Intertemporal Substitution in Consumption

Robert E. Hall

Stanford University and National Bureau of Economic Research

One of the important determinants of the response of saving and consumption to the real interest rate is the elasticity of intertemporal substitution. That elasticity can be measured by the response of the rate of change of consumption to changes in the expected real interest rate. A detailed study of data for the twentieth-century United States shows no strong evidence that the elasticity of intertemporal substitution is positive. Earlier findings of substantially positive elasticities are reversed when appropriate estimation methods are used.

I. Introduction

A higher expected real interest rate makes consumers defer consumption, everything else held constant. The magnitude of this intertemporal substitution effect is one of the central questions of macroeconomics. If consumers can be induced to postpone consumption by modest increases in interest rates, then (1) movements of interest rates will make consumption decline whenever other components of aggregate demand rise—total output will not be much influenced by changes in those components; (2) the deadweight loss from the taxation of interest is important; (3) the burden of the national debt or unfunded social security is relatively unimportant; and (4) consumption will move along with real interest rates over the business cycle, to

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name four of the many issues that rest on the intertemporal substitutability of consumption.

This paper estimates parameters of the representative individual's utility function rather than parameters of the consumption function or savings function. As Lucas (1976) has pointed out, there may not be anything that could be properly called a consumption or savings function: the relation between consumption, income, and interest rates depends on the wider macroeconomic context and may not be stable over time, even though consumers are always trying to maximize the same utility function. The techniques of this paper are more robust with respect to this kind of instability than standard econometric models of consumption and savings.

The essential idea of the paper is that consumers plan to change their consumption from one year to the next by an amount that depends on their expectations of real interest rates. Actual movements of consumption differ from planned movements by a completely unpredictable random variable that indexes all the information available next year that was not incorporated in the planning process the year before. If expectations of real interest rates shift, then there should be a corresponding shift in the rate of change of consumption. The magnitude of the response of consumption to a change in real interest expectations measures the intertemporal elasticity of substitution. All this is set up in a formal econometric model in which the assumptions are formalized and the estimation techniques rigorously justified.

Over the postwar period, there have been downward and upward shifts in the expected real return from common stocks, Treasury bills, and savings accounts, three of the investments that govern the real interest rate for consumers. Over the same period, there have been only small shifts in the rate of growth of consumption. Consequently, all the estimates presented in this paper of the intertemporal elasticity of substitution are small. Most of them are also quite precise, supporting the strong conclusion that the elasticity is unlikely to be much above 0.1, and may well be zero.

II. Theory of the Consumer under Uncertain Real Interest Rates

Finance theory has examined the role of the consumer in an economy with one or more securities with stochastic returns. Breeden (1977, 1979) was the pioneer in what has become known as the consumption capital asset pricing model. Hansen and Singleton (1983) provide an application of the model to macroeconomic consumption data. Mankiw, Rotemberg, and Summers (1985) have extended the model to include labor supply. The basic model of the joint distribution of consumption and the return earned by one asset that has emerged from that research is the following: The joint distribution of the log of consumption in period t, c_t, and the return earned by the asset from period $t-1$ to period t, r_{t-1}, is normal with a covariance matrix that is unchanging over time. The means obey the linear relation

$$\bar{c}_t = \sigma \bar{r}_{t-1} + c_{t-1} + k. \tag{1}$$

That is, the expected change in the log of consumption is a parameter, σ, times the expected real return plus a constant.

The simplest rationalization for this model of the joint distribution of the two variables is based on the hypothesis that the consumer maximizes the expected value of an intertemporally separable utility function

$$\Sigma e^{-\delta t + [1-(1/\sigma)]c_t}. \tag{2}$$

For the purposes of deriving the joint distribution of consumption growth and the real return, it is not necessary to make specific assumptions about the market setting of the maximization. At one extreme, the consumer could face a full set of markets in contingent commodities, and then the budget constraint would say that the sum of all the consumer's demands for the contingent claims valued at market prices would equal his endowment. At the other extreme, the consumer could be Robinson Crusoe, with a single risky investment in a real asset. Then the budget constraint would say that his holdings of the real asset could never be negative. For a further discussion of this point, see Grossman and Shiller (1982).

In any case, one of the many choices facing the consumer is to spend a little less in year $t-1$, invest the savings in one asset, and spend the stochastic proceeds in year t. Suppose that a unit investment in year $t-1$ has the stochastic return $e^{r_{t-1}}$ in year t. The first-order condition for the deferral of a small amount of consumption from period $t-1$ to period t, considered from the point of view of a consumption decision made in $t-1$, is

$$E_{t-1}[e^{r_{t-1}-(1/\sigma)c_t} - e^{\delta-(1/\sigma)c_{t-1}}] = 0. \tag{3}$$

Equation (3) is the precise mathematical formulation of the principle that the marginal rate of substitution should equal the ratio of the prices of present and future consumption. Under uncertainty, it is not true that the expected marginal rate of substitution should equal the expected price ratio (the discount function). Rather, the appropriate discount rate is the risk-adjusted one described by the first factor in equation (3); it is the expectation of the product of the discount function and the marginal stochastic utility next period. This expression is related to the "consumption beta" of modern finance theory.

The reallocation condition of equation (3) is the generalization of the proposition investigated in my earlier paper (Hall 1978) that marginal utility should be a trended random walk when real interest rates are constant over time. Further progress in translating the reallocation condition into consequences for observed variables requires assumptions about the distributions of the random influences. A set of assumptions related to those introduced by Breeden (1977) seems a natural approach. First, assume that the real interest rate, r_{t-1}, conditional on information available in year $t - 1$, obeys the normal distribution with mean $\bar{r}_{t-1}$. Because interest rates as they are defined in this paper can be indefinitely negative, the normal distribution is a natural assumption. Second, assume that the consumer's rule for processing new information about income and interest rates makes the distribution of consumption lognormal, conditional on information available last year; that is, log c_t is normal with mean $\bar{c}_t$. Because the new information arriving in year t has a bearing both on the actual return to investments maturing in t and on the consumer's long-term well-being estimated in that year, the two random variables r_{t-1} and c_t will be correlated.

Applying the intertemporal allocation condition under the assumptions of lognormality gives the relation between the expected value of the log of consumption in period t given consumption in period $t - 1$ and the mean of the distribution of the real interest rate:

$$\bar{c}_t = \sigma\bar{r}_{t-1} + c_{t-1} + k. \tag{1}$$

Here k is a constant that depends on the variances and covariance of z, r, and c. I will assume that it does not change significantly over time.

This condition says that the mean level of consumption in period t generated by the consumer's choice as of period $t - 1$ is the level of consumption chosen for period $t - 1$ plus a constant plus an adjustment positively related to the mean of the real interest rate. A high value of σ means that, when the real interest rate is expected to be high, the consumer will actively defer consumption to the later period.

The condition is a constraint on the consumption rule. It says that an optimal rule will wind up choosing a level of consumption in period t, after the new information becomes available, whose mean obeys this restriction. The condition is not a complete description of consumption behavior under uncertainty. It does not describe the actual amount by which consumption changes when new information about income or asset returns becomes available.

The actual log of consumption in period t, c_t, differs from the mean, $\bar{c}_t$, by a completely unpredictable surprise, which I will call ϵ_t. By the hypotheses already stated, ϵ_t is a normal random variable. The two equations of interest can be put in the form of a bivariate regression:

$$c_t = \sigma\bar{r}_{t-1} + c_{t-1} + k + \epsilon_t, \tag{4}$$

$$r_{t-1} = \bar{r}_{t-1} + \nu_t. \tag{5}$$

The random variable ν_t also has the normal distribution.

If the expected real interest rate $\bar{r}_{t-1}$ is observed directly, then the key parameter σ can be estimated simply by regressing the change in the log of consumption on the expected real rate. That regression also has the property that no other variable known in period $t - 1$ belongs in the regression. The strong testable implication of the theory is that the mean of the rate of growth of consumption is shifted only by the mean of the real interest rate. Information available in year $t - 1$ is helpful in predicting the rate of growth of consumption only to the extent that it predicts the real interest rate. This testable implication is the logical extension of the one derived in my earlier paper (Hall 1978) under constancy of real interest rates. In that case, no variable known in year $t - 1$ should help predict the rate of growth of consumption.

A. *Interpretation of the Parameter* σ

In the model of finance theory, based on the maximization of the expected value of the intertemporally additive utility function of equation (2), the parameter σ is interpreted as the reciprocal of the coefficient of relative risk aversion. However, σ is the intertemporal elasticity of substitution as well. The two parameters are assumed to be reciprocals of one another. If consumers are highly risk averse, they must have low intertemporal substitution as well. For the purposes of this research, it would be desirable to eliminate this automatic connection between intertemporal substitution and risk aversion. The empirical finding that intertemporal substitution is weak or absent does not contradict any widely held beliefs about consumer behavior. But the corresponding conclusion that the coefficient of relative risk aversion is close to infinity is incompatible with the observed willingness of consumers to take on risk. Hence, it is desirable to show that the finding of this paper has no automatic bearing on risk aversion.

It is a topic of active research in the theory of the consumer as to how to characterize preferences about uncertainty and intertemporal choice. It is apparent that the relationship between the rate of growth of consumption and the expected real interest rate is governed by the intertemporal substitution aspect of preferences. That is, the parameter σ in equation (1) is on its face the intertemporal elasticity; it is literally the elasticity of the consumption ratio to the corresponding relative price. Under certainty, where only intertemporal substitution has a role, equation (1) would hold for an intertemporally separable utility function with a constant elasticity of substitution, σ.

With the maximization of the expected value of the utility function (2), it is unarguably the case that $1/\sigma$ is the coefficient of relative risk aversion as well. However, recent work has shown that equation (1) does not reveal the coefficient of relative risk aversion under less restrictive assumptions about preferences. The earliest research on finding a clean separation between intertemporal substitution and risk aversion appears in a characterization of preference orderings with only two time periods by Selden (1978) in what he calls the ordinal certainty equivalence (OCE) framework. The OCE setup departs from expected utility but retains additive separability. In the OCE framework, the relation between consumption growth and expected real interest reveals just the intertemporal elasticity of substitution and says nothing about the coefficient of relative risk aversion. Selden lets one concave function describe risk aversion in the second period. With it, risk aversion has the effect of reducing uncertain future consumption to its certainty equivalent. Then a second utility function describes intertemporal substitution between current consumption and the certainty equivalent of future consumption. Under the assumption that both utility functions have the constant-elastic form, and the same lognormal assumptions about the interest rate and future consumption, the OCE framework gives rise to exactly equation (1) (for details, see Hall [1985]). It is an unambiguous conclusion that the intertemporal elasticity of substitution alone controls the relation between consumption growth and the expected real interest rate. Unfortunately, the OCE framework does not generalize in any known way to more than two periods (see Johnsen and Donaldson 1985).

More recently, partly in response to remarks in earlier versions of this paper, a number of authors have developed representations of intertemporal preferences under uncertainty that depart from expected utility in a way originally proposed by Kreps and Porteus (1978). Attanasio (1987), Epstein and Zin (1987*a*, 1987*b*), Weil (1987), and Zin (1987*a*, 1987*b*) have all shown that, under suitable assumptions, the Kreps-Porteus setup implies that the coefficient σ is the intertemporal elasticity of substitution and not the reciprocal of the coefficient of relative risk aversion.

In the framework developed here, the bivariate relation between consumption and real interest rates does not necessarily reveal anything about risk aversion. Estimation of the risk aversion parameter would be possible in a multivariate system that considered the real returns to two or more assets. Then the magnitudes of the risk premiums together with the correlations of the returns with consumption would provide estimates of the coefficient of relative risk aversion.

In this paper, I will refer to the parameter σ as the intertemporal elasticity of substitution; I do not think this interpretation is at all controversial. Readers who have a strong prior belief that the utility function is additively separable and that consumers follow the principle of maximizing expected utility will also interpret σ as the reciprocal of the coefficient of relative risk aversion. Others, such as myself, will avoid drawing any conclusions about risk aversion from the results presented here.

B. *Relation to Hansen and Singleton*

Hansen and Singleton (1983) studied the joint distribution of the rate of growth of consumption and asset returns in the conventional expected intertemporal utility framework. They do not mention intertemporal substitution in their discussion at all. They identify the single critical parameter they estimate as the coefficient of relative risk aversion. Their statistical model is the same as the one derived here. Their estimation technique, based on maximum likelihood in a bivariate system, examines the relation between the rate of change of consumption and expected real asset returns and interprets the coefficient as the reciprocal of the coefficient of relative risk aversion. In their framework, as I mentioned above, the coefficient is also the intertemporal elasticity of substitution. The argument offered here suggests that Hansen and Singleton's estimated coefficient may not be informative about risk aversion. However, I do not offer any evidence on this question one way or the other.

Hansen and Singleton (1983), and Grossman and Shiller (1982) before them, are on firm ground in treating the differences in returns among assets as revealing something about risk aversion. Indeed, Hansen and Singleton's rejection of the cross-equation restrictions in a model combining consumption growth with returns on multiple assets may occur because the intertemporal elasticity of substitution is different from the reciprocal of the coefficient of relative risk aversion.

III. Expectations of the Real Interest Rate

I take two approaches to the measurement of the expected real interest rate. First, I study changes in consumption over a period for which survey data on expected price changes are available. The expected real interest rate is the market nominal rate for an instrument of suitable term, adjusted for taxes, less the expected rate of change of the price level. Real returns from the stock market can also be used in this framework because survey data on expected nominal stock prices are available.

The second approach relates the conditional mean of the real interest rate, $\bar{r}_{t-1}$, to observed variables known to consumers at the time that they choose c_{t-1}. Recall that $\bar{r}_{t-1}$ is the mean of the subjective distribution for the real interest rate held by the typical consumer at the time consumption decisions are made for year $t - 1$. A specification for expectations that has been employed frequently in macroeconomic models derived from rational expectations and, in particular, underlies the recent work of Hansen and Singleton is derived as follows. Let the mean of the subjective distribution be a linear combination of observed variables,

$$\bar{r}_{t-1} = x_{t-1}\beta, \tag{6}$$

and suppose that the coefficients, β, are known in advance. Under this specification, the complete model of expectations and consumption becomes a simple application of bivariate regression with parameter constraints across the equations. Alternatively, the same estimation technique can be thought of as instrumental variables applied to the consumption equation, with the determinants of the expected real rate as the instruments. The second interpretation is the one adopted in this paper, in which all estimates are obtained by instrumental variables except when expectations are observed directly.

IV. Time Aggregation

The basic equation for the rate of change of consumption,

$$\Delta c_t = \sigma \bar{r}_{t-1} + k + \epsilon_t, \tag{7}$$

refers to consumption in discrete time. From the derivation in Section II, it is also apparent that it applies to observations on the instantaneous flow of consumption measured at two points of time in a setup in which time is measured continuously. However, it does not correctly characterize the behavior of time averages of consumption. If c_t is the average flow of consumption over an interval of continuous time, then the relation of its rate of change to the real interest rate is more complex. My discussion will note the difference between time aggregation of the level of consumption and aggregation of its logarithm. The difference is trivial for consumption because it changes so little during any given year.

As with other aggregation problems in econometrics, time aggregation for the left-hand variable causes only mild problems. If the right-hand variable is observed continuously, or at least quite frequently, then the aggregation of the left-hand variable in effect defines an appropriate way to aggregate the right-hand variable. The problem of time aggregation becomes much more difficult if only a time average of the right-hand variable is available (see Grossman, Melino, and Shiller 1985). However, in the present case, interest rates and rates of inflation are measured monthly or more frequently over the whole time span for which any data at all are available for consumption, so the time aggregation problem is readily soluble.

Suppose that only a time average of consumption is observed, say once a year. Each month, the expected real interest rate is known; call it $\bar{r}_{t,m}$ with t the year and m the month. There is an unobserved $c_{t,m}$ each month, and it evolves as

$$\Delta c_{t,m} = \sigma \bar{r}_{t,m-1} + \epsilon_{t,m}. \tag{8}$$

Now write out $c_{t-1,m}$ and $c_{t,m}$ as increments over the initial value $c_{t-1,1}$. Note that, to a close approximation,

$$c_t = \frac{1}{12} \Sigma c_{t,m} + \log 12. \tag{9}$$

Then a little manipulation shows that the change in aggregate consumption is

$$\begin{aligned} \Delta c_t = {} & \frac{1}{12} \sum_{m=1}^{12} (m - 1)(\sigma \bar{r}_{t-1,m-1} + \epsilon_{t-1,m}) \\ & + \frac{1}{12} \sum_{m=1}^{12} (12 - m + 1)(\sigma \bar{r}_{t,m-1} + \epsilon_{t,m}). \end{aligned} \tag{10}$$

Define the time aggregates of the expected real interest rate and the random element as

$$\bar{r}_{t-1} = \frac{1}{12} [\Sigma(m - 1)\bar{r}_{t-1,m-1} + \Sigma(12 - m + 1)\bar{r}_{t,m-1}], \tag{11}$$

$$\epsilon_t = \frac{1}{12} [\Sigma(m - 1)\epsilon_{t-1,m} + \Sigma(12 - m + 1)\epsilon_{t,m}]. \tag{12}$$

Then the relation among the time aggregates is

$$\Delta c_t = \sigma \bar{r}_{t-1} + \epsilon_t. \tag{13}$$

Two properties of the aggregate random element ϵ_t call for note. First, as Working derived in a famous paper (1960), ϵ_t is not white noise; rather, it obeys a first-order moving average process with serial correlation .25. Second, ϵ_t is likely to be correlated with r_{t-1} or with its determinants or instruments, even if these variables are uncorrelated at the monthly level.

The combination of serial correlation in the residuals and endogenous instrumental variables calls for an estimator designed to deal with these circumstances. Hayashi and Sims (1983) have provided what seems to be the most suitable estimator for this problem. They propose that the data on the left- and right-hand variables undergo a preliminary transformation that yields a scalar covariance matrix for the disturbances. However, the transformation must also preserve the timing conditions that make the instruments and the transformed disturbances orthogonal. The standard autoregressive transformation destroys the timing conditions. On the other hand, an autoregressive transformation that subtracts future rather than past values will preserve the timing conditions and accomplish the necessary transformation. Application of the Hayashi-Sims estimator in the present case is particularly easy because the time-series process for the disturbances is prescribed by theory and does not need to be estimated in a preliminary stage. The process is first-order moving average with a parameter of 0.27. The corresponding autoregressive transformation can be closely approximated as

$$\Delta\tilde{c}_t = \Delta c_t - .27\Delta c_{t+1} + .07\Delta c_{t+2}. \tag{14}$$

This is the first two terms of the infinite autoregressive representation of the first-order moving average process. The same transformation is applied to the real interest rate variable. Then the instrumental variables estimator is applied to the transformed variables, using untransformed lagged variables as instruments. Hayashi and Sims show that the resulting estimates are consistent and that the standard estimate of the covariance matrix of the estimates is also consistent.

In estimating the time-aggregated Euler equation, the timing of the instruments turns out to be critical. If the data measured the instantaneous flow of consumption at two isolated points, any variable known at the time that c_{t-1} was chosen would be eligible as an instrument. However, when c_{t-1} is an annual average, it is apparent that any variable measured during calendar year $t - 1$ can be correlated with the disturbance ϵ_t. For annual data, the most recent permissible instrument is one measured in December of year $t - 2$. Annual aggregates for year $t - 2$ and earlier are usable, but not those for year $t - 1$. The most recent variables eligible as instruments are the change in annual log consumption in year $t - 2$, the level of the average real return over year $t - 2$, and the nominal return in December of year $t - 2$.

V. Data

Following are brief definitions of the data series used in this study: c_t = log of real consumption of nondurables (not including services) in year, quarter, or month t, from the U.S. National Income and Product Accounts; the data are available monthly from 1959, quarterly from 1947, and annually from 1919 (for derivation before 1929, see Hall [1986]); r_t = realized real return after taxes on an investment in the Standard and Poor's (S&P) 500 stock portfolio, liquidated at a later date corresponding to the consumption variable, or realized real return after taxes from a savings account earning the regulated passbook interest rate or realized real return after taxes from holding a sequence of four 90-day Treasury bills over the year; h_t = log of the S&P 500 index of share prices, deflated; d_t = dividend yield of the S&P 500; z_t = nominal yield of Treasury bills, discount basis; q_t = nominal passbook interest rate in the third quarter; p_t = log of the implicit deflator for consumption of nondurables (used as a deflator for all deflated variables).

After-tax magnitudes were calculated using the effective marginal rate under the federal personal income tax from Barro and Sahasakul (1983). The full nominal amount of dividends and interest was assumed to be taxed at this effective marginal rate. Capital gains and losses were assumed to be untaxed on the grounds that the combination of low statutory rates, taxation only at realization, and forgiveness of accrued gains at death makes the effective rate close to zero. All data for the study are available from the author on an IBM diskette.

VI. Summary of Results

Following is a brief summary of the various attempts I have made to estimate the intertemporal elasticity of substitution by estimating the relationship of the rate of change of consumption to expected real interest rates.

The first set of results uses inflation and stock price expectations recorded in the Livingston survey (see Sec. VII). In this work, the expected real return is measured directly and the elasticity of substitution estimated by simple regression. For real returns in the stock market, the results are informative: the elasticity of substitution is close to zero and the estimate has a small standard error. For savings accounts and Treasury bills, the estimates are almost useless because of large standard errors. In these cases, the lack of variation in the expected real return makes it difficult to estimate the elasticity.

A second set of results uses annual changes in consumption starting in 1924. The real return on Treasury bills is aggregated from monthly data as suggested in Section IV. Because this technique uses a longer span of data and uses all the data for each year, the standard error of the estimate of the intertemporal elasticity is much smaller. The point estimate of the elasticity is negative. All positive values lie outside the 95 percent confidence interval.

A third set of results reconciles the findings of this paper—that the intertemporal elasticity is around zero—with Hansen and Singleton's finding of large positive elasticities. The difference comes from their choice of a time period for estimation and from their use of instruments that are correlated with the innovation in the real return.

A fourth set of results examines Summers's (1982) findings of intertemporal elasticities of around one, using quarterly postwar data. Again, use of an appropriate estimator reverses his conclusion.

My overall conclusion from all four sets of results is that the evidence points in the direction of a low value for the intertemporal elasticity. The value may even be zero and is probably not above .2.

Before plunging into formal econometric results, I think it is useful to indicate why the data point toward the answer that pervades the results of this paper, namely, that the intertemporal elasticity of substitution is small. Some simple facts about the data are apparent just by taking averages over 5-year intervals. The averaging removes most of the random expectation errors but turns out to leave a good deal of variation in the real interest rate. Figure 1 shows the real after-tax return on Treasury bills and the rate of change of consumption for intervals from 1921 through 1940 and 1946–83 (the last interval is only 3 years long).

Except for three of the observations, the rate of change of consumption is close to its average value of a little below 3 percent per year. When consumption was near average, however, the real interest rate varied from -5 percent to $+5$ percent. The only observation combining a high real interest rate and rapid consumption growth was for 1921–25, in the upper right-hand corner. The other observation with high consumption growth was for 1936–40, when the real interest rate was almost exactly zero. The period 1931–35 had a high real interest rate and slightly negative consumption change. As a general matter, figure 1 makes a fairly strong case that periods of high real interest rates have not typically been periods of high consumption growth. Rather, consumption growth has generally stuck

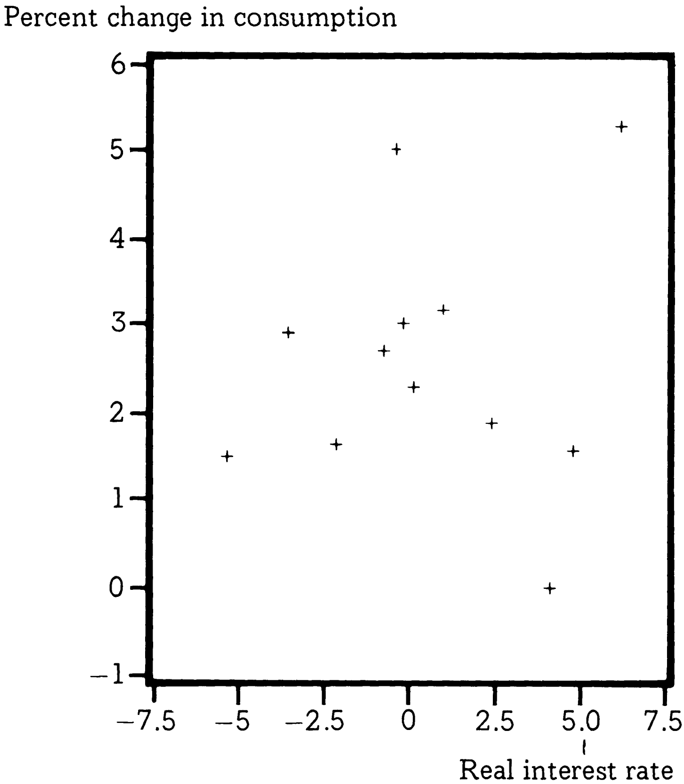

FIG. 1.—Five-year averages of the real return on Treasury bills (horizontal axis) and the rate of change of consumption (vertical axis), 1921–40 and 1946–83.

fairly close to its average value no matter what has happened to real interest rates.

VII. Results Based on the Livingston Survey

Each November, Joseph Livingston asks a panel of economists to predict the values of a long list of economic variables for the following June. Among the variables are the consumer price index and the S&P 400 stock price index. From these, it is possible to construct three measures of expected real returns that are relevant for consumers.

Treasury bills.—The starting point is the market value of a bill maturing in June as reported in November. All elements of the expected real rate are known except for the marginal tax rate, which is highly predictable. I computed the expected real return as

$$r = \log\left[\frac{1 - (7/12)mz}{1 - (7/12)z} - \frac{p_N}{p_J}\right]. \tag{15}$$

Here z is the nominal return measured in discount form at an annual rate (as a decimal), m is the marginal tax rate, p_N is the known price level in November, and p_J is the expected price level in June.

Savings accounts.—Nominal bank rates, q, are not entirely known in advance but are highly predictable. I compute the expected real after-tax return as

$$\log\left(\{e^{(7/12)q} - m[1 - e^{(7/12)q}]\}\frac{p_N}{p_J}\right). \tag{16}$$

Stocks.—I treat the dividend yield, d, as known and use the survey data for the expected share price. The expected real after-tax return is

$$\log\left\{\left[\frac{7}{12}(1 - m)d + \frac{h_J}{h_N}\right]\frac{p_N}{p_J}\right\}. \tag{17}$$

Here h_N is the known stock price index in November and h_J is the expected index for the following June.

The dependent variable, the log of the change in consumption per capita, is constructed to match the Livingston data as closely as possible. Monthly data on consumption in November and June are divided by monthly estimates of the U.S. population.

The results from regressing the log change in consumption on these three measures of the expected real return are the following (standard errors are in parentheses):

Security	Estimate of σ	Standard Error	Durbin-Watson
Treasury bills	.346 (.337)	.0169	2.13
Savings accounts	.271 (.330)	.0170	2.17
Stocks	.066 (.050)	.0166	2.36

The results for Treasury bills and savings accounts are hardly conclusive. The variation in the expected real returns over the 24 7-month periods in the data is inadequate to provide any useful information about the elasticity of substitution, σ. But for the stock market, the results are conclusive. The estimate of σ is close to zero and the standard error is small as well. The confidence interval for σ excludes all values that correspond to strong intertemporal substitution.

VIII. Results from Annual Data with Consistent Time Aggregation

Annual averages of consumption are available starting just after World War I. Monthly data on the realized return on Treasury bills can be calculated for the same period. Aggregation of the real return data to annual rates, as described in Section IV, makes it possible to estimate the intertemporal elasticity of substitution from a much longer historical record with much more variance in expected real returns. The estimate of the intertemporal elasticity of substitution obtained by applying the Hayashi-Sims estimator with the annual log of the change in consumption per capita as the dependent variable and realized real returns as the independent variable, with the change in consumption 2 years earlier, the realized real return 2 years earlier, and the nominal bill rate in December 2 years ago as instruments, for the years 1924–40 and 1950–83, is

$$\text{estimate of } \sigma: \quad \underset{(.20)}{-.40} \quad \text{D-W: } 2.09; \quad \text{SE: } 2.5 \text{ percent.}$$

The finding of a negative value of the intertemporal elasticity of substitution was not sensitive to the choice of instruments as long as endogenous variables from year $t - 1$ were excluded. Separate estimates for the pre- and postwar periods showed that the estimate was somewhat negative in the earlier period and positive for the later period. However, the pooled estimate clearly rejects all positive values of σ. It simply cannot be said that the relation between the real return and the rate of change of consumption supports strong intertemporal substitution. Of course, the finding of a negative estimate of σ cannot be taken literally since it implies nonconcave utility. Rather, the conclusion I draw is that the case for a significantly positive value of σ cannot be made in this framework.

IX. Results Based on Recent Monthly Data

Hansen and Singleton (1983) obtained results with monthly data that can be interpreted as evidence of large values for σ. Although I have not attempted to reproduce their results exactly, simple instrumental estimates do give high estimates of σ, especially over the particular time period they studied. For example, for data from October 1959 through December 1978, with the real rate lagged 1–6 months and the rate of change of consumption lagged 1, 2, and 3 months as instruments, I obtain

$$\text{estimate of } \sigma: \quad \underset{(.33)}{.98} \quad \text{D-W: } 2.59; \quad \text{SE: } .81 \text{ percent.}$$

The real return is computed from monthly averages of daily data. Following Hansen and Singleton, I have not made adjustments in the real return to take account of time aggregation nor have I used the Hayashi-Sims estimator. Incorporating data through December 1983, I get a somewhat lower value with the same procedure:

$$\text{estimate of } \sigma\text{:} \quad \underset{(.22)}{.48} \quad \text{D-W: } 2.64\text{;} \quad \text{SE: } .79 \text{ percent.}$$

Hansen and Singleton's use of the immediately lagged change in log consumption as an instrument does not give rise to a consistent estimate of σ when the dependent variable is the change in a time aggregate. Section IV showed that last year's change in consumption depends on some of the same random disturbances as this year's change. The most recent change in consumption admissible as an instrument is the one lagged 2 years.

Because data on the price level are compiled no more frequently than monthly, it is not possible to apply the full apparatus developed earlier in this paper for monthly data. However, it is possible to come close. A good approximation to the correct time aggregate of the real return on Treasury bills can be computed, with respect to the change in consumption between last month and this month, by using the simple average of the Treasury bill yields in each of the months, adjusted for taxes, less a moving average of price changes. The moving average gives a weight of .125 to next month's price change, .75 to this month's, and .125 to last month's. These weights can be derived by combining a simple interpolation formula with the aggregation process derived in Section IV. Then the Hayashi-Sims estimator can be applied. Because the computed real return uses an additional future month's price data, the earliest value that can be used as an instrument is from 3 months ago. However, the observed nominal yields on Treasury bills 2 and 3 months ago can be used as instruments. Making all these changes, including dropping Δc_{t-1} from the list of instruments, further reduces the estimate of σ:

$$\text{estimate of } \sigma\text{:} \quad \underset{(.38)}{-.03} \quad \text{D-W: } 3.00\text{;} \quad \text{SE: } .91 \text{ percent.}$$

For the stock market, use of recent monthly data does not change the conclusion reached by Hansen and Singleton and the earlier results in this paper that the estimate of the elasticity is reliably low:

$$\text{estimate of } \sigma\text{:} \quad \underset{(.10)}{.03} \quad \text{D-W: } 3.02\text{;} \quad \text{SE: } .90 \text{ percent.}$$

Large fluctuations occurred over the period from 1959 through 1983 in the expected real return from the stock market, not matched by corresponding changes in the rate of change of consumption. The monthly results for the stock market strongly confirm the results from 7-month changes in the earlier study with the Livingston data.

All the results for monthly data show negative serial correlation of the first difference of consumption after the small adjustments for the effects of changes in the expected real interest rate and after the orthogonalizing transformation. This negative serial correlation suggests that there is a transitory element in monthly consumption that is not accounted for by the model. Similar but weaker evidence is found for quarterly data but not for annual data.

X. Results Based on Postwar Quarterly Data

Summers (1982) presents results to support the view that the intertemporal elasticity of consumption is substantial. In a subsequent paper (Summers 1984), he has cited his findings in making a case for the interest elasticity of saving: "available evidence tends to suggest that savings are likely to be interest elastic. I find in the more reliable estimates in my working paper [Summers 1982] values of the intertemporal elasticity of substitution which cluster at the high end of the range Evans and I considered [above one]. Similar estimates are found . . . by Hansen-Singleton. Where investigators find low estimates of intertemporal elasticity of substitution, it is usually because of the difficulty in modelling *ex ante* rates of return on corporate stock" (p. 252).

I have not tried to duplicate Summers's findings exactly. With postwar quarterly data on consumption and time averages of the real after-tax yield on Treasury bills computed as described in Section IV, I have obtained the following estimate of σ using the same inappropriate instruments as Summers, namely the real yield, the inflation rate, and the rate of change of consumption dated $t-1$ and $t-2$, and without transforming for the serial correlation induced by time aggregation:

$$\text{estimate of } \sigma\text{:} \quad \underset{(.13)}{.34} \quad \text{D-W: } 1.95; \quad \text{SE: } .87 \text{ percent.}$$

However, use of the Hayashi-Sims estimator and deletion of the instruments known to be correlated with the disturbance reverses the finding of an unambiguously positive σ:

$$\text{estimate of } \sigma\text{:} \quad \underset{(.23)}{.10} \quad \text{D-W: } 2.49; \quad \text{SE: } .88 \text{ percent.}$$

XI. Conclusions

My investigation has shown little basis for a conclusion that the behavior of aggregate consumption in the United States in the twentieth century reveals an important positive value of the intertemporal elasticity of substitution. All investigators have agreed that the covariation of stock market returns and consumption did not suggest that consumption rises more rapidly in times of high expected real returns in the stock market. Earlier evidence based on interest-bearing securities such as Treasury bills had suggested values of σ as high as one. However, use of appropriate estimation techniques taking account of time aggregation reverses this finding. Moreover, extension of the investigation to prewar years and to data from the past few years strengthens the evidence that periods of high expected real interest rates have not been periods of rapid growth of consumption.

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