Massachusetts Institute of Technology
Physics 8.03SC Fall 2016
Homework 2

Problems

Problem 2.1 (20 pts)

In the circuit shown in Figure 1, $C = 2\ \mu$F, $L = 2$ mH and $R = 20\ \Omega$. Initially at time $t = 0$, we have $V_c(t = 0) = 5$ V and current $i(t = 0) = 0.5$ A.

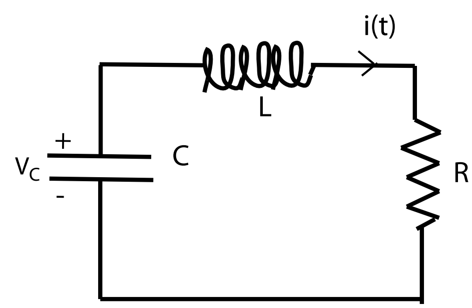


Figure 1: RLC Circuit

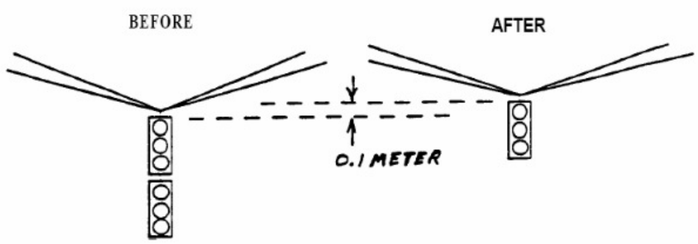
a. Now we would like to translate this physical situation into mathematics. Write down the equation of motion in terms of Q(t), the charge stored in the capacitor.
b. What kind of oscillator are we looking at here? (Undamped, underdamped, critical damped or overdamped?) Explain why do you think so.
c. Find an analytic expression for $V_C(t)$ for all $t > 0$.

Problem 2.2 (20 pts)

In a lab close to the Large Hadron Collider, a delicate crystal of mass M is supported by four massless springs in parallel, each with spring constant k. The whole setup is put on a table. When the graduate students in the lab move the table across the floor, the tabletop vibrates, producing an effective vertical force $F = MA_0 \cos(\omega_d t)$ on the mass M in the tabletop reference frame.

a. Let x be the vertical displacement of the crystal from its equilibrium position. Write down the equation of motion of the instrument. (You can assume that there is no drag force in this part of the question)
b. Find the vibration amplitude of the crystal (the mass M) in the steady state. (Assuming that there are very small energy losses in this system such that the homogeneous solution die out as $t \to \infty$.)
c. To reduce this vibration amplitude of the crystal in (a) by a factor of ten, how would you propose to modify the four springs, i.e. how much longer (shorter) should the new springs be? Assume $k/M \gg \omega_d^2$. (Hint: the spring constant of a spring is proportional to its area and inversely proportional to its length.)
d. A better way to reduce this vibration amplitude by a factor of ten from what it originally was is to insert some kind of a soft massless cushion between the crystal and the table, in parallel to the springs. Assuming that the cushion produces a resistive force $-b$ times the velocity of M, derive an equation that allows you to determine the value of b in terms of k, M, ω_d and solve for b, for $k/M \gg \omega_d^2$.

Problem 2.3 (20 pts)



During a recent hurricane, a physicist was waiting at an intersection. She watched the traffic signal, suspended above it by wires, oscillate up and down in the wind. Since she took 8.03 before when she was a MIT undergrad, she immediately noticed that the system behaved like a damped mass-spring oscillator with the wire suspension playing the role of the spring. She observed that amplitude disturbances took 4 seconds (the e^{-1} time) to damp out (i.e, it takes 4 seconds for the amplitude $A \to e^{-1}A$). Suddenly, the lower half of the signal broke off and crashed to the pavement. The subsequent equilibrium position of the remaining half was 0.1 meter higher than it had been.

a. What is the undamped natural frequency (in Hertz) of the traffic light after the separation?
b. Assume that the dissipation is entirely in the suspension system. What is the decay time (the e^{-1} time) of the traffic light's amplitude oscillations after the separation?
c. Write down an analytic expression for the vertical displacement, $y(t)$, of the light from its new equilibrium position after the separation. Assume that the traffic light was stationary ($\dot{y} = 0$) at the previous equilibrium position at the time of the separation, t = 0. Give numerical values, with units, for any constants which appear in the expression.

Problem 2.4 (20 pts)

Consider a mass m moving on a horizontal air track. The mass is attached on both sides to two identical springs each with spring constant k and relaxed length ℓ_0 (see Figure 2). The end of the first spring is fixed. The end of the second spring is attached to an electrical motor that causes it to undergo harmonic motion with amplitude Δ and angular frequency ω. At $t = 0$ the springs are relaxed at $\ell = \ell_0$ and the mass is not moving $\dot{x}(0) = 0$. Define the origin of the coordinate system to be at the position of the mass at $t = 0$ such that $x(0) = 0$

At $t = 0$ the motor has been switched on such that the end of the spring starts to move according to the following equation:

$$x_{end} = \Delta \sin(\omega t) + \ell_0$$

Assume that the motion of the mass is affected by a small velocity dependent air friction, $-b\dot{x}$.

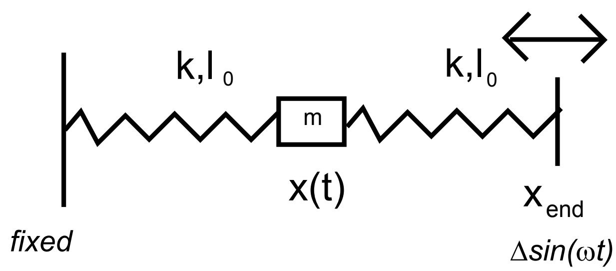


Figure 2: Classroom demonstration: mass on the air track

a. Set up carefully a one dimensional equation of motion for mass m including all the forces. Organize your equation neatly to clearly indicate the oscillator terms and the terms related to the external force.

b. Postulate the complete solution for the motion of the mass $x(t) = x_{free}(t) + x_{driven}(t)$ without solving. Indicate which constants in the solution depend only on oscillator properties, which on the properties of external force and which need to be determined using the initial conditions.

c. Find the amplitude and phase of the steady state motion of the mass. Sketch their dependence on the frequency of the driving force ω and the given parameters.

d. Find the frequency ω_{max} for which the amplitude is maximum.

e. Use the initial conditions to find the specific solution including both the free oscillator and the steady state solution with the appropriate choice of all parameters.

Problem 2.5 (20 pts)

Consider a system of three springs and two masses as shown in Figure 3, where the masses are constrained to move only in the vertical direction The spring constants are $K_A = 78$, The system is prepared in a lab on Earth and the gravitation force is pointing downward. $K_B = 60$ and $K_C = 24$ measured in $[N/m]$. The two masses are $m_1 = 4$ and $m_2 = 12$ measured in $[kg]$. The unstretched lengths of spring A and B equal the unstretched length C.

a. Define coordinate system(s) and write equations of motion for the two masses.

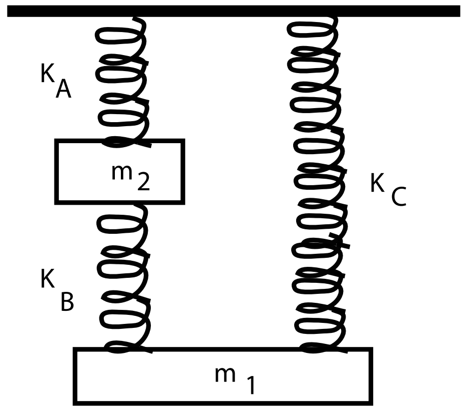


Figure 3: Two masses hanging vertically on springs

b. Write the equation of motion in matrix form. Show clearly the matrix $M^{-1}K$ as defined in the textbook.

c. Find the normal modes of oscillations and their associated angular frequencies.

d. If the mass m_1 is displaced by 1 cm up from its equilibrium position and m_2 is held at its original equlibrium position and both blocks released from rest at $t = 0$, write expressions for the subsequent motion of both blocks.