

Topics: Conductors & Capacitors

Related Reading:

Course Notes (Liao et al.): Sections 4.3-4.4; Chapter 5

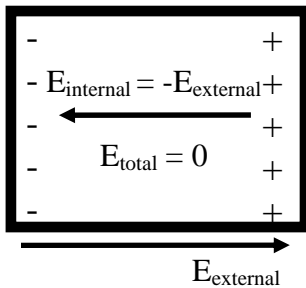
Topic Introduction

Today we introduce two new concepts – conductors & capacitors. Conductors are materials in which charge is free to move. That is, they can *conduct* electrical current (the flow of charge). Metals are conductors. For many materials, such as glass, paper and most plastics this is not the case. These materials are called insulators.

For the rest of the class we will try to understand what happens when conductors are put in different configurations, when potentials are applied across them, and so forth. Today we will describe their behavior in static electric fields.

Conductors

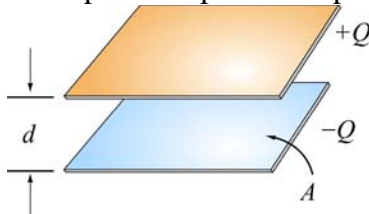
Since charges are free to move in a conductor, the electric field inside of an isolated conductor must be zero. Why is that? Assume that the field were not zero. The field would apply forces to the charges in the conductor, which would then move. As they move, they begin to set up a field in the opposite direction. An easy way to picture this is to think of a bar of metal in a uniform external electric field (from left to right in the picture below). A net positive charge will then appear on the right of the bar, a net negative charge on the left. This sets up a field opposing the original. As long as a net field exists, the charges will continue to flow until they set up an equal and opposite field, leaving a net zero field inside the conductor.



Capacitance

Using conductors we can construct a very useful device which stores electric charge: the capacitor. Capacitors vary in shape and size, but the basic configuration is two conductors carrying equal but opposite charges ($\pm Q$). In order to build up charge on the two plates, a potential difference ΔV must be applied between them. The ability of the system to store charge is quantified in its capacitance: $C \equiv Q/|\Delta V|$. Thus a large capacitance capacitor can store a lot of charge with little “effort” – little potential difference between the two plates.

A simple example of a capacitor is pictured at left – the parallel plate capacitor, consisting of



two plates of area A , a distance d apart. To find its capacitance we first arbitrarily place charges $\pm Q$ on the plates. We calculate the electric field between the plates (using Gauss’s Law) and integrate to obtain the potential difference between them. Finally we calculate the capacitance:

$$C = Q/|\Delta V| = \epsilon_0 A/d .$$

Note that the capacitance depends only on geometrical factors, not on the amount of charge stored (which is why we were justified in starting with an arbitrary amount of charge).

Energy

In the process of storing charge, a capacitor also stores electric energy. We can see this by considering how you “charge” a capacitor. Imagine that you start with an uncharged capacitor. Carry a small amount of positive charge from one plate to the other (leaving a net negative charge on the first plate). Now a potential difference exists between the two plates, and it will take work to move over subsequent charges. Reversing the process, we can release energy by giving the charges a method of flowing back where they came from (more on this in later classes). So, in charging a capacitor we put energy into the system, which can later be retrieved. Where is the energy stored? In the process of charging the capacitor, we also create an electric field, and it is in this electric field that the energy is stored. We assign to the electric field a “volume energy density” u_E , which, when integrated over the volume of space where the electric field exists, tells us exactly how much energy is stored.

Important Equations

Capacitance:

$$C \equiv Q/|\Delta V|$$

Energy Stored in a Capacitor:

$$U = \frac{Q^2}{2C} = \frac{1}{2}Q|\Delta V| = \frac{1}{2}C|\Delta V|^2$$

Energy Density in Electric Field:

$$u_E = \frac{1}{2}\epsilon_0 E^2$$