x_1 + 2.49 s_1 & \text{if } x_2 = 0.95 \end{cases}$$

For illustration, consider the base case where the actual daily water demand is 5.5 MGD and the actual raw water quality is 40 mg O_2/L CBOD (both values have 50% likelihood). For a treatment plant with capacity of 2 or 4 MGD, the objective function is:

$$\min f_1(\cdot) = \begin{cases} 30.0 + 1.42 \ln x_1 + 0.78 x_1 & \text{if } x_2 = 0.85 \\ 33.3 + 1.05 \ln x_1 + 0.29 x_1 & \text{if } x_2 = 0.90 \\ 25 - 0.22 \ln x_1 + 2.49 x_1 & \text{if } x_2 = 0.95 \end{cases}$$

Otherwise,

$$\min f_1(\cdot)=\begin{cases}30.0+1.42\ln x_1+0.78r_1 & if\ x_2=0.85\\ 33.3+1.05\ln x_1+0.29r_1 & if\ x_2=0.90\\ 25-0.22\ln x_1+2.49r_1 & if\ x_2=0.95\end{cases}$$

Table XII.1.3 and Figure XII.1.3 summarize and illustrate the cost for plant capacity with respect to three x_2 values (0.85, 0.90 and 0.95) given r_1 = 5.5 and r_2 = 40.

Table XII.1.3. Cost versus Plant Capacity (for r_1 = 5.5 and for r_2 = 40)

Plant Capacity MGD	Cost Million $		
	$x_2=0.85$	$x_2=0.90$	$x_2=0.95$
2	32.54	34.61	29.83
4	35.09	35.92	34.66
6	36.83	36.78	38.30
8	37.24	37.08	38.24
10	37.56	37.31	38.19

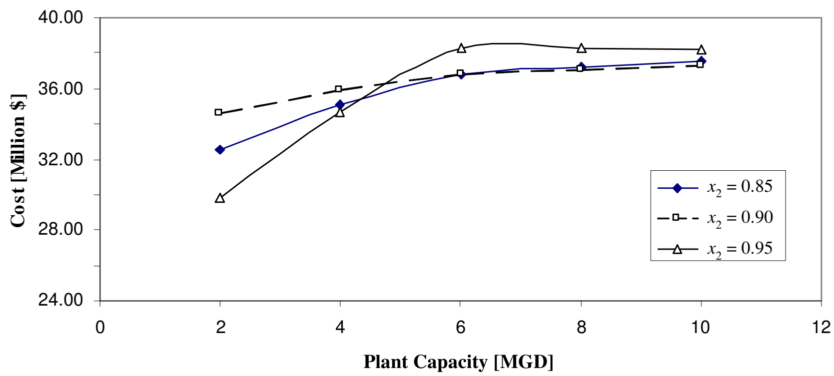

Figure XII.1.3. Cost versus Plant Capacity (for r_1 = 5.5 and for r_2 = 40)

ANALYSIS

The first objective function here is the *cost.* In general, during the decision process it is still uncertain if the design capacity of the treatment plant will correspond to the actual future water demand. If it does, the total cost will be higher for higher treatment efficiency. However, this may not be true if the treatment plant is under- or over-capacity. For the base case that is explored here, note that the design treatment capacity is 5.5 MGD. If the actual demand is also 5.5 MGD, it can be seen from Figure XII.1.3 that indeed, the least expensive is that of efficiency 0.85 followed by 0.90 and the most expensive is that of 0.95. On the other hand, if the future capacity is less than 4 MGD, the treatment plant with efficiency of 0.95 will turn out to be most economical. The reason for this is that a removal efficiency of 0.95 would be based on Reverse Osmosis (RO) desalination treatment. With current advances in this technology, the capital expenditure for RO desalination is already competitive with conventional treatment technologies. However, the disadvantage of desalination is a high operating cost because it is energy-intensive and maintenance-intensive. At the higher capacities, the cost of RO desalination (corresponding to the efficiency of 0.95) will be far higher than the two other options. Lastly, since RO desalination is modular in nature (many parallel trains), some trains can be easily taken out of service if the actual capacity turns out to be just 4 MGD or 2 MGD. This will result in a lower operating cost. This advantage is reflected in Figure XII.1.3 above.

The second objective function, *reliability,* refers to the combined probability that the treatment plant will be able to meet the volume demand and conform to the quality standards for drinking water.

$$\max\ f_2(\cdot) = 1 - \left\{ \sum P(s_1 < r_{1i}) + P(s_{2i} > 20) - \left\lfloor P(s_{2i} > 20) \sum P(s_1 < r_{1i}) \right\rfloor \right\}$$

For illustration, let us again use our base case with design capacity of 5.5 MGD. From Table XII.1.1, this capacity corresponds to the median value. Thus, there is a 50% probability that the treatment plant will not be able to supply the total demand.

$$s_1 = 5.5$$

$$\sum P(s_1 < r_{1i}) = 0.50$$

For the reliability in terms of meeting the quality standards, refer to Table XII.1.4 below. The values shaded in blue are above the maximum allowable contaminants of 20 mg O_2/L.

Table XII.1.4 Treated Water Quality vs. Raw Water Quality and Treatment Removal Efficiency

Raw Water Quality (CBOD), r2	Probability	Cum. Probability	Treated Water Quality, s2		
mgO2/L	%	%	x2=0.85	x2=0.90	x2=0.95
0	0	0	0	0	0
10	5	5	1.5	1	0.5
20	9	14	3	2	1
30	14	28	4.5	3	1.5
40	22	50	6	4	2
50	12	62	7.5	5	2.5
60	7	69	9	6	3
70	7	76	10.5	7	3.5
80	6	82	12	8	4
90	3	85	13.5	9	4.5
100	1	86	15	10	5
110	0	86	16.5	11	5.5
120	3	89	18	12	6
130	2	91	19.5	13	6.5
140	0	91	21	14	7
150	1	92	22.5	15	7.5
160	0	92	24	16	8
170	0	92	25.5	17	8.5
180	2	94	27	18	9
190	3	97	28.5	19	9.5
200	1	98	30	20	10
210	0	98	31.5	21	10.5
220	1	99	33	22	11
230	0	99	34.5	23	11.5
240	1	100	36	24	12
250	0	100	37.5	25	12.5

The probability of not meeting the required water quality can then be computed as shown below:

Table XII.1.5. Reliability vs. Treatment Removal Efficiency

	$x_2 = 0.85$	$x_2 = 0.90$	$x_2 = 0.95$
$\sum P(s_1 < r_{1i})$	0.50	0.50	0.50
$P(s_{2i} > 20)$	0.09	0.02	0.00
Reliability	0.455	0.49	0.5

In general, this two-objective problem can be solved using the ε -constraint formulation.

$$\min f_1(\cdot)$$

subject to

$$\begin{aligned}
&f_2(\cdot) \geq \varepsilon_2 \\
&x_1 \in \{2, \quad 4, \quad 6, \quad 8, \quad 10\} \\
&x_2 \in \{0.85, \quad 0.90, \quad 0.95\} \\
&0 \leq r_1 \leq 10 \\
&0 \leq r_2 \leq 250
\end{aligned}$$

Consider that the decisionmaker set the epsilon constraint to be 0.90 (or 90% reliability level).

$$\min f_1(\cdot) = \begin{cases} 30.0 + 1.42 \ln x_1 + 0.78 s_1 & \textit{if } x_2 = 0.85 \\ 33.3 + 1.05 \ln x_1 + 0.29 s_1 & \textit{if } x_2 = 0.90 \\ 25 - 0.22 \ln x_1 + 2.49 s_1 & \textit{if } x_2 = 0.95 \end{cases}$$

subject to:

$$\sum P(x_1 < r_{1i}) + P((1 - x_{2i})r_2 > 20) - P((1 - x_{2i})r_2 > 20) \sum P(x_1 < r_{1i}) \leq 0.1$$

$$\begin{aligned}
&x_1 \in \{2, \quad 4, \quad 6, \quad 8, \quad 10\} \\
&x_2 \in \{0.85, \quad 0.90, \quad 0.95\} \\
&0 \leq r_1 \leq 10 \\
&0 \leq r_2 \leq 250
\end{aligned}$$

where:

$$s_1(\cdot) = \begin{cases} r_1 & \textit{if } r_1 \leq x_1 \\ x_1 & \textit{if } r_1 > x_1 \end{cases}$$

$$s_2(\cdot) = (1 - x_2) r_2$$

Simulation was done by generating random numbers and determining (by interpolation) the corresponding values of the two random variables. Table XII.1.6 shows some of the Pareto-optimal solutions.

Table XII.1.6. Sample of Pareto-Optimal Solutions

	Run		
	1	2	3
x_1	8	8	8
x_2	0.85	0.9	0.95
random no.	0.110	0.416	0.940
r_1	3.163	5.178	8.000
random no.	0.538	0.648	0.541
r_2	43.136	54.069	43.404
s_1	3.163	5.178	8.000
s_2	6.470	5.407	2.170
f_1	35.41981	36.98505	44.46252
f_2	0.8554	0.9212	0.94

Note: By relating random numbers to Tables XII.1.1 and XII.1.2, we can get r_1 and r_2 respectively.

Reference:

Santos-Borja, Adelina and Nepomuceno, Dolora (2004). *Laguna de Bay: Experience and Lessons Learned Brief*. Laguna Lake Development Authority, Philippines.

PROBLEM XII.2: Shipping Cars to Multiple Dealerships

In the Southeastern region of the United States, a car manufacturing company has three manufacturing plants and must ship the cars to five regional dealerships.

DESCRIPTION

In this multiobjective problem, the three car manufacturing plants are in:

- i) Lexington, KY,
- ii) Huntsville, AL, and
- iii) Columbia, SC.

They supply five regional dealerships located in:

- i) Memphis, TN,
- ii) Atlanta, GA,
- iii) Jackson, MS,
- iv) Louisville, KY, and
- v) Raleigh, NC.

The objectives of the car company are to minimize transportation costs (fuel, tolls, fees) while maximizing the number of cars shipped between the manufacturing plant and regional dealerships. Operational costs (insurance, depreciation, and others) are assumed calculated into the per-mile costs of transport per truck. The company also has a policy to ship cars only when the 9-car capacity of the car carrier is reached.

METHODOLOGY

The Multiobjective Statistical Method (MSM) is used to solve this problem.

Verbal Objectives, Constraints, and Decisions

We begin by stating the basic objective functions:

$f_1(\cdot)$ = minimize transportation costs from manufacturing plant to dealership
$f_2(\cdot)$ = maximize the number of cars shipped between manufacturing plant and dealership

Next, we determine the *decision, input, exogenous, random, state,* and *output variables* to gain understanding of the transportation problem. These are as follows:

Decision variables:

- $x_{i,j}$ = shipment of new car from manufacturer *i* to dealership *j*

Input variables:

- u_1 = average miles per gallon of fuel per truck
- u_2 = truck capacity/car carrying limit
- $u_{i,j}$ = tolls/fees per route (from manufacturer *i* to dealership *j*)
- $u_{m,i}$ = production capacity at manufacturing plant (i)

- $u_{d,j}$ = dealership (j) demand

Exogenous variables:

- α_1 = operational costs of transportation
- α_2 = insurance costs of transportation
- α_3 = depreciation costs of transportation

Random variables:

- r_1 = diesel costs per gallon
- r_2 = weather/route availability
- r_3 = parts shortages affecting production
- r_4 = worker strikes affecting production

State variables:

- s_1 = current miles driven in the week from manufacturing plant (i) to dealership (j)
- s_2 = current number of car shipments

Output variables:

- y_1 = total transportation costs in the week
- y_2 = total number of car shipments made

Next, to get important modeling information, the company drivers and management fill out the following questionnaire:

1) Where are each of the manufacturing plants located?
2) Where are each of the dealerships located?
3) What is the best approximation of mileage for travel between each manufacturing plant and destination?
4) What is the approximate fuel efficiency (miles/gallon) for each truck?
5) How many vehicles are transported on each shipping truck?
6) For each of the routes, what tolls and fees apply?
7) What is the dollar amount per mile for operations associated with each truck (insurance, depreciation, maintenance)?
8) Any additional information not stated above?

This questionnaire was completed by the company with the following details:

Table XII.2.1. Mileage between Manufacturer and Dealerships with Production Outputs and Dealership Demand

Miles between:	Dealership (j)					
Manufacturer (i)	1: Memphis	2: Atlanta	3: Jackson	4: Louisville	5: Raleigh	Output
1: Lexington	422	384	638	79	492	*100*
2: Huntsville	216	196	335	306	587	*250*
3: Columbia	651	214	594	506	227	*200*
Cars Needed (Demand)	*125*	*60*	*80*	*175*	*110*	

Table XII.2.2. Route Tolls and Fees between Manufacturer and Dealership Locations

Tolls/Fees ($) between:	Dealership (j)				
Manufacturer (i)	1: Memphis	2: Atlanta	3: Jackson	4: Louisville	5: Raleigh
1: Lexington	25	40	50	0	30
2: Huntsville	0	20	0	20	40
3: Columbia	40	20	30	50	20

Average fuel efficiency per truck (u_1): 8 miles per gallon.

Number of cars shipped per truck (u_2): 9. There are 5 on top and 4 on the bottom. Cars are shipped only when the trucks are fully loaded.

Operational, insurance, and depreciation costs per mile driven is approximately $0.55, in addition to fuel costs.

With this survey information, we proceed to solving the MSM problem.

SOLUTION

Quantify Objective Functions

First, we define the two objective functions along with their constraints. Let x_{ij} = shipment from manufacturer i to dealership j.

min $f_1(x_{i,j})$ = transportation costs from manufacturing plant to dealership

= {[miles from manufacturing plant (i) to dealership (j)] x ([operational costs/mile] + [fuel cost/gallon] / [fuel efficiency (mile/gallon)]) + [route tolls and fees from manufacturing plant (i) to dealership (j)]} / [carrying truck capacity]

= {[miles from manufacturing plant (i) to dealership (j)] x ($\alpha_{1,2,3}$ + [r_1] / [u_1]) + [route tolls and fees from manufacturing plant (i) to dealership (j)]} / [u_2]

= {[$422x_{11} + 384x_{12} + 638x_{13} + 79x_{14} + 492x_{15} + 216x_{21} + 196x_{22} + 335x_{23} + 306x_{24} + 587x_{25} + 651x_{31} + 214x_{32} + 594x_{33} + 506x_{34} + 227x_{35}$] x (0.55 + normal (2.50, 0.5) / 8) + [$25x_{11} + 40x_{12} + 50x_{13} + 0x_{14} + 30x_{15} + 0x_{21} + 20x_{22} + 0x_{23} + 20x_{24} + 40x_{25} + 40x_{31} + 20x_{32} + 30x_{33} + 50x_{34} + 20x_{35}$]} / 9

Subject to:

$f_2(x_{i,j})$ = the number of cars shipped between manufacturing plant and dealership

$$= x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{31} + x_{32} + x_{33} + x_{34} + x_{35} \geq \varepsilon$$

$$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = 100$$
$$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} = 250$$
$$x_{31} + x_{32} + x_{33} + x_{34} + x_{35} = 200$$
$$x_{11} + x_{21} + x_{31} = 125$$
$$x_{12} + x_{22} + x_{32} = 60$$
$$x_{13} + x_{23} + x_{33} = 80$$
$$x_{14} + x_{24} + x_{34} = 175$$
$$x_{15} + x_{25} + x_{35} = 110$$
$$x_{ij} \geq 0,\ i = 1,2,3 \ \ j = 1,2,3,4,5$$

f_1 $(x_{i,j})$ can be described as the costs for sending each fully-loaded truck from the manufacturing location to a dealership. An assumption is that any cars not shipped in a one-week period (due to waiting for a fully-loaded carrier truck) will be shipped the following week. This remainder cost for shipping, however, will be factored into the originating week's total, which produces a meaningful shipping cost representation for weekly comparison.

f_2 $(x_{i,j})$ is described as the total number of new cars transported to dealerships. The ε-constraint can be varied to determine the tradeoffs between costs (f_1) and maximizing the cars shipped (f_2). An assumption is that the focus is only on truck costs and shipments from manufacturer to dealership.

We choose to model the random variable r_1, the fuel cost/gallon, from data given by the Department of Transportation website. Based upon recent data trends, we choose a *normal* (\$2.50, \$0.50) representation of diesel fuel pricing.

Figure XII.2.1 graphically shows the diesel price changes between years 2004-2005 and 2005-2006. This data was gathered from Gasoline and Diesel Fuel Update provided by the Energy Information Administration (EIA), a statistical agency of the United States Department of Energy.

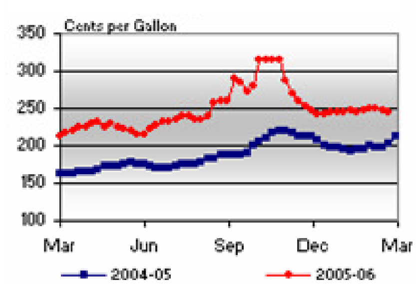

Figure XII.2.1. Diesel Fuel Prices between 2004 and 2006

Simulation

We simulate five trial runs representing individual weeks of operation. They solve the Linear Programming (LP) problem and determine the tradeoff values.

Simulated fuel prices for the five trials are given as follows:

Table XII.2.3. Simulated Fuel Prices

	Run				
	1	2	3	4	5
r_1 Cost/Gallon	$2.22	$2.78	$2.49	$2.91	$3.27

Since the total production capacity of the three plants is 550 vehicles, they set ε equal to 550, which is the maximum number of cars that can be shipped. Ideally, this is the weekly basis on which the needed supply reaches the dealerships—where it can be moved and sold—not sitting in a factory parking lot, which is why we choose to ship the maximum cars possible. Though not modeled for this problem, the ε-constraint can be reduced to lower levels, such as 500 car shipments per week. Doing so will allow the analyst to calculate the minimum transportation costs and choose the best routes.

Simulation results are given as follows:

Table XII.2.4. Results for Transportation Costs and Cars Shipped

	Run				
	1	2	3	4	5
r_1 Cost/Gallon	$2.22	$2.78	$2.49	$2.91	$3.27
f_1 Transportation costs	$12,455.63	$13,454.76	$12,937.35	$13,686.70	$14,329.00
f_2 Cars shipped	550	550	550	550	550

Table XII.2.5. Results for Car Shipments from Manufacturer (i) to Dealership (j)

Manufacturer (i)	Memphis	Atlanta	Jackson	Louisville	Raleigh	Output
Lexington	0	0	0	100	0	100
Huntsville	125	0	80	45	0	250
Columbia	0	60	0	30	110	200
Output	125	60	80	175	110	

Each of the five runs produced the exact results for the routes $x_{i,j}$. This shows that the route patterns are consistent for the range of diesel prices and the constraint on number of cars shipped.

Compute Expected Values

Arithmetic means were calculated from the five trials to determine the expected minimum transportation costs *(f_1)* and cars shipped *(f_2)*.

With the average diesel fuel cost equal to $2.73 per gallon, the expected minimum transportation costs *(f_1)* were $13,373 per week. This allows 550 cars to be transported to the dealerships, thus meeting their sales demands.

One interesting ratio is f_1 / f_2. For this optimization, the minimum transportation cost/car shipped is $24.31.

In summary:

$\bar{f}_1 {}^*(x_{i,j})$ = $13,373/week

$\bar{f}_2 {}^*(x_{i,j})$ = 550 cars/week

where $x^X_{i,j}$ = (0,0,0,100,0,125,0,80,45,0,0,60,0,30,110)

Use the Surrogate Worth Tradeoff (SWT) Method

This part of the MSM yields some interesting relationships. First, we find the tradeoff values ($\lambda_{12} = \dfrac{-\partial f_1}{\partial f_2}$), which is the dollar amount for an additional car shipped. The results are shown in Table XII.2.6 below.

Table XII.2.6. Tradeoffs ($/Additional Car Shipped) for Shipments from Manufacturer (i) to Dealership (j)

	Run				
	1	2	3	4	5
r_1 Cost/Gallon	$2.22	$2.78	$2.49	$2.91	$3.27
f_1 Transportation costs	$12,455.63	$13,454.76	$12,937.35	$13,686.70	$14,329.00
f_2 Cars shipped	550	550	550	550	550
Lambda$_{12}$ Dollars per additional car shipped	$52.52	$56.68	$54.53	$57.65	$60.33

The average tradeoff value (λ_{12}) for the optimization is $56.34 per additional car shipped.

Though this is an important relationship at a "global" level, the tradeoffs in terms of cost for the decision variables $x_{i,j}$ are equally revealing at a "local" level. As each $x_{i,j}$ is a decision variable, it possesses a cost tradeoff for deciding whether a car should be shipped on that route. The solutions provided (Table XII.2.5) represent Pareto-optimality. These choices of routes minimize costs while meeting shipment demand. Changing one of these choices can only result in a negative effect (increased cost) on the objective function (f_1).

Table XII.2.7 below presents the output for the reduced costs of each decision variable. Reduced cost is interpreted as the amount of penalty to be paid to introduce one unit of that variable into the solution. Examining the problem in this way allows the decisionmaker to see how changing either the minimized cost solution or the shipment requirements would affect weekly shipping costs. Perhaps the company prefers one particular route over another due to social or political considerations. Table XII.2.7 allows them to quickly determine the increased costs due to changing the optimization.

Table XII.2.7. Average Tradeoffs/Reduced Costs ($/Car Shipped) for 5 Runs

Average Reduced Costs	Memphis	Atlanta	Jackson	Louisville	Raleigh
Lexington	$47.9	$66.9	$60.3	$	$75.3
Huntsville	$	$21.4	$-	$	$61.0
Columbia	$24.4	$	$5.8	$	$

For example, if the company wished to add to the optimization 9 cars from Lexington to Memphis, a route that is currently not taken, the additional average weekly cost would become $47.9 x 9 = $431.10 greater.

The surrogate worth function (denoted by W_{12}) represents the decisionmakers' assessment as to how much they are willing to trade in dollars for shipping one additional unit (or vehicle). If $W_{12} = 0$, this means the company would be satisfied to spend $56.34 ($\lambda_{12}$) to ship an additional car, or to save that amount by not shipping one. This is the case, or the "indifference band," when the proper decision has been made. We are assuming that $W_{12} = 0$ for our optimization, since we have satisfied the objective functions of minimizing weekly transportation costs and maximizing car shipments. If the W_{12} was less than or greater than zero, a solution set would be preferred over the model's solution set, and we would need to reevaluate our model.

ANALYSIS

In sum, the goals of the car manufacturing company were to minimize cost and increase the number of cars shipped to various dealerships. Below are three summary plots of the analysis.

Figure XII.2.2 displays the Pareto-optimal value for the minimized cost objective function (f_1) vs. tradeoffs λ_{12}. This shows that as transportation costs increase, which is due primarily to increased diesel pricing, the tradeoff costs for increasing shipments goes up. This makes sense intuitively. The same relationship is true in Figure XII.2.3, which shows that as diesel costs increase, so do the tradeoff costs for increasing shipments. Figure XII.2.4 is along the same interpretation—as diesel costs increase, so do the minimized transportation costs.

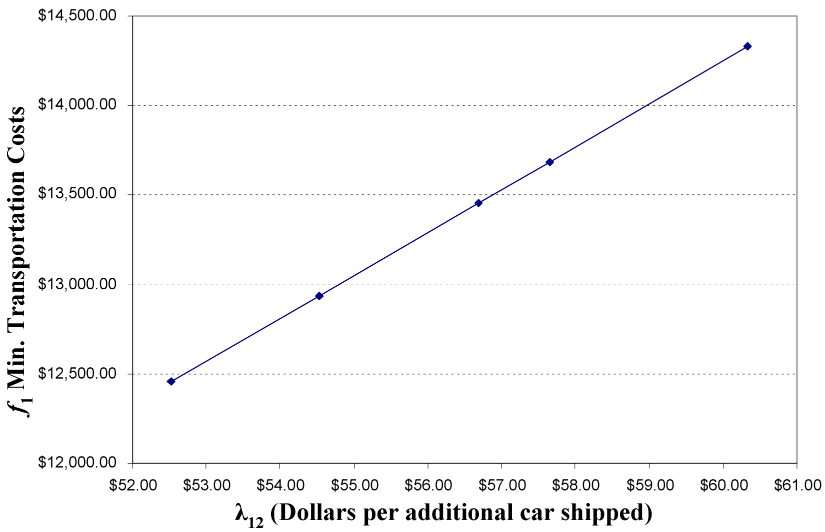

Figure XII.2.2. Transportation Costs (f_1) vs. Dollars per Additional Car Shipped (λ_{12})

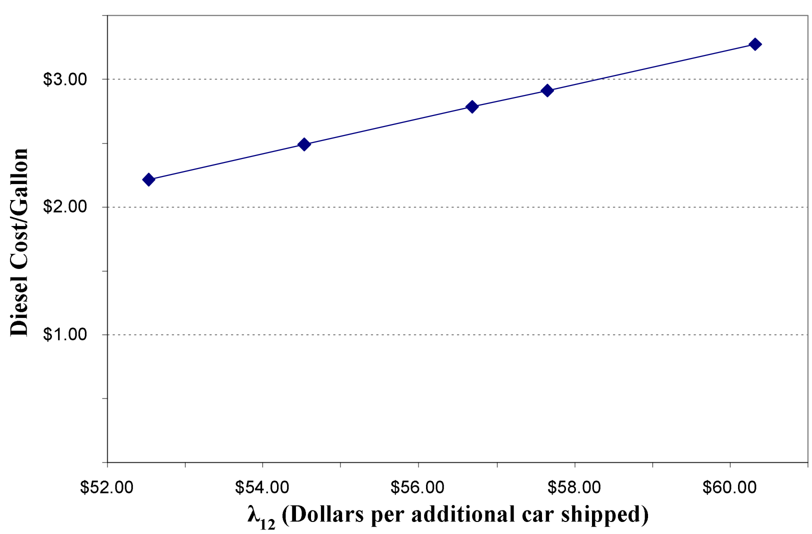

Figure XII.2.3. Diesel Cost/Gallon (r_1) vs. Dollars per Additional Car Shipped (λ_{12})

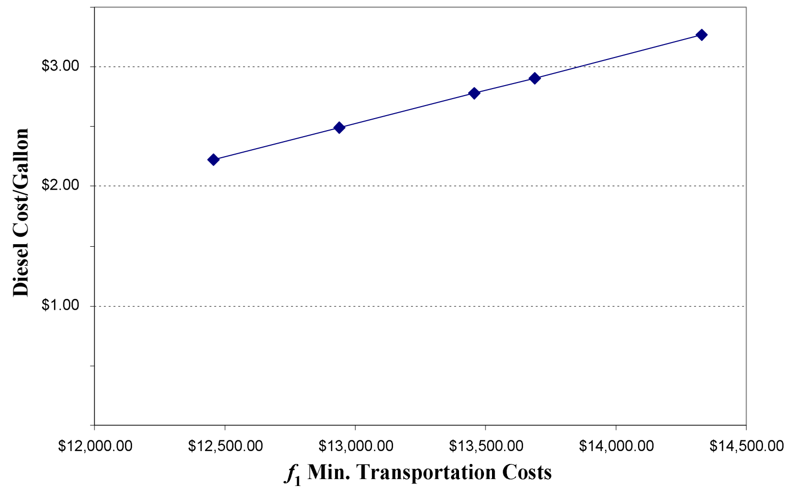

Figure XII.2.4. Diesel Cost/Gallon (r_1) vs. Transportation Costs (f_1)

Regarding a plot of f_1 vs. f_2, we chose not to show this in the report since f_2 is held constant at 550 cars for the ε constraint. One analysis that would be interesting to perform as a next step would be the change in λ_{12} as epsilon is decreased. This would simulate the effect of reduced demand with steady production, and tradeoff costs associated with new optimized routes.

Applying the Multiobjective Statistical Method (MSM), we incorporated system objectives, constraints, and decisions. This included the six system variables to understand their effects on the transportation process, a questionnaire to determine the constraints, previous and current decisions, and additional information to gain further perspective into the transportation problem.

We then determined the objective functions and mathematical representation of constraints to best analyze the transportation system. The simulation to compute the optimum shipping routes allowed us to find the optimal solutions for minimizing transportation costs while maximizing cars shipped among the five scenarios simulated for gas pricing. With this, we determined the optimal number of cars to ship from manufacturer to dealership, thus satisfying the goals of the decisionmaker. This allowed the car company to specify the expected performance values for the entire system.

We determined the surrogate worth tradeoff (SWT), which allows for a combination of multiple λs. In this case, the value was found to be zero, which is a result of the solution set being optimal.

Using MSM, the analyst and outside individuals are able to better understand and justify the development process from problem statement to final solution. MSM gives us a framework to follow, ensuring that all pertinent and necessary information is examined. In the same respect, the process allows the analyst to explore proper methods to solve the problem, without having to use one predefined technique or method.

PROBLEM XII.3: Developing a Wetlands Mitigation Plan

A state Department of Environmental Quality (DEQ) needs to develop a wetlands mitigation plan for an important river. A subproject task is to design and develop anti-erosion measures.

DESCRIPTION

Increases in construction, both residential and commercial, as well as in recreational use have led to noticeable erosion along the river's banks; adversely affecting the wetlands. The objectives for the anti-erosion plan must minimize the number of acres in the wetlands *i*:

- that are lost to erosion, and
- that are disturbed by anti-erosion activities

METHODOLOGY

The Multiobjective Statistical Method (MSM) will be used to compare and contrast the impact of different anti-erosion plans. The MSM allows an analyst to express risk in two formats: 1) as state variables, such as in the initial modeling stages; and 2) as a function of decision variables, such as during the optimization/tradeoff analysis phase. MSM also allows assessing the different combinations of possible system configurations. An analyst is rarely confronted with an absolute configuration to assess; plan dimensions and amounts can always change.

In this problem, there are three anti-erosion options to consider incorporating into the mitigation plan. Each of these will save a given number of acres from erosion; in turn, saving this acreage will affect another given number of acres. Policy guidelines mandate choosing the plan that will save the most acres and adversely affect the least acres for the least amount of money.

Reformulate the objectives into state variables:

> Define $W1(\mathbf{x};\ \eta_i,\ r_m)$ as the number of acres in Wetlands 1 involved in Plan x_k
> Define $W2(\mathbf{x};\ \eta_i,\ r_m)$ as the number of acres in Wetlands 2 involved in Plan x_k
>
> x_k, $k = 1, 2$, or 3 represents a mitigation plan
> η_i represents the acreage saved by Plan x_k
> r_m represents the acreage affected by Plan x_k

The number of acres in wetlands *i* can be expressed as a function of the two state variables:

- that are lost to erosion, and
- that are disturbed by anti-erosion activities

The objective functions are:

$\min f_1(x) = \text{cost}$

$\min f_2(r_m) = \text{acreage affected by Plan } x_k$

$\max f_3(\eta_i,) = \text{acreage saved by Plan } x_k$

(Alternatively, $\min f_3(x)$ = acreage lost to erosion by implementing Plan x_k)

The decision variables are of the following forms:

x_{ij} use Plan i for Wetlands j (j = 1, 2)

x_{ijk} acres saved (k = 1) or affected (k = 2) by using Plan i for Wetlands j

The x_{ij} variables are indicator (0, 1) variables.

Data from a similar river project last year was used as the cost basis for creating the objective functions and constraints. The data are shown in Table XII.3.1.

Table XII.3.1. Base Data from Past River Project

	Plan 1	Plan 2	Plan 3
# Acres Saved - W1	17	8	15
Cost/Acre	$2,231	$4,477	$1,289
Cost - Saved W1	$37,927	$35,816	$19,335
# Acres Affected - W1	13	7	1
Cost/Acre	$2,135	$3,997	$2,685
Cost - Affected W1	$27,755	$27,979	$2,685
Total Cost - W1	$65,682	$63,795	$22,020
# Acres Saved - W2	9	3	20
Cost/Acre	$4,594	$2,360	$2,213
Cost - Saved W2	$41,346	$7,080	$44,260
# Acres Affected - W2	14	18	16
Cost/Acre	$4,857	$3,517	$3,457
Cost - Affected W2	$67,998	$63,306	$55,312
Total Cost - W2	$109,344	$70,386	$99,572
Total Cost	$175,026	$134,181	$121,592
Total Acres Saved	26	11	35
Total Acres Affected	27	25	17

where W1 = ‘Wetlands 1’ and W2 = ‘Wetlands 2’

Stating the objectives in the ε-constraint form results in:

$$\min f_1\ (\mathrm{x})$$

subject to

$$f_2\ (\mathrm{r_m}) <= \varepsilon_2$$
$$f_3\ (\eta_{\mathrm{i},}) >= \varepsilon_3$$

The linear equation for f_1 is:

$$f_1(x) = (2231x_{111} + 2135x_{112})x_{11} + (4477x_{211} + 3997x_{212})x_{21} + (1289x_{311} + 2685x_{312})x_{31} + (4594x_{121} + 4857x_{122})x_{12} + (2360x_{221} + 3517x_{222})x_{22} + (2213x_{321} + 3457x_{322})x_{32}$$

Note: f_2 and f_3(x) are dependent on f_1.

The overall problem is thus formulated as:

$$\begin{aligned}\min f_1(\mathrm{x}) = {} & (2231x_{111} + 2135x_{112})x_{11} + (4477x_{211} + 3997x_{212})x_{21} \\ & + (1289x_{311} + 2685x_{312})x_{31} + (4594x_{121} + 4857x_{122})x_{12} \\ & + (2360x_{211} + 3517x_{212})x_{22} + (2213x_{311} + 3457x_{312})x_{32}\end{aligned}$$

Subject to:

$$x_{112} + x_{122} \leq \varepsilon_2$$
$$x_{212} + x_{222} \leq \varepsilon_2$$
$$x_{312} + x_{322} \leq \varepsilon_2$$

(f_2 - minimize acres affected in Wetlands j under Plan i)

$$x_{111} + x_{121} \geq \varepsilon_3$$
$$x_{211} + x_{221} \geq \varepsilon_3$$
$$x_{311} + x_{321} \geq \varepsilon_3$$

(f_3 - maximize acres saved in wetlands j under plan i)

$$x_{\mathrm{i}11} + x_{\mathrm{i}12} \leq 20$$
$$x_{\mathrm{i}21} + x_{\mathrm{i}22} \leq 25$$

(Wetlands 1 is 20 acres in size; Wetlands 2 is 25 acres in size.)

$x_{\mathrm{ijk}} \geq 0$ for all i, j, and k

$x_{\mathrm{ij}} = 0$ or 1 integer constraints on indicator variables

SOLUTION

The initial set of scenarios can now be solved, given the following additional constraints:

- cost cannot exceed $200,000,
- total acreage affected must be no more than 10% of the total acreage, and
- total acreage saved must be at least 50% of the total acreage.

The optimal solutions for saving a certain number of acres are found given these parameters. The total cost target is varied to find out which plan can save the most acreage and affect the least acreage. The results are presented in Table XII.3.2.

Table XII.3.2. Pareto-Optimal Solutions—Scenario 1

Plan	Cost	Acres Saved
3	$50,000.00	23.4415883
3	$75,000.00	34.9026209
1	$100,000.00	26.74304711
1	$125,000.00	32.32133528
1	$150,000.00	38.18099695
1	$160,653.50	40.5

Note that each solution is based on affecting the maximum allowed acreage, 4.5 acres. The non-Pareto-optimal solutions in Table XII.3.3 were found by setting the amount of acreage saved under the optimal figures and solving for the implementation cost.

Table XII.3.3. Non-Pareto-Optimal Solutions—Scenario 1

Plan	Cost	Acres Saved
3	$36,037.50	15
3	$47,102.50	20
1	$68,250.00	20
1	$79,405.00	25
2	$84,196.50	20

ANALYSIS

Figure XII.3.1 shows the Pareto Optimal Curve for the Scenario 1. The “stars” on the graph indicate positions of non-optimal solutions. The turquoise lines indicate the initial “band of indifference.” The tradeoff vector, λ, was determined by ocular inspection and a review of Table XII.3.2. Spending approximately $11,000 more results in saving approximately 2 more acres of wetlands; at that point, the project is in the realm of “diminishing returns.”

Note that Plan 2 was not selected in this iteration. Plan 2 is the most expensive plan to implement ($6,837 to save one acre and affect another).

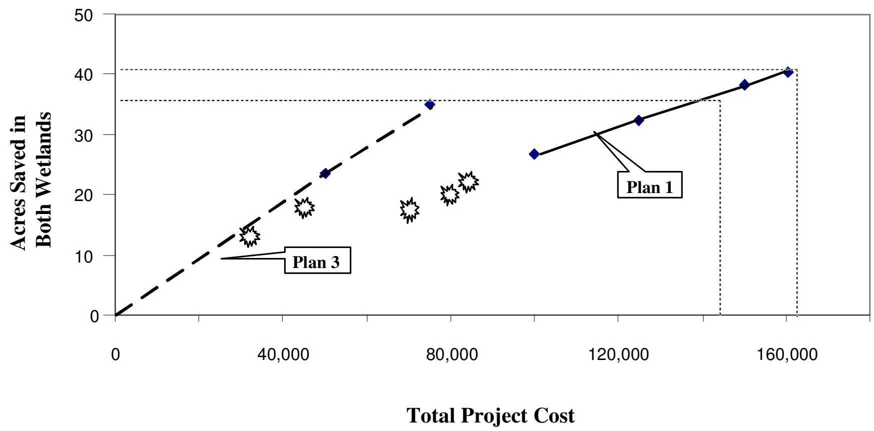

Figure XII.3.1. Pareto-Optimal Curve—Scenario 1

How much impact does varying the epsilon values have on the number of acres saved given an expenditure of $150,000? (This is the amount that an initial analysis found will provide the best value for the money.)

The result of varying ε_2 between .05 and .25, the range of the percentages of affected acreage in similar projects in the past, and varying ε_3 between .25 and .75, the range of the percentages of saved acreage in the past, is presented in Table XII.3.4.

Table XII.3.3. Acreage Saved

Epsilon 2	Epsilon 3	Acres Saved - W1	Acres Saved - W2	Total Acres Saved	Plan Used
0.12	0.55	19.155	23.333	42.488	1
0.05	0.75	19.155	21.009	40.164	1
0.25	0.75	19.155	21.009	40.164	1
0.25	0.6	19.134	20.965	40.099	1
0.05	0.6	19.157	21.011	40.168	1
0.05	0.7	19.157	21.011	40.168	1
0.2	0.8	19.157	21.011	40.168	1

Plan 1 is selected for use when given a budget of $150,000.

The different values of ε_2 and ε_3 were determined through simulation based on the MSM method. The expected values of ε_2 and ε_3 according to this simulation, .13857 and .67857, respectively, resulted in a solution of W1 acres saved of 19.155 and W2 acres saved of 21.009—data points already found by the simulation.

Each of the above simulations resulted in .01063 W1 acres affected and 2.208 W2 acres affected. Thus, the next decision is based solely on the number of acres saved. Figure XII.3.2 illustrates the findings of Table XII.3.4.

Figure XII.3.2. Acres Saved (MSM)

Given the current constraints, the best solution is to implement Plan 1 and spend \$150,000 to save 19.155 Wetlands 1 acres and 23.333 Wetlands 2 acres.

Note: The data in Table XII.3.1 was from the Pamunky, VA river project, and the epsilon values were created by using the simulation technique RANDBETWEEN in Excel.

PROBLEM XII.4: Football Team Success Strategy

The coach of a professional football team wants to know when and how often to rush the ball in order to gain the most points.

DESCRIPTION

We refer to the records of the past two football seasons to derive the figures for total points earned, turnovers, rush plays, and conversion attempts.

METHODOLOGY

We use the Multiobjective Statistical Method (MSM) to analyze the total points earned $f_1(\cdot)$ and total turnover $f_2(\cdot)$ during the last 2 years of games between a professional football team and its opponents.

SOLUTION

The mathematical model for this problem is shown as follows:

$$\min f_1(x_1,x_2;r_1,r_2) = -1\{5.8-38x_1+104x_1^2+4.88x_2^2-11.5x_2+20.2r_1-0.49r_2+1.4x_1x_2\} \tag{XII.4.1}$$

$$\min\ f_2(x_1,x_2;r_1,r_2) = 0.9-0.7x_1+0.5x_2+0.5r_1+0.3r_2 \tag{XII.4.2}$$

where the objective functions are:

$f_1(x_1,x_2;r_1,r_2)$ = -1 × total points earned
$f_2(x_1,x_2;r_1,r_2)$ = total turnover

The decision variables are:

x_1 = ratio of total rush plays
x_2 = 4th down conversion attempts

The random variables are given as follows:

r_1 = ratio of total rush plays by other teams
r_2 = 4th down conversion attempt by other teams

These variables are normally distributed as shown in Figures XII.4.1 and XII.4.2 below:

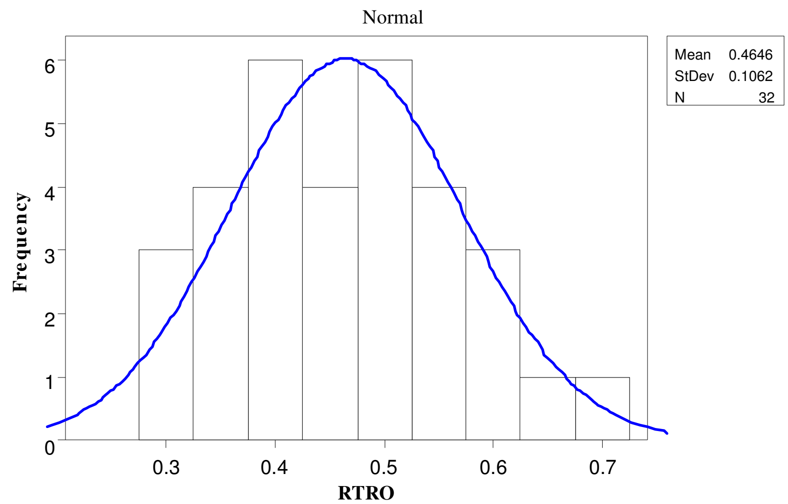

Figure XII.4.1. Histogram of Ratio of Total Rush Plays by Other Teams

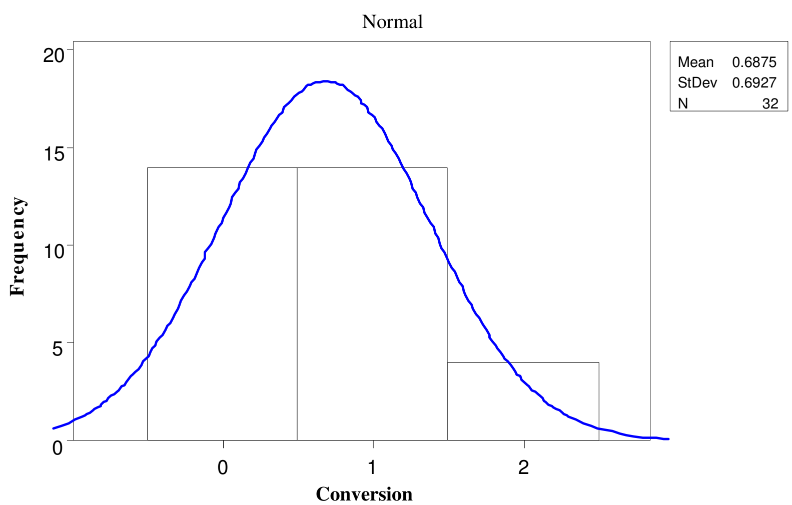

Figure XII.4.2. Histogram of 4th Down Conversion Attempts by Other Teams

For the purpose of implementing the MSM, we need to calculate the expected values of two objective functions, substitute the expected values of the random variables into the objective functions, and then obtain their expected values:

$$\min f_1(x_1, x_2; r_1, r_2) = -1\{5.8 - 38x_1 + 104x_1^2 + 4.88x_2^2 - 11.5x_2 + 20.2(0.46) - 0.49(0.69) + 1.4x_1x_2\} \quad \text{(XII.4.3)}$$

$$\min f_2(x_1, x_2; r_1, r_2) = 0.9 - 0.7x_1 + 0.5x_2 + 0.5(0.46) + 0.3(0.69) \quad \text{(XII.4.4)}$$

With Eqs. (XII.4.3) and (XII.4.4), we can formulate the Lagrangian method:

$$L = -14.75 + 38x_1 - 104x_1^2 - 4.88x_2^2 + 11.5x_2 - 1.4x_1x_2 + \lambda_{12}\{0.9 + 0.5(0.46) + 0.3(0.69) - 0.7x_1 + 0.5x_2 - \varepsilon_2\} \quad \text{(XII.4.5)}$$

The non-negativity condition of the decision variables, x_1, x_2, simplifies the Kuhn-Tucker conditions:

$$\frac{\partial L}{\partial x_1} = 38 - 208x_1 - 1.4x_2 + \lambda_{12}(-0.7) = 0 \quad \text{(XII.4.6)}$$

$$\frac{\partial L}{\partial x_2} = 11.5 - 9.76x_2 - 1.4x_1 + \lambda_{12}(0.5) = 0 \quad \text{(XII.4.7)}$$

$$\lambda_{12} = \frac{38 - 208x_1 - 1.4x_2}{0.7} = \frac{-11.5 + 9.76x_2 - 1.4x_1}{0.5} \quad \text{(XII.4.8)}$$

After solving Eq. (XII.4.8) for x_1, we get:

$$x_1 = -0.072x_2 + 0.2577 \quad \text{(XII.4.9)}$$

Using the Eq. (XII.4.8) and (XII.4.9), we can calculate the noninferior solutions for this problem. Table XII.4.1 summarizes the noninferior solutions and corresponding tradeoff values. In Figures XII.4.3 and XII.4.4, those Pareto optimal solutions are plotted in functional space and in decision space, respectively.

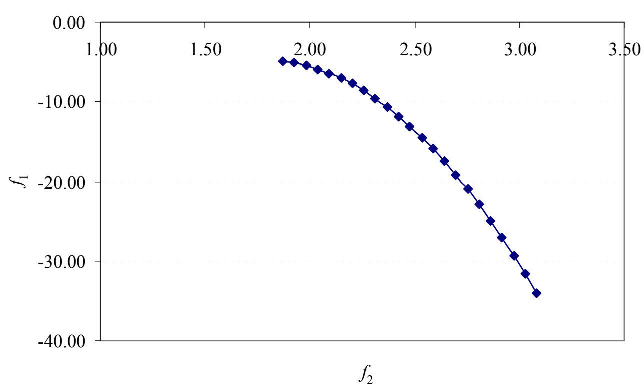

Figure XII.4.3. Pareto-Optimal Solutions in the Functional Space

Table XII.4.1. Noninferior Solutions and Tradeoff Values

x_1	x_2	λ_{12}	f_1	f_2
0.16	1.30	2.92	-4.91	1.87
0.16	1.40	4.86	-5.12	1.93
0.15	1.50	6.80	-5.44	1.98
0.14	1.60	8.74	-5.86	2.04
0.14	1.70	10.68	-6.39	2.09
0.13	1.80	12.62	-7.03	2.15
0.12	1.90	14.56	-7.77	2.20
0.11	2.00	16.50	-8.62	2.26
0.11	2.10	18.44	-9.57	2.31
0.10	2.20	20.38	-10.63	2.37
0.09	2.30	22.32	-11.80	2.42
0.08	2.40	24.26	-13.07	2.48
0.08	2.50	26.20	-14.45	2.53
0.07	2.60	28.14	-15.94	2.59
0.06	2.70	30.08	-17.53	2.64
0.06	2.80	32.02	-19.23	2.70
0.05	2.90	33.96	-21.03	2.75
0.04	3.00	35.89	-22.95	2.81
0.03	3.10	37.83	-24.96	2.86
0.03	3.20	39.77	-27.09	2.92
0.02	3.30	41.71	-29.32	2.97
0.01	3.40	43.65	-31.66	3.03
0.01	3.50	45.59	-34.10	3.08

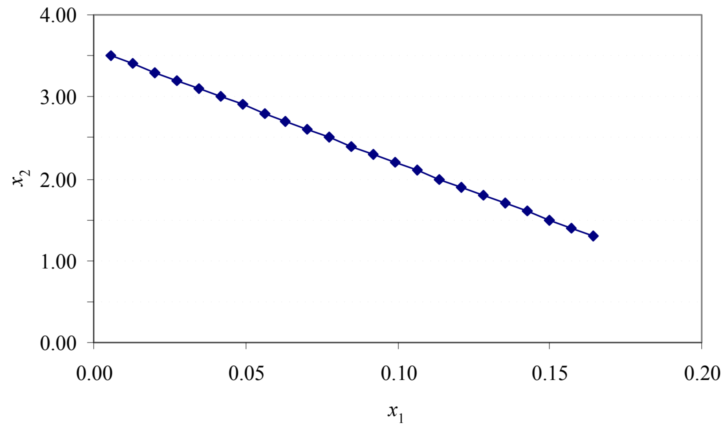

Figure XII.4.4. Pareto-Optimal Solutions in the Decision Space

ANALYSIS

According to our findings, it would be optimal to rush the ball somewhere between 1% and 16% of the time, which would mean going for it on the 4th down 1.5 to 3.5 times, respectively. This would result in anywhere between 5 and 34 points scored, and 1.87 to 3.08 turnovers per game. All of these conclusions can be seen above in Table XII.4.1, and in Figures XII.4.3 and XII.4.4.

State variables, total playing time, and total yards earned are here the functions of decision variables given the random variables. Thus, in this problem, the state variables are not represented in the objective functions; however, they are strongly related to the decision variables.

PROBLEM XII.5: Renovating Manufacturing Assembly Lines

A consulting firm is in charge of renovating an outdated manufacturing process for an automobile manufacturer.

DESCRIPTION

There are three identical and independent manufacturing lines. Renovating is an imperative as the current process takes too much time and costs too much money. The consulting firm has determined the five best remodeling policies. These are to renovate:

A – one line at time
B – A first, then B and C simultaneously
C – B first, then A and C simultaneously
D – C first, then A and B simultaneously
E – all lines simultaneously

In addition to choosing the policy, the consulting group can dictate how many people should work on the renovation. Some of these workers will be drawn from another process line, which will reduce the production on that line (i.e., $150/person)/hour). Based on the responses obtained in a survey, no fewer than 10 and no more than 20 workers should be taken from the other lines for renovation purposes. Also, because renovation is a different skill set, there is a $7000/person training cost associated with moving a worker to renovation. The objectives are to reduce the cost of renovating the process while minimizing the number of customers the company loses by not being able to supply enough cars.

METHODOLOGY

We solve this problem using the Multiobjective Statistical Method (MSM). The different policies have different project overrun risks. Table XII.5.1 outlines the explicit values. Basically, choosing to renovate one line at a time is the least risky (i.e., P(10000 hours of delay) = .1) but has the smallest chance to be completed on time (i.e., P(1000 hours of delay) = .1). Likewise, renovating all 3 lines simultaneously is the riskiest (i.e., P(10000 hours of delay) = .3) but also has the largest chance to finish on time (i.e., P(1000 hours of delay) = .3).

Decision variables

1. policy overhaul - { p_a p_b p_c p_d p_e }
2. N - number of people to pull off assembly line - {$10 \leq$ # of people ≤ 20 }

Random variable

1. delay = delay for completing the project (hours)

Table XII.5.1 represents the probabilities of hours of delay. For example, for Policy A, there is a 0.1 probability of 1000 hours of delay. This data was obtained from a series of similar projects completed in the past. This data was then validated through a questionnaire.

Table XII.5.1. Summary of Overrun Risk Probabilities

policy		Hours of delay				
		0	1000	2000	5000	10000
A	1 at a time	0	0.1	0.4	0.4	0.1
B	a then bc	0	0.15	0.35	0.35	0.15
C	b then ac	0	0.2	0.3	0.3	0.2
D	c then ab	0	0.25	0.25	0.25	0.25
E	all lines	0	0.3	0.2	0.2	0.3

State variables

The state variable is overhaul time= 10000 man-hours (this is the baseline estimate for the project) plus any hours of delay (the random input based on our policy selection).

Overhaul Time → OT(policy, delay, N) = {10000 + delay(policy) }/N

Based on our study the following two objective functions were developed.
Objective functions:

1. minimize f_1, \$ renovation loss →
   $f_1(\cdot)$ = (\$7000 training/worker) • N
   + (\$150/hour prodution. lost)•OT(policy, delay, N)

2. minimize f_2, lost customers →
   $f_2(\cdot)$ = 0.25 * OT(policy, delay,N)/20000

SOLUTION

The *cumulative distribution functions* (*CDF's*) for the five policy options:

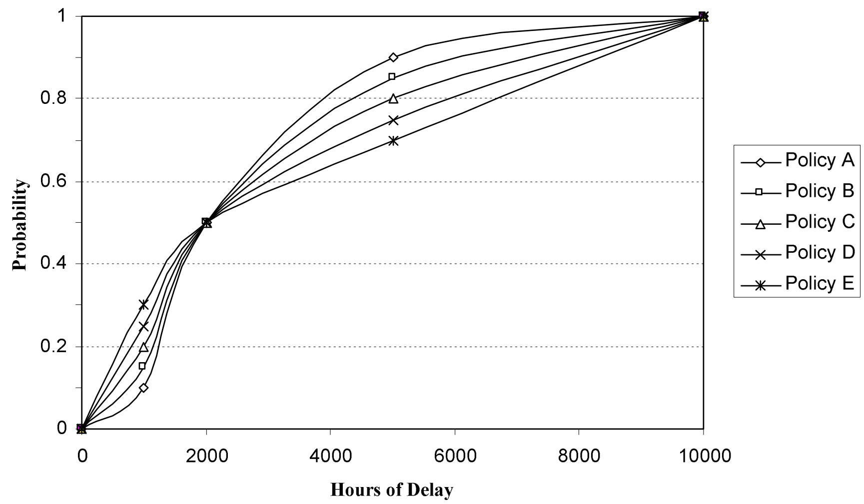

Figure XII.5.1. CDF's for the Five Policy Options

Figure XII.5.2 show *Exceedence probability functions* for the five policies:

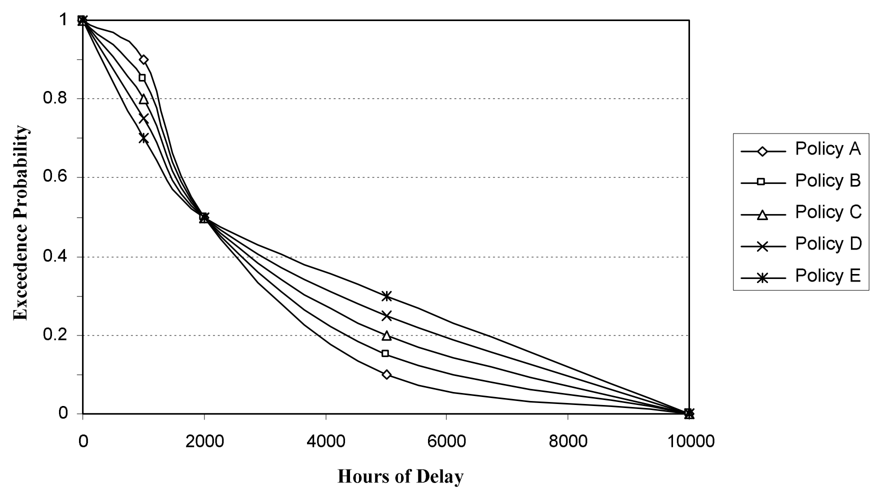

Figure XII.5.2. Exceedance probabilities for the five policy options.

The consulting company calculates the expected $ loss of each policy. Figure XII.5.3 shows the results.

Figure XII.5.3. Expected loss ($) for each policy

As shown in Table XII.5.2, the minimum values for each policy are

Table XII.5.2. Expected Loss by Policies and Number of Workers

Number of workers	Expected Loss ($)				
	A	B	C	D	E
10	278,500	281,500	284,500	287,500	290,500
11	266,545	269,273	272,000	274,727	277,455
12	257,750	260,250	262,750	265,250	267,750
13	251,385	253,692	256,000	258,308	260,615
14	246,929	249,071	251,214	253,357	255,500
15	244,000	246,000	248,000	250,000	252,000
16	242,313	244,188	246,063	247,938	249,813
17	**241,647**	**243,412**	245,176	246,941	248,706
18	241,833	243,500	**245,167**	**246,833**	**248,500**
19	242,737	244,316	245,895	247,474	249,053
20	244,250	245,750	247,250	248,750	250,250
min value	**241,647**	**243,412**	**245,166**	**246,833**	**248,500**

The consulting firm calculates the **conditional $ expected loss** of each policy. They choose alpha = 0.9, so they look for the worst 10% of the outcome. Table XII.5.3 shows the worst 10% scenario values for overhaul time for each policy.

Table XII.5.3. Overhaul Time in the Worst 10% Scenario

	Worst 10% scenario (man-hours)
Policy A	15,000
Policy B	16,050
Policy C	17,250
Policy D	17,925
Policy E	18,300

Figure XII.5.4 shows the *conditional expected loss* ($) based on the worst 10% case for each policy, and the minimum values for conditional expectation for each policy are shown in Table XII.5.4.

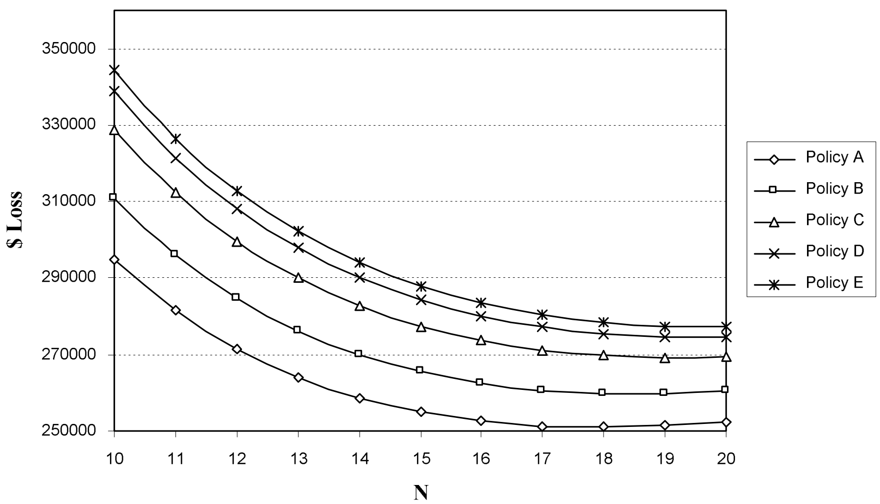

Figure 3. Conditional expected loss ($)

Table XII.5.4. Conditional Expected Loss by Policies and Number of Workers

Number of workers	Conditional Expected Loss ($)				
	A	B	C	D	E
10	295,000	310,750	328,750	338,875	344,500
11	281,545	295,864	312,227	321,432	326,545
12	271,500	284,625	299,625	308,063	312,750
13	264,077	276,192	290,038	297,827	302,154
14	258,714	269,964	282,821	290,054	294,071

Number of workers	Conditional Expected Loss ($)				
	A	B	C	D	E
15	255,000	265,500	277,500	284,250	288,000
16	252,625	262,469	273,719	280,047	283,563
17	251,353	260,618	271,206	277,162	280,471
18	**251,000**	259,750	269,750	275,375	278,500
19	251,421	**259,711**	**269,184**	274,513	277,474
20	252,500	260,375	269,375	**274,438**	**277,250**
min value	**251,000**	**259,711**	**269,184**	**274,438**	**277,250**

The consulting company calculates the **expected percentage of customers lost** for each policy:

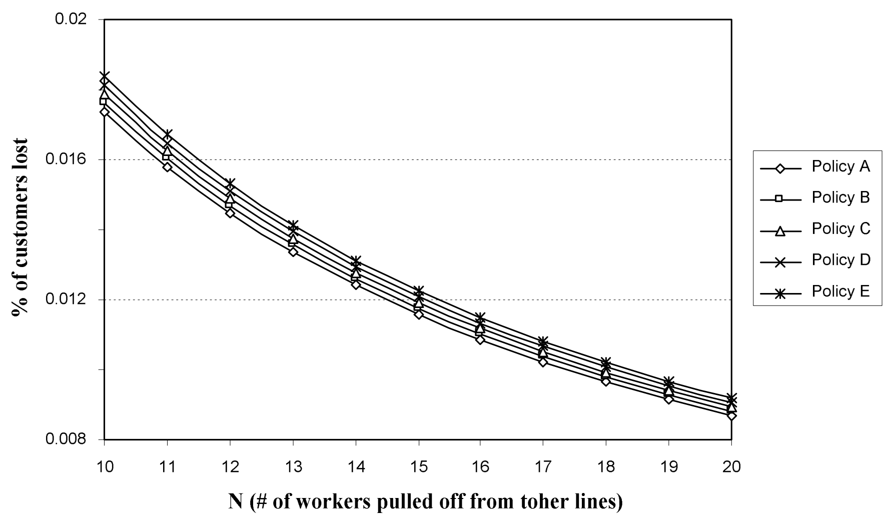

Figure XII.5.5. Expected percentage of customer lost

Table XII.5.5 shows the minimum expected percentage of customers lost:

Table XII.5.5. Expected Percentage of Customers Lost by Policies and Number of Workers

Number of workers	Expected Percentage of Customers Lost				
	A	B	C	D	E
10	0.017375	0.017625	0.017875	0.018125	0.018375
11	0.015795	0.016023	0.016250	0.016477	0.016705
12	0.014479	0.014688	0.014896	0.015104	0.015313
13	0.013365	0.013558	0.013750	0.013942	0.014135
14	0.012411	0.012589	0.012768	0.012946	0.013125
15	0.011583	0.011750	0.011917	0.012083	0.012250
16	0.010859	0.011016	0.011172	0.011328	0.011484
17	0.010221	0.010368	0.010515	0.010662	0.010809
18	0.009653	0.009792	0.009931	0.010069	0.010208
19	0.009145	0.009276	0.009408	0.009539	0.009671
20	**0.008688**	**0.008813**	**0.008938**	**0.009063**	**0.009188**
min value	**0.008688**	**0.008813**	**0.008938**	**0.009063**	**0.009188**

The consulting company calculates the *conditional expected percentage of customers lost* for each policy. The firm chooses alpha = 0.9. This refers to the 10% worst-case scenario. Figure XII.5.6 shows this expected value.

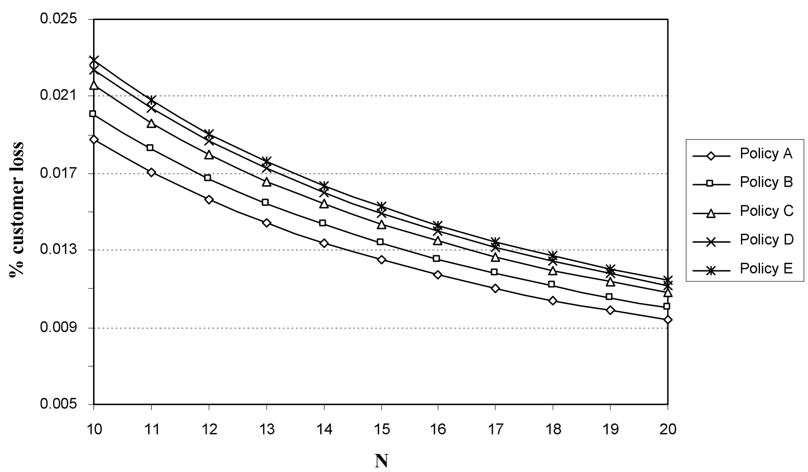

Figure XII.5.6. Conditional expected percentage of customers lost

Table XII.5.6 shows the minimum expected percentage of customers lost:

Table XII.5.6. Conditional Expected Percentage of Customers Lost by Policies and Number of Workers

Number of workers	Conditional Expected Percentage of Customers Lost				
	A	B	C	D	E
10	0.018750	0.020063	0.021563	0.022406	0.022875
11	0.017045	0.018239	0.019602	0.020369	0.020795
12	0.015625	0.016719	0.017969	0.018672	0.019063
13	0.014423	0.015433	0.016587	0.017236	0.017596
14	0.013393	0.014330	0.015402	0.016004	0.016339
15	0.012500	0.013375	0.014375	0.014938	0.015250
16	0.011719	0.012539	0.013477	0.014004	0.014297
17	0.011029	0.011801	0.012684	0.013180	0.013456
18	0.010417	0.011146	0.011979	0.012448	0.012708
19	0.009868	0.010559	0.011349	0.011793	0.012039
20	**0.009375**	**0.010031**	**0.010781**	**0.011203**	**0.011438**
min value	**0.009375**	**0.0100313**	**0.0107813**	**0.011203**	**0.011438**

ANALYSIS

The consulting company judges by four criteria. As seen from Table XII.5.7, Policy A is the best choice for all four criteria, hence the company is recommending Policy A.

Table XII.5.7. Summary of Policy Evaluation

Criteria	Best Policy
Min Expected Loss ($)	**A**
Min Expected Percentage of Customers Lost	**A**
Min Conditional Expected Loss ($)	**A**
Min Conditional Expected Percentage of Customers Lost	**A**

PROBLEM XII.6: Determining Optimal Fuel Dispensing Capacity

A gas station wants to find out how many gas dispensing units (pumps) it needs to install at its new location. It is interested in minimizing the installation cost while maintaining a low waiting time for incoming drivers.

DESCRIPTION

Assume that the gas station can choose from one of four types of pumps, each differing from the rest in the average speed at which it can pump gas. The number of people who arrive at the station at any given time affects the total number of units needed at the station.

METHODOLOGY

Every real-life system that any decisionmaker wishes to address can be characterized by three important components: 1) the factors that affect the system (either random or deterministic), 2) the state variables, and 3) the decisions which the decisionmaker would want to carry out (also called decision variables). Obtaining a mathematical relationship between these components is in general a difficult problem. The Multiobjective Statistical Method (MSM) is one procedure which allows a decisionmaker to integrate those factors affecting the system (input variables) and the decision variables through the state variables. Once this relationship is established, the decisionmaker can use any mathematical tool to optimize the desired objective function(s). It is important to note that the objective functions chosen for this problem are not necessarily exhaustive, and the idea is to merely illustrate the MSM.

SOLUTION

For this system, the number of people who arrive within any given interval of time is the input variable, and the number of people who wait while others pump the gas is a state variable. Assume that the time interval is fixed, say 1 unit of time. The decision variables for this system are the number and type of dispensing units (gas pumps). Installation costs and waiting time are the objective functions.

Let d be the number of pumps at the gas station. The objective functions in this case are the cost of installing the pumps, f_1, and the average waiting time for each customer, f_2. The cost of installation, f_1, is given by:

$$f_1(d,s) = 4000 + 115d^3 - 10s^3 \tag{XII.6.1}$$

where s denotes the average service time of the pumps chosen from the set {1, 3, 5, 7} (in units of time), depending on the gas station's choice of pumps.

Applying the MSM

The gas station owner follows the key steps of MSM to solve the problem:

Step 1. The feasible set of decisions for which f_1 and f_2 are minimized are $d \geq 1$ $(d \in \mathrm{Z})$ and $s \in \{1, 3, 5, 7\}$.

Step 2. The average waiting time, f_2, depends on the number of drivers waiting, *m,* and the average arrival rate of customers, λ¸ as follows:

$$f_2(d,s) = \frac{E[m]}{\lambda} \tag{XII.6.2}$$

where $E[\cdot]$ indicates the expected value, and the cost of installation f_1 is given by (XII.6.1).

Step 3. The average waiting time depends on the rate of customer arrival at the gas station and is denoted by λ. Thus, the arrival process can be modeled as a Poisson random variable with mean arrival rate λ. Since there is a random number in the queue waiting for gas, the number of people serviced by any pump in a given time is also random and is also modeled as a Poisson random variable parameterized by $1/s$. This type of modeling is known as *M*/*M*/*d* queue where *d* is the number of gas pumps.

Let *N* denote the random variable; then the probability of exactly *n* customers arriving during a unit time interval is given by

$$P_N(n) = \frac{\lambda^n e^{-\lambda}}{n!} \tag{XII.6.3}$$

Note that the average number of customers arriving within unit time is $E[N] = \sum_{n=0}^{\infty} nP_n = \lambda$

Step 4. Using the expressions for queue length (i.e., the number of people in the queue, say m) for a Poisson random variable with parameter, the owner obtains:

$$f_2(d,s) = \frac{\rho}{d-\rho}\left[\sum_{n=0}^{d} \frac{\rho^{n-d} d!}{n!} + 1\right]^{-1} \tag{XII.6.4}$$

where $\rho = s\lambda$ is the ratio of the arrival rate to the service rate and is usually less than 1 for practical situations.

Step 5. The objective is to find the values of d and s that minimize the functions f_1 and f_2. Figures XII.6.1 and XII.6.2 show Pareto-optimal solutions in functional space and in decision space, respectively.

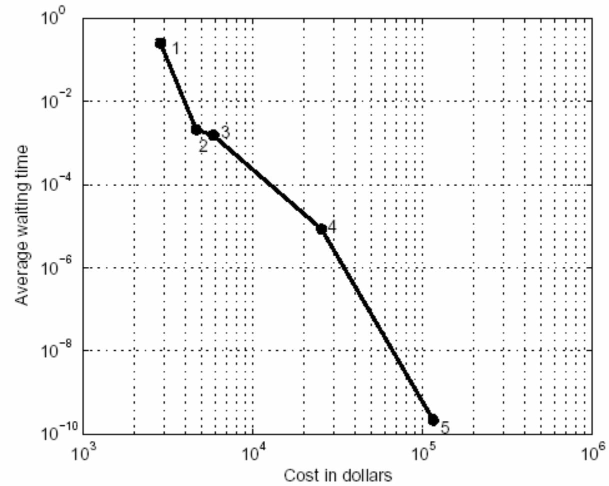

Figure XII.6.1. Pareto-optimal curve for minimizing f_1 and f_2 in functional space

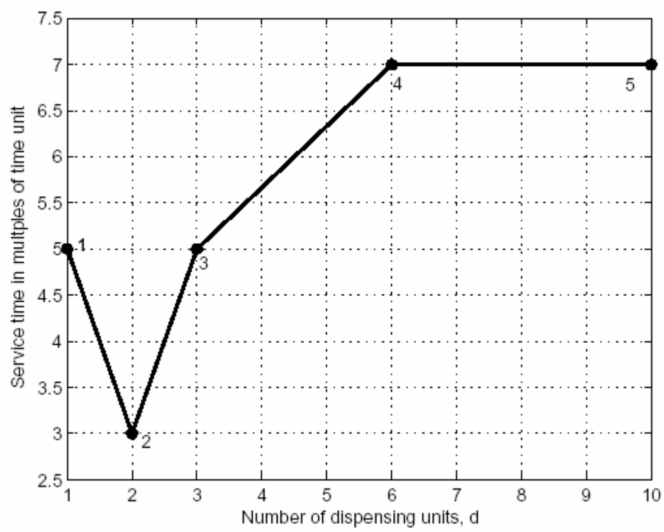

Figure XII.6.2. Pareto-optimal curve for minimizing f_1 and f_2 in decision space

ANALYSIS

The optimization problem is difficult since the constraint region is not continuous (the values of d and s are discrete). To simplify the problem, the objective functions are assumed to be piece-wise continuous. However, the optimization problem cannot be solved analytically. Hence the solution is calculated by numerical optimization, first by simulating the objective functions for various values of *d* and *s,* and then by using multiobjective optimization to find the Pareto-optimal set of solutions.

The Pareto-optimal curve for this optimization in function space is shown in Figure XII.6.1. In Figure XII.6.2 the Pareto-optimal curve is plotted in the decision space. It is now left to the decisionmaker to judiciously choose the values of d and s using the Pareto-optimal curve. The indifference band would probably be around the point marked 2 in Figure XII.6.1, since a small change around that point in one objective function yields a small change in the other objective function.

Similarly, if we look at the decision space, the change of one variable around Point 2 (i.e., (2; 2)) does not change the other variable significantly. Hence, around 2 gas pumps, one with service of around 3 units of time, would be beneficial to the gas station. On the other hand if we look at Points 1, 3, 4, or 5 in the function space, a slight change in cost would result in a huge change in the waiting time, which may or may not be advantageous for the gas station.

PROBLEM XII.7: Ordering Newspapers for Multiple Newsstands

A newsstand company wants to know how many copies of two major newspapers to purchase for its entire chain.

DESCRIPTION

A newsstand company operates numerous newsstands in a large city. They are identical in that all carry a certain volume of the two leading local newspapers. These newspapers will have the same distribution in all the newsstands. How many copies of each newspaper should the newsstand company purchase? And at what cost, in order for the company to make a profit?

METHODOLOGY

We employ the Multiobjective Statistical Method (MSM) to solve this problem.

SOLUTION

The wholesale cost of the newspaper (to the newsstand company) is a function of the state variable. Further assume that, based on past history; all the newspapers that are purchased for the newsstands are sold.

Decision variables

x_1 = number of Newspaper A purchased by the company.
x_2 = number of Newspaper B purchased.

State variables

$s(r)$ = number of city residents/workers that are interested in obtaining news = $16000e^{r/10}$

Random variables

r = events that would cause a fluctuation in the state variable—for instance, number and popularity of news stories that would increase/decrease the interest in reading the news. r is uniformly distributed between 10 and 35 major local and national news stories (where popular stories count as multiple stories).

Questionnaire

To obtain the figures on news stories, we queried a plethora of stakeholders—readers, newsstand owners, newspaper editors, and editorial writers. From this questionnaire, we derived the above probability distributions to fit our findings.

Objective functions

Maximize profit = f_1 = $[0.5 - 3*10^{-8} rs(r)]x_1 + [0.5 - 2*10^{-8} rs(r)]x_2$

Newspapers A and B both charge the newsstand company variable rates depending on the amount of interest in the news. In other words, the price paid is a function of the state variables. The price of Newspaper A is $\$2.6*10^{-8} r\, s(r)$ and the Newspaper B price is $\$1.8*10^{-8} r\, s(r)$. The cost to the consumer of either newspaper is \$0.50. Not included are labor or other costs associated with operating the newsstand.

Minimize shelf space = f_2 = volume taken up by each newspaper =
(volume of A)x_1 + (volume of B)x_2

> volume of A = (0.09)(10)(15) = 13.5 in^3
> volume of B = (0.097)(10)(15) = 14.5 in^3

Constraints

Demand must be met $\Rightarrow x_1 + x_2 \geq (r/350)*s(r) = (r/350)(16000e^{r/10})$

> Note that demand is considerably less than buyer interest as the company has several newsstands and other media outlets with which the store competes.

Number of A purchased by newsstand company $\Rightarrow x_1 \geq (r/350)*s(r)/6 = (r/350)(16000e^{r/10})/6$

> The company cannot purchase less than one-sixth of demand for A.

Number of B purchased by newsstand company $\Rightarrow x_2 \geq (r/350)*s(r)/3 = (r/350)(16000e^{r/10})/3$

> The company cannot purchase less than one-third of demand for B..

Simulation

In order to perform the above analysis, a simulation of 1000 randomly generated r values uniformly distributed on [10, 35] was performed to achieve the expected value of r to calculate f_1, demand, and the minimum number of Newspapers A and B. These numerical values are found in Table XII.7.1.

Table XII.7.1. Simulation Results from Uniformly Distributed *r*

E[r]	22
E[$s(r)$]	193522
E[Profit]	$0.34x_1 + 0.39x_2$
E[Demand]	15052
E[Minimum Posts]	2509
E[Minimum Times]	5107

Another simulation of 1000 r values representing the extreme 10% of values was performed, showing demand in the upper 10% of its distribution. Calculating the conditional expectation of r given the extreme 10% of values produces the following table. This conditional expectation is referred to here as E'[r]. Table XII.7.2 summarizes the extreme value simulation result.

Table XII.7.2. Simulation Results from Extreme Values of *r*

E'[r]	33.8
E'[$s(r)$]	469354
E'[Profit]	$0.02x_1 + 0.18x_2$
E'[Demand]	45343
E'[Minimum Posts]	7557
E'[Minimum Times]	15114

Tables XII.7.1 and XII.7.2 provide information for two multiobjective optimization formulations: one for E[r] and one for E'[r]. Figure XII.7.1 shows the cumulative distribution of the random variable r with the shaded area portraying the extreme 10% of the distribution.

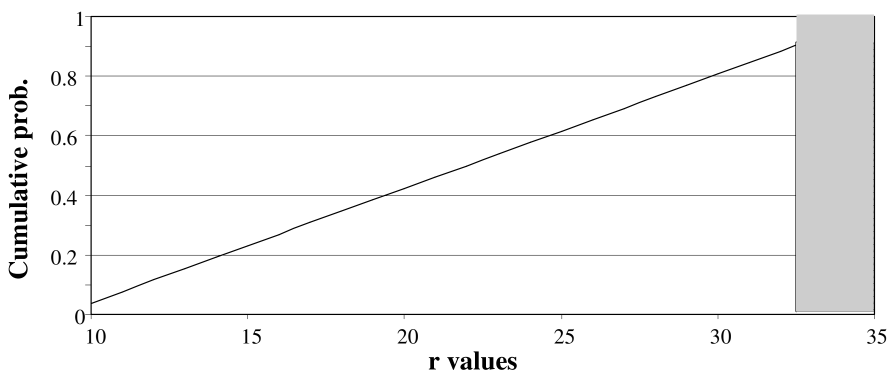

Figure XII.7.1. Cumulative Distribution Function (*cdf*) of *r*

Multiobjective Optimization Problem – Expected Value

$$\begin{aligned} &\max\ 0.39x_1 + 0.42x_2 \\ &\min\ 13.5x_1 + 14.5x_2 \\ &\text{subject to} \quad x_1 + x_2 \geq 15183 \\ &\qquad\qquad\quad x_1 \geq 2530 \\ &\qquad\qquad\quad x_2 \geq 5060 \end{aligned}$$

$$\begin{aligned} &\min —(0.39x_1 + 0.42x_2) \\ &\text{subject to} \quad 13.5x_1 + 14.5x_2 \leq \varepsilon_{21} \\ &\qquad\qquad\quad x_1 + x_2 \geq 15183 \\ &\qquad\qquad\quad x_1 \geq 2530 \\ &\qquad\qquad\quad x_2 \geq 5060 \end{aligned}$$

Table XII.7.3 shows the expected values for 1,350,000 in^3 $\leq \varepsilon_{21} \leq$ 1,500,000 in^3

Table XII.7.3. Expected Values for ε 21

x_1	x_2	f_1	f_2
6535	3226	4066.53	135000
6609	3226	4096.87	136000
6683	3226	4127.21	137000
6757	3226	4157.55	138000
6831	3226	4187.89	139000
6905	3226	4218.23	140000
6979	3226	4248.57	141000
7053	3226	4278.91	142000
7128	3226	4309.66	143000
7202	3226	4340.00	144000
7259	3242	4370.50	145000
7259	3311	4399.92	146000
7259	3380	4429.59	147000
7259	3449	4459.26	148000
7259	3517	4488.50	149000
7259	3586	4518.17	150000

These values produce the following tradeoff graph which shows that there is only one optimal point regardless of the volume size.

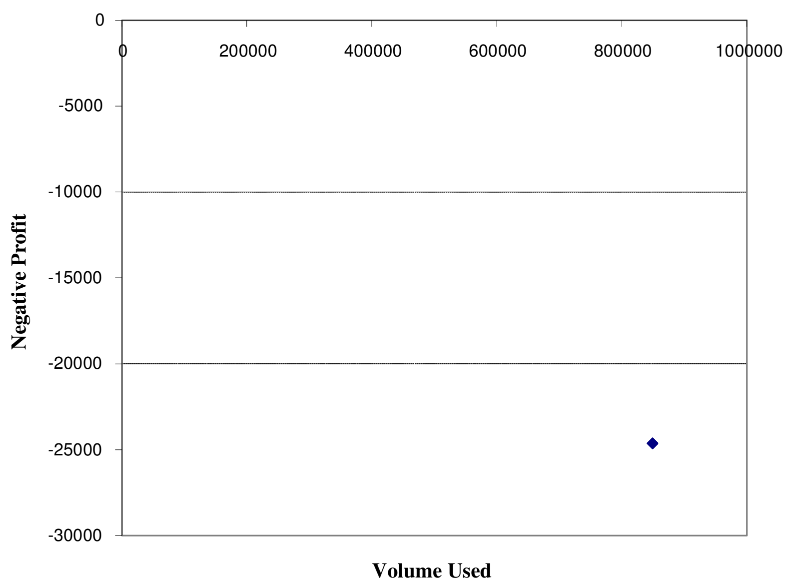

Figure XII.7.2. Tradeoff between Competing Objectives with Expected Values

Multiobjective Optimization Problem—Conditional Expected Value

max $0.18x_1 + 0.26x_2$		min—$(0.18x_1 + 0.26x_2)$	
min $13.5x_1 + 14.5x_2$		subject to	$13.5x_1 + 14.5x_2 \leq \varepsilon_{21}$
subject to	$x_1 + x_2 \geq 45250$		$x_1 + x_2 \geq 45250$
	$x_1 \geq 7542$		$x_1 \geq 7542$
	$x_2 \geq 15083$		$x_2 \geq 15083$

Table XII.7.4 shows the expected values for 1350000 in$^3 \leq \varepsilon_{21} \leq$ 1500000 in^3

Table XII.7.4. Conditional Expected Values for ε_{21}

x_1	x_2	f_1	f_2
7542	86082	-23739	1350000
7542	86771	-23918	1360000
7542	87461	-24097	1370000
7542	88151	-24277	1380000
7542	88840	-24456	1390000
7542	89530	-24635	1400000
7542	90220	-24815	1410000
7981	90500	-24967	1420000
8722	90500	-25100	1430000
9463	90500	-25233	1440000
10204	90500	-25367	1450000
10944	90500	-25500	1460000
11685	90500	-25633	1470000
12426	90500	-25767	1480000
13167	90500	-25900	1490000
13907	90500	-26033	1500000

These values produce the following tradeoff graph. Figure XII.7.3 shows the tradeoff between two competing objectives for conditional expected value analysis. The line represents the Pareto Optimum. We can see that improving one objective definitely degrades another. Also, there is a cusp around (f_1, f_2) = (24815, 14100).

ANALYSIS

From the results, we are able to give the policymaker two distinct Pareto-Optimal frontiers to decide upon. Obviously they are dependent on each other. The Multiobjective Statistical Method (MSM) gives us a holistic thought process to analyze this work from many different angles.

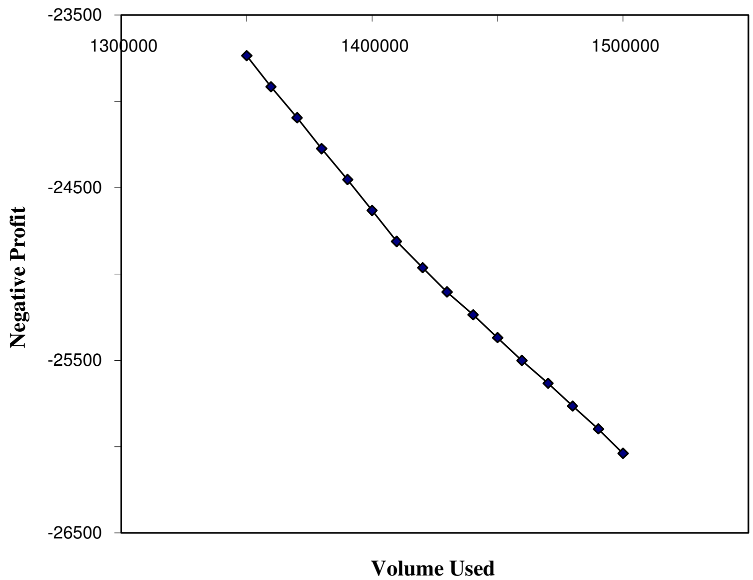

Figure XII.7.3. Tradeoff between Competing Objectives with Conditional Expected Values

PROBLEM XII.8: Machine/Manpower Resource Determination

The purpose of this problem is to minimize cycle time and unit cost of a production system. A critical problem in production is resource determination. Resources are the manpower, equipment, materials, power and other input factors used in the production of goods. To simplify the system, this study is limited to two types of resources—manpower and machine.

Use 1) the Multiobjective Statistical Method (MSM) and 2) the Surrogate Worth Tradeoff (SWT) method to arrive at a production decision.

Figure XII.8.1 describes the production system setup:

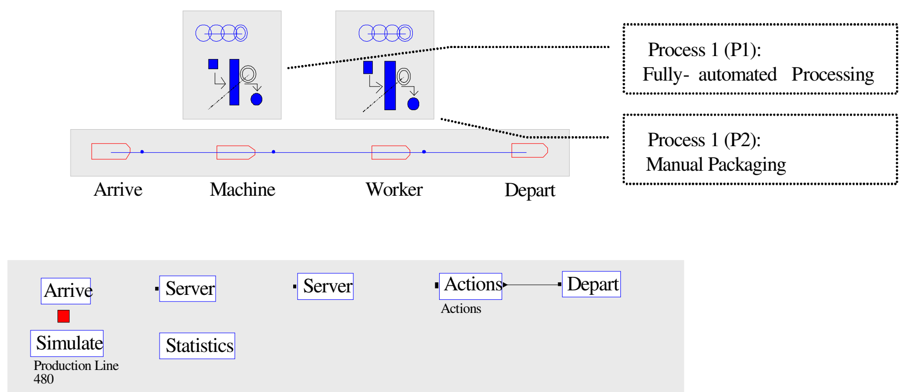

Figure XII.8.1. Simulated System Setup

The system variables are as follows:

Decision Variables	X_1: No. of machines to be used for Process 1 (fully-automated processing) X_2: No. of workers/manpower for Process 2 (packaging)
State Variables	Among the various state variables for production, those relevant to this problem are: S_1: (WIP) Work-in-process units (unfinished units due to bottlenecks, unfinished daily production S_2: (FGs) Finished goods produced by the system at the end of the day, as a function of system capacity and machine/manpower reliability
Random Variables	Machine/Worker rates: Machine: exp(2 units/minute) Worker: exp(1 unit/minute) Machine Breakdown: exp(1 b/down per day)
Objectives	f_1: Min average cycle time f_2: Min average unit cost

From the previous description of the production system, we used ARENA to construct a simulation model to generate the data that will be used for analysis. Table XII.8.1 shows the results.

Table XII.8.1. Output Statistics on the ARENA Simulation Model

Design Inputs		Model Outputs			
Machines (X_1)	Workers (X_2)	Cycle Time (f_1)	WIP (S_1)	Fin. Goods (S_2)	B/down (b)
1	1	155.36	466.35	508	0.0208
1	2	90.24	273.51	885	0.0417
1	3	85.616	264.17	889	0.0417
1	4	83.871	260.53	891	0.0417
1	5	85.088	257.52	918	0.0417
1	6	85.148	261.89	896	0.0417
2	1	160.11	486.56	462	0.0417
2	2	77.798	239.33	940	0.0208
2	3	9.9573	30.848	1350	0.0208
2	4	2.6637	7.9814	1428	0.0417
2	5	2.1004	6.2969	1435	0.0208
2	6	1.9964	5.9858	1435	0.0208
3	1	149.48	474.49	467	0.0208
3	2	84.941	252.14	947	0.0208
3	3	6.8959	21.446	1369	0.0208
3	4	1.8131	5.4368	1434	0.0417
3	5	1.6212	4.8635	1436	0.0208
3	6	1.5884	4.7652	1436	0.0208

Cycle Time

Cycle time is defined as the time it takes for a unit to be produced. This was computed as a function of WIP(S_1) and FG(S_2). From simulation and regression analysis (using MINITAB), the f_1 function is defined as:

$$f_1(S_{1,}S_2) = 0.457S_1 + 0.0647S_2 - 92.8$$

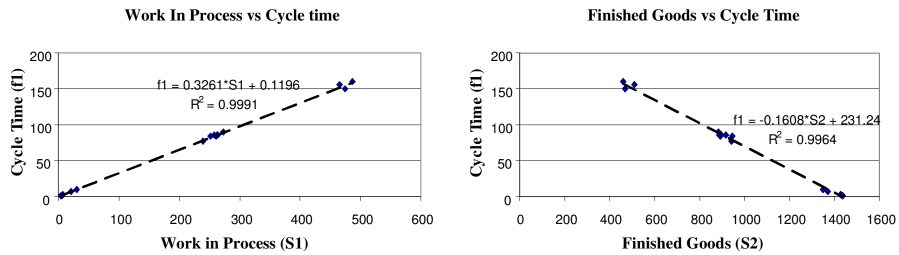

Figure XII.8.2. Superposition of Relationship between f_1, S_1, and S_2

Unit Cost

The unit cost is simply computed as the total operation cost (attributed to machine/manpower and maintenance costs) divided by the number of finished goods (S_2) produced for the period. The assumed salary and operation costs used in the computation are:

	Cost Factor	$ Cost	Unit
(A)	Worker salary	100	$ per worker per day
(C)	Maintenance cost (fixed)	60	$ per machine per day
(D)	Breakdown cost *(lost production and repair costs)*	500 (average proportion of time machine is down per day)	$ per machine per day

The unit cost is computed as:

$$f_2(x_1, x_2, S_2) = \frac{(1500 + 60 + 500(b))x_1 + 100x_2}{S_2}$$

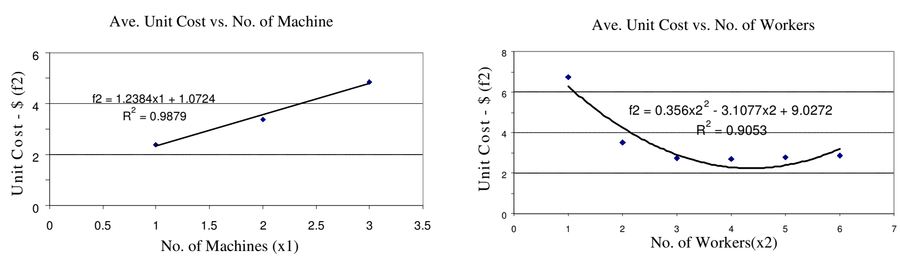

Figure XII.8.3. Superposition of Relationships between f_2, x_1, and x_2

Using Regression to Translate State Variables to Decision Variables

Using regression analysis (MINITAB), the following relationships are determined:

$$S_1 = 1158 + 82.8x_1^2 - 416x_1 + 30.4x_2^2 - 282x_2$$
$$S_2 = -169x_1^2 + 850x_1 - 59.4x_2^2 + 557x_2 - 900$$

Complete the rest of the problem using multiobjective optimization.

PROBLEM XII.9: School Bus Company Faces Growing Demand

A consulting company is tasked with assessing the risks associated with a school bus company's future needs.

The bus company is currently assessing the increased demand placed upon it. The number of schools, and hence the number of students, has grown exponentially over the past two years. The bus company anticipates the growth to continue over the next several years and is looking to:

- Minimize the costs associated with company growth - f_1
- Minimize the number of students injured - f_2

These goals will constitute the two objective functions for assessing the company's risk.

The consulting company has determined that the most convenient way to assess risk is in terms of state variables rather than the two decision variables. They will use the Multiobjective Statistical Method (MSM) to construct risk. This tool will allow them to regenerate the objective functions in terms of decision variables. With the new objective functions and decision variables, risks can then be assessed using the Surrogate Worth Tradeoff Method (SWT). Below are the details of the assessment.

The Multiobjective Statistical Method (MSM)

The bus company is looking to expand in the following ways:

1. Increase the number of times the bus will stop to pick up children to 3, 4 or 5 more stops.

2. Determine if 14, 16, or 18 will be the optimum number of buses the company will operate for highest safety.

With these two decision variables in mind, the company will try to minimize the costs of future growth, f_1, and minimize the number of students injured, f_2. Listed below are all the variables and objective functions used for assessing the bus company's risks. *For academic purposes, several variables and objectives were left out of this discussion (e.g., the time spent on each route, traffic conditions, weather conditions, and condition of buses). Other liberties were taken in calculating costs, objective functions, state variables, and assumptions.*

Assumptions:

1. Presently, each bus stops 10 times.

2. Dollar amount is in terms of millions.

3. Each bus can hold a maximum of 18 students.

4. Each stop does not have the same number of students.

5. Number of injuries given collective time on bus is at a rate of 1 injury per360hours (1 school year of riding bus 2 hours per day for 180 days).

Random variables:

1. r_1 is the random number of students per stop, with no more than 5 students per stop and a probability distribution of 0.8.

2. r_2 is the random number of injuries per bus

Decision variables:

1. Number of times each bus will stop where x1 is 3, 4, or 5 additional stops.

2. Number of buses the company will use where x_2 is 10, 12, 14, 16, or 18

State variables:

1. Number of students per bus = $N(r, x_2) = E(r_1) * x_1$

2. Number of injuries per day = $T(r_2, x_1, x_2) = E(r_2) * x_1 * x_2$

The following objective functions are assumed to be the result of simulating the above variables and assumptions.

Objective functions:

1. min [cost of company growth ($millions)] = $f_1(x_1, x_2) = (x_1 - 3)^2 + (x_2 - 5)^2 + 4$

2. min [number of students injured] = $f_2(x_1, x_2) = (x_1 - 4)^2 + (x_2 - 12)^2 + 6$

Complete the remainder of the multiobjective optimization problem.

PROBLEM XII.10: Estimating Construction Time

A construction company in Charlottesville, VA receives a contract to widen a short section of a major commuter highway. Rain and the breakdown of machinery are two elements that can delay the completion of the project. The company must decide how many workers and machines should be used to complete the construction while minimizing both the total cost and the construction time.

Since we are asked to minimize two objective functions, we must estimate parameters, s_1 and s_2, in the objective functions (equations (1) and (2)). These values will be calculated based on the Multiobjective Statistical Method (MSM) and then a linear regression can be applied.

Mathematically, this problem can be described as follows:

Assume the maximum number of workers = 80, and the maximum number of machines = 135.

Decision variables:

x_1 = number of workers needed
x_2 = number of machines needed

Random variables:

r_1 = number of rainy days per month
r_2 = number of machine downtimes per day

State variables:

$s_1 = s(r_1)$ = status index of rain conditions. The relationship describing s_1 and r_1 is:
$s_1 = 50r_1$

$s_2 = s(r_2)$ = status index of machine breakdown. The relationship describing s_2 and r_2 is: $s_2 = 0.2r_2$

Objective functions:

$$f_1(x_1, x_2, s_1) = 500x_1 + 100x_2 + s_1 \qquad \text{(XII.10.1)}$$

$$f_2(x_1, x_2, s_1, s_2) = \left[\frac{100}{(x_1 - 30)} + \frac{100}{(x_2 - 20)}\right] \times \left(1 + \frac{s_2}{7.6}\right) \times \left(1 + \frac{s_1}{3000}\right) \qquad \text{(XII.10.2)}$$

where:

$f_1(x_1, x_2, s_1)$ = total cost (USD) of construction
$f_2(x_1, x_2, s_1, s_2)$ = total time (months) for the project

Minimize the objectives f_1 and f_2 after obtaining the values of parameters by simulating random variables. Refer to Tables XII.10.1 and XII.10.2 for the sample values of the two random variables, r_1 and r_2

For r_1: The historical rain data (February 2007) for the city of Charlottesville is listed in Table XII.10.1.

Table XII.10.1. Precipitation in Charlottesville, VA for February 2007[1]

Date	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Precip.(inches)	0	0	0	0	0	0.1	0	0	0	0	0	0.02	0.75	0.51
Date	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Precip.(inches)	0	0	0	0	0	0.04	0	0	0	0	0.76	0	0.01	0

Assume the number of rainy days per month is Poisson-distributed. For example, if the total number of rainy days is 6, we can say r_1~Poisson(6) for the month.

For r_2 the distribution for machine downtime, known from experience, is listed in Table XII.10.2:

Table XII.10.2. Probability Distribution of Machine Downtime

Downtime (per day)	Probability
0	0.05
1	0.12
2	0.14
3	0.15
4	0.17
5	0.13
6	0.11
7	0.08
8	0.05

[1] The Weather Channel, Monthly Weather for Charlottesville, VA. Available online: <http://www.weather.com/outlook/recreation/ski/monthly/22903?month=-1>

PROBLEM XII.11: Paper Mill Pollution

A paper plant on a river needs to be minimized its disposal costs (f_1) as well as the amount of pollution the disposal of its wastes dumps into the river (f_2).

New state regulations are forcing a reevaluation of how much waste a paper mill can typically discard into a river. These new regulations impose a financial penalty based on the level of pollution in that section of the river. The pollution level is a function of the amount that the plant discards into the river as well as the amount of rainfall, which is a random variable. The plant has the option of reprocessing the wastes at a cost; therefore it must decide how much to discard into the river and how much to reprocess.

The paper plant manager wants to know 1) how much waste can be discarded into the river to conform to the new regulations, and 2) the costs of reprocessing the waste.

Perform Multiobjective Statistical Method (MSM) for the above pollution control problem using the following functions and variable definitions below.

The cost to reprocess one gallon of waste is ($19/gallon)^2 and the cost for a specific pollution level is ($12.8/level of pollution)^2.

Objective functions: $\min f_1(D,R,P) = 19*R^2 + 12.8*P^2$

$$\min f_2 = P = \left[\frac{D^2}{1000000}\right]^{1/3} * \frac{1}{L}$$

subject to: $D+R = 250$

$D, R, L >= 0$

State variable: P = level of pollution in the river = $\left[\frac{D^2}{1000000}\right]^{1/3} * \frac{1}{L}$

In this case, the state variable is also one of the objective functions.

Decision variables: D = Amount of waste to discard into the river (gallons)
R = Amount of waste to reprocess (gallons)

Random variables: L = amount of precipitation per month in a two-year period (inches)

L is a random variable that has a uniform distribution of between 0 and 10 inches ~ U(0,10).