IX. Multiobjective Risk Impact Analysis Method

PROBLEM IX.1: Controlling Cholesterol through Diet

An individual with high cholesterol does not want to take a cholesterol-reducing medication. The individual plans to control this problem through dietary changes.

DESCRIPTION

It is widely accepted that high cholesterol can significantly increase the risk of heart attack. The logical solution would seemingly be to simply lower one's cholesterol levels by changing the diet. In practice, however, this is more easily said then done. The problem is that high-cholesterol foods are usually the most tempting. How can an individual choose a low-cholesterol diet that will be fully enjoyable?

METHODOLOGY

The Multiobjective Risk Impact Analysis Method (MRIAM) can help shape the solution to this health problem.

Objectives:

In choosing a dietary policy, the decisionmaker must decide which of the two objectives, cholesterol level or food enjoyment, is more important.

State Variable:

For this problem, the state variable at stage k, denoted by $x(k)$, is the cholesterol level, which is given as a ratio of the present cholesterol level to the initial level. It is assumed that $x(0) = 1$.

Decision Variable:

The decision variable is the amount of high-cholesterol food to include in one's diet. This involves an obvious trade-off between food enjoyment and cholesterol levels.

Food enjoyment is expressed in terms of the *level of suffering* that the decisionmaker must undergo in order to effect a change in cholesterol level. This means that the individual must choose to eat foods with lower cholesterol. The level of suffering increases as the volume of high cholesterol foods is decreased. The state equation, which is presented below, captures this effect. In particular, it reflects the fact that radical dietary modifications lead to greater suffering levels than would gradual changes implemented over longer periods of time. If, for example, the decisionmaker wanted to affect a large decrease in cholesterol level over a short period of time, then the suffering level would increase greatly.

<u>Mathematical Formulation</u>

2 Stages: $k = 1, 2$	with each stage representing a 3-month policy horizon	
State Variable: $x(k)$	cholesterol level	
Decision Variable: $u(k)$	amount of high-cholesterol food consumed	
Random Variable: $d(k)$		random factors
Objective Functions	f_1	degree of suffering (cost)
	f_4	partitioned cholesterol level (partitioned at $\mu + \sigma$)
	f_5	non-partitioned cholesterol level

The state equation is as follows:

$$x(k+1) = ax(k) + bu(k) + d(k)$$

Where $a = 0.75$, $b = 0.40$, $\mu_d = 0$, $S_d^2 = 0.05$, and $0 < u(k) \le 1$.

The cost (suffering) function is defined as:

$$f_1^0 = 100(1-u(0))^3$$

$$f_1^1 = \frac{100(1-u(0))^3 + 75(1-u(1))^2}{1.75}$$

$$f_1^1 = \frac{400}{7}(1-u(0))^3 + \frac{300}{7}(1-u(1))^2$$

The state functions for the first two stages are:

$$x(1) = ax(0) + bu(0) + d(0)$$

$$x(2) = ax(1) + bu(1) + d(1)$$

$$x(2) = a^2x(0) + abu(0) + bu(1) + ad(0) + d(1)$$

$$x(2) = a^2 + abu(0) + bu(1) + ad(0) + d(1)$$

The expected values of the cholesterol levels are determined by:

$$E[x(1)] = a + bu(0)$$

$$E[x(1)] = 0.75 + 0.40u(0)$$

$$E[x(2)] = a^2 + abu(0) + bu(1)$$

$$E[x(2)] = 0.5625 + 0.3u(0) + 0.4u(1)$$

SOLUTION

Policy 1: $u(0) = u(1) = 1$, meaning no reduction in high-cholesterol foods

Stage 1:

$$E[x(1)] = a + b = 0.75 + 0.40 = 1.15$$
$$Var[x(1)] = S_d^{\ 2} = 0.05$$
$$\Rightarrow \mu = 1.15, \sigma = \sqrt{0.05}, = 0.2236$$
$$\Rightarrow f_5 = 1.15$$
$$\Rightarrow f_4 = \mu + 1.525\sigma = 1.15 + (1.525)(0.2236) = 1.490 > f_5$$
$$\Rightarrow f_1 = 0$$

Stage 2:

$$\begin{aligned} E[x(2)] &= a^2 + abu(0) + bu(1) \\ &= 0.75^2 + (0.75)(0.4) + 0.4 \\ &= 1.2625 \\ Var[x(2)] &= (a^2 + 1)S_d^{\ 2} = (0.75^2 + 1)(0.05) = 0.07813 \end{aligned}$$
$$\Rightarrow \mu = 1.2625, \sigma = 0.2795$$
$$\Rightarrow f_5 = 1.2625$$
$$\Rightarrow f_4 = \mu + 1.525\sigma = 1.2625 + (1.525)(0.2795) = 1.6888 > f_5$$
$$\Rightarrow f_1 = 0$$

This result indicates that if the consumption of high-cholesterol food is not reduced, the cholesterol level will increase steadily. (The partitioned cholesterol level will be even higher.)

Policy 2: $u(0) = 0.8$, $u(1) = 0.5$

Stage 1:

$$E[x(1)] = a + bu(0) = 0.75 + 0.40 \times 0.8 = 1.07$$
$$Var[x(1)] = S_d^{\ 2} = 0.05$$
$$\Rightarrow \mu = 1.07, \sigma = \sqrt{0.05} = 0.2236$$
$$\Rightarrow f_5 = 1.07$$
$$\Rightarrow f_4 = \mu + 1.525\sigma = 1.07 + (1.525)(0.2236) = 1.4110 > f_5$$
$$\Rightarrow f_1 = 0.8$$

Stage 2:

$$\begin{aligned} E[x(2)] &= a^2 + abu(0) + bu(1) \\ &= 0.75^2 + (0.75)(0.4)(0.8) + (0.4)(0.5) \\ &= 1.0025 \end{aligned}$$

$$Var[x(2)] = (a^2 + 1)S_d^{\,2} = (0.75^2 + 1)(0.05) = 0.07813$$
$$\Rightarrow \mu = 1.0025, \sigma = 0.2795$$
$$\Rightarrow f_5 = 1.0025$$
$$\Rightarrow f_4 = \mu + 1.525\sigma = 1.0025 + (1.525)(0.2795) = 1.4288 > f_5$$
$$\Rightarrow f_0 = 11.1714$$

This example indicates that if the amount of high-cholesterol food is cut down, then the expected cholesterol level can be reduced compared to the previous policy. On the other hand, the high f_4 suggests that there is still a risk of reaching a high-cholesterol level, which is likely to be ignored if one looks at the expected level only.

Policy 3: $u(0) = 0.5$, $u(1) = 0.4$

Stage 1:

$$E[x(1)] = a + bu(0) = 0.75 + (0.40)(0.5) = 0.95$$
$$Var[x(1)] = S_d^{\,2} = 0.05$$
$$\Rightarrow \mu = 0.95, \sigma = \sqrt{0.05} = 0.2236$$
$$\Rightarrow f_5 = 0.95$$
$$\Rightarrow f_4 = \mu + 1.525\sigma = 0.95 + (1.525)(0.2236) = 1.2910 > f_5$$
$$\Rightarrow f_1 = 12.5$$

Stage 2:

$$\begin{aligned} E[x(2)] &= a^2 + abu(0) + bu(1) \\ &= 0.75^2 + (0.75)(0.4)(0.5) + (0.4)(0.4) \\ &= 0.8725 \\ Var[x(2)] &= (a^2 + 1)S_d^{\,2} = (0.75^2 + 1)(0.05) = 0.07813 \end{aligned}$$
$$\Rightarrow \mu = 0.8725, \sigma = 0.2795$$
$$\Rightarrow f_5 = 0.8725$$
$$\Rightarrow f_4 = \mu + 1.525\sigma = 0.8725 + (1.525)(0.2795) = 1.2988 > f_5$$
$$\Rightarrow f_1 = 22.5715$$

This example indicates that if the high-cholesterol food is cut down further, then the expected cholesterol level can be reduced further. On the other hand, again the high f_4 suggests that there is still a risk of reaching a high cholesterol level, which is likely to be ignored if one looks at the expected level.

General Result:

Stage 1:

$$f_1 = 100(1-u(0))^3$$
$$f_5 = a + bu(0) = 0.75 + 0.40u(0)$$
$$f_4 = a + bu(0) + 1.525\sigma = 0.75 + 0.40u(0) + 0.3410 = 1.0910 + 0.40u(0)$$

Stage 2:

$$f_1 = \frac{1}{7}(400(1-u(0))^3 + 300(1-u(1))^2)$$
$$f_5 = a^2 + abu(0) + bu(1) = 0.5625 + 0.30u(0) + 0.40u(1)$$
$$f_4 = f_5 + 1.525\sigma = 0.9887 + 0.3u(0) + 0.40u(1)$$

The non-inferior Pareto solutions are displayed in the following four graphs.

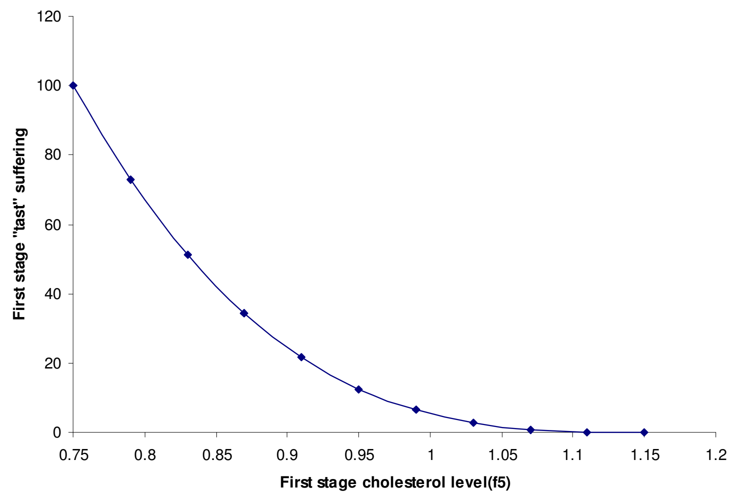


Figure IX.1.1. f_1 **vs** f_5 **for the first stage**

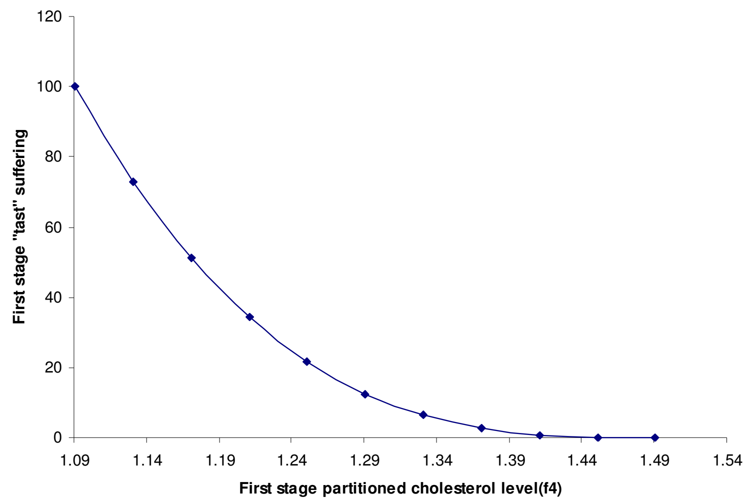


Figure IX.1.2. f_1 vs f_4 for the first stage

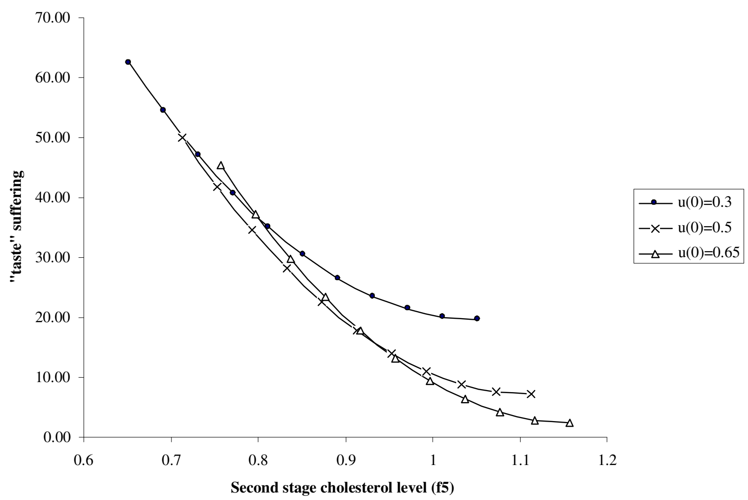


Figure IX.1.3. f_1 vs f_5 for the second stage

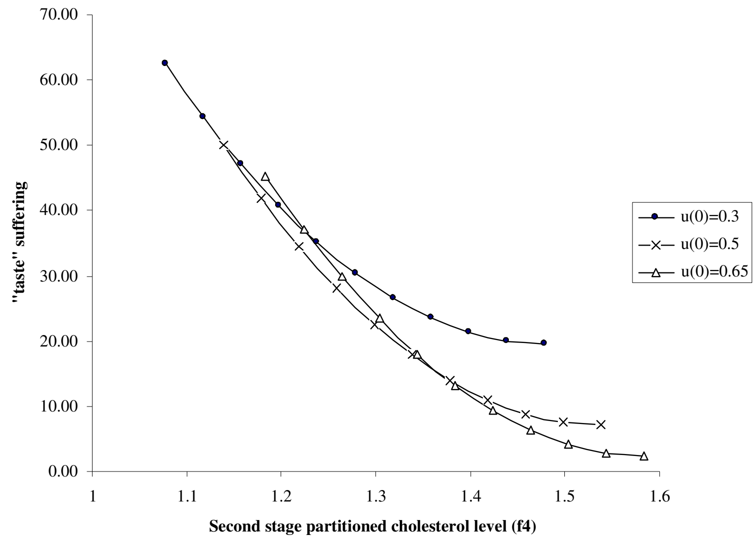


Figure IX.1.4. f_1 **vs** f_4 **for the second stage**

From these graphs, the individual can identify whatever Pareto solutions best suit the individual's needs. As an illustrative example, we give the following Pareto solutions:

Table IX.1.1. First-Stage Pareto Solutions

Policy $u(0)$	f_1	f_5	f_4
0	100	0.75	1.091
0.1	72.9	0.79	1.131
0.2	51.2	0.83	1.171
0.3	34.3	0.87	1.211
0.4	21.6	0.91	1.251
0.5	12.5	0.95	1.291
0.6	6.4	0.99	1.331
0.7	2.7	1.03	1.371
0.8	0.8	1.07	1.411
0.9	0.1	1.11	1.451
1	0	1.15	1.491

Table IX.1.2. Second-Stage Pareto Solutions

Policy $u(1)$	f_1	f_5	f_4
0.3	24.5714	0.8634	1.2896
0.4	19	0.9034	1.3296
0.5	14.2857	0.9434	1.3696
0.6	10.4285	0.9834	1.4096
0.7	7.4285	1.0234	1.4496
0.8	5.2857	1.0634	1.4896
0.9	4	1.1034	1.5296
1	3.5714	1.1434	1.5696

Note: Assume that Policy u(0) = 0.603 for Table IX.1.4

ANALYSIS

In this project a cholesterol control problem is solved by using the MRIAM. The model addresses a two-stage cholesterol control problem. The result indicates that by cutting down the consumption of high-cholesterol food at each stage (a 3-month period) one can reduce the expected cholesterol level (f_5) considerably. However, by partitioning at $\mu + \sigma$ we discovered that there is still a risk of ending up with a high cholesterol level.

The insight provided by this partitioned risk analysis should receive special attention. Pareto solutions are generated for both stages. The second stage presented a family of Pareto solutions corresponding to different policies. Because of the increase in dimensionality, one policy at the first stage (one point) maps into a curve (corresponding to a set of different $u(1)$) at the second stage. This suggests an even higher complexity at an ensuing stage.

PROBLEM IX.2: Reducing River Channel Overflow

An important waterway supports the economic livelihoods and social activities of the surrounding communities. However, river flooding poses a great challenge to the local government in terms of controlling channel overflow, particularly during monsoon seasons. The region is visited by an average of 23 typhoons annually, beginning as early as May and stretching up to November. Deforestation and poor waste disposal practices contribute in a major way to the situation. The effects of channel overflow include flooded roads, destroyed lives and properties, and disrupted basic services (electric power, transportation, and communication), among others.

DESCRIPTION

Suppose a total budget of $800 million is allocated over a three-year horizon to be used for flood-control management (e.g., river dredging, constructing river-flow control infrastructures such as dikes and floodways, and others). Three policy options (A, B, and C) have been identified and are summarized in Table IX.2.1. Each of these indicates the amount of funds to be released *prior* to each period (k = IX.2.1, IX.2.2, and IX.2.3).

Table IX.2.1. Policies Options with given u(k) Values

	$u(k\text{-}1)$ = amount spent on flood control management at Stage k-1 (in million dollars)		
Policy	$u(0)$	$u(1)$	$u(2)$
A	560	160	80
B	320	320	160
C	160	240	400

METHODOLOGY

The Multiobjective Risk-Impact Analysis Method (MRIAM) is used to evaluate the performance of each policy option, in terms of shrinking the volume of channel overflow.

The following MRIAM formulas are used:

$$m(k) = CA^k x_0 + \sum_{i=0}^{k-1} CA^i Bu(k-1-i)$$

$$s^2(k) = C^2 A^{2k} X_0 + \sum_{i=0}^{k-1} C^2 A^{2i} P + R$$

Table IX.2.2 summarizes the values of parameters in the above mean and variance formulas:

Table IX.2.2. Parameter and Value Explanations

Parameter	Value	Explanation
A	$e^{-0.5}$	Decay rate of precipitation due to predicted El Niño in the next 3 years, [unitless]
B	-0.25	Controlled volume of channel overflow per million \$ invested, [in million m^3/million \$ spent]
C	1	Proportionality constant, [unitless]
x_0	1000	Initial volume for potential overflow, [in million m^3]
P	0.001	Variance of random variable $\omega(k)$, [(in million $m^3)^2$]
R	0	Variance of random variable $\nu(k)$, [(in million $m^3)^2$]
X_0	250	Initial variance of potential overflow, [(in million $m^3)^2$]

Requirements:

(a) Calculate $f_2(k)$, $f_3(k)$, and, $f_4(k)$. Do this for all periods k = 1, 2, and 3.

(b) For each option, plot $f_2(k)$, $f_3(k)$, and, $f_4(k)$ with respect to the control variable $u(k\text{-}1)$. Do this for every period. Analyze the results. Use one standard deviation partitioning for $f_2(k)$ and $f_4(k)$.

SOLUTION

(a) Table IX.2.3 presents the calculated values of $f_2(k)$, $f_3(k)$, and, $f_4(k)$ for all periods k = 1, 2, and 3.

Table IX.2.3. Summary of Expected Values for the Three Periods

Period 1

Policy	$u(0)$	$m(1)$	$s(1)$	$f_2(1)$	$f_3(1)$	$f_4(1)$
A	560	915	250	534	915	1296
B	320	951	250	570	951	1333
C	160	976	250	594	976	1357

Period 2

Policy	$u(1)$	$m(2)$	$s(2)$	$f_2(2)$	$f_3(2)$	$f_4(2)$
A	160	924	250	543	924	1305
B	320	922	250	541	922	1303
C	240	949	250	568	949	1330

Period 3

Policy	$u(2)$	$m(3)$	$s(3)$	$f_2(3)$	$f_3(3)$	$f_4(3)$
A	80	896	250	515	896	1277
B	160	913	250	532	913	1294
C	400	931	250	550	931	1313

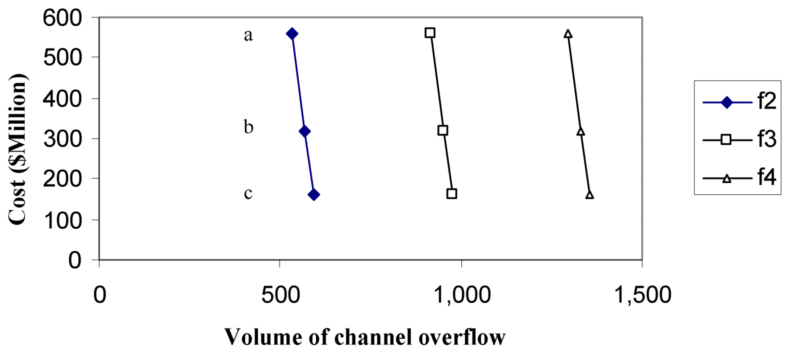


Figure IX.2.5. Conditional Expected Values for Period 1.

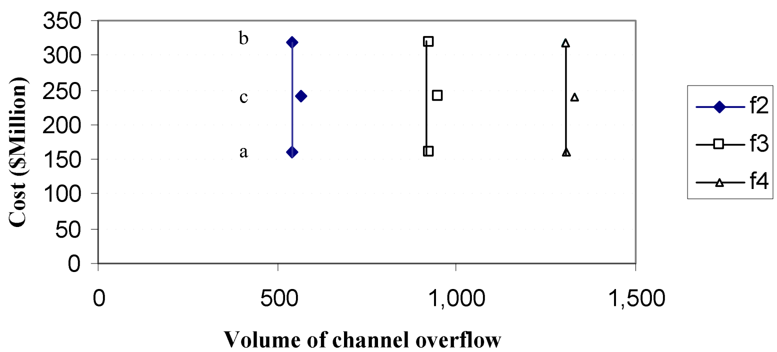


Figure IX.2.6. Conditional Expected Values for Period 2

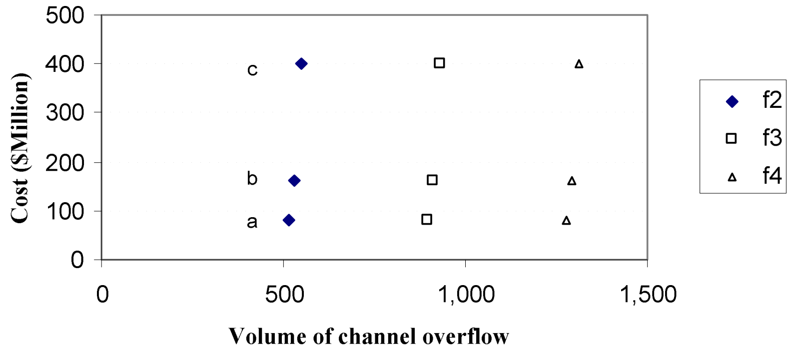


Figure IX.2.7. Conditional Expected Values for Period 3

ANALYSIS

To minimize the total volume of overflow, Policy A is recommended since it is Pareto optimal throughout all periods (Note that the effect of Policy B is a marginally better than that of Policy A in the second period). However, Policy C is inferior to the other policies in both periods 2 and 3 since its overflow volume is greater, though each policy uses the same budget.

PROBLEM IX.3: Formulating MRIAM as an Epsilon Constraint Problem

The purpose of this exercise is to derive an equivalent $\in$-constraint representation of the MRIAM.

DESCRIPTION

Consider the system defined by the following standard dynamic equation.

The objective functions are given as:

Minimize: $f_1^0 = (1-u(0))^2 + 0.5(1-u(1))^2$
(present-value cost function)
f_4^1 = high-range conditional expected damage at Stage 1
f_4^2 = high-range conditional expected damage at Stage 2

i) Formulate the multiobjective multistage risk analysis problem.

ii) Use the $\in$-constraint method to formulate the Lagrangian function. Determine $u(0)$ and $u(1)$.

METHODOLOGY

Use the Multiobjective Risk-Impact Analysis Method (MRIAM) to solve this problem.

The following MRIAM equations will be used in deriving the $\in$-constraint formulation

$$\beta_4^k = \frac{\int_{s_k'}^{\infty} \tau \frac{1}{\sqrt{2\pi}} e^{-\tau^2/2} d\tau}{\int_{s_k'}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\tau^2/2} d\tau}$$

$$f_i^k(u) = \mu(k;u) + \beta_i^k \sigma(k)$$

$$\tau^2/2 = \frac{(y(k)-\mu(k))^2}{2\sigma^2(k)}$$

$$E[v^2(k)] = R(k) = R$$

$$\mu(k) = E[y(k)] = CE[x(k)]$$

$$\sigma^2(k) = C^2 A^{2k} X_0 + \sum_{i=0}^{k-1} C^2 A^{2i} P + R$$

$$E[X(k+1)] = AE[X(k)] + Bu(k)$$

$$\sigma^2(k) = C^2 Var[X(k)] + R$$

$$s_k' = \frac{s_k - \mu(k)}{\sigma(k)}$$

SOLUTION

i) $$\beta_4^k = \frac{\int_{S_k'}^{\infty} \tau \frac{1}{\sqrt{2\pi}} e^{-\tau^2/2} d\tau}{\int_{S_k'}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\tau^2/2} d\tau}$$

$$= \frac{-\frac{1}{\sqrt{2\pi}} e^{-\tau^2/2} d\tau \Big|_2^{\infty}}{1 - F(2)}$$

$$= \frac{1}{\sqrt{2\pi}} \frac{e^{-2}}{1 - 0.9772}$$

$$= \frac{0.05399}{0.0228}$$

$$= 2.368$$

$$\sigma^2(k) = c^2 A^{2k} X_0 + \sum_{i=0}^{k-1} C^2 A^{2i} P + R$$

$$\sigma^2(1) = (1)^2 (1.5)^{2(1)} (1) + (1)^2 (1.5)^0 (1) + 2$$
$$= 5.25$$

$$\sigma^2(2) = (1)^2 (1.5)^{2(2)} (1) + (1)^2 (1.5)^0 (1) + (1)^2 (1.5)^1 (1) + 2$$
$$= 9.5625$$

$$\mu(k) = CE[x(k)]$$

$$\mu(1) = E[x(1)]$$
$$= 1.5x(0) + u(0) + 0$$
$$= 1.5 + u(0)$$

$$\mu(2) = E[x(2)]$$
$$= 1.5E[x(1)] + u(1) + 0$$
$$= 1.5\{1.5[x(0) + u(0)]\} + u(1)$$
$$= 2.25 + 1.5u(0) + u(1)$$

The multiobjective problem is given by

$$f_1^0 = \{1 - u(0)\}^2 + 0.5\{1 - u(1)\}^2$$
$$f_4^1 = 6.926 + u(0)$$
$$f_4^2 = 9.573 + 1.5u(0) + u(1)$$

ii) Rewriting for the $\in$-constraint method, we have:

$$\min \quad \{1-u(0)\}^2 + 0.5\{1-u(1)\}^2$$

$$\begin{aligned} s.t. \quad & 6.926 + u(0) \le \in_1 \\ & 9.573 + 1.5u(0) + u(1) \le \in_2 \end{aligned}$$

The Lagrangian function is given by:

$$\begin{aligned} L = & \{1-u(0)\}^2 + 0.5\{1-u(1)\}^2 + \lambda_{14}^{01}[6.926 + u(0) - \in_1] \\ & + \lambda_{24}^{01}[9.573 + 1.5u(0) + u(1) - \in_2] \end{aligned}$$

Assuming the tradeoff values are positive and taking the partial with respect to the tradeoff values, we have:

$$\frac{\partial L}{\partial \lambda_{14}^{01}} = 6.926 + u(0) - \in_1 = 0$$

$$u(0) = \in_1 - 6.926$$

$$\frac{\partial L}{\partial \lambda_{24}^{01}} = 9.573 + 1.5u(0) + u(1) - \in_2 = 0$$

$$\begin{aligned} u(1) &= \in_2 - 1.5u(0) - 9.573 \\ &= \in_2 - 1.5\in_1 + 0.636 \end{aligned}$$

ANALYSIS

Through a prespecified present-value cost function, $\{1-u(0)\}^2 + 0.5\{1-u(1)\}^2$, the expected values and conditional expected values of the damage functions for two stages have been formulated as an $\in$-constraint problem. The Pareto optimal values of the decision variables, $u(0)$ and $u(1)$ have been determined using the Lagrangian method. Given specific values of the right-hand side constraints $\in_1$ and $\in_2$, one can determine a explicit relationship between the decision variables.

PROBLEM IX.4: Discouraging Insurgent Terrorists

Determine the most cost-effective way of retraining captured insurgents to prevent their return to terrorist activities.

DESCRIPTION

The typical insurgent tries to convert his fellow citizens into joining his cause. If the insurgent is caught, he will spend approximately one year in a holding facility and then return to his terrorist activities. However, retraining can discourage him and prevent this. An individual who has not been properly retrained will return to fight again. Insurgent ages range from 12 to 90 years.

METHODOLOGY

Solve this problem using the Multiobjective Risk-Impact Analysis Method (MRIAM).

State Variables:

$x_1(k)$ = general population over age 12 at year k

$x_2(k)$ = number of insurgents at year k

$x_3(k)$ = number of insurgents undergoing retraining in year k

$x_4(k)$ = number of insurgents captured and placed in holding facility in year k

$x_5(k)$ = number of insurgents released in year k

Parameter Definitions

b_1 = rate at which members of population reach 12 years of age

d_1 = death rate of general population over 12

a = rate at which insurgents convert other members into terrorists

d_2 = death rate of insurgents (in general higher than d_1)

c = percentage rate of incarcerated insurgents

e = percentage of insurgents in retraining

f = rate at which an insurgent in retraining converts another insurgent to be retrained

g = rate of insurgents whose training was ineffective

The five state equations are:

General population:

$$x_1(k) = x_1(k) + (b_1 - d_1)x_1(k) - ax_1(k)x_2(k) \qquad \text{(IX.4.1)}$$

Insurgent population:

$$x_2(k) = x_2(k) + ax_1(k)x_2(k) - (d_2 + c + e)x_2(k) - fx_2(k)x_3(k) + gx_3(k) + x_5(k) \qquad \text{(IX.4.2)}$$

Retraining population:
$x_3(k+1) = x_3(k) + fx_3(k)x_2(k) = ex_2(k) - d_1x_3(k) - gx_3(k)$ (IX.4.3)

Incarcerated population: $x_4(k+1) = cx_2(k)$ (IX.4.4)

Released from holding facility: $x_3(k+1) = x_4(k)$ (IX.4.5)

General population over age 12: $x_1(k+1)$ (IX.4.6)

SOLUTION

Assume:

1. $x_1(k+1) = x_1(k)$ —general population is constant
2. f (the rate at which insurgents in retraining convert other insurgents to retrain) = 0
3. Neglect death rate of normal population and insurgent population (d_2, d_1)
4. $x_5(k) = 0$
5. g = 0

With these assumptions, we can simplify (IX.4.2) and (IX.4.3) as follows:

$$x_2(k+1) = (1 + ax_1(k) - c - e)x_2(k) + x_5(k)$$
$$= (1 + ax_1(k) - c - e)x_2(k) \quad (\because x_5(k) = 0 \text{ by assumption})$$
$$x_3(k+1) = x_3(k) = ex_2(k)$$

Where,

$a = 4.5\times10^{-6}$
$c = 15\%$
$e = 20\%$
$x_2(0) = 10{,}000$
$x_3(0) = 2000$
$x_1(0) = 100{,}000$
w_k = normally distributed random variable with $\mu = 0$, $s_2^2 = 625$

$f_4(\cdot)$ = high-range conditional number of insurgents
$f_5(\cdot)$ = unconditional expected number of insurgents

Y_1 = cost due to insurgent activity = \$100,000/insurgent
Y_2 = cost of incarceration = \$25,000/inmate
Y_3 = cost of retraining = \$35,000/insurgent

$$Cost = Y_1x_2(k) + Y_2cx_2(k) + Y_3ex_2(k)$$

Consider the 3 possible policy decisions:

A. Linearly increase the percentage of jailed insurgents up to 30% over the next 3 years:

$c(1) = 15$; $c(2) = 20$; $c(3) = 25$; $c(4) = 30$

All other variables remain constant.

B. Linearly increase the retraining program over the next 3 years:

e(1) = 20; e(2) = 25; e(3) = 30; e(4) = 35

All other variables remain constant.

C. Combine methods A and B over the next 3 years.

We can calculate unconditional expected number of insurgents, and in order to calculate the conditional expected number of insurgents, f_4, we assume that $w(k) \sim N(0, 25^2)$; thus $ß_4 s(k) = (1.525)(25) = 38.125$.

$$f_5(x_2(1)) = (1 + 4.5 \cdot 10^{-6}(100{,}000) - 0.15 - 0.2)10{,}000 = 11{,}000$$

$$f_5(x_2(2)) = (1 + 4.5 \cdot 10^{-6}(100{,}000) - 0.20 - 0.2)11{,}000 = 11{,}500$$

$$f_5(x_2(3)) = (1 + 4.5 \cdot 10^{-6}(100{,}000) - 0.25 - 0.2)11{,}500 = 11{,}500$$

$$f_5(x_2(4)) = (1 + 4.5 \cdot 10^{-6}(100{,}000) - 0.30 - 0.2)11{,}500 = 10{,}973$$

Table IX.4.3. Unconditional and Conditional Expected Value of Societal Cost for Four Years

Decision	Year	$f_5(x_2(k))$	$Billion Cost	$f_4(x_2(k))$	$Billion Cost
A	1	11,000	1.218	11,038	1.222
	2	11,550	1.294	11,588	1.298
	3	11,550	1.308	11,588	1.312
	4	10,973	1.256	11,011	1.261
B	1	11,000	1.218	11,038	1.222
	2	11,550	1.299	11,588	1.304
	3	11,550	1.320	11,588	1.324
	4	10,973	1.273	11,011	1.277
C	1	11,000	1.218	11,038	1.222
	2	11,000	1.251	11,038	1.256
	3	9,900	1.156	9,938	1.160
	4	7,920	0.948	7,958	0.953

ANALYSIS

Option C, where the percentages of both jailed insurgents and insurgents trained are increased, gives the better outcome for the program. Both the number of insurgents and the cost decrease after 4 years. Options A and B show a slight decrease in insurgents after 4 years, but the costs have risen compared to the cost of an initial year.

PROBLEM IX.5: Modeling of Cancer Patient Population

Should the government cover the cost of treating cancer patients?

DESCRIPTION

The following problem is an example to illustrate a study on methods and costs of treatment for cancer patients. It is a hypothetical example of treating cancer with chemotherapy in a given region.

METHODOLOGY

This problem can be analyzed using the Multiobjective Risk Impact Analysis Method (MRIAM).

$x_1(k)$ = general population at year k

$x_2(k)$ = number of cancer patients at year k

$x_3(k)$ = number of cancer patients undergoing chemotherapy at year k

$x_4(k)$ = number of cancer survivors at year k

b_1 = birthrate of healthy population

d_1 = birthrate of normal population

a = rate of cancer development

d_2 = death rate of cancer patients

e = percentage of cancer patients undergoing chemotherapy

f = rate of cancer-patient cures

g = death rate of chemotherapy patients

State Equations:

Non-cancer population: $x_1(k+1) = x_1(k) + (b - d_1)x_1(k) - ax_1(k)x_2(k)$

Cancer population not receiving treatment:

$$x_2(k+1) = x_2(k) + ax_1(k)x_2(k) - (d_2 + e)x_2(k) - fx_2(k)x_3 + gx_3(k)$$

Chemotherapy population:

$$x_3(k+1) = x_3(k) + fx_3(k)x_2(k) + ex_2(k) - d_1x_3(k) - gx_3(k)$$

Cancer survivors: $x_4(k+1) = x_3(k)$

Assumptions:

1. General population is constant { $x_1(k+1) = x_1(k)$ }
2. Rate at which a chemotherapy patient is cured of cancer = 0, { $f = 0$ }

3. Death rate due to neglect { $d_1 = d_2 = 0$ }
4. Neglect g {g =0}

With those assumptions:

$$x_2(k+1) = (1 + ax_1(k) - e)x_2(k)$$
$$x_3(k+1) = x_3(k) + ex_2(k)$$

One state equation:

$$x(k+1) = ax(k) + u(k) + w(k)$$

where,

$a = 4 \times 10^{-6}$ Rate at which people develop cancer

$e = 50$ Percent of cancer patients undergoing treatment with chemotherapy

$x_1(0) = 100{,}000$ Number of people in the region at year k

$x_2(0) = 500$ Number of cancer patients in the region at year k

$w(k) =$ Normal distributed random variable with $\mu = 0, s_2^2 = 625$

Consider two objective functions:

$f_4(\cdot) =$ Conditional expected value of annual societal cost

$f_5(\cdot) =$ Unconditional expected value of annual societal cost

Cost = $\gamma_1 x_2(k) + \gamma_2 e x_2(k)$

γ_1 = Cost due to cancer = \$50,000/cancer patient (e.g., costs associated with the productivity loss incurred by a cancer patient)

γ_2 = Cost of chemotherapy = \$10,000/chemotherapy patient

$$E[w(k)] = 0 \text{ for } f_5(\cdot)$$
$$f_4(\cdot) = m + \beta_4 s_2 = 0 + (1.525)(25) = 38.125$$

Policy decisions:

Assume the region decides to pay for the chemotherapy using tax revenues. This causes a linear increase in the percentage of cancer patients who desire treatment over the next 4 years (5% per year starting with 50% the first year).

$e(1) = 50\%$

$e(2) = 55\%$

$e(3) = 60\%$

$e(4) = 65\%$

$e(5) = 70\%$

SOLUTION

The following results are obtained in the next 5 years, assuming all other values remain constant.

$$x_2(k+1) = (1 + ax_1(k) - e)x_2(k) + w(k)$$

We can assume $w(k) = 0$

$$f_5(x_2(1)) = (1 + 0.4 - 0.50)500 + 0 = 450$$
$$f_5(x_2(2)) = (1 + 0.4 - 0.55)450 + 0 = 383$$
$$f_5(x_2(3)) = (1 + 0.4 - 0.60)383 + 0 = 306$$
$$f_5(x_2(4)) = (1 + 0.4 - 0.65)306 + 0 = 230$$
$$f_5(x_2(5)) = (1 + 0.4 - 0.70)230 + 0 = 161$$

Now we calculate $f_4(\cdot)$ and we get:

$$f_4(x_2(1)) = 450 + 38.125 = 488.125$$
$$f_4(x_2(2)) = 383 + 38.125 = 420.625$$
$$f_4(x_2(3)) = 306 + 38.125 = 344.125$$
$$f_4(x_2(4)) = 230 + 38.125 = 267.625$$
$$f_4(x_2(5)) = 161 + 38.125 = 197.775$$

We calculate cost as follows:

Cost = $\gamma_1 x_2(k) + \gamma_2 e x_2(k)$

γ_1 = Cost due to cancer = \$50,000/cancer patient

γ_2 = Cost of chemotherapy = \$10,000/chemotherapy patient

Table IX.5.1 charts the costs to society over 5 years of subsidized chemotherapy. Figure IX.5.1 illustrates this graphically.

Table IX.5.1. Unconditional and Conditional Expected Value of Societal Cost for 5 Periods

Year	e	$f_5(\cdot)$	Cost (Millions)	$f_4(\cdot)$	Cost (Millions)
1	0.50	450	\$24.75	488.125	\$26.85
2	0.55	383	\$21.04	420.625	\$23.13
3	0.60	306	\$16.83	344.125	\$18.93
4	0.65	230	\$12.62	267.625	\$14.72
5	0.70	161	\$8.84	198.775	\$10.93

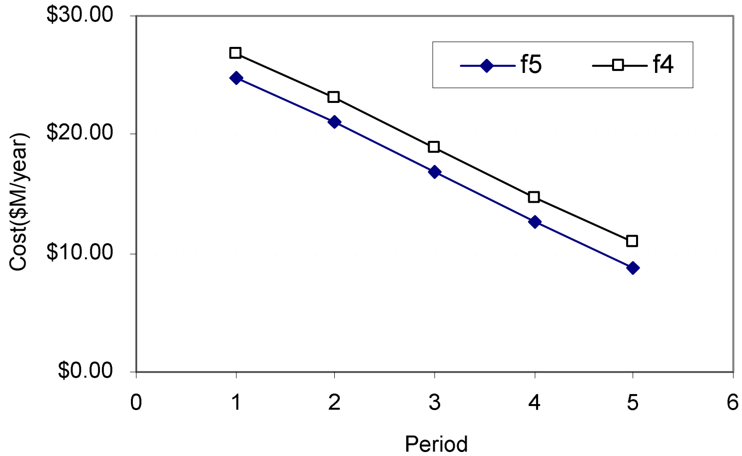


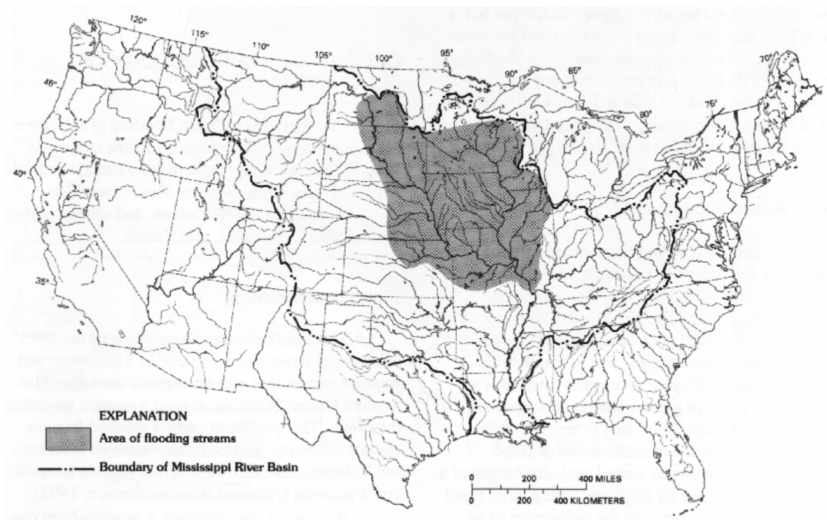
Figure IX.5.1. Annual societal costs over 5 periods

ANALYSIS

We can see that as time passes the conditional and unconditional expected value of cost decreases. The percentage of cancer patients seeking chemotherapy increases, and the survival rate is increasing faster than the rate of new people developing this disease. Thus, the expected total cost per year would eventually decrease as shown above. If the value of life is above that of money, it is obvious that the government should cover the cost of chemotherapy.

PROBLEM IX.6: River Flooding Control

Mississippi River is a main waterway in the US. It has supported the economic livelihoods and social activities of the surrounding states. River flooding poses a great challenge to the US Corps of Engineers in terms of controlling channel overflow, particularly during rainy seasons. In 1993, excessive precipitation visited the Upper Mississippi River Basin, which resulted to massive and destructive flooding in the Midwest region as (as depicted in the map below).



Map of Mississippi River Flooding, June-August 1993
Source (date accessed : April 28, 2005):
[http://www.geo.mtu.edu/department/classes/ge404/flood/background/1120-b/]

The effects of channel overflow include flooding of roads, destruction of lives and properties, and disruption of basic services (electric power, transportation, and communication), among others. In particular, the adverse effects of the 1993 flooding include the following: (i) loss of water supply, pipelines, and treatment facilities in Des Moines, Iowa, the center of the flooding, amounted to over \$700 million; (ii) agricultural damage exceeded \$20 million; (iii) river transportation was halted for two months resulting in an average loss of \$1 million per day; and (iv) about \$500 million was realized due to damage in hundreds of miles of roads.

Suppose a total budget of \$500 million dollars is allocated over a three-year horizon to be used for flood control management (e.g., river dredging, constructing river flow control infrastructures such as levees and reservoirs, etc.). Three policy options (a, b, and c) have been identified and are summarized in Table IX.6.1 below. Each of these policy options indicates the amount of funds to be released *prior* to each period (k = 1, 2, and 3).

Table IX.6.1. Policy Options with given $u(k)$ Values

Policy	$u(k$-1) = amount spent on flood control management at stage k-1 (in million dollars)		
	$u(0)$	$u(1)$	$u(2)$
a	500	0	0
b	250	150	100
c	0	250	250

Table IX.6.2 summarizes the values of parameters in the mean and variance formulas.

Table IX.6.2. Summary of Parameters

Parameter	Value	Explanation
A	0.8	Decay rate of precipitation due to predicted El Niño in the next 3 years, [unitless]
B	-0.5	Controlled volume of channel overflow per million $ invested, [in million m^3/million $ spent]
C	1	Proportionality constant, [unitless]
x_0	1000	Initial volume for potential overflow, [in million m^3]
P	1	Variance of random variable $\omega(k)$, [(in million m^3)^2]
R	0	Variance of random variable $\nu(k)$, [(in million m^3)^2]
X_0	200	Initial variance of potential overflow, [(in million m^3)^2]

The following two steps are required to solve this problem:

(a) Calculate $f_2(k)$, $f_3(k)$, and, $f_4(k)$. Use the table format shown below. Do this for all periods k = 1, 2, and 3.

Period, k						
Policy	$u(k-1)$	$m(k)$	$s(k)$	$f_2(k)$	$f_3(k)$	$f_4(k)$
a						
b						
c						

(b) For each option, plot $f_2(k)$, $f_3(k)$, and, $f_4(k)$ with respect to the control variable $u(k$-1). Do this for every period. Analyze the results.

Note: Use one standard deviation partitioning for $f_2(k)$ and $f_4(k)$.

PROBLEM IX.7: Road Project Construction

This problem is concerned with scheduling city road construction projects within a region that comprises four districts.

Three projects are under consideration:

- *Project A*—Adding a new interchange to a downtown expressway to ease traffic issues due to new economic development in the area,
- *Project B*—Resurfacing the region's roadways in four districts to reduce accidents and lawsuits due to the roadway infrastructure, and
- *Project C*—Synchronizing traffic lights to increase traffic throughput and reduce traffic accidents/fatalities.

A two-year time period will be used for each of the projects to be constructed. They can be completed concurrently or individually. For this exercise, we introduce three possible scenarios:

Policy 1: All projects start at year zero,
Policy 2: Project A starts at year zero with Projects B and C starting at year two
Policy 3: Project A starts at year zero with Project B starting at year two and Project C starting at year four.

Use the Multiobjective Risk-Impact Analysis Method (MRIAM) to allow the decisionmakers to look at the scenarios from a cost versus traffic-throughput standpoint and analyze their options for a six-year period (two-year intervals, three periods) and provide insights into making decisions with public funds.

Expected benefits by:

$f_1(k)$ = cumulative costs (CC)
$f_2(k)$ = μ - σ conditional expected value for benefits (measured in estimation of uninterrupted travel/delays)
$f_3(k)$ = μ unconditional expected value for benefits
$f_4(k)$ = μ + σ conditional expected value for benefits

Three projects:

A: Add a downtown expressway interchange
B: Resurface roads in four city districts
C: Synchronize traffic lights to aid traffic flow

Initial Costs:

Project A: \$7.4 Million
Project B: \$6.5 Million
Project C: \$5.8 Million

Operating costs including road repair, inspections, and traffic-light monitoring are factored into the initial cumulative costs at roughly \$0.4 million per project per 2-year period. (see Tables IX.7.1, IX.7.2, and IX.7.3 below).

Table IX.7.1. Parameter Values and Descriptions

Parameter	Value	Description
A	1	Multiplier effect for initial benefits
B	1	Multiplier effect for control variable
C	1	Proportionality constant for system input and output
x_0	0.5	Initial level of optimal transportation conditions (throughput/uninterrupted travel/no delays)
P	0.005	Variance of random variable $\omega(k)$
R	0	Variance of random variable $\nu(k)$
X_0	0.0025	Initial variance

Table IX.7.2. u(k) Values for each Policy

	$u(k)$, level of traffic improvement at stage from period k to k+1		
Policy	$u(0)$	$u(1)$	$u(2)$
1	-5.0%	25.0%	5.0%
2	-3.0%	8.0%	18.0%
3	-3.0%	9.0%	9.0%

Table IX.7.3. Cumulative Costs for each Policy

	Cumulative cost (CC) at period k–1 in \$millions		
Policy	$CC(0)$	$CC(1)$	$CC(2)$
1	19.7	20.9	22.1
2	7.4	20.1	21.3
3	7.4	14.3	20.9

Since u(k) is the level of traffic improvement in the time period, it can be between −1.0 and +1.0. Traffic throughput will degrade during construction, as lanes will be closed, detours are created, and traffic is slowed.

PROBLEM IX.8: Funding Skin Cancer Research

Scientists in Switzerland are considering methods for reducing the number of cases of a certain type of skin cancer that currently affects 10,000 people in the country. Fifty million dollars has been allocated for a 3-year endeavor to help prevent and cure cases of skin cancer.

Three policies have been devised to allocate the funds. With each policy, there is an associated cost related to the present value of the funds being used for each year as well as the implied costs associated with administering each option. The Multiobjective Risk Impact Analysis Method (MRIAM) is used to arrive at a solution.

Variable Definitions

$x(k)$ = number of cases of skin cancer in year k, $x(0) = x_0 = 10{,}000$

The simplified model is

$$x(k) = A^k x(0) + \sum_{i=0}^{k-1} A^i$$

$$x(k+1) = Ax(k) + Bu(k) + w(k)$$

$$y(k) = Cx(k)$$

$u(k)$ = amount of money spent on skin cancer research at year k.

$w(k)$ = random variations due to risk factors such as sun intensity, general knowledge of preventative measures (sunscreen, sun time, etc.)

a = rate of diagnosis of new skin cancer patients = 0.05

A = growth rate of population afflicted with skin cancer = 1.05

B = number of cases of skin cancer per dollar spent = 1 case per \$10,000 = -0.0001

C = coefficient parameter = 1

P = variance of random variable $w(k)$ = $\mathrm{E}[w^2(k)] = P(k) = 600$

R = variance of random variable $v(k)$ = 0

X_0 = initial variance = 500

Table IX.8.1 lists the costs associated with skin-cancer research policies (strategies) *a, b,* and *c:*

Table IX.8.1. Comparative Costs of Cancer Prevention Strategies

	u(k)=funds to be used		
Strategy	$u(0)$	$u(1)$	$u(2)$
a	\$20,000,000	\$20,000,000	\$10,000,000
b	\$17,000,000	\$17,000,000	\$16,000,000
c	\$28,000,000	\$5,000,000	\$17,000,000

Given the model and data, conduct the Multiobjective Risk Impact Analysis method to evaluate the Pareto-optimal solution(s).

PROBLEM IX.9: Controlling River Channel Overflow

Policymakers in a metropolitan city in the Philippines are challenged with solving the chronic flooding problem caused by overflow of a major river.

The Marikina River is a main waterway that supports commodity transport and other economic livelihood activities in a major city in the Philippines. However, heavy rainfall events that visit the area every year cause the river levels to rise above limits resulting in severe flooding with catastrophic consequences.

Most policy decisions and infrastructure investments involve long-range impacts to other decisions and concerns. The objective of this problem is to model and identify the impact of the current policy decision on future concerns using the Multiobjective Risk Impact Analysis Method (MRIAM).

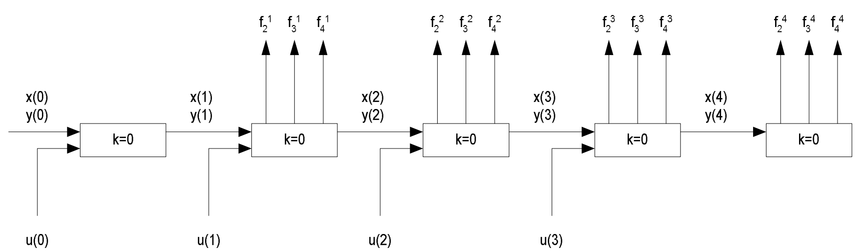


Figure IX.9.1. System model

where,

- $x(k)$ represents the state variables, defined as the number of affected localities when the river overflows for Period k;
- $y(k)$ represents the output variables, defined as the damages (in Philippine Pesos—PhP) for period k;
- $u(k)$ represents the control variable, which is the amount of money spent on an option at period k;
- f_i^k represents the conditional expected value of risk calculated for each period k

The general form of the state equation is:

$$x(k+1) = Ax(k) + Bu(k) + \omega(k)$$

and the output equation is:

$$y(k) = Cx(k) + \upsilon(k)$$

where,

$A = 1.25$, growth rate of the population in affected localities

B = (-)50, number of residents protected per unit (1,000 PhP) investment (residents/ 1,000 PhP), that is -0.05

C = PhP 2000, average damage cost per resident in a flood event

ω (k) random

υ (k) random

Perform Multiobjective Risk Impact Analysis method to evaluate the Pareto-optimal solution(s).

Table IX.9.1 summarizes the three policies being evaluated that define the schedule of budget release for the control of river overflow.

Table IX.9.1. Budget Allocation Policies

	u(k-1) = Amount Spent on Flood Control Project (in million PhP)			
Policy	k=1	k=2	k=3	k=4
A	0.5	0.5	1.5	1.5
B	4	0	0	0
C	1	1	1	1

Note: It is assumed that the funding is released at the beginning of a period (i.e., Period k-1), and the effect would be realized in Period k. Use one standard deviation partitioning for $f_2(k)$ and $f_4(k)$.

PROBLEM IX.10: Modeling the Design Phases of a New Automobile

A design review looks at the progress of the project and analyzes it compared to past projects and current expert opinion. As the design is partially completed, the review reduces uncertainty in the final cost.

There are three phases in the design process for a new automobile: These are:

- The *Concept* phase (k =1), a very abstract brainstorming session where no concrete design specifications are used. At the completion of the conceptual phase, you should have the vehicle class, feature packages, and a general idea of what makes your design different from all other vehicles in its class.
- The *Pre-prototyping* phase (k = 2), where the detailed design work begins. The emphasis is on accurate cost analysis, manufacturing feasibility, and production scheduling.
- The *Prototyping* phase (k = 3), where the selected alternative is designed and built in full, and any more minor improvements are made.

In keeping with the 3-phase design process, there are several strategies to conducting a design review. The more money spent on the design review, the more accurate the reduction in uncertainty of the final project, but at a cost of adding to vehicle cost and production delay. Likewise, a more accurate result is derived from a review in a later design phase, but at the cost of reduced flexibility in design changes based on suggestions from the design review board. Reviewing an earlier design is less costly. The company has $5 million available for the design review process, and we must decide how much and when to spend the available funds. Initially, there is an expected uncertainty of 90% in sales after introducing the new design.

We use the Multiobjective Risk-Impact Analysis Method (MRIAM) to decide which strategy is best to reduce uncertainty in the new design.

For all periods (k = 1, 2, and 3) and strategies (a, b, and c), Tables IX.10.1 through IX.10.3 provide the data on proportion of funds, level of fund utilization, and cumulative cost, respectively.

Table IX.10.1. Proportion of Funds for each Period

	$p(k-1)$ = proportion of funds used for design review		
Strategy	$p(0)$	$p(1)$	$p(2)$
A	0.8	0.1	0.1
B	0.1	0.7	0.2
C	0.1	0.3	0.6

Table IX.10.2. Level of Fund Utilization for each Period

	$u(k\text{-}1) = \ln[1\text{-}p(k-1)]$		
Strategy	$u(0)$	$u(1)$	$u(2)$
A	ln(0.2)	ln(0.9)	ln(0.9)
B	ln(0.9)	ln(0.3)	ln(0.8)
C	ln(0.9)	ln(0.7)	ln(0.4)

TableIX.10.3. Cumulative Cost at Period *k*–1

	Cumulative cost (CC) at period $k-1$ (PV in millions)		
Strategy	$CC(0)$	$CC(1)$	$CC(2)$
A	2.4	2.7	3.0
B	0.4	3.2	4.0
C	0.3	2.2	5.0

Table IX.10.4 summarizes the values of parameters in the above mean and variance formulas:

Table IX.10.4. Summary of Parameters

Parameter	Value	Description
A	1	Multiplier effect for initial sales uncertainty
B	.5	Multiplier effect for control variable
C	1	Proportionality constant for system input and output
x_0	ln(0.9)	ln(Initial sales uncertainty)
P	0.1	Variance of random variable $\omega(k)$
R	0	Variance of random variable $\nu(k)$
X_0	0.03	Initial variance

Perform Multiobjective Risk Impact Analysis method to evaluate the Pareto-optimal solution(s). Use one standard deviation partitioning for $f_2(k)$ and $f_4(k)$.