Star-Forming Region NGC 3324

Hubble Heritage

NASA, ESA, and The Hubble Heritage Team (STScI/AURA) • *Hubble Space Telescope* WFPC2 • STScI-PRC08-34

Star-forming region in Carina, NGC 3582, from Astronomy Picture of the Day: http://apod.nasa.gov/apod/ap130611.html

Star-forming region in Cassiopeia, Heart and Soul nebula, IC1805 & IC1848, from Astronomy Picture of the Day: http://apod.nasa.gov/apod/ap100601.html

Star-forming region in Cassiopeia, IC1795, from Astronomy Picture of the Day: http://apod.nasa.gov/apod/ap091210.html

Star-forming region in Carina NGC3372, from Astronomy Picture of the Day: http://apod.nasa.gov/apod/ap100226.html

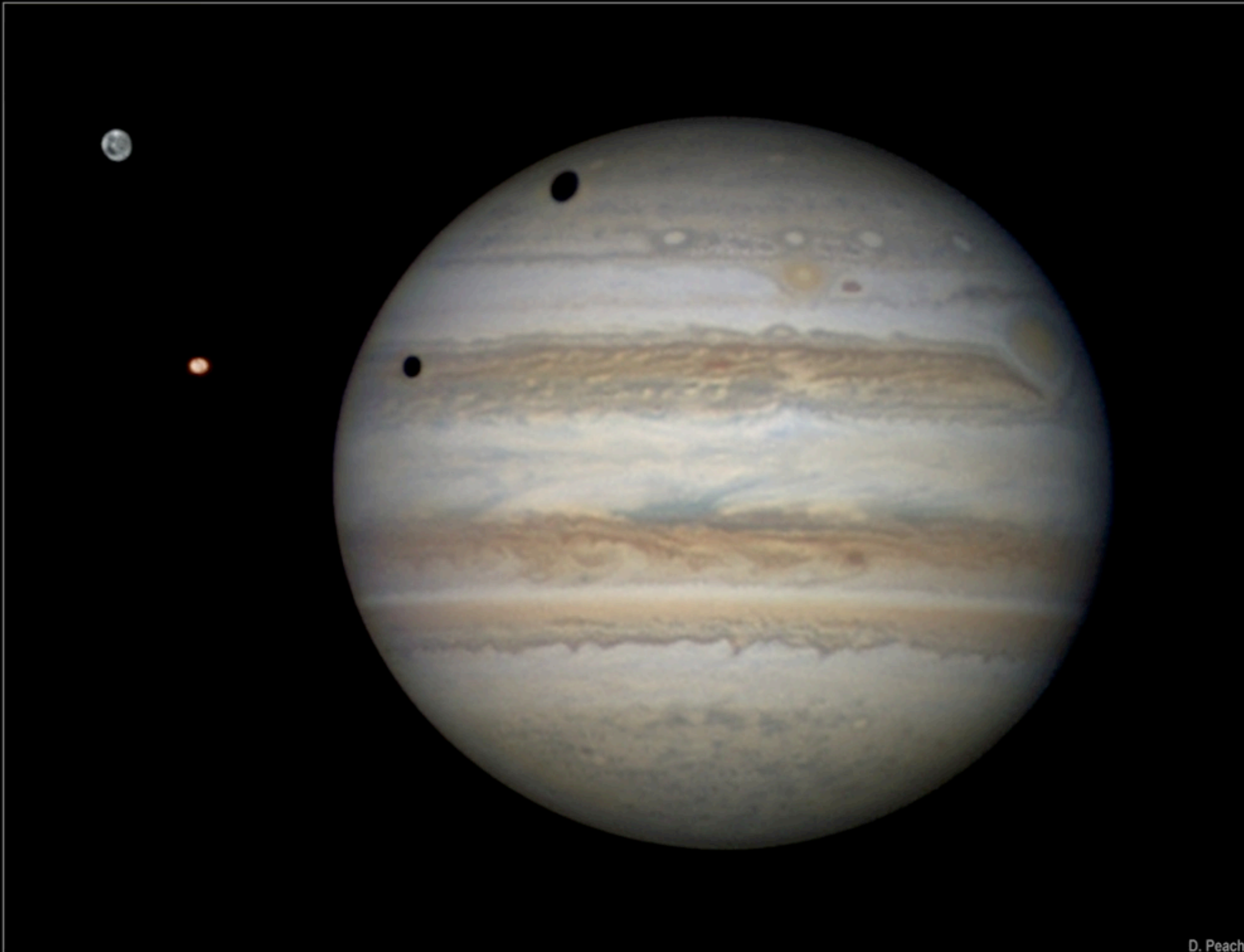
Stellar Configurations

- Self gravitating
- Self-consistent solution needed
- Different processes resist collapse

Planets

- Gravity weak because of small M

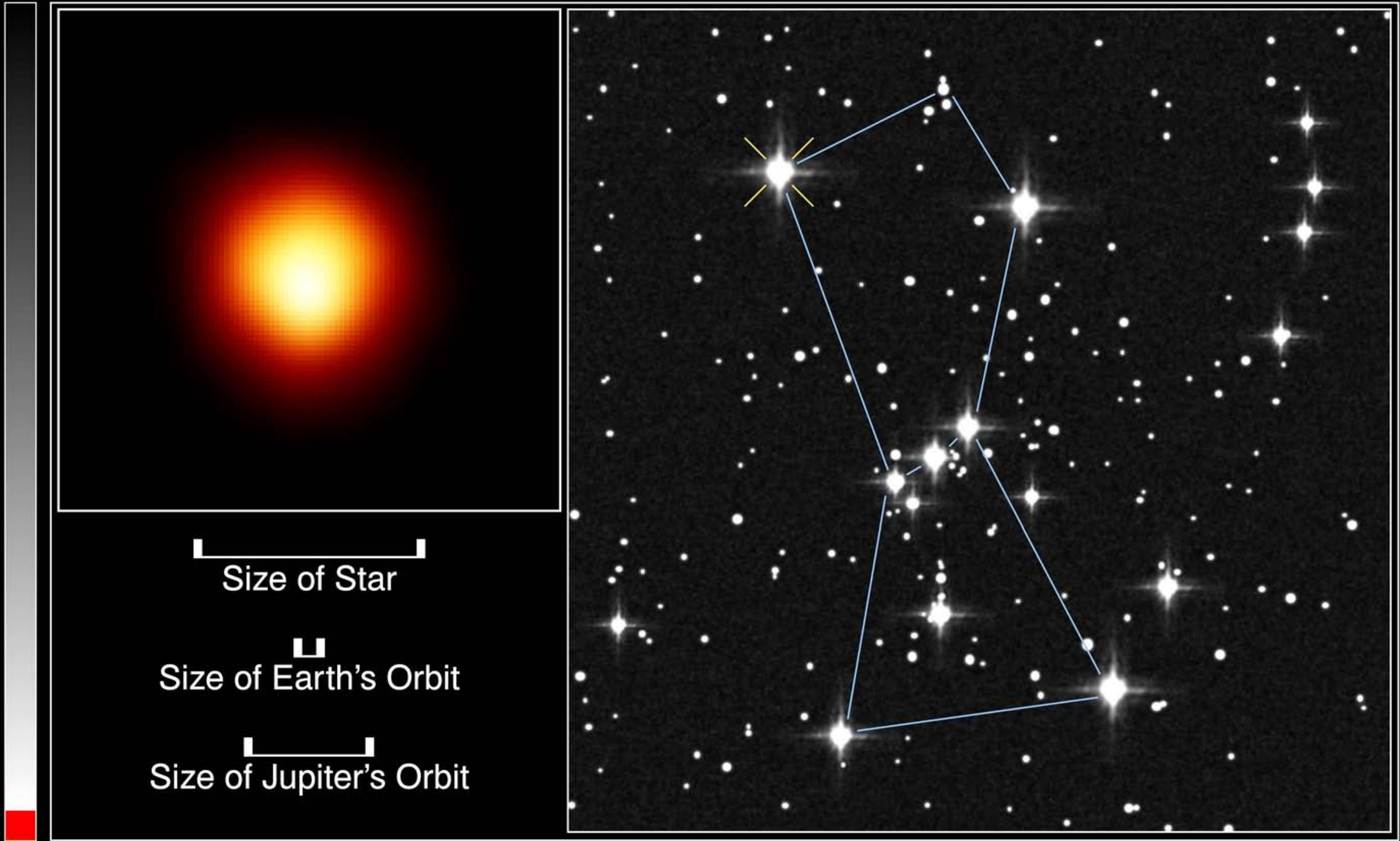
- Atomic forces provide balancing pressure



Jupiter with moons Ganymede (upper) and Io (lower), from Astronomy Picture of the Day: http://apod.nasa.gov/apod/ap130215.html

Normal Stars

- Gravitational energy starts process
- Fusion then supplies energy
- Plasma of electrons and nuclei
- Kinetic pressure, $P = nkT$
- Radiation pressure, $P = \frac{1}{3}u(T)$, helps and dominates above about $10M_\odot$

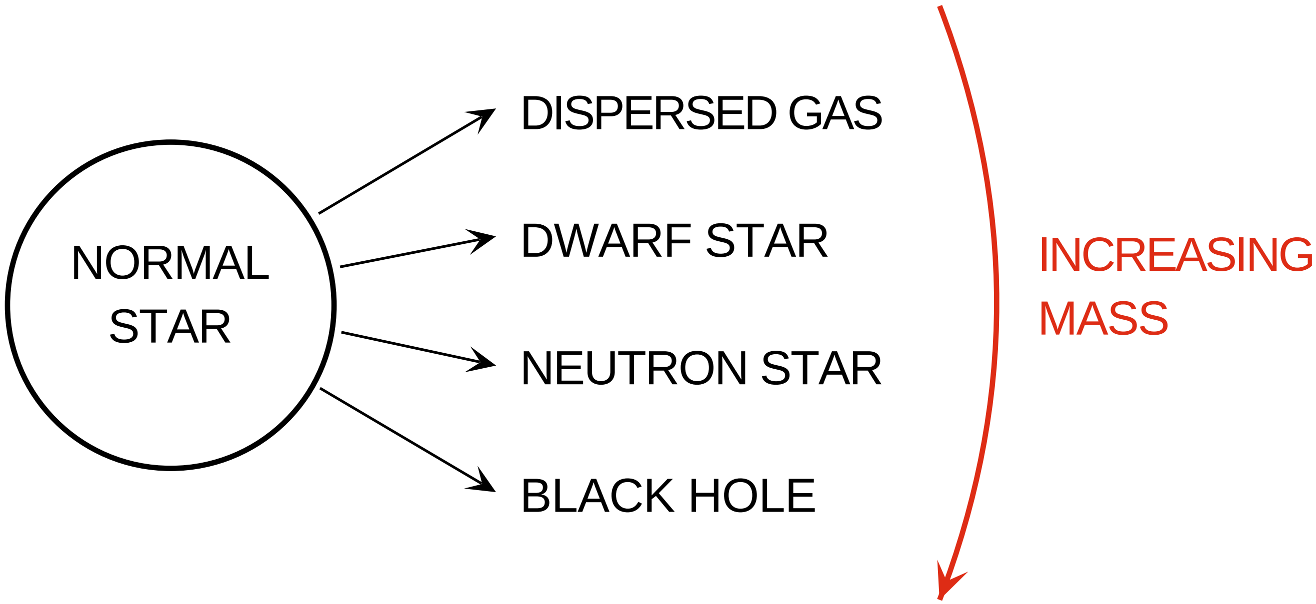


Atmosphere of Betelgeuse · Alpha Orionis

Hubble Space Telescope · Faint Object Camera

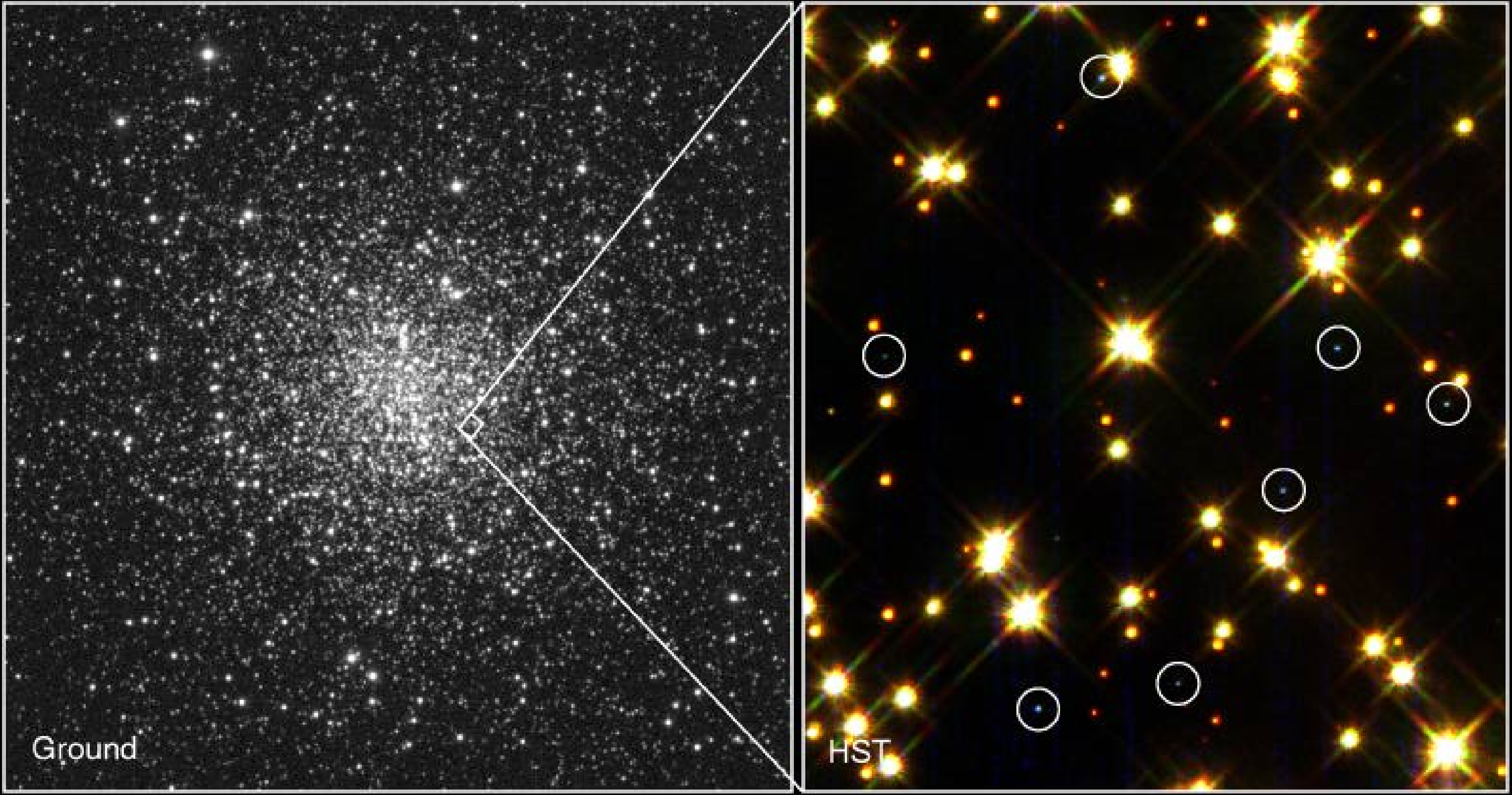
PRC96-04 · ST ScI OPO · January 15, 1996 · A. Dupree (CfA), NASA, ESA

FOUR POSSIBLE END STATES OF STARS



White Dwarf

- Fusion has stopped
- Collapses to a small size, nuclear spacing $\sim 1/100$ that of a solid
- Electron degeneracy pressure supports it, $P \propto \frac{1}{m_e} n^{5/3}$
- White $\rightarrow$ gray $\rightarrow$ brown (dead, cold)



White Dwarf Stars in M4

HST • WFPC2

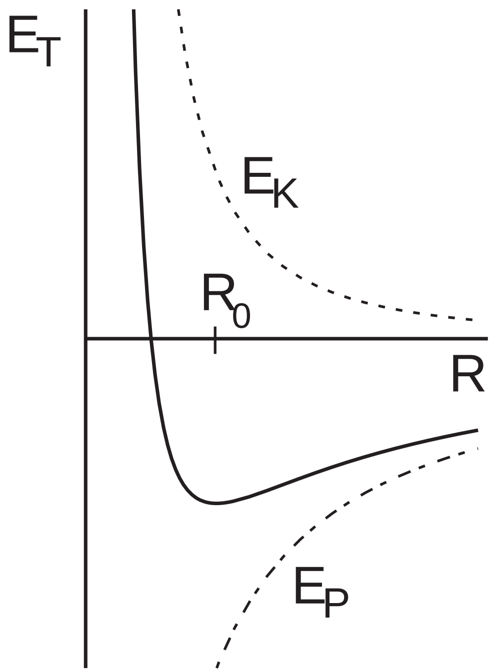
PRC95-32 • ST ScI OPO • August 28, 1995 • H. Bond (ST ScI), NASA

Assume uniform density of α^{++} and e^{-}

$$E_K = \underbrace{E_K^{(\alpha)}}_{\text{small}} + E_K^{(e)} = \frac{3}{5}\, N_e \epsilon_F = \frac{3}{5}\, N_e \,\frac{\hbar^2}{2m_e}\left(3\pi^2 (N_e/V)\right)^{2/3}$$

$$V = \frac{4}{3}\pi R^3 \qquad M \approx N_\alpha m_\alpha = (N_e/2) m_\alpha \Rightarrow N_e = 2M/m_\alpha$$

$$E_K = \frac{3}{5}\left(\frac{9\pi}{2}\right)^{2/3} \frac{\hbar^2}{m_e}\left(\frac{M}{m_\alpha}\right)^{2/3} \frac{1}{R^2} \qquad\qquad E_P = -\frac{3}{5}\, G\, \frac{M^2}{R}$$

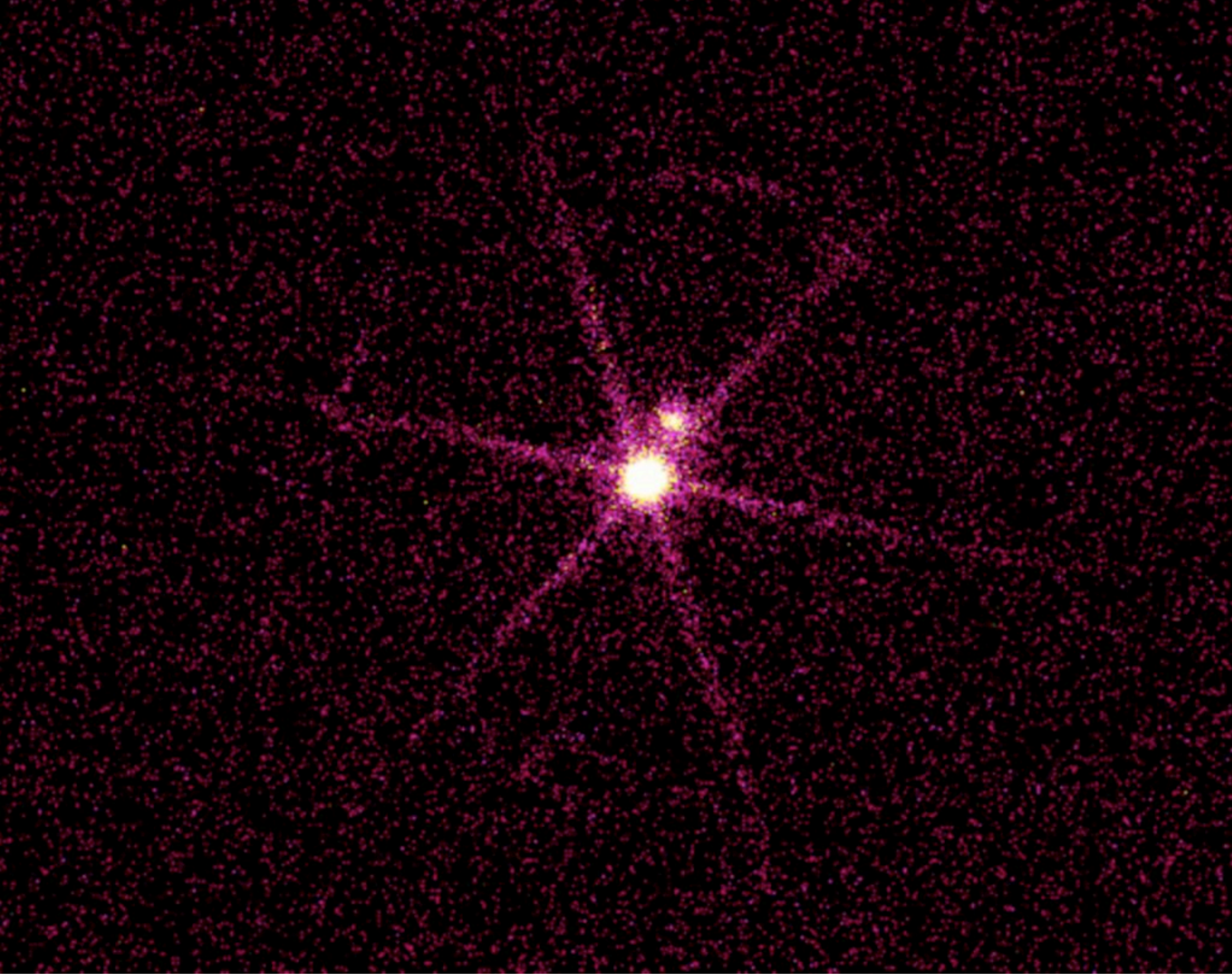


$$E_T = E_K + E_P$$

$$MR_0^3 = 2(9\pi)^2 \frac{\hbar^6}{G^3 m_e^3 m_\alpha^5}$$

$$= 0.74 \times 10^{51} \text{ kg-m}^3$$

$R_0 \propto 1/M^{1/3}$ <u>Stable for any M</u>.



X-ray image of Sirius B (brighter) and Sirius A (less bright) , from Astronomy Picture of the Day: http://apod.nasa.gov/apod/ap001006.html

Sirius B: $M = 2.1 \times 10^{30}$ kg

R

observed	5.6×10^6 m	
our model	7.1×10^6 m	(good)
better model	8.6×10^6 m	($\Rightarrow$ a problem)

Our model of Sirius B implies

$n_e = 8.6 \times 10^{29}\ \text{cm}^{-3}$

$\epsilon_F = 4.7 \times 10^{-7}\ \text{ergs} \rightarrow 3.4 \times 10^9\text{K}$

$(T_{\text{surface}} \sim 2 \times 10^7\ \text{K})$

<u>But</u> $m_e c^2 = 8.2 \times 10^{-7}$ ergs $\Rightarrow$ relativity needed

$$D_{\text{wavevectors}}(k) = \frac{V}{(2\pi)^3} \qquad D_{\text{states}}(k) = \frac{2V}{(2\pi)^3}$$

$$N = \left(\frac{4}{3}\pi k_F^3\right)\frac{2V}{(2\pi)^3} \Rightarrow k_F = (3\pi^2 N/V)^{1/3}$$

$$\epsilon = c\hbar|\vec{k}| \Rightarrow \underline{\epsilon_F = c\hbar(3\pi^2 N/V)^{1/3}}$$

$$\#(\epsilon) \;=\; \left(\frac{4}{3}\pi\, \underbrace{k^3(\epsilon)}_{(\epsilon/c\hbar)^3}\right)\left(\frac{2V}{(2\pi)^3}\right) = \frac{1}{3\pi^2} V \left(\frac{1}{c\hbar}\right)^3 \epsilon^3$$

$$D(\epsilon) \;=\; \frac{d\#}{d\epsilon} = \underline{\frac{V}{\pi^2}\left(\frac{1}{c\hbar}\right)^3 \epsilon^2}$$

$$D(\epsilon) = a\epsilon^2$$

$$N = \int_0^{\epsilon_F} D(\epsilon)\, d\epsilon = \int_0^{\epsilon_F} a\epsilon^2\, d\epsilon = \frac{1}{3}a\epsilon_F^3$$

$$E = \int_0^{\epsilon_F} \epsilon D(\epsilon)\, d\epsilon = \int_0^{\epsilon_F} a\epsilon^3\, d\epsilon = \frac{1}{4}a\epsilon_F^4$$

$$= \underline{\frac{3}{4}N\epsilon_F}$$

$$P = -\underbrace{\left(\frac{\partial E}{\partial V}\right)_{N,S}}_{dS=0 \text{ at } T=0} = -\frac{3}{4}N\left(\frac{\partial \epsilon_F}{\partial V}\right)_N$$

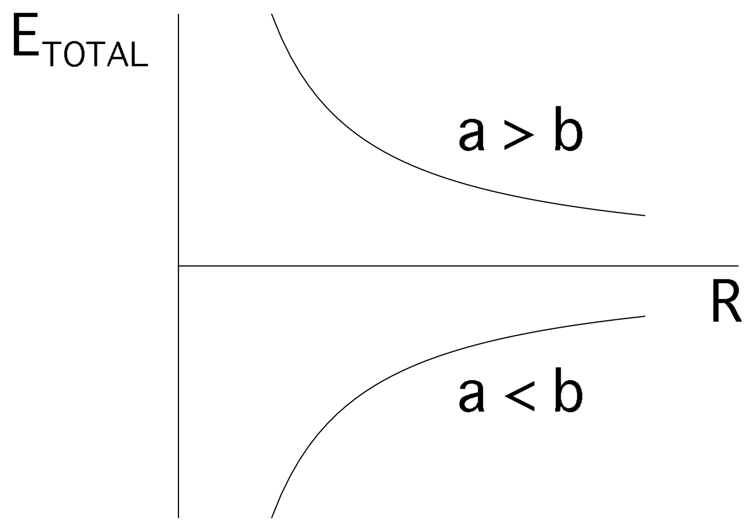
$$= \underline{\frac{1}{4}(N/V)\epsilon_F \propto (N/V)^{4/3}}$$

This pressure rises less steeply with density, $(N/V)^{4/3}$, than is the case for the non-relativistic gas, $(N/V)^{5/3}$.

For a white dwarf composed of α particles and electrons,

$$
\begin{aligned}
V &= \frac{4}{3}\pi R^3 \\
M &\approx N_\alpha m_\alpha = \frac{1}{2} N_e m_\alpha \Rightarrow N_e = 2(M/m_\alpha) \\
E_K &= \frac{3}{4} N_e \epsilon_F = \frac{3}{4} N_e c\hbar (3\pi^2 N_e/V)^{1/3} \\
&= \frac{3}{2} c\hbar \left(\frac{M}{m_\alpha}\right) \left(\frac{9\pi}{2}\frac{M}{m_\alpha}\frac{1}{R^3}\right)^{1/3} \\
&= \underline{\frac{3}{2}\left(\frac{9\pi}{2}\right)^{1/3} c\hbar \left(\frac{M}{m_\alpha}\right)^{4/3} \frac{1}{R}}
\end{aligned}
$$

The R dependence of the two contributions to the total energy is straight forward: $E_K = a/R$ and $E_P = -b/R$ where a and b are known expressions. Then $E_{\text{TOTAL}} = (a-b)/R$ which is never stable. The condition $a = b$ is a special case, a dividing line between collapse and infinite expansion.



$$c\hbar\left(\frac{M}{m_\alpha}\right)^{4/3} \sim GM^2$$

$$\frac{c\hbar}{Gm_\alpha^{4/3}} \sim M^{2/3}$$

$$M \sim \underline{\left(\frac{c\hbar}{Gm_\alpha^2}\right)^{3/2} m_\alpha}$$

The Chandrasekhar limit for the maximum possible mass of a white dwarf is

$$M_{\text{Ch}} = 0.20 \left(\frac{Z}{A}\right) \left(\frac{ch}{Gm_p^2}\right)^{3/2} m_p$$

where Z/A is the average ratio of atomic number to atomic weight of the stellar constituents. Note that it has the same form as our expression. For $Z/A = 0.5$ (α particles) this gives $M_{\text{Ch}} = 1.4 M_{\text{Sun}}$.

Neutron Star

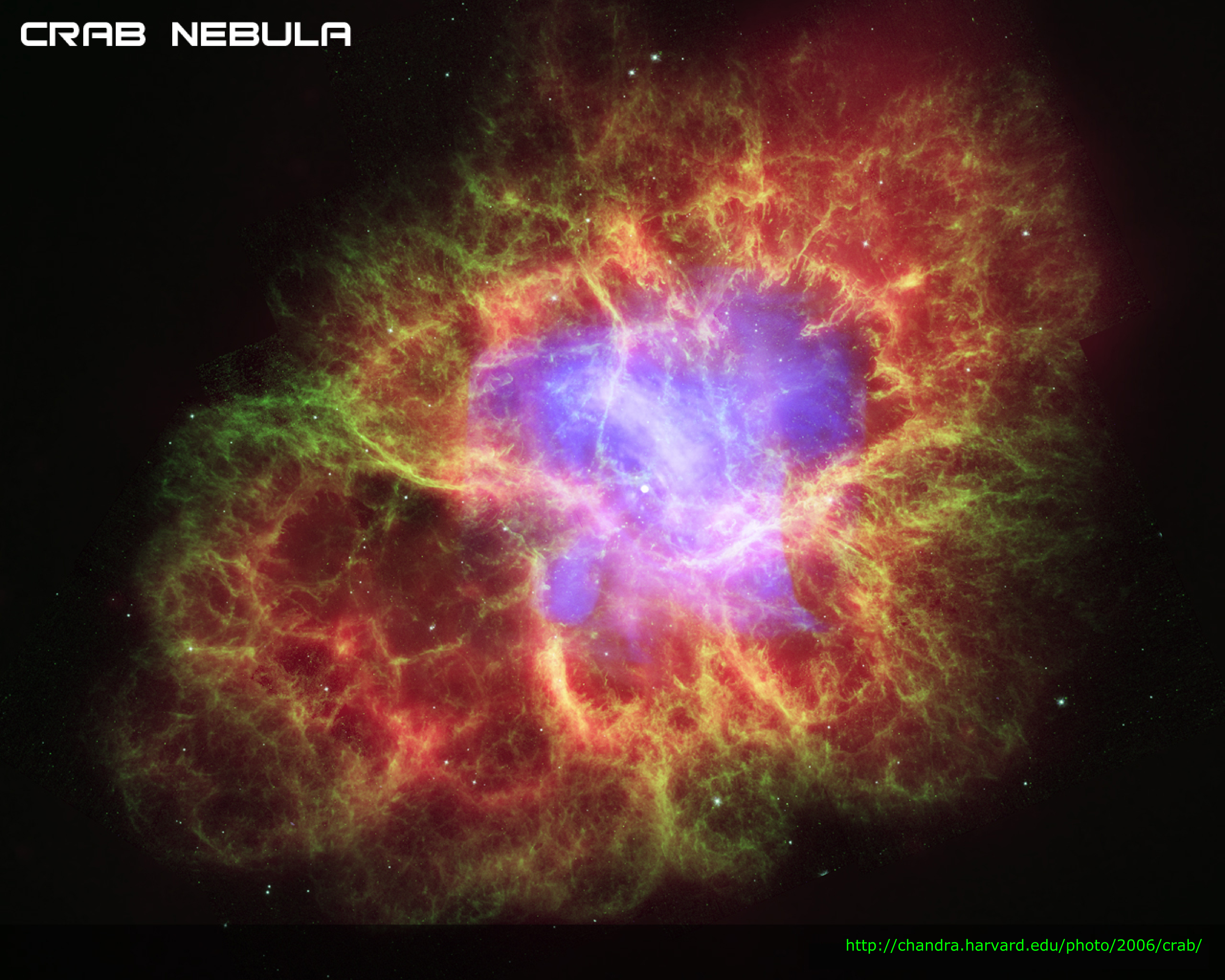
- $p^+ + e^- \rightarrow n$ to lower coulomb energy

- Degeneracy pressure of neutrons
  $MR_0^3 \propto \hbar^6/G^3 m_n^8 \;\; \Rightarrow \;\; R_0 \sim 15$ km if $M = 1.4 M_\odot$

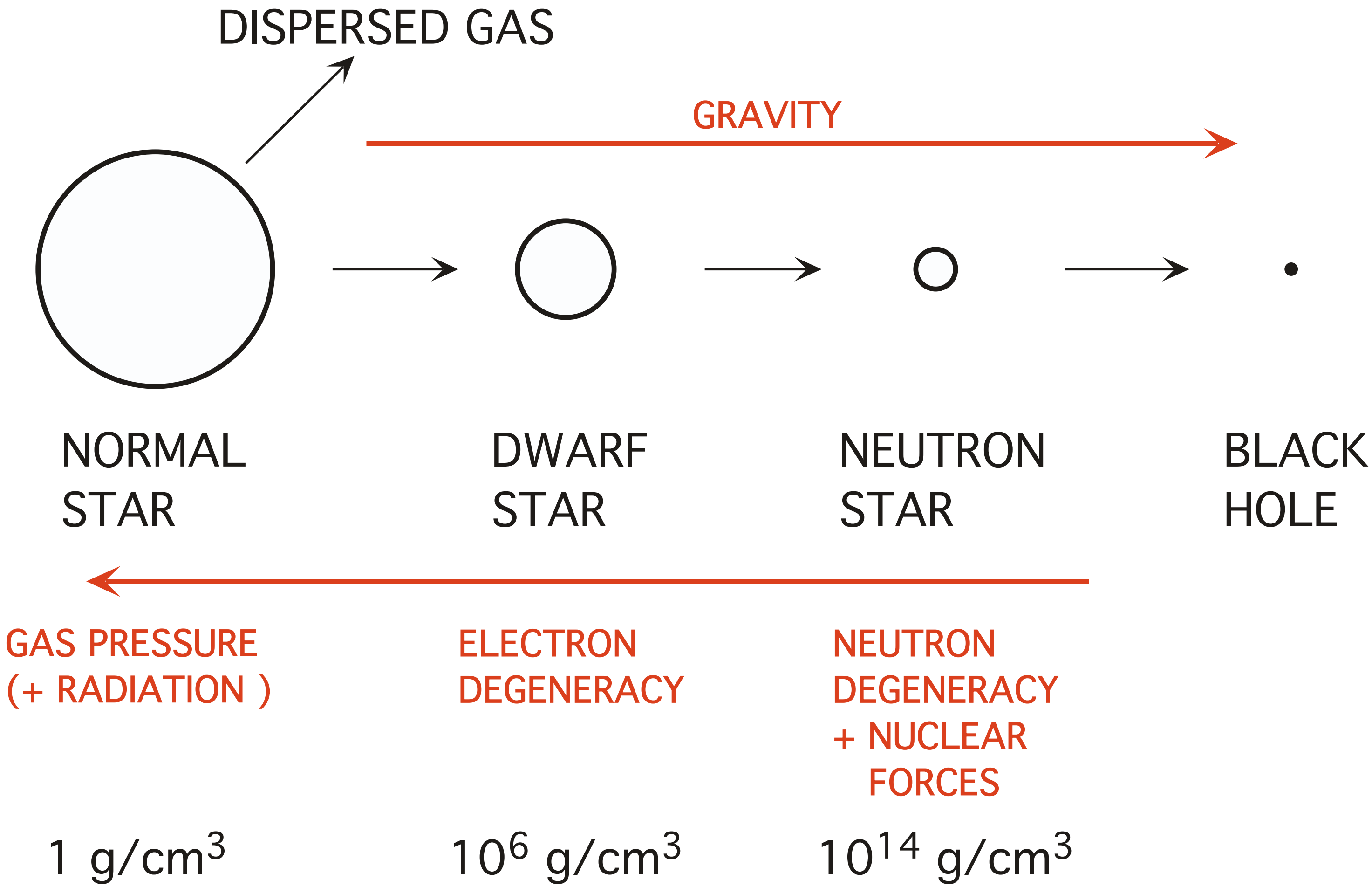
- Nuclear forces also contribute to P

- Rotating neutron stars seen as pulsars

- Also subject to stability limit, $M \sim 2M_\odot$


CRAB NEBULA
http://chandra.harvard.edu/photo/2006/crab/

STELLAR CONFIGURATIONS



Artist's conception of accretion disk around a black hole , from Astronomy Picture of the Day: http://apod.nasa.gov/apod/ap130312.html

CYGNUS–X1 *Black hole*

http://commons.wikimedia.org/wiki/File:Cygnus_X-1.png

8.044 Statistical Physics I
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