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Electromechanical Dynamics

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Appendix D

GLOSSARY OF COMMONLY USED SYMBOLS

Section references indicate where symbols of a given significance are introduced; grouped symbols are accompanied by their respective references. The absence of a section reference indicates that a symbol has been applied for a variety of purposes. Nomenclature used in examples is not included.

| Symbol | Meaning | Section |
|---------------------------------|---|----------------------|
| A | cross-sectional area | |
| A_i | coefficient in differential equation | 5.1.1 |
| (A_n^+, A_n^-) | complex amplitudes of components of n th mode | 9.2.1 |
| A_w | cross-sectional area of armature conductor | 6.4.1 |
| a | spacing of pole faces in magnetic circuit | 8.5.1 |
| $a, (a_c, a_s)$ | phase velocity of acoustic related waves | 13.2.1, 11.4.1 |
| a_b | Alfvén velocity | 12.2.3 |
| (a, b, c) | Lagrangian coordinates | 11.1 |
| a_i | constant coefficient in differential equation | 5.1.1 |
| \mathbf{a}_p | instantaneous acceleration of point p fixed in material | 2.2.1c |
| B, B_r, B_s | damping constant for linear, angular and square law dampers | 2.2.1b, 4.1.1, 5.2.2 |
| $\mathbf{B}, \mathbf{B}_i, B_0$ | magnetic flux density | 1.1.1a, 8.1, 6.4.2 |
| B_i | induced flux density | 7.0 |
| $(B_r, B_{ra}, B_{rb}, B_{rm})$ | radial components of air-gap flux densities | 4.1.4 |
| $[B_{rf}, (B_{rf})_{av}]$ | radial flux density due to field current | 6.4.1 |
| b | width of pole faces in magnetic circuit | 8.5 |
| b | half thickness of thin beam | 11.4.2b |
| C | contour of integration | 1.1.2a |
| $C, (C_a, C_b), C_o$ | capacitance | 2.1.2, 7.2.1a, 5.2.1 |
| C | coefficient in boundary condition | 9.1.1 |
| \mathbf{C} | the curl of the displacement | 11.4 |
| (C^+, C^-) | designation of characteristic lines | 9.1.1 |

| Symbol | Meaning | Section |
|---|---|--|
| c_p | specific heat capacity at constant pressure | 13.1.2 |
| c_v | specific heat capacity at constant volume | 13.1.2 |
| D | electric displacement | 1.1.1a |
| d | length | |
| da | elemental area | 1.1.2a |
| $d\mathbf{f}_n$ | total elemental force on material in rigid body | 2.2.1c |
| $d\mathbf{l}$ | elemental line segment | 1.1.2a |
| $d\mathbf{T}_n$ | torque on elemental volume of material | 2.2.1c |
| dV | elemental volume | 1.1.2b |
| E | constant of motion | 5.2.1 |
| E | Young's modulus or the modulus of elasticity | 9.1 |
| \mathbf{E}, E_0 | electric field intensity | 1.1.1a, 5.1.2d |
| E_f | magnitude of armature voltage generated by field current in a synchronous machine | 4.1.6a |
| E_i | induced electric field intensity | 7.0 |
| e_{11}, e_{ij} | strain tensor | 9.1, 11.2 |
| \dot{e}_{ij} | strain-rate tensor | 14.1.1a |
| F | magnetomotive force (mmf) | 13.2.2 |
| F | force density | 1.1.1a |
| \hat{F} | complex amplitude of $f(t)$ | 5.1.1 |
| F_0 | amplitude of sinusoidal driving force | 9.1.3 |
| f | equilibrium tension of string | 9.2 |
| f | driving function | 5.1.1 |
| $f, \mathbf{f}, f^e, f^s, f_j, f_i, f_1$ | force | 2.2.1, 2.2.1c, 3.1, 5.1.2a, 3.1.2b, 8.1, 9.1 |
| f | arbitrary scalar function | 6.1 |
| f' | scalar function in moving coordinate system | 6.1 |
| f | three-dimensional surface | 6.2 |
| f | integration constant | 11.4.2a |
| G | a constant | 5.1.2c |
| G | shear modulus of elasticity | 11.2.2 |
| G | speed coefficient | 6.4.1 |
| G | conductance | 3.1 |
| g | air-gap length | 5.2.1 |
| g, \mathbf{g} | acceleration of gravity | 5.1.2c, 12.1.3 |
| $(\mathbf{H}, H_x, H_y, H_z)$ | magnetic field intensity | 1.1.1a |
| h | specific enthalpy | 13.1.2 |
| I, I, (I_r, I_s), I_f | electrical current | 10.4.3, 12.2.1a, 4.1.2, 6.4.1 |
| $(i, i_1, i_2, \dots, i_k), (i_{ar}, i_{as}, i_{br}, i_{bs}), i_a, (i_a, i_b, i_c), (i_f, i_t), (i_r, i_s)$ | electrical current | 2.1, 4.1.3, 6.4.1, 4.1.7, 6.4.1, 4.1 |

| Symbol | Meaning | Section |
|--|---|---|
| i_n | unit vector perpendicular to area of integration | 6.2.1 |
| i_s | unit vector normal to surface of integration | 6.2.1 |
| $(i_x, i_y, i_z), (i_1, i_2, i_3)$ | unit vectors in coordinate directions | 2.2.1c |
| J, J_f | current density | 7.0, 1.1.1a |
| $J, J_r, (J_x, J_y, J_z)$ | moment of inertia | 5.1.2b, 4.1.1, 2.2.1c |
| J_{xx}, J_{yz} | products of inertia | 2.2.1c |
| j | $\sqrt{-1}$ | 4.1.6a |
| K | loading factor | 13.2.2 |
| K, K_f | surface current density | 7.0, 1.1.1a |
| K | linear or torsional spring constant | 2.2.1a |
| K_i | induced surface current density | 7.0 |
| $k, k_c, (k_r, k_t)$ | wavenumber | 7.1.3, 10.1.3, 10.0 |
| k | summation index | 2.1.1 |
| k | maximum coefficient of coupling | 4.1.6b |
| k_n | n th eigenvalue | 9.2 |
| $(L, L_1, L_2), (L_a, L_f),$ $L_m, (L_0, L_2),$ $(L_r, L_s, L_{sr}), L_{ss}$ | inductance | 2.1.1, 6.4.1, 2.1.1, 4.2.1, 4.1.1, 4.2.4 |
| L | length of incremental line segment | 6.2.1 |
| l | value of relative displacement for which spring force is zero | 2.2.1a |
| l, l_w, l_y | length | |
| M | Hartmann number | 14.2.2 |
| M | mass of one mole of gas in kilograms | 13.1.2 |
| M | Mach number | 13.2.1 |
| M | mass | 2.2.1c |
| M | number of mechanical terminal pairs | 2.1.1 |
| M, M_s | mutual inductance | 4.1.1, 4.2.4 |
| M | magnetization density | 1.1.1a |
| m | mass/unit length of string | 9.2 |
| N | number of electrical terminal pairs | 2.1.1 |
| N | number of turns | 5.2.2 |
| n | number density of ions | 12.3.1 |
| n | integer | 7.1.1 |
| n | unit normal vector | 1.1.2 |
| P | polarization density | 1.1.1a |
| P | power | 12.2.1a |
| p | number of pole pairs in a machine | 4.1.8 |
| p | power per unit area | 14.2.1 |
| p | pressure | 5.1.2d and 12.1.4 |
| P_e, P_g, P_m, P_r | power | 4.1.6a, 4.1.6b, 4.1.2, 4.1.6b |
| Q | electric charge | 7.2.1a |
| q, q_i, q_k | electric charge | 1.1.3 and 2.1.2, 8.1, 2.1.2 |
| R, R_i, R_o | radius | |

| Symbol | Meaning | Section |
|---|--|--|
| $R, R_a, R_b, R_f, R_r, R_s$ | resistance | |
| (R, R_ρ) | gas constant | 13.1.2 |
| R_e | electric Reynolds number | 7.0 |
| R_m | magnetic Reynolds number | 7.0 |
| r | radial coordinate | |
| \mathbf{r} | position vector of material | 2.2.1c |
| \mathbf{r}' | position vector in moving reference frame | 6.1 |
| \mathbf{r}_m | center of mass of rigid body | 2.2.1c |
| S | reciprocal modulus of elasticity | 11.5.2c |
| S | surface of integration | 1.1.2a |
| S | normalized frequency | 7.2.4 |
| S | membrane tension | 9.2 |
| S_z | transverse force/unit length acting on string | 9.2 |
| s | complex frequency | 5.1.1 |
| (s, s_{mT}) | slip | 4.1.6b |
| s_i | i th root of characteristic equation, a natural frequency | 5.1.1 |
| T | period of oscillation | 5.2.1 |
| T | temperature | 13.1.2 |
| $\mathbf{T}, T, T^e, T_{em}, T_m, T_0, T_1$ | torque | 2.2.1c, 5.1.2b, 3.1.1, 4.1.6b, 4.1.1, 6.4.1, 6.4.1 |
| \mathbf{T} | surface force | 8.4 |
| T_{ij}^m | mechanical stress tensor | 13.1.2 |
| T_{mn} | the component of the stress-tensor with the m th-direction on a cartesian surface with a normal vector in the n th-direction | 8.1 |
| T_{or} | constant of coulomb damping | 4.1.1 |
| T_0 | initial stress distribution on thin rod | 9.1.1 |
| T | longitudinal stress on a thin rod | 9.1.1 |
| T_z | transverse force per unit area on membrane | 9.2 |
| T_2 | transverse force per unit area acting on thin beam | 11.4.2b |
| t | time | 1.1.1 |
| t' | time measured in moving reference frame | 6.1 |
| U | gravitational potential | 12.1.3 |
| U | longitudinal steady velocity of string or membrane | 10.2 |
| u | internal energy per unit mass | 13.1.1 |
| u | surface coordinate | 11.3 |
| $u_0(x - x_0)$ | unit impulse at $x = x_0$ | 9.2.1 |
| u | transverse deflection of wire in x -direction | 10.4.3 |
| $u_{-1}(t)$ | unit step occurring at $t = 0$ | 5.1.2b |
| V, V_m | velocity | 7.0, 13.2.3 |
| V | volume | 1.1.2 |
| V, V_a, V_f, V_o, V_s | voltage | |
| V | potential energy | 5.2.1 |

| Symbol | Meaning | Section |
|--|--|--------------------|
| v, \mathbf{v} | velocity | |
| (v, v_1, \dots, v_k) | voltage | 2.1.1 |
| $v', (v_a, v_b, v_c),$ v_f, v_{oc}, v_t | voltage | |
| v_n | velocity of surface in normal direction | 6.2.1 |
| v_o | initial velocity distribution on thin rod | 9.1.1 |
| v_p | phase velocity | 9.1.1 and 10.2 |
| \mathbf{v}^r | relative velocity of inertial reference frames | 6.1 |
| v_s | $\sqrt{f/m}$ for a string under tension f and having mass/unit length m | 10.1.1 |
| v | longitudinal material velocity on thin rod | 9.1.1 |
| v | transverse deflection of wire in y -direction | 10.4.3 |
| (W_e, W_m) | energy stored in electromechanical coupling | 3.1.1 |
| (W'_e, W'_m, W') | coenergy stored in electromechanical coupling | 3.1.2b |
| W'' | hybrid energy function | 5.2.1 |
| w | width | 5.2.2 |
| w | energy density | 11.5.2c |
| w' | coenergy density | 8.5 |
| X | equilibrium position | 5.1.2a |
| $(x, x_1, x_2, \dots, x_k)$ | displacement of mechanical node | 2.1.1 |
| x | dependent variable | 5.1.1 |
| x_p | particular solution of differential equation | 5.1.1 |
| $(x_1, x_2, x_3), (x, y, z)$ | cartesian coordinates | 8.1, 6.1 |
| (x', y', z') | cartesian coordinates of moving frame | 6.1 |
| (α, β) | constants along C^+ and C^- characteristics, respectively | 9.1.1 |
| (α, β) | see (10.2.20) or (10.2.27) | |
| α | transverse wavenumber | 11.4.3 |
| (α, β) | angles used to define shear strain | 11.2 |
| (α, β) | constant angles | 4.1.6b |
| α | space decay parameter | 7.1.4 |
| α | damping constant | 5.1.2b |
| α | equilibrium angle of torsional spring | 2.2.1a |
| γ | ratio of specific heats | 13.1.2 |
| γ | piezoelectric constant | 11.5.2c |
| $\gamma, \gamma_0, \gamma'$ | angular position | |
| $\Delta_a(t)$ | slope excitation of string | 10.2.1b |
| Δ_0 | amplitude of sinusoidal slope excitation | 10.2.1b |
| Δr | distance between unstressed material points | 11.2.1a |
| Δs | distance between stressed positions of material points | 11.2.1a |
| $\delta()$ | incremental change in () | 8.5 |
| $\delta, \delta_1, \delta_0$ | displacement of elastic material | 11.1, 9.1, 11.4.2a |
| δ | thickness of incremental volume element | 6.2.1 |
| δ | torque angle | 4.1.6a |

| Symbol | Meaning | Section |
|--|---|----------------------|
| δ_{ij} | Kronecker delta | 8.1 |
| (δ_+, δ_-) | wave components traveling in the $\pm x$ -directions | 9.1.1 |
| ϵ | linear permittivity | 1.1.1b |
| ϵ_0 | permittivity of free space | 1.1.1a |
| η | efficiency of an induction motor | 4.1.6b |
| η | second coefficient of viscosity | 14.1.1c |
| $\theta, \theta_i, \theta_m$ | angular displacement | 2.1.1, 3.1.1, 5.2.1 |
| θ | power factor angle; phase angle between current and voltage | 4.1.6a |
| θ | equilibrium angle | 5.2.1 |
| $\dot{\theta}$ | angular velocity of armature | 6.4.1 |
| θ_m | maximum angular deflection | 5.2.1 |
| $(\lambda, \lambda_1, \lambda_2, \dots, \lambda_k)$ | magnetic flux linkage | 2.1.1, 6.4.1, 4.1.7, |
| λ_a | | 4.1.3, 4.1 |
| $(\lambda_a, \lambda_b, \lambda_c)$ | | |
| $(\lambda_{ar}, \lambda_{as}, \lambda_{br}, \lambda_{bs})$ | | |
| (λ_r, λ_s) | | |
| λ | Lamé constant for elastic material | 11.2.3 |
| λ | wavelength | 7.1.4 |
| μ | linear permeability | 1.1.1a |
| $\mu, (\mu_+, \mu_-)$ | mobility | 12.3.1, 1.1.1b |
| μ | coefficient of viscosity | 14.1.1 |
| μ_d | coefficient of dynamic friction | 2.2.1b |
| μ_0 | permeability of free space | 1.1.1a |
| μ_s | coefficient of static friction | 2.2.1b |
| ν | Poisson's ratio for elastic material | 11.2.2 |
| ν | damping frequency | 10.1.4 |
| (ξ, ξ) | continuum displacement | 8.5 |
| ξ_0 | initial deflection of string | 9.2 |
| ξ_d | amplitude of sinusoidal driving deflection | 9.2 |
| $(\xi_n(x), \xi_n(x))$ | n th eigenfunctions | 9.2.1b |
| (ξ_+, ξ_-) | amplitudes of forward and backward traveling waves | 9.2 |
| $\dot{\xi}_0(x)$ | initial velocity of string | 9.2 |
| ρ | mass density | 2.2.1c |
| ρ_f | free charge density | 1.1.1a |
| ρ_s | surface mass density | 11.3 |
| Σ | surface of discontinuity | 6.2 |
| σ | conductivity | 1.1.1a |
| σ_f | free surface charge density | 1.1.1a |
| σ_m | surface mass density of membrane | 9.2 |
| σ_o | surface charge density | 7.2.3 |
| σ_s | surface conductivity | 1.1.1a |
| σ_u | surface charge density | 7.2.3 |
| τ | surface traction | 8.2.1 |
| τ, τ_d | diffusion time constant | 7.1.1, 7.1.2a |
| τ | relaxation time | 7.2.1a |

| Symbol | Meaning | Section |
|--------------------------------|--|---------------|
| τ_e | electrical time constant | 5.2.2 |
| τ_m | time for air gap to close | 5.2.2 |
| τ_o | time constant | 5.1.3 |
| τ_t | traversal time | 7.1.2a |
| ϕ | electric potential | 7.2 |
| ϕ | magnetic flux | 2.1.1 |
| ϕ | cylindrical coordinate | 2.1.1 |
| ϕ | potential for H when $J_f = 0$ | 8.5.2 |
| ϕ | flow potential | 12.2 |
| χ_e | electric susceptibility | 1.1.1b |
| χ_m | magnetic susceptibility | 1.1.1a |
| ψ | the divergence of the material displacement | 11.4 |
| ψ | angle defined in Fig. 6.4.2 | 6.4.1 |
| ψ | angular position in the air gap measured from stator winding (a) magnetic axis | 4.1.4 |
| ψ | electromagnetic force potential | 12.2 |
| ψ | angular deflection of wire | 10.4.3 |
| Ω | equilibrium rotational speed | 5.1.2b |
| Ω | rotation vector in elastic material | 11.2.1a |
| Ω_n | real part of eigenfrequency (10.1.47) | 10.1.4 |
| $\omega, (\omega_r, \omega_s)$ | radian frequency of electrical excitation | 4.1.6a, 4.1.2 |
| ω | natural angular frequency (Im s) | 5.1.2b |
| ω, ω_m | angular velocity | 2.2.1c, 4.1.2 |
| ω_c | cutoff frequency for evanescent waves | 10.1.2 |
| ω_d | driving frequency | 9.2 |
| ω_n | n th eigenfrequency | 9.2 |
| ω_o | natural angular frequency | 5.1.3 |
| (ω_r, ω_t) | real and imaginary parts of ω | 10.0 |
| ∇ | nabla | 6.1 |
| ∇_Σ | surface divergence | 6.2.1 |