Chapter 3

A Consumer's Constrained Choice

If this is coffee, please bring me some tea; but if this is tea, please bring me some coffee. —Abraham Lincoln

Microeconomics provides powerful insights into the myriad questions and choices facing consumers. For example, does U.S. consumers' purchases of relatively fewer SUVs and more small vehicles in the past few years reflect a change in tastes or a response to higher prices? How can we use information about consumers' allocations of their budgets across various goods in the past to predict how a price change will affect their demands for goods today? Are consumers better off receiving cash or a comparable amount in food stamps? Should you buy insurance or save your money? Work at home or in the marketplace? Have children? Invest in bonds or in stocks?

To answer these and other questions about how consumers allocate their income over many goods, we use a model that lets us look at an individual's decision making when faced with limited income and market-determined prices. This model allows us to derive the market demand curve that we used in our supply-and-demand model and to make a variety of predictions about consumers' responses to changes in prices and income.

Our model of consumer behavior is based on the following premises:

- Individual *tastes* or *preferences* determine the amount of pleasure people derive from the goods and services they consume.
- Consumers face *constraints* or limits on their choices.
- Consumers *maximize* their well-being or pleasure from consumption, subject to the constraints they face.

Consumers spend their money on the bundle of products that gives them the most pleasure. If you like music and don't have much of a sweet tooth, you spend a lot of your money on concerts and CDs and relatively little on candy.[1] By contrast, your chocoholic friend with the tin ear may spend a great deal on Hershey's Kisses and very little on music.

All consumers must choose which goods to buy because limited incomes prevent them from buying everything that catches their fancy. In addition, government rules restrict what they may buy: Young consumers cannot buy alcohol or cigarettes legally, and laws prohibit people of all ages from buying crack cocaine and other recreational drugs (although, of course, enforcement is imperfect). Therefore, consumers buy the goods that give them the most pleasure, subject to the constraints that they cannot spend more money than they have and that they cannot spend it in ways that the government prevents.

In economic analyses that are designed to explain behavior (positive analysis—see Chapter 1) rather than to judge it (normative statements), economists assume that *the*

[1]Microeconomics is the study of trade-offs: Should you save your money or buy that Superman *Action Comic* Number 1 you always wanted? Indeed, an anagram for *microeconomics* is *income or comics*.

consumer is the boss. **If your brother gets pleasure from smoking, economists wouldn't argue with him that it's bad for him any more than they'd tell your sister, who likes reading Stephen King, that she should read Adam Smith's *Wealth of Nations* instead.[2] Accepting each consumer's tastes is not the same as condoning the resulting behaviors. Economists want to predict behavior. They want to know, for example, whether your brother will smoke more next year if the price of cigarettes decreases 10%. The following prediction is unlikely to be correct: "He shouldn't smoke; therefore, we predict he'll stop smoking next year." A prediction based on your brother's actual tastes is more likely to be correct: "Given that he likes cigarettes, he is likely to smoke more of them next year if the price falls."**

In this chapter, we examine four main topics:

1. **Preferences:** We use five properties of preferences to predict which combinations, or bundle, of goods an individual prefers to other combinations.
2. **Utility:** Economists summarize a consumer's preferences using a utility function, which assigns a numerical value to each possible bundle of goods, reflecting the consumer's relative ranking of these bundles.
3. **Budget Constraint:** Prices, income, and government restrictions limit a consumer's ability to make purchases by determining the rate at which a consumer can trade one good for another.
4. **Constrained Consumer Choice:** Consumers maximize their pleasure from consuming various possible bundles of goods given their income, which limits the amount of goods they can purchase.

3.1 Preferences

Do not do unto others as you would that they would do unto you. Their tastes may not be the same. —George Bernard Shaw

We start our analysis of consumer behavior by examining consumer preferences. Using four assumptions, we can make many predictions about preferences. Once we know about consumers' preferences, we can add information about the constraints that consumers face so that we can answer many questions, such as the ones posed at the beginning of the chapter, or derive demand curves, as we do in the next chapter.

As a consumer, you choose among many goods. Should you have ice cream or cake for dessert? Should you spend most of your money on a large apartment or rent a single room and use the money you save to pay for trips and concerts? In short, you must allocate your money to buy a *bundle* (*market basket* or combination) of goods.

[2] As the ancient Romans put it: "De gustibus non est disputandum"—there is no disputing about (accounting for) tastes. Or, as it was put in the movie *Grand Hotel* (1932), "Have caviar if you like, but it tastes like herring to me."

How do consumers choose the bundle of goods they buy? One possibility is that consumers behave randomly and blindly choose one good or another without any thought. However, consumers appear to make systematic choices. For example, you probably buy the same specific items, more or less, each time you go to the grocery store.

To explain consumer behavior, economists *assume* that consumers have a set of tastes or preferences that they use to guide them in choosing between goods. These tastes differ substantially among individuals. Three out of four European men prefer colored underwear, while three out of four American men prefer white underwear.[3] Let's start by specifying the underlying assumptions in the economist's model of consumer behavior.

PROPERTIES OF CONSUMER PREFERENCES

I have forced myself to contradict myself in order to avoid conforming to my own taste. —Marcel Duchamp, Dada artist

We start by making five assumptions about the properties of consumers' preferences. For brevity, these properties are referred to as *completeness*, *transitivity*, *more is better*, *continuity*, and *strict convexity*.

Completeness. The completeness property holds that, when facing a choice between any two bundles of goods, a consumer can rank them so that one and only one of the following relationships is true: The consumer prefers the first bundle to the second, prefers the second to the first, or is indifferent between them. This property rules out the possibility that the consumer cannot decide which bundle is preferable.

It would be very difficult to predict behavior if consumers' rankings of bundles were not logically consistent. The next property eliminates the possibility of certain types of illogical behavior.

Transitivity. The transitivity (or what some people refer to as rationality) property is that a consumer's preferences over bundles is consistent in the sense that, if the consumer *weakly prefers* Bundle z to Bundle y—that is, likes z at least as much as y—and weakly prefers Bundle y to Bundle x, the consumer also weakly prefers Bundle z to Bundle x.[4]

If your sister told you that she preferred a scoop of ice cream to a piece of cake, a piece of cake to a candy bar, and a candy bar to a scoop of ice cream, you'd probably think she'd lost her mind. At the very least, you wouldn't know which of these desserts to serve her.

[3]L. M. Boyd, "The Grab Bag," *San Francisco Examiner*, September 11, 1994, p. 5.

[4]The assumption of transitivity of weak preferences is sufficient for the following analysis. However, it is easier (and plausible) to assume that other preference relations—strict preference and indifference between bundles—are also transitive.

More Is Better. The more-is-better property (the economics jargon is *nonsatiation*) holds that, all else the same, more of a commodity is better than less of it.[5] Indeed, economists define a **good** as a commodity for which more is preferred to less, at least at some levels of consumption. In contrast, a **bad** is something for which less is preferred to more, such as pollution. We now concentrate on goods.

Although the completeness and transitivity properties are crucial to the analysis that follows, the more-is-better property is included to simplify the analysis—our most important results would follow even without this property.

So why do economists assume that the more-is-better property holds? The most compelling reason is that it appears to be true for most people.[6] A second reason is that if consumers can freely dispose of excess goods, consumers can be no worse off with extra goods. (We examine a third reason later in the chapter: We observe consumers buying goods only where this condition is met.)

Continuity. Loosely, the continuity property holds that if a consumer prefers Bundle *a* to Bundle *b*, then the consumer prefers Bundle *c* to *b* if *c* is very close to *a*. The purpose of this assumption is to rule out sudden preference reversals in response to small changes in the characteristics of a bundle. This assumption is technical and allows us later in this chapter to develop the mathematical theory concerning utility functions.

Strict Convexity. Strict convexity of preferences means that consumers prefer averages to extremes. For example, if Bundle *a* and Bundle *b* are distinct bundles and the consumer prefers both of these bundles to Bundle *c*, then the consumer prefers a weighted average of *a* and *b*, $\beta a + (1 - \beta)b$ (where $0 < \beta < 1$), to Bundle *c*. This condition is a technical one, which we will usually assume holds.[7]

PREFERENCE MAPS

Surprisingly enough, with just the first three properties, we can tell a lot about a consumer's preferences. One of the simplest ways to summarize information about a consumer's preferences is to create a graphical interpretation—a map—of them. For simplicity, we concentrate on choices between only two goods, but the model can be generalized to handle any number of goods.

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[5]When teaching microeconomics to Wharton MBAs, I told them about a cousin of mine who had just joined a commune in Oregon. His worldly possessions consisted of a tent, a Franklin stove, enough food to live on, and a few clothes. He said that he didn't need any other goods—that he was *satiated*. A few years later, one of these students bumped into me on the street and said, "Professor, I don't remember your name or much of anything you taught me in your course, but I can't stop thinking about your cousin. Is it really true that he doesn't want *anything* else? His very existence is a repudiation of my whole way of life." Actually, my cousin had given up his ascetic life and was engaged in telemarketing, but I, for noble pedagogical reasons, responded, "Of course he still lives that way—you can't expect everyone to have the tastes of an MBA."

[6]How much wealth do you need to live comfortably? In a survey of wealthy people (*Business Week*, February 28, 2005, p. 13), those with a net worth of over \$1 million said that they needed \$2.4 million to live comfortably, those with at least \$5 million in net worth said that they need \$10.4 million, and those with at least \$10 million wanted \$18.1 million. Apparently, people never have enough.

[7]This condition helps ensure that the second-order condition holds in the consumer maximization problem.

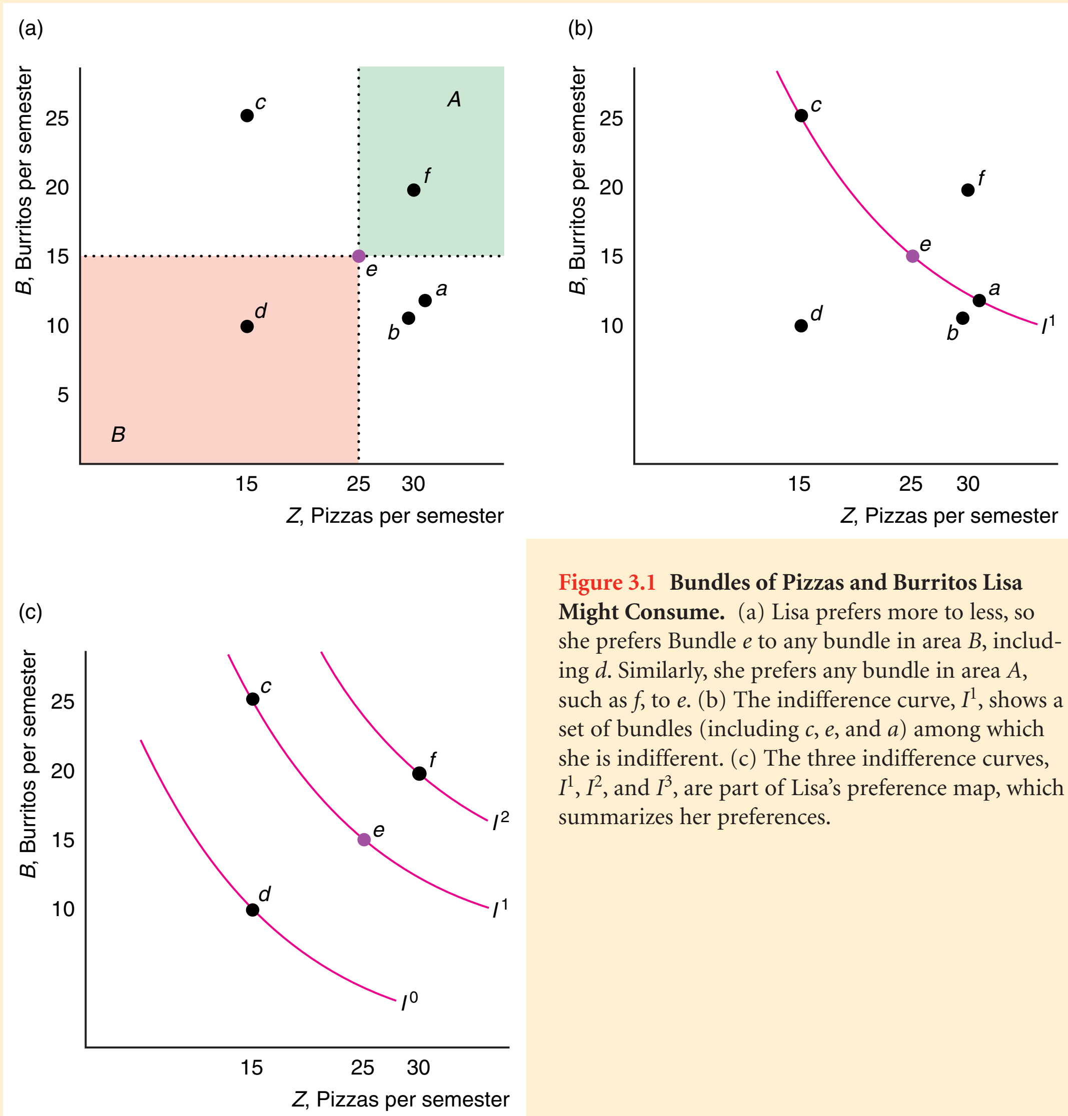

Figure 3.1 Bundles of Pizzas and Burritos Lisa Might Consume. (a) Lisa prefers more to less, so she prefers Bundle *e* to any bundle in area *B*, including *d*. Similarly, she prefers any bundle in area *A*, such as *f*, to *e*. (b) The indifference curve, I^1, shows a set of bundles (including *c*, *e*, and *a*) among which she is indifferent. (c) The three indifference curves, I^1, I^2, and I^3, are part of Lisa's preference map, which summarizes her preferences.

Each semester, Lisa, who lives for fast food, decides how many pizzas and burritos to eat. The various bundles of pizzas and burritos she might consume are shown in panel a of Figure 3.1, with (individual-size) pizzas per semester, q_1, on the horizontal axis and burritos per semester, q_2, on the vertical axis.

At Bundle *e*, for example, Lisa consumes 25 pizzas and 15 burritos per semester. By the more-is-better property, all the bundles that lie above and to the right (area *A*) are preferred to Bundle *e* because they contain at least as much of both pizzas and burritos as Bundle *e*. Thus Bundle *f* (30 pizzas and 20 burritos) in that region is preferred to *e*. By the same reasoning, Lisa prefers *e* to all the bundles that lie in area *B*, below and to the left of *e*, such as Bundle *d* (15 pizzas and 10 burritos).

Bundles such as *b* (30 pizzas and 10 burritos), in the region below and to the right of *e*, or *c* (15 pizzas and 25 burritos), in the region above and to the left, may or may not be preferred to *e*. We can't use the more-is-better property to determine which bundle is preferred because these bundles each contain more of one good and less of the other than *e* does.

INDIFFERENCE CURVES

Suppose we asked Lisa to identify all the bundles that give her the same amount of pleasure as consuming Bundle *e*. Using her answers, we draw curve *I* in panel b of Figure 3.1 through all the bundles she likes as much as *e*. Curve *I* is an **indifference curve:** the set of all bundles of goods that a consumer views as being equally desirable.

Indifference curve *I* includes Bundles *c*, *e*, and *a*, so Lisa is indifferent about consuming Bundles *c*, *e*, and *a*. From this indifference curve, we also know that Lisa prefers *e* (25 pizzas and 15 burritos) to *b* (30 pizzas and 10 burritos). How do we know that? Because Bundle *b* lies below and to the left of Bundle *a*, Lisa prefers Bundle *a* to Bundle *b* by the more-is-better property. Both Bundle *a* and Bundle *e* are on indifference curve *I*, so Lisa likes Bundle *e* as much as Bundle *a*. Because Lisa is indifferent between *e* and *a* and she prefers *a* to *b*, she must prefer *e* to *b* by transitivity.

If we asked Lisa many, many questions, in principle we could draw an entire set of indifference curves through every possible bundle of burritos and pizzas. Lisa's preferences are summarized in an **indifference map** or *preference map*, which is a complete set of indifference curves that summarize a consumer's tastes. Panel c of Figure 3.1 shows three of Lisa's indifference curves, I^1, I^2, and I^3.

The figure shows indifference curves that are continuous (have no gaps). The indifference curves are parallel in the figure, but they need not be. Given our assumptions, all indifference curve maps must have five important properties:

1. Bundles on indifference curves farther from the origin are preferred to those on indifference curves closer to the origin.
2. There is an indifference curve through every possible bundle.
3. Indifference curves cannot cross.
4. Indifference curves slope downward.
5. Indifference curves cannot be thick.

First, we show that bundles on indifference curves farther from the origin are preferred to those on indifference curves closer to the origin. By the more-is-better property, Lisa prefers Bundle *f* to Bundle *e* in panel c of Figure 3.1. She is indifferent among all the bundles on indifference curve I^3 and Bundle *f*, just as she is indifferent among all the bundles, such as Bundle *c* on indifference curve I^2 and Bundle *e*. By the transitivity property, she prefers Bundle *f* to Bundle *e*, which she likes as much as Bundle *c*, so she prefers Bundle *f* to Bundle *c*. By this type of reasoning, she prefers all bundles on I^3 to all bundles on I^2.

Second, we show that there is an indifference curve through every possible bundle as a consequence of the completeness property: The consumer can compare any bundle to another bundle. Compared to a given bundle, some bundles are preferred, some are enjoyed equally, and some are inferior. Connecting the bundles that give the same pleasure produces an indifference curve that includes the given bundle.

Third, we show that indifference curves cannot cross: A given bundle cannot be on two indifference curves. Suppose that two indifference curves crossed at Bundle *e* as in panel a of Figure 3.2. Because Bundles *e* and *a* lie on the same indifference curve I^0, Lisa is indifferent between *e* and *a*. Similarly, she is indifferent between *e* and *b* because

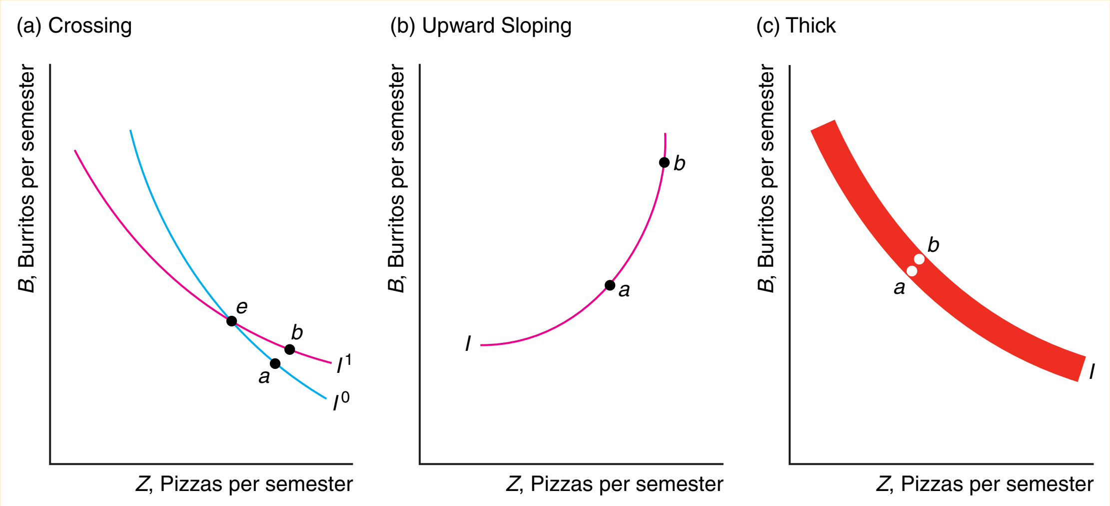

Figure 3.2 Impossible Indifference Curves.
(a) Suppose that the indifference curves cross at Bundle *e*. Lisa is indifferent between *e* and *a* on indifference curve I^0 and between *e* and *b* on I^1. If Lisa is indifferent between *e* and *a* and she is indifferent between *e* and *b*, she must be indifferent between *a* and *b* by transitivity. But *b* has more of both pizzas and burritos than *a*, so she *must* prefer *a* to *b*. Because of this contradiction, indifference curves cannot cross. (b) Suppose that indifference curve *I* slopes upward. The consumer is indifferent between *b* and *a* because they lie on *I* but prefers *b* to *a* by the more-is-better assumption. Because of this contradiction, indifference curves cannot be upward sloping. (c) Suppose that indifference curve *I* is thick enough to contain both *a* and *b*. The consumer is indifferent between *a* and *b* because both are on *I* but prefers *b* to *a* by the more-is-better assumption because *b* lies above and to the right of *a*. Because of this contradiction, indifference curves cannot be thick.

both are on I^1. By transitivity, if Lisa is indifferent between *e* and *a* and she is indifferent between *e* and *b*, she must be indifferent between *a* and *b*. But that's impossible! Bundle *b* is above and to the right of Bundle *a*, so Lisa *must* prefer *b* to *a* by the more-is-better property. Thus because preferences are transitive and more is better than less, indifference curves cannot cross.

Fourth, we show that indifference curves must be downward sloping. Suppose, to the contrary, that an indifference curve sloped upward, as in panel b of Figure 3.2. The consumer is indifferent between Bundles *a* and *b* because both lie on the same indifference curve, *I*. But the consumer prefers *b* to *a* by the more-is-better property: Bundle *a* lies strictly below and to the left of Bundle *b*. Because of this contradiction—the consumer cannot both be indifferent between *a* and *b* and strictly prefer *b* to *a*—indifference curves cannot be upward sloping.

SOLVED PROBLEM 3.1

Can indifference curves be thick?

Answer

Draw an indifference curve that is at least two bundles thick, and show that a preference property is violated: Panel c of Figure 3.2 shows a thick indifference curve, *I*, with two bundles, *a* and *b*, identified. Bundle *b* lies above and to the right of *a*:

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Bundle b has more of both burritos and pizzas. Thus by the more-is-better property, Bundle b must be strictly preferred to Bundle a. But the consumer must be indifferent between a and b because both bundles are on the same indifference curve. Because both relationships between a and b cannot be true, there is a contradiction. Consequently, indifference curves cannot be thick. (We illustrate this point by drawing indifference curves with very thin lines in our figures.)

3.2 Utility

Underlying our model of consumer behavior is the belief that consumers can compare various bundles of goods and decide which bundle gives them the greatest pleasure. We can summarize a consumer's preferences by assigning a numerical value to each possible bundle to reflect the consumer's relative ranking of these bundles.

Following the terminology of Jeremy Bentham, John Stuart Mill, and other nineteenth-century British utilitarianism economist-philosophers, economists apply the term **utility** to this set of numerical values that reflect the relative rankings of various bundles of goods. The statement that "Bonnie prefers Bundle x to Bundle y" is equivalent to the statement that "Consuming Bundle x gives Bonnie more utility than consuming Bundle y." Bonnie prefers x to y if Bundle x gives Bonnie 10 *utils*—units of utility—and Bundle y gives her 8 utils.

UTILITY FUNCTION

The **utility function** is the relationship between utility measures and every possible bundle of goods. If we know the utility function, we can summarize the information in indifference maps succinctly.

Suppose that the utility, U, that Lisa gets from pizzas and burritos is

$$U = \sqrt{q_1 q_2}.$$

From this function, we know that the more Lisa consumes of either good, the greater her utility. Using this function, we can determine whether she would be happier if she had Bundle x with 16 pizzas and 9 burritos or Bundle y with 13 of each. The utility she gets from x is $12\ (= \sqrt{16 \times 9})$ utils. The utility she gets from y is $13\ (= \sqrt{13 \times 13})$ utils. Therefore, she prefers y to x.

The utility function is a concept that economists use to help them think about consumer behavior; utility functions do not exist in any fundamental sense. If you asked your mother what her utility function is, she would be puzzled—unless, of course, she is an economist. But if you asked her enough questions about her choices of bundles of goods, you could construct a function that accurately summarizes her preferences. For example, by questioning people, Rousseas and Hart (1951) constructed indifference curves between eggs and bacon, and MacCrimmon and Toda (1969) constructed indifference curves between French pastries and money (which can be used to buy all other goods).

Typically, consumers can easily answer questions about whether they prefer one bundle to another, such as "Do you prefer a bundle with one scoop of ice cream and two pieces of cake to another bundle with two scoops of ice cream and one piece of cake?" But they have difficulty answering questions about how much more they prefer one bundle to another because they don't have a measure to describe how their pleasure from two goods or bundles differs. Therefore, we may know a consumer's rank-ordering of bundles, but we are unlikely to know by how much more that consumer prefers one bundle to another.

ORDINAL PREFERENCES

If we know only consumers' relative rankings of bundles, our measure of pleasure is *ordinal* rather than *cardinal*. An ordinal measure is one that tells us the relative ranking of two things but does not tell us how much more one rank is than another.

If a professor assigns only letter grades to an exam, we know that a student who receives a grade of A did better than a student who received a B, but we can't say how much better from that ordinal scale. Nor can we tell whether the difference in performance between an A student and a B student is greater or less than the difference between a B student and a C student.

A cardinal measure is one by which absolute comparisons between ranks may be made. Money is a cardinal measure. If you have \$100 and your brother has \$50, we know not only that you have more money than your brother but also that you have exactly twice as much money as he does.

Because utility is an ordinal measure, we should not put any weight on the absolute differences between the utility number associated with one bundle and that associated with another. We care only about the relative utility or ranking of the two bundles.

Because preference rankings are ordinal and not cardinal, utility measures are not unique. Let $U(q_1, q_2)$ be the original utility function that assigns numerical values corresponding to any given combination of q_1 and q_2. Let F be an *increasing function* (in jargon: a *positive monotonic transformation*): an order-preserving function that is strictly increasing in the sense that if $x > y$, then $F(x) > F(y)$. By applying this transformation to the original utility function, we obtain a new function, $V(q_1, q_2) = F(U(q_1, q_2))$, which is a utility function with the same ordinal-ranking properties as $U(q_1, q_2)$. Economists often express this idea by using the mellifluous statement that a *utility function is unique only up to a positive monotonic transformation*. As an example, suppose that the transformation is linear: $F(x) = a + bx$, where $b > 0$. Then, $V(q_1, q_2) = a + bU(q_1, q_2)$. The rank-ordering is the same for these utility functions because $V(q_1, q_2) = a + bU(q_1, q_2) > V(q_1^*, q_2^*) = a + bU(q_1^*, q_2^*)$ if and only if $U(q_1, q_2) > U(q_1^*, q_2^*)$.

Thus when we talk about utility numbers, we need to remember that these numbers are not unique and that we place little meaning on the absolute numbers. We care only whether one bundle's utility value is greater than that of another.

UTILITY AND INDIFFERENCE CURVES

We can use Lisa's utility function to construct a three-dimensional diagram that shows how utility varies with changes in the consumption of q_1 and q_2. Imagine that you are standing with your back against a corner of a room. Walking away from the corner along the wall to your left, you are tracing out the q_2 axis: The farther you get from the corner, the more burritos Lisa has. Similarly, starting back at the corner and walking along the wall to your right, you are moving along the q_1 axis. When you stand in the corner, you are leaning against the utility axis, where the two walls meet. The higher the point along your back, the greater Lisa's utility. Because her utility is increasing (more is preferred to less) in both q_1 and q_2, her utility rises as you walk away from the corner (origin) along either wall or into the room, where Lisa has more q_1 or q_2 or both. Lisa's utility or *hill of happiness* rises as you move away from the corner.

What is the relationship between Lisa's utility and one of her indifference curves, those combinations of q_1 and q_2 that give Lisa a particular level of utility? Imagine that the hill of happiness is made of clay. If you were to cut the hill parallel to the floor at a particular height on the wall—a given level of utility—you'd get a smaller hill above the cut. Now suppose that you place that smaller hill directly on the floor and trace the outside edge of the hill. Looking down at the floor, the traced outer edge of the hill represents an indifference curve on the two-dimensional floor. Making other parallel cuts in the hill of happiness, placing the smaller hills on the floor, and tracing their outside edges, you could obtain a map of indifference curves on which each indifference curve reflects a different level of utility.

In short, an indifference curve consists of all those bundles that correspond to a particular utility measure. If Lisa's utility function is $U(q_1, q_2)$, then the expression for one of her indifference curves is

$$\overline{U} = U(q_1, q_2). \tag{3.1}$$

This expression determines all those bundles of q_1 and q_2 that give her $\overline{U}$ utils of pleasure. For example, if the utility function is $U = \sqrt{q_1 q_2}$, then the indifference curve $4 = \overline{U} = \sqrt{q_1 q_2}$ includes any (q_1, q_2) bundles such that $q_1 q_2 = 16$, including the bundles (4, 4), (2, 8), (8, 2), (1, 16), and (16, 1).

WILLINGNESS TO SUBSTITUTE BETWEEN GOODS

To analyze how consumers make choices when faced with limited resources, it is useful to know the slope of an indifference curve at a particular bundle of goods. Economists call the slope at a point of an indifference curve the **marginal rate of substitution** (*MRS*), because it is the maximum amount of one good that a consumer will sacrifice (trade) to obtain one more unit of another good.

Lisa is willing to trade one good for more of another good. The downward slope of her indifference curve in Figure 3.3 shows that Lisa is willing to give up some burritos for more pizzas and vice versa. Because the indifference curve is downward sloping, the *MRS* is a negative number.

We can use calculus to determine the *MRS* at a point on Lisa's indifference curve in Equation (3.1). We will show that the *MRS* depends on how much extra utility Lisa gets from a little more of each good. We call the extra utility that a consumer gets from consuming the last unit of a good the **marginal utility.** Given that Lisa's utility function is $U(q_1, q_2)$, the extra or *marginal utility* that she gets from a little more pizza, holding the quantity of burritos fixed, is

$$\text{marginal utility of pizza} = \frac{\partial U}{\partial q_1} = U_1.$$

Similarly, the marginal utility from more burritos is $U_2 = \partial U/\partial q_2$, where we hold the amount of pizza constant.

We determine the slope of Lisa's indifference curve, the *MRS*, by ascertaining the changes in q_1 and q_2 that leave her utility unchanged, keeping her on her original indifference curve: $\overline{U} = U(q_1, q_2)$. Let $q_2(q_1)$ be the implicit function that shows how much q_2 it takes to keep Lisa's utility at $\overline{U}$ given that she consumes q_1. We want to know how

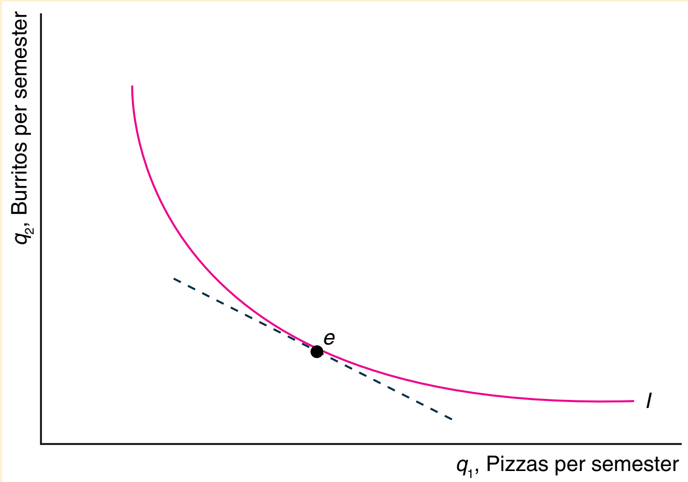


Figure 3.3 Marginal Rate of Substitution. Lisa's marginal rate of substitution, $MRS = dq_2/dq_1$, at Bundle e is the slope of indifference curve I at that point. The marginal rate of substitution, at e is the same as the slope of the line that is tangent to I at e.

much q_2 must change if we increase q_1, dq_2/dq_1, given that we require her utility to remain constant. To answer this question, we differentiate $\overline{U} = U(q_1, q_2(q_1))$ with respect to q_1:

$$\frac{d\overline{U}}{dq_1} = 0 = \frac{\partial U(q_1, q_2(q_1))}{\partial q_1} + \frac{\partial U(q_1, q_2(q_1))}{\partial q_2}\frac{dq_2}{dq_1} = U_1 + U_2\frac{dq_2}{dq_1}. \qquad (3.2)$$

Because $\overline{U}$ is a constant, $d\overline{U}/dq_1 = 0$.

Since Lisa derives pleasure from both goods, if we increase one of the goods, we must decrease the other to hold her utility constant and keep her on her indifference curve. Rearranging the terms in Equation (3.2), we find that her marginal rate of substitution is

$$MRS = \frac{dq_2}{dq_1} = -\frac{\partial U/\partial q_1}{\partial U/\partial q_2} = -\frac{U_1}{U_2}. \qquad (3.3)$$

Thus the slope of her indifference curve is the negative of the ratio of her marginal utilities.

We can give a graphical interpretation of the slope of the indifference curve. The slope of her indifference curve I at Bundle e is the same as the slope of the line that is tangent to the indifference curve at that point.

SOLVED PROBLEM 3.2

Suppose that Jackie has what is known as a *Cobb-Douglas utility function:*[8]

$$U = q_1^a q_2^{1-a}, \qquad (3.4)$$

where a is a positive constant, q_1 is the number of music CDs she buys a year, and q_2 is the number of movie DVDs she buys. What is her marginal rate of substitution?

[8]The Cobb-Douglas utility function may be written more generally as $U = Aq_1^c q_2^d$. However, we can always transform that utility function into this simpler one through a monotonic transformation: $q_1^a q_2^{1-a} = F(A\, q_1^c\, q_2^d)$, where $F(x) = x^{1/(c+d)}/A$, so that $a = c/(c + d)$.

Answer

1. *Determine Jackie's marginal utilities of CDs and DVDs*: Her marginal utility of CDs is

$$U_1 = aq_1^{a-1}q_2^{1-a} = a\frac{U(q_1, q_2)}{q_1},$$

and her marginal utility of DVDs is

$$U_2 = (1 - a)q_1^a q_2^{-a} = (1 - a)\frac{U(q_1, q_2)}{q_2}.$$

2. *Express her marginal rate of substitution in terms of her marginal utilities*: Using Equation 3.3, we find that her marginal rate of substitution is

$$MRS = \frac{dq_2}{dq_1} = -\frac{U_1}{U_2} = -\frac{aU/q_1}{(1 - a)U/q_2} = -\frac{a}{1 - a}\frac{q_2}{q_1}. \tag{3.5}$$

APPLICATION

MRS Between Music CDs and Movie DVDs

In 2005, a typical owner of a home theater (a television and a DVD player) bought 12 music CDs (q_1) per year and 6 top-20 movie DVDs (q_2) per year. We estimate this average consumer's Cobb-Douglas utility function as

$$U = q_1^{0.6}q_2^{0.4}. \tag{3.6}$$

That is, in the more general Cobb-Douglas equation 3.4, $a = 0.6$.

Continuing our analysis of Solved Problem 3.2, given that Jackie's Cobb-Douglas utility function is that of the typical consumer, we can determine her marginal rate of substitution by substituting $q_1 = 12$, $q_2 = 6$, and $a = 0.6$ into Equation 3.5:

$$MRS = -\frac{a}{1 - a}\frac{q_2}{q_1} = -\frac{0.6}{0.4}\frac{6}{12} = -0.75.$$

CURVATURE OF INDIFFERENCE CURVES

Unless the indifference curve is a straight line, the marginal rate of substitution varies along the indifference curve. Because the indifference curve in Figure 3.3 is convex to the origin, as we move to the right along the indifference curve, the *MRS* becomes smaller in absolute value: Lisa will give up fewer burritos to obtain one pizza. This willingness to trade fewer burritos for one more pizza as we move down and to the right along the indifference curve reflects a *diminishing marginal rate of substitution*: The *MRS* approaches zero—becomes flatter or less sloped—as we move down and to the right along an indifference curve.

So far, we have drawn indifference curves as convex to the origin. An indifference curve doesn't have to be convex, but casual observation suggests that most people's

Figure 3.4 Perfect Substitutes, Perfect Complements, Imperfect Substitutes. (a) Ben views Coke and Pepsi as perfect substitutes. His indifference curves are straight, parallel lines with a marginal rate of substitution (slope) of –1. Ben is willing to exchange one can of Coke for one can of Pepsi. (b) Maureen likes pie à la mode but does not like pie or ice cream by itself: She views ice cream and pie as perfect complements. She will not substitute between the two; she consumes them only in equal quantities. (c) Lisa views burritos and pizza as imperfect substitutes. Her indifference curve lies between the extreme cases of perfect substitutes and perfect complements.

indifference curves over commodities are convex. When people have a lot of one good, they are willing to give up a relatively large amount of it to get a good of which they have relatively little. However, after that first trade, they are willing to give up less of the first good to get the same amount more of the second good.

It is hard to imagine that Lisa's indifference curves are *concave* to the origin. If her indifference curve were strictly concave, Lisa would be willing to give up more burritos to get one more pizza, the fewer the burritos she has. Two extreme versions of downward-sloping, convex indifference curves are plausible: straight-line or right-angle indifference curves.

One extreme case is **perfect substitutes,** goods that a consumer is completely indifferent as to which to consume. Because Ben cannot taste any difference between Coca-Cola and Pepsi-Cola, he views them as perfect substitutes: He is indifferent between one additional can of Coke and one additional can of Pepsi. His indifference curves for these two goods are straight, parallel lines with a slope of –1 everywhere along the curve, as in panel a of Figure 3.4. Thus Ben's MRS is -1 at every point along these indifference curves. (His marginal utility from each good is identical, so the $MRS = -U_1/U_2 = -1$.)

The slope of indifference curves of perfect substitutes need not always be –1; it can be any constant rate. For example, Amos knows from reading the labels that Clorox bleach is twice as strong as a generic brand. As a result, Amos is indifferent between one cup of Clorox and two cups of the generic bleach. Amos's utility function over Clorox, C, and the generic bleach, G, is

$$U(C, G) = iC + jG, \tag{3.7}$$

where both goods are measured in cups, $i = 2$, and $j = 1$. His indifference curves are straight lines with a slope or *MRS* of $-i/j = -2$, where the generic bleach is on the vertical axis.[9]

The other extreme case is **perfect complements**: goods that a consumer is interested in consuming only in fixed proportions. Maureen doesn't like apple pie, *A*, by itself or vanilla ice cream, *V*, by itself but loves apple pie à la mode: a slice of pie with a scoop of vanilla ice cream on top. Her utility function is

$$U(A, V) = \min(iA, jV), \tag{3.8}$$

where $i = j = 1$ and the min function says that the utility equals the smaller of the two arguments, iA or jV. Her indifference curves have right angles in panel b of Figure 3.4. If she has only one piece of pie, she gets as much pleasure from it and one scoop of ice cream, Bundle *a*, as from one piece and two scoops, Bundle *d*, or as from one piece and three scoops, Bundle *e*. For example, if she were at *b*, she would be unwilling to give up an extra slice of pie to get, say, two extra scoops of ice cream, as at point *e*. That is, she won't eat the extra scoops because she does not have pieces of pie to go with the ice cream. The only condition where she doesn't have an excess of either good is when $iA = jV$, or $V/A = i/j = 1$. Therefore, she consumes only bundles like *a*, *b*, and *c* in which pie and ice cream are in fixed (here, equal) proportions, because she is unwilling to substitute more of one good for less of another. (The marginal utility is zero for each good, because increasing that good while holding the other one constant does not increase Maureen's utility.)

The standard-shaped, convex indifference curve in panel c of Figure 3.4 lies between these two extreme examples. Convex indifference curves show that a consumer views two goods as imperfect substitutes. A consumer with a Cobb-Douglas utility function 3.4 has convex indifference curves.

APPLICATION

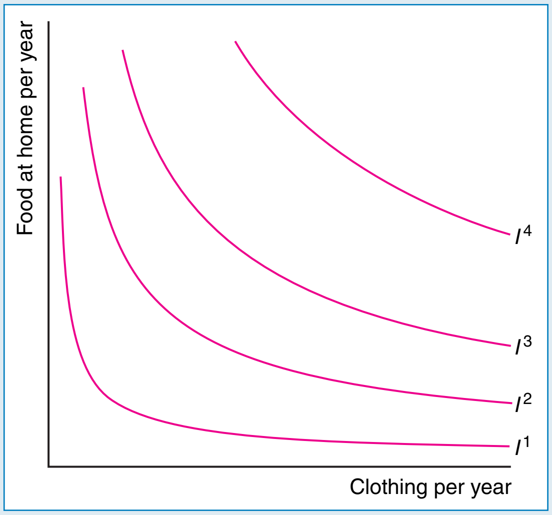
Indifference Curves Between Food and Clothing

Using the estimates of Eastwood and Craven (1981), the figure shows the indifference curves of the average U.S. consumer between food consumed at home and clothing. The food and clothing measures are weighted averages of various goods. At relatively low quantities of food and clothing, the indifference curves, such as I^1, are nearly right angles: perfect complements. As we move away from the origin, the indifference curves become flatter: closer to perfect substitutes.

One interpretation of these indifference curves is that there are minimum levels of food and clothing necessary to support life. The consumer cannot trade one good for the other if it means having less than those critical levels. As the consumer obtains more of both goods, however, the consumer is increasingly willing to trade between

[9] Sometimes it is difficult to guess which goods are close substitutes. According to Harper's Index 1994, flowers, perfume, and fire extinguishers rank 1, 2, and 3 among Mother's Day gifts that Americans consider "very appropriate."

the two goods. According to Eastwood and Craven's estimates, food and clothing are perfect complements when the consumer has little of either good, and perfect substitutes when the consumer has large quantities of both goods.



3.3 Budget Constraint

You can't have everything. . . . Where would you put it? —Steven Wright

Knowing an individual's preferences is only the first step in analyzing that person's consumption behavior. Consumers maximize their well-being subject to constraints. The most important constraint most of us face in deciding what to consume is our personal budget constraint.

If we cannot save and borrow, our budget is the income we receive in a given period. If we can save and borrow, we can save money early in life to consume later, such as when we retire; or we can borrow money when we are young and repay those sums later in life. Savings is, in effect, a good that consumers can buy. For simplicity, we assume that each consumer has a fixed amount of money to spend now, so we can use the terms *budget* and *income* interchangeably.

For graphical simplicity, we assume that consumers spend their money on only two goods. If Lisa spends all her budget, Y, on pizza and burritos, then

$$p_1q_1 + p_2q_2 = Y, \tag{3.9}$$

where p_1q_1 is the amount she spends on pizza and p_2q_2 is the amount she spends on burritos. Equation 3.9 is her **budget line** or *budget constraint*: the bundles of goods that can be bought if the entire budget is spent on those goods at given prices.

In Figure 3.5, we plot Lisa's budget line in pizza-burrito space, just as we did with her indifference curves. How many burritos can Lisa buy? Using algebra, we can

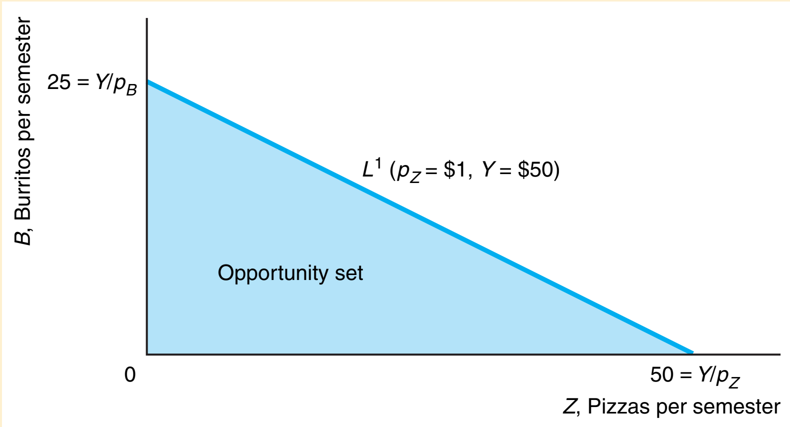


Figure 3.5 Budget Constraint. If Y = \$50, p_1 = \$1, and p_2 = \$2, Lisa can buy any bundle in the opportunity set, the shaded area, including points on the *budget line*, *L*, which has a slope of $-^1/_2$.

rewrite her budget constraint, Equation 3.9, as

$$q_2 = \frac{Y - p_1 q_1}{p_2}. \tag{3.10}$$

According to Equation 3.10, she can buy more burritos with a higher income ($dq_2/dY = 1/p_2 > 0$), the purchase of fewer pizzas ($dq_2/dq_1 = -p_1/p_2 < 0$), or a lower price of burritos or pizzas [$dq_2/dp_2 = -(Y - p_1q_1)/p_2^2 = -q_2/p_2 < 0$, $dq_2/dp_1 = -q_1/p_2 < 0$]. For example, if she has one more dollar of income (Y), she can buy $1/p_2$ more burritos.

If p_1 = \$1, p_2 = \$2, and Y = \$50, Equation 3.10 is

$$q_2 = \frac{\$50 - (\$1 \times q_1)}{\$2} = 25 - \frac{1}{2}q_1.$$

This equation is plotted in Figure 3.5. This budget line shows the combinations of burritos and pizzas that Lisa can buy if she spends all of her \$50 on these two goods. As this equation shows, every two pizzas cost Lisa one burrito. How many burritos can she buy if she spends all her money on burritos? By setting $q_1 = 0$ in Equation 3.10, we find that $q_2 = Y/p_2 = \$50/\$2 = 25$. Similarly, if she spends all her money on pizzas, $q_2 = 0$ and $q_1 = Y/p_1 = \$50/\$1 = 50$.

The budget constraint in Figure 3.5 is a smooth, continuous line. Implicitly, Lisa can buy fractional numbers of burritos and pizzas. Is that true? Do you know of a restaurant that will sell you a quarter of a burrito? Probably not. Why, then, don't we draw the opportunity set and the budget constraint as points (bundles) of whole numbers of burritos and pizzas? The reason is that Lisa can buy a burrito at a *rate* of one-half per time period. If Lisa buys one burrito every other week, she buys an average of one-half burrito every week. Thus it is plausible that she could purchase fractional amounts over time, and this diagram concerns her behavior over a semester.

Lisa could, of course, buy any bundle that costs less than \$50. The **opportunity set** consists of all the bundles a consumer can buy, including all the bundles inside the budget constraint and on the budget constraint (all those bundles of positive q_1 and q_2 such that $p_1q_1 + p_2q_2 \le Y$). Lisa's opportunity set is the shaded area in the figure. For example, she could buy 10 burritos and 15 pizzas for \$35, which falls inside her budget constraint. However, she can obtain more of the foods she loves by spending all of her budget and picking a bundle on the budget constraint rather than a bundle below the constraint.

We call the slope of the budget line the **marginal rate of transformation** (*MRT*): the trade-off the market imposes on the consumer in terms of the amount of one good the consumer must give up to obtain more of the other good. It is the rate at which Lisa can trade burritos for pizzas in the marketplace, where the prices she pays and her income are fixed.

Holding prices and income constant and differentiating Equation 3.10 with respect to q_1, we find that the slope of the budget constraint, or the marginal rate of transformation, is

$$MRT = \frac{dq_2}{dq_1} = -\frac{p_1}{p_2}. \tag{3.11}$$

Because the price of a pizza is half that of a burrito ($p_1 = \$1$ and $p_2 = \$2$), the marginal rate of transformation that Lisa faces is

$$MRT = -\frac{p_1}{p_2} = -\frac{\$1}{\$2} = -\frac{1}{2}.$$

An extra pizza costs her half an extra burrito—or, equivalently, an extra burrito costs her two pizzas.

3.4 Constrained Consumer Choice

My problem lies in reconciling my gross habits with my net income. —Errol Flynn

Were it not for the budget constraint, consumers who prefer more to less would consume unlimited amounts of at least some goods. Well, they can't have it all! Instead, consumers maximize their well-being subject to their budget constraints. To complete our analysis of consumer behavior, we have to determine the bundle of goods that maximizes well-being subject to the budget constraint. We first take a graphical approach and then use calculus.

THE CONSUMER'S OPTIMAL BUNDLE

Veni, vidi, Visa. (We came, we saw, we went shopping.) —Jan Barrett

We want to determine which bundle within the opportunity set gives the consumer the highest level of utility. That is, we are trying to solve a constrained maximization problem, where a consumer maximizes utility subject to a budget constraint.

To determine which bundle in the opportunity set gives Lisa the highest level of pleasure, we use her indifference curves in panel a of Figure 3.6. We will show that her optimal bundle lies on an indifference curve that touches the budget constraint, L, at only one point (e on I^2)—hence that indifference curve does not cross the constraint.

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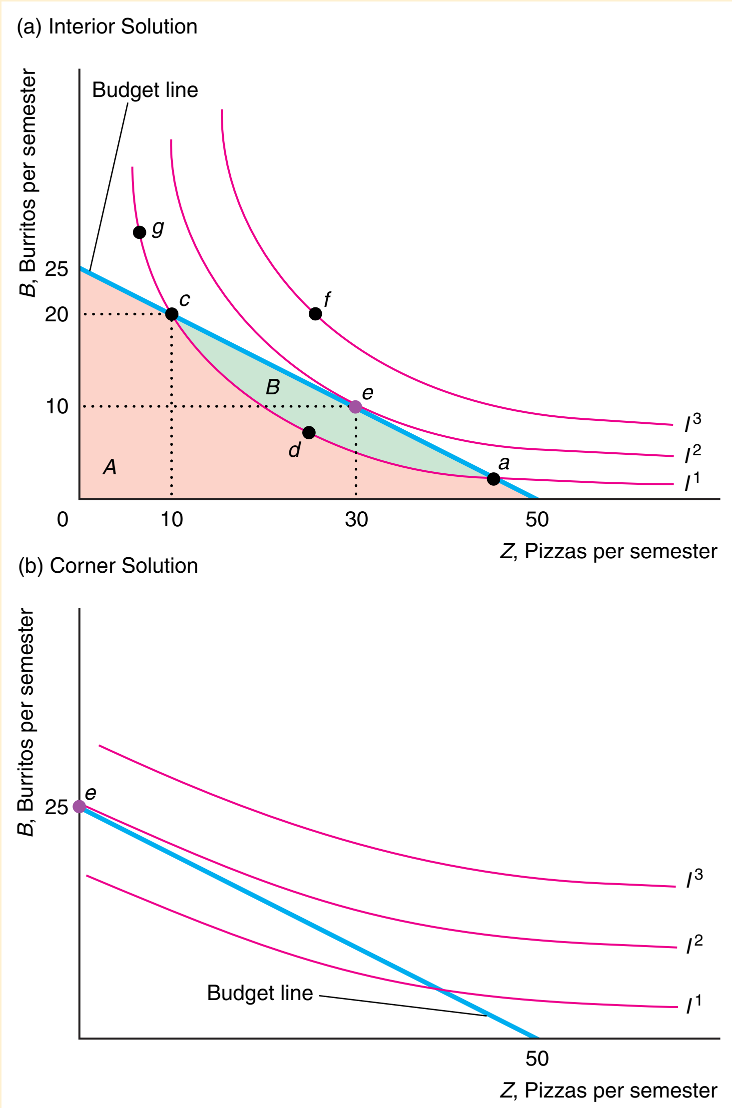


Figure 3.6 Consumer Maximization. (a) *Interior solution:* Lisa's optimal bundle is *e* (10 burritos and 30 pizzas) on indifference curve I^2. Any bundle that is preferred to *e* (such as points on indifference curve I^3) lies outside of the opportunity set—it can't be purchased. Bundles inside the opportunity set, such as *d*, are less desirable than *e*. (b) *Corner solution:* Spenser's indifference curves are relatively flat (he'll give up many pizzas for one more burrito), so his optimal bundle occurs at a corner of the opportunity set at Bundle *e*: 25 burritos and 0 pizzas.

We show this result by rejecting the possibility that the optimal bundle could be located off the budget constraint or that it lies on an indifference curve that intersects the budget constraint.

The optimal bundle must be *on* the budget constraint. Bundles that lie on indifference curves above the constraint, such as those on I^3, are not in the opportunity set. So even though Lisa prefers *f* on indifference curve I^3 to *e* on I^2, *f* is too expensive and she can't purchase it. Although Lisa could buy a bundle inside the budget constraint, she does not want to do so, because more is better than less: For any bundle inside the constraint (such as *d* on I^1), there is another bundle on the constraint with more of at least one of the two goods, and hence she prefers that bundle. Therefore, the optimal bundle must lie on the budget constraint.

Bundles that lie on indifference curves that cross the budget constraint (such as I^1, which crosses the constraint at *a* and *c*) are less desirable than certain other bundles on the constraint. Only some of the bundles on indifference curve I^1 lie within the opportunity set: Bundles *a* and *c* and all the points on I^1 between them, such as *d*, can be purchased. Because I^1 crosses the budget constraint, the bundles between *a* and *c* on I^1 lie strictly inside the constraint, so there are bundles in the opportunity set (area $A + B$) that are preferable to these bundles on I^1 and are affordable. By the more-is-better property, Lisa prefers *e* to *d* because *e* has more of both pizzas and burritos than *d*. By transitivity, Lisa prefers *e* to *a*, *c*, and all the other points on I^1—even those, like *g*, that Lisa can't afford. Because indifference curve I^1 crosses the budget constraint, area *B* contains at least one bundle that is preferred to—lies above and to the right of—at least one bundle on the indifference curve.

Thus the optimal bundle must lie on the budget constraint and be on an indifference curve that does not cross it. Such a bundle is the *consumer's optimum*. If Lisa is consuming this bundle, she has no incentive to change her behavior by substituting one good for another.

There are two ways for an optimal bundle to lie on an indifference curve that touches the budget constraint but does not cross it. The first is an *interior solution*, in which the optimal bundle has positive quantities of both goods: The optimal bundle is on the budget line rather than at one end or the other. The other possibility is called a *corner solution*, where the optimal bundle is at one end or the other of the budget line: It is at a corner with one of the axes.

Interior Solution. In panel a of Figure 3.6, Bundle *e* on indifference curve I^2 is the optimum bundle. It lies in the interior of the budget line away from the corners. Lisa prefers consuming a balanced diet, *e*, of 10 burritos and 30 pizzas, to eating only one type of food.

For the indifference curve I^2 to touch the budget constraint but not cross it, it must be *tangent* to the budget constraint: The budget constraint and the indifference curve have the same slope at the point *e* where they touch. The slope of the indifference curve, the marginal rate of substitution, measures the rate at which Lisa is *willing* to trade burritos for pizzas: $MRS = -U_1/U_2$, Equation 3.3. The slope of the budget line, the marginal rate of transformation, measures the rate at which Lisa *can* trade her money for burritos or pizza in the market: $MRT = -p_1/p_2$, Equation 3.11. Thus Lisa's utility is maximized at the bundle where the rate at which she is willing to trade burritos for pizzas equals the rate at which she can trade in the market:

$$MRS = -\frac{U_1}{U_2} = -\frac{p_1}{p_2} = MRT. \tag{3.12}$$

Rearranging terms, this condition is equivalent to

$$\frac{U_1}{p_1} = \frac{U_2}{p_2}. \tag{3.13}$$

Equation 3.13 says that U_1/p_1, the marginal utility of pizzas divided by the price of a pizza—the amount of extra utility from pizza per dollar spent on pizza—equals U_2/p_2, the marginal utility of burritos divided by the price of a burrito. Thus Lisa's utility is maximized if the last dollar she spends on pizzas gets her as much extra utility as the last dollar she spends on burritos. If the last dollar spent on pizzas gave Lisa more extra utility than the last dollar spent on burritos, Lisa could increase her happiness by spending more on pizzas and less on burritos. Her cousin Spenser is a different story.

Corner Solution. Spenser's indifference curves in panel b of Figure 3.6 are flatter than Lisa's. His optimal bundle lies on an indifference curve that touches the budget line, *L*, only once, at the upper-left corner of the opportunity set, *e*, where he buys only burritos (25 burritos and 0 pizzas).

Bundle *e* is the optimal bundle because the indifference curve does not cross the constraint into the opportunity set. If it did, another bundle would give Spenser more pleasure.

Spenser's indifference curve is not tangent to his budget line. It would cross the budget line if both the indifference curve and the budget line were continued into the "negative pizza" region of the diagram, on the other side of the burrito axis.

SOLVED PROBLEM 3.3

Nigel, a Brit, and Bob, a Yank, have the same tastes, and both are indifferent between a sport-utility vehicle (SUV) and a luxury sedan. Each has a budget that will allow him to buy and operate one vehicle for a decade. For Nigel, the price of owning and operating an SUV is greater than that for the car. For Bob, an SUV is a relative bargain because he benefits from an SUV tax break. Use an indifference curve–budget line analysis to explain why Nigel buys and operates a car while Bob chooses an SUV.

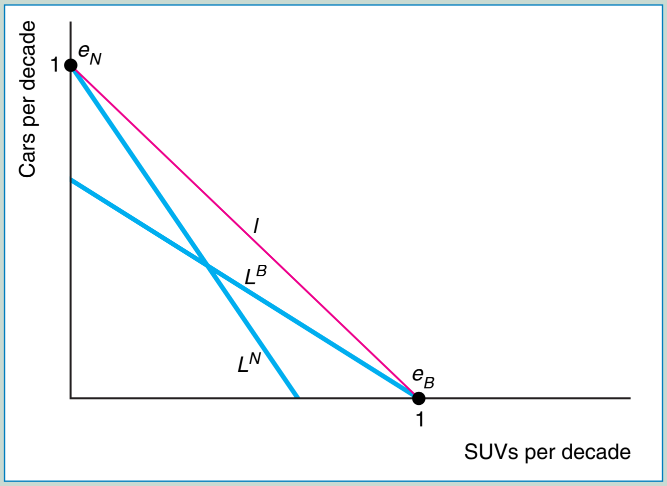
Answer

1. *Describe their indifference curves*: Because Nigel and Bob view the SUV and the car as perfect substitutes, each has an indifference curve for buying one vehicle that is a straight line with a slope of –1 and that hits each axis at 1 in the figure.

2. *Describe the slopes of their budget lines*: Nigel faces a budget line, L^N, that is flatter than the indifference curve, and Bob faces one, L^B, that is steeper.

3. *Use an indifference curve and a budget line to show why Nigel and Bob make different choices*: As the figure shows, L^N hits the indifference curve, *I*, at 1 on the car axis, e_N, and L^B hits *I* at 1 on the SUV axis, e_B. Thus Nigel buys the relatively inexpensive car and Bob scoops up a relatively cheap SUV.

Comment: If Nigel and Bob were buying a bundle of cars and SUVs for their large families or firms, the analysis would be similar—Bob would buy relatively more SUVs than would Nigel.



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APPLICATION

U.S. Versus EU SUVs

If you believe what newspapers report, Americans have a love affair with sport-utility vehicles (SUVs), and Europeans see no reason to drive a vehicle nearly the size of Luxembourg. SUVs are derided as "Chelsea tractors" in England and "Montessori wagons" in Sweden. News stories point to this difference in tastes to explain why SUVs account for less than a twentieth of total car sales in Western Europe but a quarter in the United States. Maybe the narrower European streets or Europeans' greater concern for the environment is the explanation. The analysis in Solved Problem 3.3 provides an alternative explanation: The price of owning and operating an SUV is much lower in the United States than it is in Europe, so people with identical tastes are more likely to buy an SUV in the United States than in Europe.

Higher European gasoline taxes make gas-guzzling SUVs more expensive to operate in Europe than in the United States. In 2005, gas taxes as a percentage of the final gas price were 22% in the United States, 54% in Canada, 85% in Japan, 130% in Spain, 216% in France, and 235% in Britain. After-tax gas prices in Europe can be two to three times that in the United States.

Europeans are calling for taxes against SUVs. The French government is considering raising taxes by up to $3,900 on heavy vehicles while giving discounts on smaller, lighter cars. London's mayor slammed SUV drivers as "complete idiots" and proposed doubling the $9 daily congestion fee for the privilege of driving around the city. A top adviser to the U.K. Department of Transport said that the current average tax on SUVs of £165 ($300) annually is too low and should be raised to three or four times that amount.

In contrast, the U.S. government subsidizes SUV purchases. Under the 2003 Tax Act, people who use a vehicle that weighs more than 6,000 pounds—such as the biggest, baddest SUVs and Hummers—in their business at least 50% of the time could deduct the purchase price up to $100,000 from their taxes. They might get a state tax deduction, too. Originally intended to help self-employed ranchers, farmers, and contractors purchase a heavy pickup truck or van necessary for their businesses, the SUV tax loophole was quickly exploited by accountants, lawyers, and doctors.

When the maximum deduction in this boondoggle was reduced from $100,000 to $25,000 in October 2004 and the price of gas rose, sales plummeted for many brands of SUVs and behemoths such as Hummers. (Sales continued to fall in 2005 and 2006.) The *Boston Globe* concluded that this drop in relative SUV sales proves that U.S. consumers' "tastes are changing again." But a more plausible alternative explanation for the difference in SUVs' share of sales in Europe and the United States (or over time in the United States) is variations in the relative costs of owning and operating SUVs.[10]

[10]See **www.aw-bc.com/perloff,** Chapter 3, "Substitution Effects in Canada," which provides a similar example concerning purchases in the United States or in Canada.

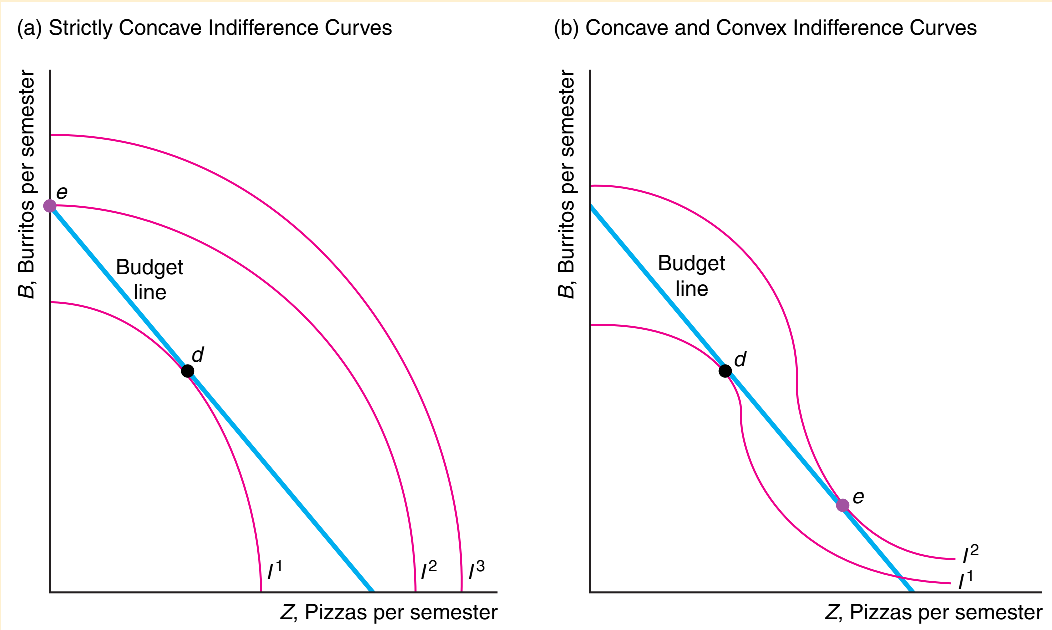


Figure 3.7 Optimal Bundles on Convex Sections of Indifference Curves. (a) Indifference curve I^1 is tangent to the budget line at Bundle *d*, but Bundle *e* is superior because it lies on a higher indifference curve, I^2. If indifference curves are strictly concave to the origin, the optimal bundle, *e*, is at a corner. (b) If indifference curves have both concave and convex sections, a bundle such as *d*, which is tangent to the budget line in the concave portion of indifference curve I^1, cannot be an optimal bundle because there must be a preferable bundle in the convex portion of a higher indifference curve, *e* on I^2 (or at a corner).

Optimal Bundles on Convex Sections of Indifference Curves. Earlier we argued, on the basis of introspection (and consistent with our assumption of strict convexity of preferences), that most indifference curves are convex to the origin. Now that we know how to determine a consumer's optimal bundle, we can give a more compelling explanation about why we assume that indifference curves are convex. We can show that if indifference curves are smooth, optimal bundles lie either on convex sections of indifference curves or at the point where the budget constraint hits an axis.

Suppose that indifference curves were strictly concave to the origin as in panel a of Figure 3.7. Indifference curve I^1 is tangent to the budget line at *d*, but that bundle is not optimal. Bundle *e* on the corner between the budget constraint and the burrito axis is on a higher indifference curve, I^2, than *d* is. Thus if a consumer had strictly concave indifference curves, the consumer would buy only one good—here, burritos. Similarly, as we saw in Solved Problem 3.3, consumers with straight-line indifference curves buy only the cheapest good. Thus if consumers are to buy more than a single good, indifference curves must have convex sections.

If indifference curves have both concave and convex sections as in panel b of Figure 3.7, the optimal bundle lies in a convex section or at a corner. Bundle *d*, where a concave section of indifference curve I^1 is tangent to the budget line, cannot be an optimal bundle. Here *e* is the optimal bundle and is tangent to the budget constraint in the convex portion of the higher indifference curve, I^2. If a consumer buys positive quantities of two goods, the indifference curve is convex and tangent to the budget line at that optimal bundle.

Buying Where More Is Better. A key assumption in our analysis of consumer behavior is that more is preferred to less: Consumers are not satiated. We now show that if both goods are consumed in positive quantities and their prices are positive, more of either good must be preferred to less. Suppose that the opposite were true and that Lisa prefers fewer burritos to more. Because burritos cost her money, she could increase her well-being by reducing the quantity of burritos she consumes until she consumes no burritos—a scenario that violates our assumption that she consumes positive quantities of both goods.[11] Though it is possible that consumers prefer less to more at some large quantities, we do not observe consumers making purchases where that occurs.

In summary, we do not observe consumer optima at bundles where indifference curves are concave or consumers are satiated. Thus we can safely assume that indifference curves are convex and that consumers prefer more to less in the ranges of goods that we actually observe.

MAXIMIZING UTILITY SUBJECT TO A CONSTRAINT USING CALCULUS

The individual choice of garnishment of a burger can be an important point to the consumer in this day when individualism is an increasingly important thing to people.
—Donald N. Smith, president of Burger King

Lisa's objective is to maximize her utility, $U(q_1, q_2)$, subject to (s.t.) her budget constraint:

$$\begin{aligned} &\max_{q_1, q_2} U(q_1, q_2) \\ &\text{s.t. } Y = p_1q_1 + p_2q_2. \end{aligned} \tag{3.14}$$

This mathematical statement of her problem shows that her *control variables*—those that she chooses—are q_1 and q_2, which appear under the "max" term in the equation. We assume that Lisa has no control over the prices she faces, p_1 and p_2, or her income, Y.

Because this problem is a constrained maximization, we cannot use the standard unconstrained maximization approach. However, we can transform this problem into one that we can solve. There are at least two approaches that we can use if we know that Lisa buys both goods, so that we are looking for an interior solution: substitution and the Lagrangian method.

[11]Similarly, at her optimal bundle, Lisa cannot be *satiated*—indifferent between consuming more or fewer burritos. Suppose that her budget is obtained by working and that Lisa does not like working at the margin. Were it not for the goods she can buy with what she earns, she would not work as many hours as she does. Thus if she were satiated and did not care if she consumed fewer burritos, she would reduce the number of hours she worked, thereby lowering her income, until her optimal bundle occurred at a point where more was preferred to less or where she consumed none.

Substitution. First, we can substitute the budget constraint into the utility function. Using algebra, we can rewrite the budget constraint as $q_1 = (Y - p_2q_2)/p_1$. If we substitute this expression for q_1 in the utility function, $U(q_1, q_2)$, we can rewrite Lisa's problem as

$$\max_{q_2} U\left(\frac{Y - p_2q_2}{p_1}, q_2\right). \tag{3.15}$$

Equation 3.15 is an unconstrained problem, so we can use standard maximization techniques to solve it. The first-order condition is obtained by setting the derivative of the utility function with respect to q_2 equal to zero:

$$\frac{dU}{dq_2} = \frac{\partial U}{\partial q_1}\frac{dq_1}{dq_2} + \frac{\partial U}{\partial q_2} = \left(-\frac{p_2}{p_1}\right)\frac{\partial U}{\partial q_1} + \frac{\partial U}{\partial q_2} = \left(-\frac{p_2}{p_1}\right)U_1 + U_2 = 0, \tag{3.16}$$

where $\partial U/\partial q_1 = U_{1\ldots}$ is the partial derivative of the utility function with respect to q_1 (the first argument) and dq_1/dq_2 is the derivative of $q_1 = (Y - p_2q_2)/p_1$ with respect to q_2.

By rearranging these terms in Equation 3.16, we get the same condition for an optimum that we obtained using a graphical approach, Equation 3.12, which is that the marginal rate of substitution equals the marginal rate of transformation:[12]

$$MRS = -\frac{U_1}{U_2} = -\frac{p_1}{p_2} = MRT.$$

To be sure that we have a maximum, we need to check that the second-order conditions hold (see the Calculus Appendix). These conditions hold if the utility function is quasi-concave, which implies that the indifference curves are convex to the origin: The *MRS* is diminishing as we move down and to the right along the curve. If we combine the *MRS* = *MRT* (first-order) condition with the budget constraint, we have two equations in two unknowns, q_1 and q_2, so we can solve for the optimal q_1 and q_2 as functions of prices, p_1 and p_2, and income, Y.

Lagrangian Method. A second approach to solving this constrained maximization problem is to use the Lagrangian method, where we write the equivalent Lagrangian problem as

$$\max_{q_1, q_2, \lambda} \mathcal{L} = U(q_1, q_2) + \lambda(Y - p_1q_1 - p_2q_2), \tag{3.17}$$

where λ (the Greek letter lambda) is the Lagrange multiplier. For values of q_1 and q_2 such that the constraint holds, $Y - p_1q_1 - p_2q_2 = 0$, so the functions $\mathcal{L}$ and U have the same values. Thus if we look only at values of q_1 and q_2 for which the constraint holds, finding the constrained maximum value of U is the same as finding the critical value of $\mathcal{L}$.

[12]Had we substituted for q_2 instead of for q_1 (which you should do to make sure that you understand how to solve this type of problem), we would have obtained the same condition.

The conditions for a critical value of q_1, q_2, and λ—the first-order conditions—for an interior maximization are[13]

$$\frac{\partial \mathcal{L}}{\partial q_1} = \frac{\partial U}{\partial q_1} - \lambda p_1 = U_1 - \lambda p_1 = 0, \tag{3.18}$$

$$\frac{\partial \mathcal{L}}{\partial q_2} = U_2 - \lambda p_2 = 0, \tag{3.19}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = Y - p_1 q_1 - p_2 q_2 = 0. \tag{3.20}$$

Equation 3.18 shows that—at the optimal levels of q_1, q_2, and λ—the marginal utility of pizza, $U_1 = \partial U/\partial q_1$, equals its price times λ. Equation 3.19 provides an analogous condition for burritos. Equation 3.20 restates the budget constraint.

These three first-order conditions can be solved for the optimal values of q_1, q_2, and λ. Again, we should check that we have a maximum (see the Calculus Appendix).

What is λ? If we equate Equations 3.19 and 3.18 and rearrange terms, we find that

$$\lambda = \frac{U_1}{p_1} = \frac{U_2}{p_2}. \tag{3.21}$$

That is, the optimal value of the Lagrangian multiplier, λ, equals the marginal utility of each good divided by its price—or the extra pleasure one gets from the last dollar of expenditure on either good.[14] This optimality condition is the same as the one that we derived using a graphical approach, Equation 3.13.

SOLVED PROBLEM 3.4

If Julia has a Cobb-Douglas utility function $U = q_1^a q_2^{1-a}$, what are her optimal values of q_1 and q_2 in terms of income, prices, and the positive constant a? (*Note*: We can solve this problem using either substitution or the Lagrangian approach. We use the Lagrangian approach here.)

Answer

1. *Show Julia's Lagrangian function and her first-order conditions*: Given that Julia's Lagrangian function is $\mathcal{L} = q_1^a q_2^{1-a} + \lambda(Y - p_1 q_1 - p_2 q_2)$, the first-order conditions for her to maximize her utility subject to the constraint are

$$\mathcal{L}_1 = U_1 - \lambda p_1 = a q_1^{a-1} q_2^{1-a} - \lambda p_1 = a \frac{U}{q_1} - \lambda p_1 = 0, \tag{3.22}$$

[13]To make our presentation as simple as possible, we assume that we have an interior solution, that q_1 and q_2 are infinitely divisible, and that $U(q_1, q_2)$ is continuously differentiable at least twice (so that the second-order condition is well defined). The first-order conditions determine an interior solution in which positive quantities of both goods are consumed. If these conditions do not predict that both quantities are nonnegative, the consumer is at a corner solution. One approach to solving the consumer-maximization problem allowing for a corner solution is to use a Kuhn-Tucker analysis (see the Calculus Appendix).

[14]More generally, the Lagrangian multiplier is often referred to as a *shadow value* that reflects the marginal rate of change in the objective function as the constraint is relaxed (see the Calculus Appendix).

$$\mathcal{L}_2 = U_2 - \lambda p_2 = (1-a)q_1^a q_2^{-a} - \lambda p_2 = (1-a)\frac{U}{q_2} - \lambda p_2 = 0, \quad (3.23)$$

$$\mathcal{L}_\lambda = Y - p_1 q_1 - p_2 q_2 = 0. \quad (3.24)$$

2. *Solve these three first-order equations for* q_1 *and* q_2: By equating the right-hand sides of the first two conditions, we obtain an equation—analogous to Equation 3.21—that depends on q_1 and q_2 but not on λ:

$$(1-a)p_1 q_1 = a p_2 q_2. \quad (3.25)$$

Equations 3.25 and 3.24 are two equations in q_1 and q_2. Substituting $p_2 q_2 = Y - p_1 q_1$ (from the budget constraint, which is the third first-order condition) into Equation 3.25, we can rewrite this expression as $a(Y - p_1 q_1) = (1-a)p_1 q_1$. Rearranging terms, we find that

$$q_1 = a\frac{Y}{p_1}. \quad (3.26)$$

Similarly, by substituting $p_1 q_1 = Y - p_2 q_2$ into Equation 3.25 and rearranging, we find that

$$q_2 = (1-a)\frac{Y}{p_2}. \quad (3.27)$$

Thus we can use our knowledge of the form of the utility function to solve the expression for the q_1 and q_2 that maximize utility in terms of income, prices, and the utility function parameter a. Equations 3.26 and 3.27 are Julia's demand functions for q_1 and q_2, respectively.

SOLVED PROBLEM 3.5

Given that Julia's utility function is $U = q_1^a q_2^{1-a}$, what share of her budget does she spend on q_1 and q_2 in terms of her income, prices, and the positive constant a?

Answer

Use Equation 3.26 and 3.27 *to determine her budget shares*: The share of her budget that Julia spends on pizza is her expenditure on pizza, $p_1 q_1$, divided by her budget, Y, or $p_1 q_1/Y$. By multiplying both sides of Equation 3.26, $q_1 = aY/p_1$, by p_1, we find that $p_1 q_1/Y = a$. Thus a is both her budget share of pizza and the exponent on the units of pizza in her utility function. Similarly, from Equation 3.27, we find that her budget share of burritos is $p_2 q_2/Y = 1 - a$.

Comment: The Cobb-Douglas functional form was derived to have this property. If an individual has a Cobb-Douglas utility function, we can estimate a and hence the utility function solely from information about the individual's budget shares. Indeed, that is how we obtained our estimate of Jackie's Cobb-Douglas utility function for CDs and DVDs.

APPLICATION

Utility Maximization for Music CDs and Movie DVDs

We return to our typical consumer, Jackie, who has an estimated Cobb-Douglas utility function of $U = q_1^{0.6}q_2^{0.4}$ over music CDs and movie DVDs. The average price of a CD is about $p_1 = \$15$, and the average price of a DVD is roughly $p_2 = \$20$, so her budget constraint for purchasing these entertainment goods is

$$p_1q_1 + p_2q_2 = 15q_1 + 20q_2 = 300 = Y,$$

given that Jackie, like the average consumer, spends about \$300 per year on these goods.

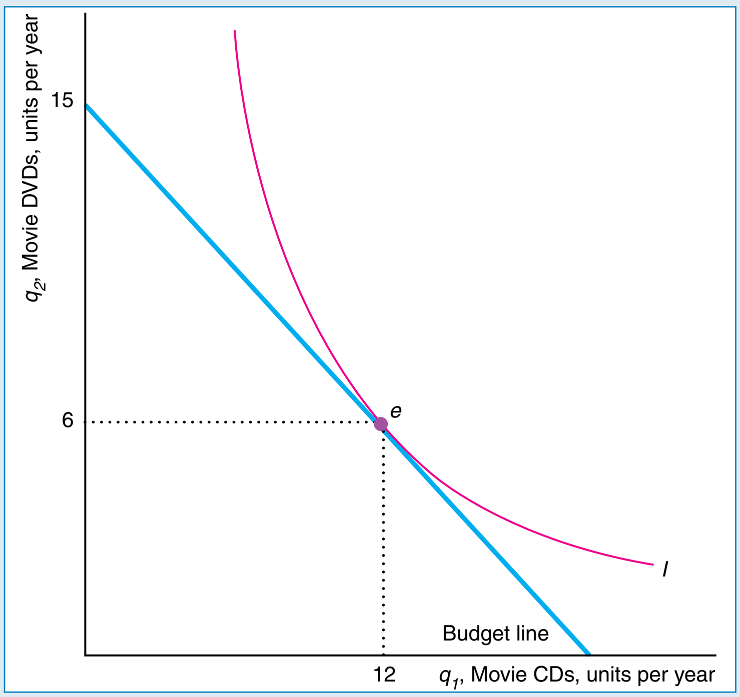
Using Equations 3.26 and 3.27 from Solved Problem 3.4, we can solve for Jackie's optimal numbers of CDs and DVDs:

$$q_1 = 0.6\frac{Y}{p_1} = 0.6 \times \frac{300}{15} = 12,$$

$$q_2 = 0.4\frac{Y}{p_2} = 0.4 \times \frac{300}{20} = 6.$$

These quantities are the average purchases for 2005. The figure shows that the optimal bundle is e where the indifference curve is tangent to the budget line.

We can use the result in Solved Problem 3.5 to confirm that the budget shares equal the exponents in Jackie's utility function. The share of Jackie's budget devoted to music CDs is $p_1q_1/Y = (15 \times 12)/300 = 0.6$, which is the exponent on music CDs in her utility function. Similarly, the budget share she allocates to DVDs is $p_2q_2/Y = (20 \times 6)/300 = 0.4$, which is the DVD exponent.



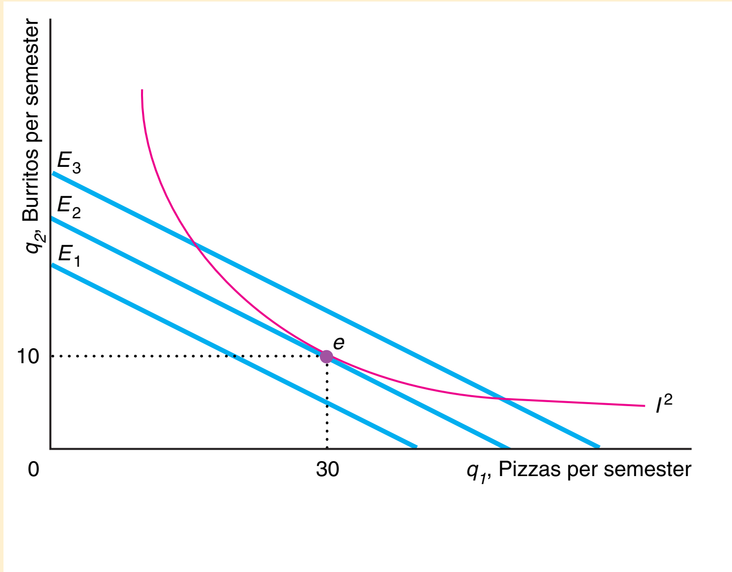


Figure 3.8 Minimizing Expenditure. The lowest expenditure that Lisa can make that will keep her on indifference curve I^2 is E_2. She buys 30 pizzas and 10 burritos.

MINIMIZING EXPENDITURE

Earlier we showed how Lisa chooses quantities of goods so as to maximize her utility subject to a budget constraint. There is a related or *dual* constrained minimization problem where she finds the combination of goods that achieves a particular level of utility for the least expenditure.

In panel a of Figure 3.6, we showed that, given the budget constraint that she faced, Lisa maximized her utility by picking a bundle of $q_1 = 30$ and $q_2 = 10$. She did that by choosing the highest indifference curve, I^2, that touched—was tangent to—the budget constraint.

Now let's consider the alternative problem where we ask how Lisa can make the lowest possible expenditure to maintain her utility at a particular level, $\overline{U}$, which corresponds to indifference curve I^2. Figure 3.8 shows three possible budget lines corresponding to budgets or expenditures of E_1, E_2, and E_3. The lowest of these budget lines with expenditure E_1 lies everywhere below I^2, so Lisa cannot achieve the level of utility on I^2 for such a small expenditure. Both the other budget lines cross I^2; however, the budget line with expenditure E_2 is the least expensive way for her to stay on I^2. The rule for minimizing expenditure while achieving a given level of utility is to choose the lowest expenditure such that the budget line touches—is tangent to—the relevant indifference curve.

The slope of all the expenditure or budget lines is $-p_2/p_1$—see Equation 3.11—which depends only on the market prices and not on income or expenditure. Thus the point of tangency in Figure 3.8 is the same as in panel a of Figure 3.6. Lisa purchases $q_1 = 30$ and $q_2 = 10$ because that is the bundle that minimizes her expenditure conditional on staying on I^2.

Thus solving either of the two problems—maximizing utility subject to a budget constraint or minimizing expenditure subject to maintaining a given level of utility—

yields the same optimal values. It is sometimes more useful to use the expenditure-minimizing approach because expenditures are observable and utility levels are not.

We can use calculus to solve the expenditure-minimizing problem. Lisa's objective is to minimize her expenditure, E, subject to the constraint that she hold her utility constant at $\overline{U} = U(q_1, q_2)$:

$$\min_{q_1, q_2} E = p_1 q_1 + p_2 q_2 \\ \text{s.t. } \overline{U} = U(q_1, q_2). \tag{3.28}$$

The solution of this problem is an expression of the minimum expenditure as a function of the prices and the specified utility level:

$$E = E(p_1, p_2, \overline{U}). \tag{3.29}$$

We call this expression the **expenditure function:** the relationship showing the minimal expenditures necessary to achieve a specific utility level for a given set of prices.

SOLVED PROBLEM 3.6

Given that Julia has a Cobb-Douglas utility function $U = q_1^a q_2^{1-a}$, what is her expenditure function?

Answer

1. *Show Julia's Lagrangian function and derive her first-order conditions*: Julia's Lagrangian function is $\mathcal{L} = p_1 q_1 + p_2 q_2 + \lambda(\overline{U} - q_1^a q_2^{1-a})$. The first-order conditions for her to minimize her expenditure subject to remaining on a given indifference curve are obtained by differentiating the Lagrangian with respect to q_1, q_2, and λ and setting each derivative equal to zero:

$$\frac{\partial \mathcal{L}}{\partial q_1} = p_1 - \lambda a q_1^{a-1} q_2^{1-a} = p_1 - \lambda a \frac{U}{q_1} = 0, \tag{3.30}$$

$$\frac{\partial \mathcal{L}}{\partial q_2} = p_2 - \lambda(1 - a) q_1^a q_2^{-a} = p_2 - \lambda(1 - a)\frac{U}{q_2} = 0, \tag{3.31}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \overline{U} - q_1^a q_2^{1-a} = 0. \tag{3.32}$$

2. *Solve these three first-order equations for q_1 and q_2*: By equating the right-hand sides of the first two conditions, we obtain an equation—analogous to Equation 3.21—that depends on q_1 and q_2 but not on λ: $p_1 q_1/(aU) = p_2 q_2/[(1 - a)U]$, or

$$(1 - a)p_1 q_1 = a p_2 q_2. \tag{3.33}$$

This condition is the same as Equation 3.25, which we derived in Solved Problem 3.4 when we were maximizing Julia's utility subject to the budget constraint.

Equations 3.33 and 3.32 are two equations in q_1 and q_2. From Equation 3.33, we know that $p_2 q_2 = p_1 q_1 (1 - a)/a$. If we substitute this expression into the expenditure definition, we find that $E = p_1 q_1 + p_2 q_2 = p_1 q_1 + p_1 q_1 (1 - a)/a = p_1 q_1 / a$. Rearranging terms, we learn that

$$q_1 = a\frac{E}{p_1}. \tag{3.34}$$

Similarly, by substituting $p_1 q_1 = Y - p_2 q_2$ into Equation 3.33 and rearranging, we learn that

$$q_2 = (1 - a)\frac{E}{p_2}. \qquad (3.35)$$

By substituting the expressions in Equations 3.34 and 3.35 into the indifference curve expression, Equation 3.32, we observe that

$$\overline{U} = q_1^a q_2^{1-a} = \left(a\frac{E}{p_1}\right)^a \left[(1 - a)\frac{E}{p_2}\right]^{1-a} = E\left(\frac{a}{p_1}\right)^a \left(\frac{1 - a}{p_2}\right)^{1-a}. \qquad (3.36)$$

Solving this expression for E, we can write the expenditure function as

$$E = \overline{U}\left(\frac{p_1}{a}\right)^a \left(\frac{p_2}{1 - a}\right)^{1-a}. \qquad (3.37)$$

Equation 3.37 shows the minimum expenditure necessary to achieve utility level $\overline{U}$ given prices p_1 and p_2. For example, if $a = 1 - a = {}^1/_2$, then $E = 2\overline{U}\sqrt{p_1 p_2}$.

Summary

Consumers maximize their utility (well-being) subject to constraints based on their income and the prices of goods.

1. **Preferences:** To predict consumers' responses to changes in these constraints, economists use a theory about individuals' preferences. One way of summarizing consumers' preferences is with a family of indifference curves. An indifference curve consists of all bundles of goods that give the consumer a particular level of utility. On the basis of observations of consumers' behavior, economists assume that consumers' preferences have three properties: completeness, transitivity, and more is better. Given these three assumptions, indifference curves have the following properties:
   - Consumers get more pleasure from bundles on indifference curves the farther from the origin the curves are.
   - Indifference curves cannot cross.
   - There is an indifference curve through any given bundle.
   - Indifference curves have no thickness.
   - Indifference curves slope downward.
   - Consumers are observed purchasing positive quantities of all relevant goods only where their indifference curves are convex to the origin.

   We also assume that consumers' preferences are continuous, and we use this assumption in our utility function analysis.

2. **Utility:** *Utility* is the set of numerical values that reflect the relative rankings of bundles of goods. Utility is an ordinal measure: By comparing the utility a consumer gets from each of two bundles, we know that the consumer prefers the bundle with the higher utility although we can't tell by how much the consumer prefers that bundle. The utility function is unique only up to a positive monotonic transformation. The marginal utility from a good is the extra utility a person gets from consuming one more unit of that good, holding the consumption of all other goods constant. The rate at which a consumer is willing to substitute Good 1 for Good 2, the marginal rate of substitution, *MRS*, depends on the relative amounts of marginal utility that the consumer gets from each of the two goods.

3. **Budget Constraint:** The amount of goods consumers can buy at given prices is limited by their income. As a result, the greater their income and the lower the prices of goods, the better off consumers are. The rate at which they can exchange Good 1 for Good 2 in the market, the marginal rate of transformation, *MRT*, depends on the relative prices of the two goods.

4. **Constrained Consumer Choice:** Each person picks an affordable bundle of goods to consume so as to maximize his or her pleasure. If an individual consumes both Good 1 and Good 2 (an interior solution), the individual's utility is maximized when the following four equivalent conditions hold:
   - The consumer buys the bundle of goods that is on the highest obtainable indifference curve.
   - The indifference curve between the two goods is tangent to the budget constraint.
   - The consumer's marginal rate of substitution (the slope of the indifference curve) equals the marginal rate of transformation (the slope of the budget line).
   - The last dollar spent on Good 1 gives the consumer as much extra utility as the last dollar spent on Good 2.

   However, consumers do not buy some of all possible goods (corner solutions). The last dollar spent on a good that is actually purchased gives more extra utility than would a dollar's worth of a good the consumer chose not to buy.

We can use our model where a consumer maximizes his or her utility subject to a budget constraint to predict the consumer's optimal choice of goods as a function of the consumer's income and market prices. The same bundle is chosen if we look at the dual problem of minimizing the consumer's expenditure while holding the consumer's utility fixed.

Questions

** = answer at the back of this book;* **W** *= audio-slide show answers by James Dearden at* **www.aw-bc.com/perloff.**

1. Which of the following pairs of goods are complements (people like to consume them together), and which are substitutes (people are willing to trade off one good for the other)? Are the goods that are substitutes likely to be perfect substitutes for some or all consumers?
   a. A popular novel and a gossip magazine
   b. A camera and film
   c. An economics textbook and a mathematics textbook
   d. A Panasonic CD player and a JVC CD player
2. Don is altruistic. Show the possible shape of his indifference curves between charity and all other goods.

*3. Arthur spends his income on bread and chocolate. He views chocolate as a good but is neutral about bread, in that he doesn't care if he consumes it or not. Draw his indifference curve map.

4. Miguel considers tickets to the Houston Grand Opera and to Houston Astros baseball games to be perfect substitutes. Show his preference map. What is his utility function?

*5. Sofia will consume hot dogs only with whipped cream. Show her preference map. What is her utility function?

6. Give as many reasons as you can why economists believe that indifference curves are convex.

7. Fiona requires a minimum level of consumption, a *threshold*, to derive additional utility: $U(X, Z)$ is 0 if $X + Z \leq 5$ and is $X + Z$ otherwise. Draw Fiona's indifference curves. Which of our usual assumptions does this example violate?

*8. Gasoline was once less expensive in the United States than in Canada, but now gasoline costs less in Canada than in the United States due to a change in taxes. How will the gasoline-purchasing behavior of a Canadian who lives equally close to gas stations in both countries change? Answer using an indifference curve and budget line diagram.

*9. Governments frequently limit how much of a good a consumer can buy. During emergencies, governments may ration "essential" goods such as water, food, and gasoline rather than let their prices rise. Suppose that the government rations water, setting quotas on how much a consumer can purchase. If a consumer can afford to buy 12 thousand gallons a month but the government restricts purchases to no more than 10 thousand gallons a month, how do the consumer's budget line and opportunity set change?

10. What happens to a consumer's optimal choice of goods if all prices and income double? (*Hint:* What happens to the intercepts of the budget constraint?)

11. Suppose that Boston consumers pay twice as much for avocadoes as they pay for tangerines, whereas San Diego consumers pay half as much for avocadoes as they pay for tangerines. Assuming that consumers maximize their utility, which city's consumers have a higher marginal rate of substitution of avocadoes for tangerines? Explain your answer.

12. Suppose that Solved Problem 3.3 were changed so that Nigel and Bob are buying a bundle of several cars and SUVs for their large families or businesses and have identical tastes, with the usual-shaped indifference curves. Use a figure to discuss how the different slopes of their budget lines affect the bundles of SUVs and cars that each chooses. Can you make any unambiguous statements about the quantity each can buy? Can you make an unambiguous statement if you know that Bob's budget line goes through Nigel's optimal bundle?

13. If a consumer has indifference curves that are convex to the origin but that have a kink in them (similar to the perfect complements example, but the angle at the kink is greater than 90°), how can we determine the optimal bundle? Use a graph to illustrate your answer. Can we use all the conditions that we derived for determining an interior solution?

*14. What is the effect of a 50% income tax on Dale's budget line and opportunity set?

15. Goolsbee (2000) finds that people who live in high sales tax areas are much more likely than other consumers to purchase over the Internet, where they are generally exempt from the sales tax if the firm is located in another state. The National Governors Association (NGA) proposed a uniform tax of 5% on all Internet sales. Goolsbee estimates that the NGA's flat 5% tax would lower the number of online customers by 18% and total sales by 23%. Alternatively, if each state imposed its own taxes (which average 6.33%), the number of buyers would fall

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by 24% and spending by 30%. Use an indifference curve–budget line diagram to illustrate the reason for his results.

16. In 2006, Michigan passed legislation that provides greater incentives to drivers who buy ethanol by lowering the state tax on each gallon of ethanol-blended fuel to 12¢, down from the 19¢ per gallon on regular gas. Show the effects of such a subsidy on a consumer who is indifferent between using ethanol-blended fuel and regular gasoline and on another consumer who views the two types of gasoline as imperfect substitutes.

Problems

17. Elise consumes cans of anchovies, A, and boxes of biscuits, B. Each of her indifference curves reflects strictly diminishing marginal rates of substitution. Where $A = 2$ and $B = 2$, her marginal rate of substitution between cans of anchovies and boxes of biscuits equals -1 ($= MU_A/MU_B$). Will she prefer a bundle with three cans of anchovies and a box of biscuits to a bundle with two of each? Why?

*18. Andy purchases only two goods, apples (a) and kumquats (k). He has an income of $40 and can buy apples at $2 per pound and kumquats at $4 per pound. His utility function is $U(a, k) = 3a + 5k$. What are his marginal utility for apples and his marginal utility for kumquats? What bundle of apples and kumquats should he purchase to maximize his utility? Why?

*19. David's utility function is $U = B + 2Z$. Describe the location of his optimal bundle (if possible) in terms of the relative prices of B and Z.

20. Mark consumes only cookies and books. At his current consumption bundle, his marginal utility from books is 10 and from cookies is 5. Each book costs $10, and each cookie costs $2. Is he maximizing his utility? Explain. If he is not, how can he increase his utility while keeping his total expenditure constant?

*21. Nadia likes spare ribs, R, and fried chicken, C. Her utility function is

$$U = 10R^2C.$$

Her weekly income is $90, which she spends on only ribs and chicken.

a. If she pays $10 for a slab of ribs and $5 for a chicken, what is her optimal consumption bundle? Show her budget line, indifference curve, and optimal bundle, e_1, in a diagram.
b. Suppose the price of chicken doubles to $10. How does her optimal consumption of chicken and ribs change? Show her new budget line and optimal bundle, e_2, in your diagram.

22. Steve's utility function is $U = BC$, where B = veggie burgers per week and C = packs of cigarettes per week. Here $MU_B = C$ and $MU_C = B$. What is his marginal rate of substitution if veggie burgers are on the vertical axis and cigarettes are on the horizontal axis? Steve's income is $120, the price of a veggie burger is $2, and that of a pack of cigarettes is $1. How many burgers and how many packs of cigarettes does Steve consume to maximize his utility? When a new tax raises the price of a burger to $3, what is his new optimal bundle? Illustrate your answers in a graph.

23. Linda loves buying shoes and going out to dance. Her utility function for pairs of shoes, S, and the number of times she goes dancing per month, T, is $U(S, T) = 2ST$. What are her marginal utility of shoes and her marginal utility of dancing? It costs Linda $50 to buy a new pair of shoes or to spend an evening out dancing. Assume that she has $500 to spend on clothing and dancing.

a. What is the equation for her budget line? Draw it (with T on the vertical axis), and label the slope and intercepts.
b. What is Linda's marginal rate of substitution? Explain.
c. Solve mathematically for her optimal bundle. Show in a diagram how to determine this bundle using indifference curves and a budget line.

24. Diogo has a utility function $U(q_1, q_2) = q_1^{3/4} q_2^{1/4}$, where q_1 is pizza and q_2 is burritos. If the price of burritos, p_2, is $2, the price of pizzas, p_1, is $1, and Y is $100, what is Diogo's optimal bundle?

25. Vasco's utility function is $U = 10q_1q_2^2$. The price of pizza, q_1, is $p_1 = \$5$, the price of burritos, q_2, is $p_2 = \$10$, and his income is $Y = \$150$. What is his optimal consumption bundle? Show it in a graph.

26. If José Maria's utility function is $U(q_1, q_2) = q_1 + Aq_1^aq_2^b + q_2$, what is his marginal utility of q_2? What is his marginal rate of substitution between these two goods?

27. Ann's utility function is $U = q_1q_2/(q_1 + q_2)$. Solve for her optimal values of q_1 and q_2 as function of p_1, p_2, and Y.

*28. Suppose we calculate the MRS at a particular bundle for a consumer whose utility function is $U(q_1, q_2)$. If we use a positive monotonic transformation, F, to obtain a new utility function, $V(q_1, q_2) = F(U(q_1, q_2))$, then this new utility function contains the same information about the consumer's rankings of bundles. Prove that the MRS is the same as with the original utility function.

29. The application "Indifference Curves Between Food and Clothing" postulates that there are minimum levels of food and clothing necessary to support life. Suppose that

the amount of food one has is F, where the minimum level to sustain life is $\underline{F}$, and the amount of clothing is C, where the minimum necessary is $\underline{C}$. We can then modify the Cobb-Douglas utility function to reflect these minimum levels: $U(C, F) = (C - \underline{C})^a(F - \underline{F})^{1-\alpha}$, where $C \geq \underline{C}$ and $F \geq \underline{F}$. Using the approach similar to that in Solved Problem 3.4, derive the optimal amounts of food and clothing as a function of prices and income. To do so, introduce the idea of *extra income*, Y^*, which is the income remaining after paying for the minimum levels of food and clothing: $Y^* = Y - p_C\underline{C} - p_F\underline{F}$. Show that the demand for clothing is $C = \underline{C} + aI^*/p_C$ and that the demand for food is $F = \underline{F} + (1 - a)I^*/p_F$. Derive formulas for the share of income devoted to each good.

30. Use the substitution approach rather than the Lagrangian method in Solved Problem 3.4 to obtain expressions for the optimal levels of q_1 and q_2. (*Hint*: It may help to take logarithms of both sides of the utility expression before you differentiate.)

31. We argued earlier that if all prices and income doubled, we would not expect an individual's choice of an optimal bundle to change. We say that a function $f(X, Y)$ is homogeneous of degree γ if, when we multiply each argument by a constant α, we have $f(\alpha X, \alpha Y) = \alpha^\gamma f(X, Y)$. Thus if a function is homogeneous of degree zero, $f(\alpha X, \alpha Y) = \alpha^0 f(X, Y) = f(X, Y)$, because $\alpha^0 = 1$. Show that optimality conditions that we derived based on the Cobb-Douglas utility function in Solved Problem 3.4 are homogeneous of degree zero. Explain why that result is consistent with our intuition about what happens if we double all prices and income.

32. In 2005, Americans bought 9.1 million home radios for $202 million and 3.8 million home-theater-in-a-box units for $730 million (**www.twice.com/article/CA6319031.html**, March 27, 2006). Suppose that the average consumer has a Cobb-Douglas utility function and buys only these two goods. Given the results in Solved Problem 3.5, estimate a plausible Cobb-Douglas utility function such that the consumer would allocate income in the proportions actually observed.

*33. The constant elasticity of substitution (CES) utility function is $U(q_1, q_2) = (q_1^\rho + q_2^\rho)^{1/\rho}$, where ρ is a positive constant. Show that there is a positive monotonic transformation such that there is an equivalent utility function (one with the same preference ordering) $U(q_1, q_2) = q_1^\rho + q_2^\rho$.

*34. What is the *MRS* for the CES utility function $U(q_1, q_2) = q_1^\rho + q_2^\rho$?

35. For the CES utility function $U(q_1, q_2) = q_1^\rho + q_2^\rho$, derive the expressions for the optimal levels of q_1 and q_2.

36. Jim spends most of his time in Jazzman's, a coffee shop on the south side of Bethlehem, Pennsylvania. Jim has $12 a week to spend on coffee and muffins. Jazzman's sells muffins for $2 each and coffee for $1.20 per cup. Jim consumes q_c cups of coffee per week and q_m muffins per week. His utility function for coffee and muffins is $U(q_c, q_m) = q_c^{1/2}q_m^{1/2}$.
    a. Draw Jim's budget line.
    b. Use the Lagrange technique to find Jim's optimal bundle.
    c. Now Jazzman's has introduced a frequent-buyer card: For every five cups of coffee purchased at the regular price of $1.20 per cup, Jim receives a free sixth cup. Draw Jim's new budget line. Is Jim's new budget line actually composed of more than one straight line?
    d. With the frequent-buyer card, does Jim consume more coffee? **W**

37. Jen's utility for chocolate, q_1, and coffee, q_2, is $U = q_1^{0.5} + q_2^{0.5}$. Does more money make Jen better off, and does less money reduce her well-being? (*Hint*: To answer the question, derive Jen's expenditure function.) **W**