

MITOCW | Investigation 1, Part 4

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MARK HARTMAN: Build a well-- let's just say build a model of how angular size-- or let's just say angular width-- linear width, and distance relate. All right? What we've spent doing most of today is just figuring out how do we measure angular width, right? We learned about pixels. We looked at how to convert between one unit and another. And then we looked at how to use scientific notation to make that work.

So now let's put everything together. Because we want to be able to predict if we can get pictures of objects, either in the room or in outer space. We want to be able to predict how big they actually are, how wide, or how tall they actually are if we know how far away they are, or maybe the other way around.

So I want everybody to flip back in their notebook. And I want somebody to come up, and I want you to reproduce from yesterday when we looked at our relationship of the distance away from the camera. And then we actually said the actual width of the object, all right? And that was our diameter of those different balls.

But instead of saying actual width now-- that's kind of what we said yesterday-- we're going to say the linear width as opposed to the angular width. The angular width we measured from the pixels on the image So who would like to come up? Some of you may have the width out here and the distance up this way.

But who would like to come up and just say in words what relationship they found for objects that have the same angular size? Because remember, how wide they look in the image is our way of saying angular size. That's what we looked at this morning. So this is for objects with the same angular size. This was the relationship.

So who would like to come up and put up the graph that they saw, and then explain that pattern in words? It's a chance to get up in front of your peers and put your ideas out there. [? Asif, ?] you want to go for it? OK, so bring up your plot. And it doesn't have to be perfect, but just show us the relationship between those two, and then explain to us in words.

[? ASIF:

?] [INAUDIBLE] when I first plotted my point for my first object, the distance of an object was [INAUDIBLE] from the camera. And the actual width of the object-- or the angular width of the object-- was [? 5. ?]

MARK HARTMAN: Hang on. Is it the actual width or is it the angular width?

[? ASIF: ?] Angular width. Angular width. [INAUDIBLE] Yeah, angular width of the object.

MARK HARTMAN: OK, what does that say right there?

ASIF: It's actual width. [INAUDIBLE] I meant to say angular.

MARK HARTMAN: OK. Did we measure yesterday anything to do with angular widths?

[? ASIF: ?] No, it's actual width.

MARK HARTMAN: No, so it wasn't. We didn't measure angular width. Instead of calling it angular and actual-- because that's kind of confusing-- [? Asif, ?] can you change that so that it says linear width of the object? Because the linear width, you could measure with a ruler. An angular width, you have to have a camera, and you measure the number of pixels.

[? ASIF: ?] [INAUDIBLE]?

MARK HARTMAN: Yep, perfect.

[? ASIF: ?] The linear width of the object was [? 5, ?] and as it proceeded, [INAUDIBLE] the second object was a [? 50 ?] mark. And the linear width of it was [? 6. ?] And as it proceeded to the last one, which was [INAUDIBLE] distant mark [INAUDIBLE] linear width was [? 9. ?]

As I plotted it, I noticed that each time the bigger object came and I used it, I had to move the object far more. And the linear width of the object was really [? hard. ?] So it just kept on increasing as the object got bigger and bigger.

MARK HARTMAN: OK. That's a perfectly good explanation in words of what you saw from your experiment. Great, thanks, [? Asif. ?] Let's do a--

[APPLAUSE]

So we're going to continue to try to have opportunities for you guys to stand up in front of everybody. But let's take a look at this. Because today we said anything that has the same angular size, which means the same angle coming out from the camera-- if you looked at those two lines, and you put an object that was very, very large but you put it really far away, that would be like saying let's put something even further. But let's predict how wide that object

would have to be the same angular size.

It would probably be up there. If we wanted to predict where we would need to put the linear-- if we had an object that was, let's say 2 centimeters wide, where do you think we'd need to put it? So maybe be like 20, right? It's going to be along this line. So let's say 2 and 20, right about there. Well, it's close enough. All right?

So this understanding of this relationship between angular width, linear width, and distance-- we can create a mathematical model to help us predict where we should put an object of a certain linear width so that it always comes out to be the same angular size. Our mathematical model in this case-- we don't extend it down past zero-- says that if you have a really big object, you have to put it really far away. If you have a really small object, you can put it pretty close. If you had a zero-sized object, where would you need to put it in order to take up some space?

AUDIENCE: [INAUDIBLE]

AUDIENCE: At the zero? [INAUDIBLE]

MARK HARTMAN: Yeah, you'd have to put it really, really close so that it would take up the same angular size. Well, things don't work particularly well all the way down to zero. But what do we see here? What kind of a relationship is this? Again?

AUDIENCE: Linear.

MARK HARTMAN: It's what we call a linear relationship. We are going to call this in the [? CAI. ?] And I want you guys to sketch this up, or maybe even on your graph. This is a direct relationship, all right? That means as one thing goes up, another thing goes up.

Can you put this on the whiteboards? Put it on the projector? So a direct relationship means as one quantity goes up, another goes up. In this case, as our linear width of the object gets bigger, the distance that we have to put it from the camera also gets bigger.

We can represent this as an equation. How many people have seen an equation like y equals mx plus b before? OK? Has anybody not seen y equals mx plus b ? OK?

So in first-year algebra, a lot of the times if you have two points, you can fit a line between them. If you're given a slope and a y -intercept, you can find the line between there. In this

case, our y value is the linear width. All right? And you'll find whenever I write equations, I'm always going to write out the whole word.

So here you want to say the linear width is equal to the slope, all right? So our slope here-- we don't know what that is just yet. So we're just going to call it m , because just like in this form.

Now what is our x variable as we increase the distance from the camera? Say distance from detector. Now in this case, what is our y intercept? If we have x equals zero, what is the value of y ? Zero. So in this case, we don't have any b value.

So our mathematical relationship or our mathematical model is going to say in the case where an object is the same angular size, if I know the linear width, that is equal to some number-- the slope of that line-- times the distance from the detector. As the distance gets bigger, this linear width has to get bigger. This number m , the slope, is the angular size of that object in radians.

So we're going to check this out. So I'm going to say linear width is equal to angular width in radians times the distance from the detector. Now does this make sense? Let's think about it. I just kind of identified this angular width in radians with the slope.

In each case, when we looked at these dots-- this morning, when we looked at the different balls that were set up different distances away, the angle between the two sides, the two lines of sights, was always the same, right? So the angular width was the same. In the case of our mathematical model, at each of these points, the slope is always the same. So it kind of makes sense.

So we're going to say our mathematical model for the relationship between angular width, linear width, and distance-- I'm just going to abbreviate angular width, linear width, and distance-- is this. All right? We're just going to-- Now if this mathematical model is any good-- just like any model, it has to make some predictions. It has to make some good predictions. So what I want you to do with your group is I want you to sit and think about how could I use this mathematical model to predict the size of an object in that picture that we took?