

PAPPUS' COMMENTARY ON EUCLID

The Commentary of Pappus on Book X of Euclid's Elements. Arabic text and translation. By William Thomson, with introductory remarks, notes, and a glossary of technical terms by Gustav Junge and William Thomson. Cambridge, Mass., Harvard University Press, 1930. 294 pp.

Of all the mathematical writers of the world the one whose name has been known to the greatest number of people is doubtless Euclid. Considering the circumstances under which he composed his classic, he was the most remarkable textbook writer that ever lived. In spite of all this, however, what is probably the most scholarly part of his writings is relatively unknown, and the works of Pappus, the best of his early commentators, have come down to us only in fragments. It is therefore a matter of special interest to scholars that Mr. Thomson has given to the English-reading world an opportunity of learning more about Book X of the *Elements*—the text of which might indeed have been found in Sir Thomas Heath's well known edition—and of learning the nature of the hitherto almost unknown commentary of Pappus, who lived some six centuries after the great classic was written.

The commentary itself is preserved, so far as known, in only a single Arabic manuscript, MS. 2457 of the Bibliothèque Nationale in Paris. It was described by Woepcke in 1856 and the Arabic text was later published by him. This text was translated into German by Suter and was published in 1922, after the latter's death, evidently without consulting the original Arabic text. Woepcke had assigned the commentary to one Valens, probably the astronomer Vettius Valens, a contemporary of Ptolemy (c. 150), basing his contention upon his reading of the abridgment of the author's name (the consonants only) as *Bs*. Suter, however, sensed the fact that this was an error in reading the Arabic, and that it should be *Bbs*, and therefore stated that it might stand for Pappus. From the text, however, he felt that this was doubtful, suggesting that it might more probably have been Proclus. Heiberg (*Euklid-Studien*, p. 169), than whom we have no better authority, felt that Pappus was the author, an opinion in which Suter seems finally to have concurred (*Abhandlungen zur Geschichte der Mathematik*, vol. 10, pp. 49, 211), which met the approval of Sir Thomas Heath, and which seems fully confirmed in the present edition.

Book X of the *Elements* sums up the work of the Pythagoreans with respect to commensurable magnitudes and hence to rational and irrational lines. It was a result of the discovery of incommensurability that Eudoxus was led to write, or at any rate prepare the ground for, Euclid's masterly Book II, on proportion. As Sir Thomas Heath has said, this discovery "must have necessitated a great recasting of the whole fabric of elementary geometry, pending the discovery of the general theory of proportion applicable to incommensurable as well as to commensurable magnitudes." The same writer has also called attention (*The Thirteen Books of Euclid's Elements*, III (2), p. 8) to the interest shown in Book X by the early algebraists, an interest which evidently led, in the 19th century, to a renewed desire to investigate the whole theory of irrational numbers.

It is for these reasons that the publication of the Arabic text and a careful translation of a work hitherto known only imperfectly, is so important. Pappus was not a great mathematician but he was the best of the Greek commentators on Euclid, and the parts of his *Collections* that have come down to us have been carefully studied and highly esteemed by such scholars as Commandino in the 16th century, Wallis in the 17th, Halley in the 18th, and Hultsch and Heiberg in the 19th, with the recent appreciation by Junge and Thomson in the book under review.

The work consists of (1) a series of "Bemerkungen zur dem vorliegenden Kommentar" by Dr. Junge, with notes and a statement of the contents of the commentary; (2) an introduction, describing the manuscript and giving a history of its translation by Abū 'Uthmān al-Dimishqī (10th century), one of the leading physicians of Bagdad and a translator of several important Greek treatises; a discussion of the sources of Pappus's conception of rational quantities; and collation of the Arabic text with the Greek scholia as given in volume V of Heiberg's edition of Euclid; (3) a translation of the text by Mr. Thomson, with numerous notes containing an extensive bibliography; (4) the Arabic text; and (5) a glossary of the technical terms as they appear in the Arabic.

The parts of the work which will be found of greatest value to scholars who do not read Arabic are, of course, the translation and the notes. As to the nature of the former, Mr. Thomson remarks that it "is avowedly of a philological and historical nature and does not pretend to render the thought of Pappus into the terms and signs of modern mathematics." It was therefore desirable that this phase of the mathematical problems should be considered independently, and this has been very acceptably done in the *Bemerkungen* of Dr. Junge already mentioned. Fortunately, Mr. Thomson had, in his translation, the added assistance of the technical terminology of Sir Thomas Heath's translation of the tenth book of the *Elements*, which he duly acknowledges.

As a whole the book is one of the most encouraging of recent evidences of a new type of scholarship in America and of a desire to clothe the products of such scholarship in a dignified and worthy dress. While we may regret that the Harvard Press was not prepared to print such a treatise in this country, it is to be congratulated that it secured the aid of such a well-equipped organization as the J. J. Augustin Buchdruckerei of Glückstadt and Hamburg with its notable history and its ability to print in no less than twenty-five of the most important of the languages, besides those of Europe and the Americas which use the Roman fonts.

DAVID EUGENE SMITH