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# Fractal Image Coding Using Luminance-Based Partitioning

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**Abstract:** In order to accelerate the procedure of capturing the similarity of image data in fractal image coding, a partitioning scheme, based on the spatial luminance distribution, is proposed. In the first step the image is partitioned using quad-tree technique, until the pre-selected size of coding unit (i.e. the *range block*) is met. According to the *luminance distribution* within the range blocks, the *midrange* and the *edge* blocks are further classified as *Z*, *O* or *Y* blocks. During the search for similar image parts, only the *range* and *domain* blocks of same type (i.e. *shade*, *midrange* or *edge*) and of the same *Z*, *O*, *Y* class are considered.

The algorithm of fractal image coding has been implemented for intraframe coding in a predictive interframe video coding scheme. It has been found that the time required for fractal coding using luminance-based partitioning is significantly shorter than the time required for other partitioning schemes, that is of fundamental importance in video coding.

**Keywords:** *fractals, fractal image coding, image compression, image partitioning, contractive image transformations*

# Scale Relativity: Many-Particle Schrödinger Equation

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**Abstract:** After having stated the principle of scale relativity and summarized the new scale-dependent mathematical tool that we use in order to implement it, we first recall how giving up the hypothesis of differentiability of space-time implies its fractal structure and allows one to recover the one-particle quantum mechanics. Then we obtain the Hamiltonian form of the Schroedinger equation from the same grounds, and finally apply this result to the many-identical particle problem.

# The Use of the Fractal Dimension in Images for Plant Recognition

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**Abstract:** Fractal information from images can be used as a tool for plant recognition. Depending on the application plant images at various scales can be used. For the distinction between certain weeds in row crops images with a low number of plants (excluding total canopy cover) are used. To determine the fractal Hausdorff dimension links with the Fourier transform are exploited. However, scatter in the power spectrum is reduced by integration in the spatial frequency domain over band-limited spatial filters. Using this approach the Hausdorff dimension is determined for whole images and subsets of these images. The usefulness of the resulting fractal dimension information as a tool for image segmentation and crop versus weed recognition are discussed.

**Keywords:** *Plant images, Hausdorff dimension, spatial frequency integration, plant discrimination.*

# Tumour Shape and Local Connected Fractal Dimension Analysis in Oral Cancer and Pre-cancer

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**Abstract:** The local complexity of the epithelial-connective tissue interface (ECTI) in normal tissues, epithelial dysplasia and squamous cell carcinoma of the floor of the mouth was investigated using the concept of local connected dimension. It was found that the distribution of the local dimensions of the ECTI alone can classify the three types of histopathological diagnoses (normal, dysplasia and carcinoma) with 85% to produce a colour-coded dimensional image of the ECTI that permits the automatic location of supposedly "higher risk" areas.

# The Solution of the Inverse Fractal Problem Applied to a Self-affine Fractal with the Help of Wavelet Decomposition

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**Abstract:** We solve the inverse fractal problem for self-affine functions in  $\mathbf{R}^2$  by means of recovering the maps with which the function was created. A suitable representation is used - wavelet maxima bifurcation representation - derived from the continuous wavelet decomposition which possesses translational and scale invariance. It allows us to uncover the invariance with respect to position and scale in the case of the self-affine fractal, and may also prove itself useful in other applications involving analysis of scale dependent features. Two algorithms are presented which give satisfactory results for the self-affine fractal and which potentially can be applied to a variety of fractal types in order to solve the related inverse fractal problem.

**Keywords:** *inverse fractal problem, wavelet transform, wavelet maxima bifurcation,*

*representation, self-affine fractals*

# Qualitative Analysis of Human Blood Based on Elements of Fractals

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**Abstract:** It has been shown experimentally that healthy red blood cells can create fractal clusters with the dimension 2,5 after the process of mophilisation of the blood. It was found that the blood of some children suffer against blood diseases acquired after Chernobyl accident has a fairly uniform distribution of the red blood cells.

# Fractal Properties of Relaxation Clusters and Phase Transition in a Stochastic Sandpile Automaton

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**Abstract:** We study numerically the spatial properties of relaxation clusters in a two dimensional sandpile automaton with dynamic rules depending stochastically on a parameter  $p$ , which models the effects of static friction. In the limiting cases  $p = 1$  and  $p = 0$  the model reduces to the critical height model and critical slope model, respectively. At  $p = p_c$ , a continuous phase transition occurs to the state characterized by a nonzero average slope. Our analysis reveals that the loss of finite average slope at the transition is accompanied by the loss of fractal properties of the relaxation clusters.

**Keywords:** *Self-organized criticality, sandpile automata, relaxation cluster, fractal dimension*



# Absorption Spectra and Photomodification of Silver Fractal Clusters

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**Abstract:** Absorption spectra shape of silver colloid aggregates are studied in combination with electron microscopic investigation of the samples. The experimental results, concerned with the selective photomodification of silver fractal clusters are reported. A correlation was made between the experimental results and the scaling theory of the optical absorption of fractal clusters. It was found that absorption spectrum shape of quasi-percolated aggregates in a silver colloid corresponds to the power law. The optical spectral dimension was found to be  $d_O \approx 0.5 \div 0.7$  for different realizations of colloid. The results of the spectral hole burning experiments are in a qualitative agreement with the estimations, based on the concept of localized collective dipolar states in a cluster.

**Keywords:** *fractal clusters, absorption, photomodification, scaling*

# Fractal Growth of Bacterial Colonies

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**Abstract:** Complex growth patterns of bacteria on solid agar were studied. The colonies were found to be not self similar objects, although the contrary was believed. Their two-fold fractal structure was revealed. Two different mechanisms governing the pattern formation below and above a certain critical scale were specified. A scenario of a colony development was suggested.

# Stochastic L-System Applied to the Calculation of the Leaf Area of a Shrubby Legume for Forage (*Chamaecytisus ruthenicus*, F. ex Wol.)

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**Abstract:** A stochastic and deterministic L-system was developed to calculate the total leaf area of a shrubby legume used for forage at the end of each growing season. Several tables were defined to describe the structure (topology) and the physiology of the plant for each life cycle. In order to include the influence of the field temperature, several parameters were related to it. Also a growth function for the deterministic model was defined and applied for a bracketed L-system.

**Keywords:** *TL-systems, stochastic, branching pattern, growth function*

# Some Remarks on Phase Transitions and Their Disappearance in the Descriptions of Multifractal Sets

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**Abstract:** For a family of skew Ulam maps, the cycle expansion theory guarantees more accurate location of transition points. Using this approach, we have found that the disappearance of phase transitions has an interesting scenario.

**Keywords:** *cycle expansions, multifractal*

# Chemically-Controlled Reaction Kinetics on Fractals: Application to Proton Exchange in Proteins

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**Abstract:** The effect of fractal diffusion on chemically- controlled reactions in solutions is considered. A general mechanism is examined that consists of a two step process. First the reactants diffuse together to form an "encounter complex." This is followed by the collapse of the complex to the final product. The first step is diffusion controlled and the second step is chemically controlled. For reactions on fractals the rate constants associated with the diffusive process will scale with time as  $t^{-h}$  where  $h$  is a constant between 0 and 1. The chemical processes are assumed to have time-independent rate constants. For reactions in which the encounter complex achieves a steady state, the differential equations governing the time course of the reaction can be solved exactly. At short times, the concentration of the reactants decays exponentially, reflecting the time constant of the chemical processes. At longer times, the decreasing diffusive rate constants result in the process being diffusion controlled. A stretched exponential of the form,  $\exp(-kt^{1-h})$ , is observed. Approximate solutions for the pre-steady state behavior of the system are also determined. These theoretical results are applied to the analysis of proton exchange kinetics in proteins. Exchange kinetics are modeled as a reaction in the boundary volume of a fractal.

# Variogram and Wavelet analysis of a cloud field advected by a baroclinic flow

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**Abstract:** In this paper scaling laws in a relatively homogeneous cloud system generated and advected by a strong baroclinic instability have been investigated. The observations consist in an infrared satellite image with a spatial (horizontal) resolution of about 1 km. The presence of two sizeable and unmistakable scaling regions, one extending from 4 to 15 km and characterized by a power law with an exponent close to 1, the other stretching from 20 km up to 100 km and characterized by a power law with exponent close to 1/3, have been revealed by variogram analysis. These two scaling laws are in agreement with the idea of universality of the turbulent motion and also suggest the presence of a self-similar structure. To explore this possibility, wavelet transform analysis at different spatial scales has been used. Our findings are that self-similarity is present at the smallest scales, but this universal characteristic is masked by non-universal effects which influence the homogeneity of the underlying turbulent motion.

# Some Experimental Results in Viscous Fingering

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**Abstract:** We show experimental results carried out to analyze viscous instabilities in an isotropic porous medium consisting of glass spheres packings. In all cases the sample was first saturated with a polymer dispersion (xanthan) at different concentration and then flooded with dyed water. A rich variety of patterns was found. The results obtained have shown the crucial influence of the fluid viscosity ratio in the resulting structure. For values larger than 100 a fractal behavior is observed. A modification to the density-density correlation function is also proposed to take into account the dependence of the fractal dimension with the relative position on the structure. The fractal dimension was measured on bands along the structure and values different are found.

# Interface roughening in magnetic systems with quenched disorder

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**Abstract:** The kinetic roughening of an interface between spin-up and spin-down domains in a model with non-conserved scalar order parameter with quenched disorder is studied numerically within a discrete time dynamics at zero temperature. Starting from a flat interface in a two dimensional system the time evolution of the height correlation function and of the width of the interface is analyzed. It is found that the interface is rough on length scales smaller than a characteristic length  $\Delta$  which depends on the degree of disorder. A novel scaling ansatz for this situation is proposed and the data are analyzed within this framework.

**Keywords:** *interface growth, kinetic roughening, model A dynamics*



# Fractal properties and scaling exponents of the Barkhausen effect

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**Abstract:** Using the results of a theory of the Barkhausen effect based on a Langevin approach of the domain wall motion in a medium with brownian properties, we calculate the scaling exponents of the durations and sizes distributions of the Barkhausen jumps. The Barkhausen signal is found to be related to a random Cantor dust having a Hausdorff dimension which is a linear function of the applied field rate. Using simple properties of fractal geometry, the distributions are easily calculated. The predictions are then verified for a Si-Fe alloy. The fractal dimension  $\Delta$  of the signal is also studied using four different methods, all giving  $\Delta = 1.5$ . The relation between the above results and the power spectra data are analyzed in the frame of self organized-criticality.

# Temporal Scaling Regions and Capacity Dimensions for Microearthquake Swarms in Greece

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**Abstract:** The characteristics of temporal scaling ranges and capacity dimensions are studied in detail for microearthquakes recorded over a two year period in three regions in central Greece. The three regions are centred on Volos, Pavliani and Patras respectively and have very different long-term seismicity and seismotectonic characteristics. The scaling range ( $2^7 - 2^{12}$  s) exists for both the Volos and Patras swarms and these have a tectonic background of strong seismicity including earthquakes of magnitude  $M_s \geq 6.0$ . This might be an important indicator that distinguishes them from the Pavliani swarm which has no historical evidence of large earthquake occurrence. Under the condition of an identical threshold magnitude, the capacity dimension values of temporal distributions for both the Pavliani and Patras swarms are higher than that for the Volos swarm. Such differences in dimension value might relate to the local complexity of the seismotectonic environmental and its characteristic seismicity.

# Object Identification in Greyscale Imagery Using Fractal Dimension

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**Abstract:** A fractal dimension operator which assigns a local fractal dimension to each pixel in a greyscale image can be used to help identify objects from a number of imaging sources, to include aerial photography, satellite imagery, radar, and x-rays. In this paper, we identify a problem with the consistency of such operators, demonstrate a shortcoming in some existing algorithms, and propose a general method for computing local fractal dimension which can be tailored to different image sources. We do this in the context of our specific application: identification of man-made objects in images produced by aerial synthetic aperture radar.

# Object Instancing Graphtals

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**Abstract:** This paper describes an application of fractals in computer graphics. It is well known that fractal theory leads to algorithms for rendering many objects in nature, like trees, mountains and seashells. It is however our experience that strictly self-similar figures are of limited use for everyday rendering: one often wants to render figures that exhibit *some* self-similarity, but are not fractals strictly according to the definition.

It is shown in this paper how by extending the notion of “object instancing”, a well known notion in computer graphics, a formalism for describing such figures is obtained. The main topic in this article is however a raytracing algorithm that was developed for rendering such figures. The advantages of our approach are its flexibility and its easiness to understand and to use.

**Keywords:** *fractals, formal languages, raytracing*

# Study of Fractal Properties in Lichtenberg Figures

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**Abstract:** The fractal stochastic model for dielectric breakdown is used to simulate Lichtenberg figures produced in a point to plane geometry. Four different models for the growth probability of the discharge are applied. Solving the Laplace equation is on the basis of all the models. For the simulated figures the fractal dimension was calculated using the box counting method. The influence of the several computation parameters in the fractal dimension of the patterns was studied. Simultaneously an experimental setup was used to obtain discharges of the same kind. The experimental figures were recorded with a photographic camera and digitalized. These images undergo an image treatment process in order to estimate the fractal dimension. A method of counting the number of branches is implemented for this purpose.

**Keywords:** *dielectric breakdown, fractal dimension, leader discharges, Lichtenberg figures, simulation*

# Tori-in-a-Torus Fractals

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**Abstract:** We present an algorithm of an iteration rule capable to generate attractors with dragon-, snowflake-, sponge-, or Swiss-flag-like cross-sections. The idea behind is the mapping of a torus into two (or more) shrunk twisted tori located inside the previous one. Upon intersecting the attractor, we simultaneously obtain both connected and disconnected fractal structures.

**Keywords:** *Fractals, Dynamical Systems, Algorithms*

# Fractal Parameters of Soil Pore Surface Area Under a Developing Crop

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**Abstract:** Fractal parameters of soils become increasingly important in understanding and quantifying of transport and adsorption phenomena in soils. It is not known how soil plant development may affect fractal characteristics of soil pores. We estimated pore surface area fractal parameters from mercury porosimetry data on gray forest soil before and during crop development in samples both containing and not containing soil carbohydrates known to be important structure-forming agents. Two distinct intervals with different fractal dimensions were found in the range of pore radii from 4 nm to 1  $\mu\text{m}$ . This could be attributed to differences in mineral composition of soil particles of different sizes. The interval of the smallest radii had the highest average fractal dimension close to 3. Smaller surface area fractal dimensions corresponding to low surface irregularity were found in the next interval of radii. The plant development affected neither fractal dimensions nor the cutoff values of soil samples. The carbohydrate oxidation caused a significant increase of the fractal dimension in the interval of larger radii, but did not affect fractal dimension in the interval of small radii. The cutoff values decreased after carbohydrate oxidation.

**Keywords:** *Soil pore surface, Fractal dimension, Structure-forming agents, Temporal variations*

# Similarity in the Problem of Discrete Contact between Fractal Surfaces

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**Abstract:** Contact problems for rough surfaces described by fractal parametric-homogeneous functions are under consideration. It is assumed that the contact is discrete. Material of contacting surfaces may have both linear and non-linear stress-strain relationships. The qualitative conclusions on the character of changes of the contact region and approaching of surfaces are described. The results are derived using a similarity method, without solving the field equations.

**Keywords:** *fractal surfaces, contact problem, similarity*



# A Graphic Tool to Determine the Period of an Antenna Component of the Mandelbrot Set

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**Abstract:** Determining the period of a cardioid-like component of the real part of the Mandelbrot set, the antenna, can be a tedious task. To avoid it, an easy graphic tool to obtain the period is introduced. We point out that the main filament of every component of period  $n$  attracts the escape lines from  $n$  to  $n$  and does not attract the other ones. Then, the period  $n$  of a component can directly be determined by knowing the last escape line attracted by its main filament.

# Fractal Geometry of Quantum Paths and the Fractal Wilson Loop

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**Abstract:** We suggest that the fractal nature of quantum mechanical paths should be seen in non-local order parameters in gauge theory. e.g., the Wilson loop, and hence should play a role for the question of confinement. We present results of numerical simulations on the lattice for imaginary time quantum mechanics, measuring the length of paths and its critical exponent (Hausdorff dimension). For local potentials we find results compatible with  $d_H = 2$ . Secondly, we consider the length of a quark propagator in lattice *QCD*. We present a definition, discuss its properties and present some numerical results for a simplified free propagator. Finally, we consider a fractal Wilson loop  $\langle F_P \rangle$ . We show for non-compact *SU(2)* lattice gauge theory in the next to leading order of strong coupling expansion that  $\langle F_P \rangle$  obeys an area law behavior and is gauge invariant.

# Analysis of an Image of the Human Brain obtained by Positron Emission Tomography in Terms of Fractal Geometry

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**Abstract:** We introduce the concept of fractal geometry to describe the irregular spatial distribution of [18F]2-fluoro-2-deoxy-D-glucose (FDG) in a section of a human brain. FDG is labeled with a positron emitting isotope that is measured by Positron Emission Tomography (PET) after intravenous injection. At first, we construct binary subsets from the image data for different ranges of radioactivity. Such subsets contain only spatial information, which allows the determination of self-similar properties. For a subset generated from low to high radioactivity we find a fractal dimension,  $D$ , of 1.9. For a subset representing basically high radioactivity we find a  $D$  of 1.2. This indicates a multifractal behavior of the set, which will be analyzed in detail in a future study. Further, the data is analyzed as a "landscape" where two dimensions are given by the spatial coordinates of the cross section of the brain and the third dimension, the height of the "landscape", is defined by the radioactivity at the spatial position on the slice. The analysis of this self-affine set with the mass radius relation gives a  $D$  of 2.3. We show in this preliminary study that  $D$  can serve as a mathematically well-defined measure to describe the global irregularity of a PET image. We assume that  $D$  will be of clinical relevance.

**Keywords:** *Fractals, Positron Emission Tomography, Brain Research*

# New Approach to Synthesis of Fractal Materials With a Given Fractal Dimension

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**Abstract:** A method for the synthesis of fractal films of CuS is proposed. An approach is based on a possibility of fractal films formation which is induced by spatial patterns formed in self-organized chemically reactive media. We used a pattern formation in 'oxygen' oscillating chemical reaction - oxidation of ascorbic acid by air oxygen in the presence of hydrogen sulfide ions, methylene blue, and copper(II) coordination compounds. A CuS film formation is observed for this system owing to the slow reaction between copper(II) coordination compound and hydrogen sulfide ions. A fractal dimension of CuS films correlates with the fractal dimension of chemical patterns. The value of fractal dimension for CuS films is always higher than for chemical patterns. This difference might indicate that the chemical patterns only induce the creation of a fractal film through the formation of the growth centers for CuS, i.e. chemical patterns are responsible only for the birth of fractal clusters. The growth of these clusters is provided by the independent mechanism of the CuS slow precipitation from the solution. The electrical resistivity of fractal films and the sensitivity of copper(II) ions selective electrodes with fractal CuS surface to the copper(II) ions were measured. Our experiments indicate that the behavior of Cu(II)-selective electrodes with fractal surface is independent on the value of a fractal dimension. The difference exists only between electrodes with  $D = 2$  and  $D > 2$ .

**Keywords:** *fractal dimension, fractal film, copper sulfide, oscillating chemical reaction, patterns formation*

# $1/f^\beta$ -Fluctuations in Bipolar Affective Illness

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**Abstract:** Temporal fluctuations which cannot be explained as consequences of statistically independent random events are found in a variety of physical and biological phenomena. These fluctuations can be characterized by a power spectrum density  $S(f)$  decaying as  $1/f^\beta$  at low frequencies with an exponent  $0.5 \leq \beta \leq 1.5$ . We present a new approach to reveal  $1/f^\beta$ -fluctuations in manic and depressive episodes in bipolar affective illness using published data from patients for whom daily records were obtained applying a 7-point magnitude category scale. This time series  $\{R(t_i)\}$  was described as a point process by introducing discriminating rating levels  $r$  and  $s$  for the occurrence of  $R(t_i) \geq r$  ('mania') and  $R(t_i) \leq s$  ('depression'). For  $\beta < 1$  a new method to estimate the low frequency part of  $S(f)$  was applied using counting statistics without applying Fast Fourier Transform. The method reliably discriminates these types of fluctuations from a random point process with  $\beta = 0.0$ . It is very tempting to speculate that the neuronal/humoral mechanisms at various levels of the nervous system underlying the manic and depressive episodes in bipolar affective illness are expressions of a self-organized critical state. But the most important result of the present study is the finding of a scaling region  $1d \leq \Delta t \leq 200d$  for the 'manias' and 'depressions' where  $S(f)$  is decaying as  $1/f^\beta$  with  $\beta \approx 0.8$ . Therefore, based on the monitored ratings for a given time period it should be possible to predict future episodes with a certain probability by applying methods of nonlinear time series analysis or modified feed-forward neural networks learning with the back-propagation algorithm. This could result in an improvement of the treatment of patients.

# Fractal Surfaces as Self Organization of Microfractures

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**Abstract:** Fracture mechanisms are formulated on the grounds of physical concepts on crack propagation in solid structure. The fracture is treated as loss of stability in stretched structure, for which the fluctuations of state are inevitable and lead to creation of zones with disturbed continuity. The fact that stability is lost implies the fractal surfaces growth phenomenon should be treated as a self - organization of microfractures in non-equilibrium system which can not be considered in terms of infinitesimal variations. Cracks have the typical zigzag fractal microconfiguration depending on the structure of loading the domain.

**Keywords:** *crack, propagation, microconfiguration, directions, regularity*

# An Aggregation Model for Town Growth

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**Abstract:** A new aggregation model based on the Eden model and its properties is presented. The local density  $\rho(r)$  exhibits scaling properties and can be written as  $\rho(r) = N^\alpha p^{-b} f(r/N^c p^d)$  when  $N$  is the total number of particles and  $p$  an integer characteristic of the model. The exponents are determined:  $a \approx d = 0.15 + - + 0.02$ ,  $b = 0.32 + -0.01$ ,  $c = 0.425 - +0.005$ . Application of the model to London and the Rhine valley towns is discussed.

# The ‘Smoothing Dimension’ - a New Fractal Analysis Method

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**Abstract:** Signal analysis involved in geophysical research or in studies regarding biological systems often imposes the difficult task of significant signal classification, in order to distinguish different types of behaviour (reflected by the same parameter at different moments in time) or to help appreciating the agreement between models and modelled systems. We introduce a new fractal analysis method, which emphasizes the way in which the length of the graph associated with the time series scales with the cut-off frequency of a filter used to "smooth" the graph. The obtained "Smoothing dimension"  $D_L$  can be determined with a good accuracy and is significant even for signals not having power-law type power spectrum.  $D_L$  proves to be well correlated with the Hurst exponent  $H$  for a certain class of signals recorded in experimental research, enhancing the possibilities of signal classification.

**Keywords:** *fractal analysis, Hurst exponent, signal classification*



# A Classification of Writings Relying on Fractal Behaviour

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**Abstract:** Since the interest for fractal geometry has risen, the applications are getting more and more numerous in many domains. The aim is to show that the concepts may be applied in the domain of automated writing treatment too and that they can bring some help, to a certain extent, to ease the solution of many problems. After the fractal behaviour of writing has been shown and its stability too, either through centuries or, for a single writer, all along his life time, a fractal dimension is computed for many types of writings and a comparison is achieved. Thus, the authors come to a classification. Two classification grids show the distribution of different types of writings. This study forms an important step that enables a better orientation of the research works in the optical reading domain and specially in the recognition task. Of course, our hypothesis has to be validated by use of an even greater number of writing samples.

**Keywords:** *fractal dimension, writing classification, legibility degree*

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# Multifractality of the Giant Electric Field Fluctuations in Semicontinuous Films Close to the Percolation Threshold

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**Abstract:** We show that the intensities of the local electric field in metal-dielectric semicontinuous films may exhibit giant fluctuations in the frequency range dominated by cluster plasmon absorption when the dissipation in metallic grains is low. It is found that these fluctuations are highly correlated in space and that for length scales smaller than the corresponding correlation length, the distribution of these local fields is multifractal.

**Keywords:** *Multifractality, percolation, thin films, optical properties, real space renormalization*

# Computing Orthogonal Polynomials for Fractal Measures and the Dynamics of Quasi-Periodic Tight-Binding Models

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**Abstract:** Iterated Functions Systems are a well developed mathematical formalism and a convenient computational tool to describe a large class of fractal measures. Yet, the problem of determining the associated orthogonal polynomials has been solved satisfactorily only recently; in this paper we present the main lines of this solution and its first, immediate applications to numerical integration and modelling of solid state mesoscopic systems and chains of oscillators.

# Far Infrared and Optical Absorption of Fractal and Multiscaling Metallic Clusters

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**Abstract:** We show that the electrical deterministic fractal lattice (DFL) model and its extension to describe multiscaling fractal objects provide a good physical and theoretical understanding of the observed far-infrared (FIR) anomalous absorption properties of fractal metallic clusters as well as the cluster plasmon resonances discussed previously by means of numerical methods. Using the analytical properties of the DFL and the Decorated-DFL (DDFL) models, it is possible to calculate exactly the power law exponents of the enhancement factors  $B$ , of the cross-over localization length  $L_\omega$ , as well as the Cantor structure of the self-similar cluster plasmon resonance spectrum in terms of the fractal characteristics of the aggregate.

**Keywords:** *Infrared absorption, optical properties, fractal cluster, cluster plasmon modes, multiscaling*

# Intermittency and Fractal Behaviour of Medium Energy Particles at High Energy

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**Abstract:** The experimental data on nuclear emulsion at 800 GeV has been analyzed to study intermittency and fractal characteristics of medium energy particles, the so called "grey particles", in terms of factorial moments. The fractal dimensions and phase structure function determined are found to be in agreement with the theoretical predictions.

**Keywords:** *Multiparticle production, Intermittency, Fractal dimension, Phase transition*

# A theory for the Morphology of Laplacian Nonlinear Growth Processes via Statistics of Equivalent Many-body Systems

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**Abstract:** The nonlinear evolution of two dimensional interfaces in Laplacian fields is discussed. By mapping the growing region conformally onto the unit disk the problem is converted to the dynamics of a many-body system. This problem is shown to be Hamiltonian, with the Hamiltonian being the complex electrostatic potential. The Hamiltonian structure allows introduction of surface effects as an external field. An extension to a continuous density of particles is presented. The results are used to formulate a first-principles statistical theory for the morphology of the interface using statistical mechanics for the many-body system. The distribution of the curvature and the moments of the growth probability along the interface are calculated exactly from the distribution of the particles. In a specific approximation the distribution of the curvature is shown to develop algebraic tails which points to the onset of fractality.

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# Transition Processes on Noise-Induced Fractal Sets

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**Abstract:** Attractor structure and transition dynamics for dissipative, bistable oscillators driven by a periodic-dichotomous-noise process are described. The oscillator motion constitutes a smooth trajectory flow in phase space between noise-switching events, which are Bernoulli trials on a two-branched quartic potential. This continuous-time stochastic dynamics induces a (two-branched) smooth stochastic map of the plane in discrete time. As the oscillator mass decreases, the corresponding attractor of this planar map changes, becoming fractal or quasi-atomic for intermediate and small mass. The transitions between left and right species defined in configuration space are studied in relation to the changes in attractor structure. The reaction dynamics can be understood in terms of the attractor structure in the transition region and the short and long time behaviour of the species autocorrelation function.

**Keywords:** *Fractal attractors, noise-induced transitions*

# Application of the Lattice-Boltzmann Equation to Transport and Hydrodynamic Phenomena in Fractal and Disordered Media

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**Abstract:** Lattice Boltzmann equation is a powerful tool for studying fluid dynamics and transport phenomena in complex geometries. In this article we focus attention on two classical hydrodynamic problems: creeping flow around an array of fractal objects and permeability measurements in disordered (percolating) systems.



# Adsorption Equilibria in Microporous Materials: the role of Fractality and of Energetic Heterogeneity

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**Abstract:** We develop a thermodynamic analysis of adsorption equilibria in microporous materials, focusing on the relationship between the adsorption isotherms and the structural (energetic, geometric) parameters characterizing the adsorbate/adsorbent system. Amongst the proposed theoretical models that include fractal scaling, Keller adsorption isotherms are discussed in detail. In this model, the fractality of the adsorbent plays the role of a closure condition, which reduces the number of independent parameters. Experimental results are reported to validate this model in the case of single and multicomponent equilibria. The problems related to thermodynamic consistency and to the low pressure behaviour of adsorption isotherms are also discussed.

# Scale Invariance in Long-term Time Series

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**Abstract:** To evaluate climate variability at time scales 100-1,000,000 years, the continuous records of paleotemperatures from SPECMAP project, the extended Vostok ice-core temperature and air composition records, as well as GRIP oxygen isotope record were analyzed in terms of structure functions. It is shown that the characteristic temperature and gas concentration changes follow the scale invariant law up to time scales of about 10,000 years (smaller than formerly recognized); the parameters of structure functions depend upon site location. Although different sites yield different slopes, there is a clear evidence for a break in the scaling at time scales of about 10,000–20,000 years at high latitudes and of about 7,000 years in equatorial belt. At time scales larger than the scaling limit and up to 700,000 years the characteristic amplitudes of temperature variations are almost constant, with superposed on it series of local maxima and minima that corresponds to the Milankovitch periodicities. The break of scaling signifies the appearance of the basic periods in climate system. The time scale corresponding to the scaling limit identifies the boundary between the random and deterministic regimes and thereby is a fundamental climatic parameter. The earlier break of scaling and the scaling exponent lower than 0.5 in equatorial sites, as well as a small range of characteristic temperature fluctuations in tropics supports the existence of stabilizing mechanisms in tropical climate, the phenomenon which is largely discussed. Several breaks in structure function of the GRIP oxygen-isotope profile and a very low exponent in scaling regime suggest a significant role of regional processes in the climate fluctuations over Greenland, with characteristic periods of a few thousand years.

# IFS based on 3D Self-similar Contraction Mapping and Modeling Trees and Coral

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**Abstract:** We present an Iterated Function System (IFS) based on three-dimensional (3D) self-similar contraction mapping to model trees and corals. IFS is the fundamental mathematical method to generate fractal patterns, such as snowflake and ferns. Those patterns are essentially two-dimensional. There are many references on using IFS for fractal generation, but most are concerned with 2D IFS that are related to complex variable functions, suitable for the generation of 2D fractals. We present an IFS based on truly self-similar 3D contraction mapping functions in order to model 3D tree patterns. The method has proven particularly suited when modeling Leeuwenberg's trees and corals. It can be represented in a very simple form and is suited to high-speed computations. In order to generate realistic tree images, we also propose methods to generate leaves of various patterns.

**Keywords:** *IFS, 3D self-similar contraction mapping, fractal tree, coral*

# The Temperature Dependence of Nanocluster Interface Morphology: a Monte Carlo Study

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**Abstract:** The Monte–Carlo simulation of two–dimensional cluster within the approach of lattice–gas model is carried out. In the context of the model used the generation of a "hanging–over" type configurations was allowed and as an opportunity of vacancies, isolated atoms and their aggregates formation were considered. Over the temperature region of  $0.30 \div 0.77k_bT/J$  the linear dependence of the cluster interface fractal dimension on temperature was obtained. It has been proposed that the transition of linear  $D_p$  dependence to non–linear, some bounded overhead function, at the reduced temperature  $k_bT/J$  more then 0.8 is caused by a phase transition.

# Fractals in Biology

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**Abstract:** In the last decade it was realized that some biological systems have no characteristic length or time scale, *i.e.*, they have fractal — or, more generally, self-affine properties<sup>1</sup>. The fractal properties in different biological systems, have quite different nature and origin. In some cases, it is the geometrical shape of a biological object itself that exhibits fractal features, while in other cases the fractal properties are the time-dependent properties. We review several biological systems which can be characterized by fractal geometry. In particular, we focus on the long-range correlations and the linguistic features found in non-coding DNA sequences and suggest their possible implications. We discuss the application of fractal analysis to the dynamics of heartbeat regulations and the recent finding that the normal heart is characterized by long-range “anticorrelations” which are absent in the diseased heart. We also discuss a dynamical fractal model for spreading and migration of populations and diseases. A major part of the presented studies was done in collaboration with S. V. Buldyrev, A. Bunde, A. L. Goldberger, H. Larralde, R. N. Mantegna, M. Meyer, C.-K. Peng, M. Simons, H. E. Stanley and P. Trunfio.

<sup>1</sup>see *e.g.*, S. V. Buldyrev, A. L. Goldberger, S. Havlin, C.-K. Peng and H. E. Stanley: in *Fractals in Science*, eds. A. Bunde and S. Havlin (Springer, Berlin 1994)