Describing Functions (cont.) 16

Note: All references to Figures and Equations whose numbers are *not* preceded by an "S" refer to the textbook.

Solution 16.1 (P6.8)

Here, the describing function for the nonlinear element can be constructed as the sum of a linear gain of unity, and a nonlinear element of the form of the third entry in Table 6.1, with $E_M = 1$, and $K = 1$. Thus, for the overall nonlinear element, the describing function is

$$G_D(E) = 1 \measuredangle 0^\circ \quad E < 1 \tag{S16.1a}$$

$$G_D(E) = 1 + [1 - \frac{2}{\pi}(\sin^{-1}R + R\sqrt{1 - R^2})] \measuredangle 0^\circ \quad E > 1 \tag{S16.1b}$$

where $R = \frac{1}{E}$.

The loop of Figure 6.27 is of the form in Figure 6.9, with $a(s) = \frac{5}{(\tau s + 1)^3}$. Thus, oscillations may be possible if particular values E_1 and ω_1 exist such that

$$a(j\omega_1) = -\frac{1}{G_D(E_1)} \tag{S16.2}$$

Because the phase of $G_D(E)$ is zero for all E, the only solution of Equation S16.2 occurs where $\measuredangle\, a(j\omega_1) = -180^\circ$. For the given $a(s)$, this requires that $-3 \tan^{-1}\tau\omega_1 = -180^\circ$, which is satisfied by $\omega_1 = \frac{1.73}{\tau}$. At this frequency, the magnitude of $a(s)$ is given by $|a(j\omega_1)| = \frac{5}{(\sqrt{\tau^2\omega_1^2 + 1})^3} = 5/8$. Thus, to satisfy Equation S16.2, we must have $G_D(E_1) = 8/5$ for oscillations to exist. Note that in a describing function sense, the gain of the nonlinearity is 1 for signals less than 1 volt in amplitude. For signals of greater than 1 volt amplitude the gain varies monotonically from 1 to 2 as the signal amplitude varies from 1 to infinity. The above statement fits our intuitive view of the behavior of the nonlinearity and is stated mathematically in Equation S16.1.

With the above information, we can plot $-\frac{1}{G_D(E)}$ and $a(j\omega)$ on a gain-phase plane, without solving explicitly for numerical values of $G_D(E)$. This plot is shown in Figure S16.1, and the intersection of the two curves indicates that oscillations may exist. However, they are not stable, as the following analysis shows. Consider the system to be oscillating with frequency $\omega = \frac{1.73}{\tau}$ and an amplitude E such that $G_D(E) = 8/5$. A slight increase in E will move the $\frac{-1}{G_E(E)}$ point to the right of the $a(j\omega)$ curve implying growing amplitude oscillations. Similarly, if E decreases, decreasing amplitude oscillations result. Thus, a stable amplitude limit cycle is not possible, even though the curves intersect.

This result can be verified by performing a linearized analysis about operating points V_A. For $|V_A| < 1$, the incremental gain of the nonlinearity is 1, and the linearized system can be shown to be stable. For all operating points with $|V_A| > 1$, the incremental gain of the nonlinearity is 2 and the system can be shown to be unstable. Thus, there is no operating point that is marginally stable (poles on $j\omega$ axis), and no stable amplitude oscillations can exist.

Figure S16.1 Describing function analysis for Problem 16.1 (P6.8).