22.312 ENGINEERING OF NUCLEAR REACTORS

Monday, December 14th, 2009, 9:00am-12:00 pm

OPEN BOOK | FINAL EXAM | 3 HOURS

Problem 1 (15%) – Sizing the shell of a spherical containment

i)
The principal stresses for a thin spherical shell are:

$$\sigma_r = -(p_i+p_o)/2 \qquad (1)$$
$$\sigma_\theta = \sigma_\varphi = (p_i-p_o)R_c/(2\ t_c)$$

where p_i=1.9 MPa and p_o=0.1 MPa, R_c=12.5 m and t_c is the shell thickness. Hook's law yields:

$$\varepsilon_\theta=u/R_c=1/E[\sigma_\theta - \nu\ (\sigma_\varphi+\sigma_r)] \qquad (2)$$

where E = 184 GPa and ν = 0.33. Substituting Eq. (1) in Eq. (2), setting u=1 cm and solving for t_c, one gets:

$$t_c=R_c(1-\nu)\ (p_i-p_o)/[2E\ u/R_c-\nu(p_i+p_o)]\approx 3.7\ \text{cm}$$

Since R_c/t_c>10, the thin shell asumption of accurate.

ii)
The primary membrane general stress intensity for this case is:

$$P_m=(\sigma_\theta - \sigma_r)\approx 102\ \text{MPa}$$

σ_θ and σ_r were calculated from Eq. (1) (thin shell assumption still applies), for t_c=8 cm. The ASME limit is S_m=110 MPa. Therefore the margin is $S_m/P_m\approx 1.075$, or 7.5%.

Problem 2 (25%) – Reduction of containment pressure after LOCA

Conservation of energy for the containment:

$$\frac{\partial E_{CV}}{\partial t} = \dot{Q}_{decay} - \dot{Q}_{ss} \qquad (3)$$

where $\dot{Q}_{decay} = \dot{Q}_0 0.066 t^{-0.2}$, $\dot{Q}_0$=1000 MW, and $\dot{Q}_{ss}$=20 MW. Integrating Eq. (3):

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$$E_2 - E_1 = \dot{Q}_0 \frac{0.066}{0.8} t_2^{0.8} - \dot{Q}_{ss} t_2 \qquad (4)$$

Expanding the left-hand side, one gets:

$$M_a c_{v,a}(T_2 - T_1) + M_w\{[u_f(T_2)(1-x_2) + u_g(T_2)x_2] - [u_f(T_1)(1-x_1) + u_g(T_1)x_1]\} = \dot{Q}_0 \frac{0.066}{0.8} t_2^{0.8} - \dot{Q}_{ss} t_2 \qquad (5)$$

where M_a, $c_{v,a}$, T_1, M_w and x_1 are all known from the problem statement. The following equation holds for the control volume:

$$V_{tot} = M_w[v_f(T_2)(1-x_2) + v_g(T_2)x_2] \qquad (6)$$

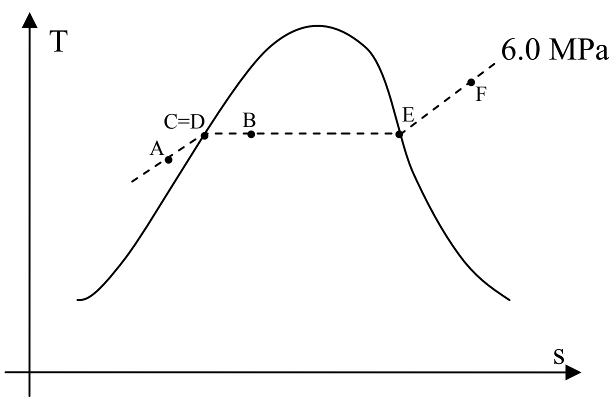
The containment pressure at t_2, P_2=0.5 MPa, is the sum of the partial pressures of water and air:

$$P_2 = P_{sat}(T_2) + \frac{M_a R_a T_2}{V_{tot} - M_w(1-x_2)v_f(T_2)} \qquad (7)$$

Therefore, Eqs. (5), (6) and (7) are 3 equations in the only unknowns t_2, T_2 and x_2. Actually solving the equations, one finds $t_2 \approx 14300$ s, $T_2 \approx 140.4$ °C and $x_2 \approx 0.035$.

Problem 3 (45%) – Superheated Boiling Water Reactor

i)
T-s diagram:



ii) Taking the whole system as a control volume, the conservation of energy yields:

$$0 = \dot{Q} + \dot{m}_{FW}(h_{FW} - h_{sup}) \quad \Rightarrow \quad \dot{m}_{FW} = \dot{Q}/(h_{sup} - h_{FW}) \qquad (8)$$

where $\dot{Q}$=1000 MW and h_{FW} and h_{sup} are the specific enthalpy of the feedwater and superheated steam, respectively. The difference h_{sup}-h_{FW} can be expressed as follows:

$$h_{sup} - h_{FW} = c_{p,g}(T_{sup} - T_{sat}) + h_{fg} + c_{p,f}(T_{sat} - T_{FW}) \approx 2936 \text{ kJ/kg}$$

where T_{FW}= 230°C and T_{sup}=510°C. Therefore, Eq. (8) yields $\dot{m}_{FW} \approx$340.6 kg/s.

iii)
The acceleration pressure drop is

$$\Delta P_{acc} = G^2 [\frac{1}{\rho^+_{m,out}} - \frac{1}{\rho^+_{m,in}}] \qquad (9)$$

where $G = \frac{\dot{m}}{A} \approx$1800 kg/m^{2}s, $\dot{m}$ =2270 kg/s, A=1.26 m^2 and

$$\rho^+_m \equiv \frac{1}{\frac{x^2}{\alpha \rho_g} + \frac{(1-x)^2}{(1-\alpha)\rho_f}} \qquad (10)$$

Since at the inlet there is only the liquid phase, it is $\rho^+_{m,in} = \rho_f$, while at the outlet x=0.15 and the void fraction can be found from the fundamental relation of two-phase flow:

$$\alpha = \frac{1}{1 + \frac{\rho_g}{\rho_f} \cdot \frac{1-x}{x} \cdot S} \approx 0.69$$

where S=2, per the problem statement. Then it is $\rho^+_{m,out} \approx$240.3 kg/m^3 from Eq. (10), and finally

Eq. (9) yields $\Delta P_{acc} \approx$ 9,235 Pa

iv)
Since the heat flux is axially constant, dryout would occur at the outlet (Point B). The critical quality at the outlet is found to be $x_{cr} \approx$0.344 from the CISE-4 correlation with L_b=3 m, and the coefficients a=0.5987 and b=2.2255, calculated for P=6 MPa, P_c=22.1 MPa, G=1800 kg/m^{2}s>G^*=1211 kg/m^{2}s, D_e=0.02 m.
Then the critical power of the A→B channels is $\dot{Q}_{cr,AB} = \dot{m}[c_{p,f}(T_{sat} - T_A) + x_{cr} h_{fg}] \approx 1311$ MW,

where T_A= 268°C. So, the $CPR = \frac{\dot{Q}_{cr,AB}}{\dot{Q}_{AB}} \approx 2.12$, with $\dot{Q}_{AB} = \dot{m}[c_{p,f}(T_{sat} - T_A) + x_B h_{fg}] \approx 618$ MW

being the operating power of the A→B channels, where x_B=0.15.

Problem 4 (15%) – Thermodynamic analysis of a new power cycle

To be thermodynamically feasible, the cycle must not violate the 1st and 2nd law of thermodynamics.

Taking the whole power cycle as the control volume, the conservation of energy (1st law) becomes:

$$0 = \dot{Q} - \dot{W} + \dot{m}_{sea}(h_{in} - h_{out}) \tag{11}$$

where steady-state was assumed and $\dot{Q}$=1000 MW, $\dot{W}$=400 MW, $\dot{m}_{sea}$=15000 kg/s, $(h_{in} - h_{out}) = c_{sea}(T_{in} - T_{out})$, c_{sea}=4000 J/kg°C and T_{in}=288 K (15°C) and T_{out}=298 K (25°C). Using these numbers, Eq. 11 is identically satisfied. Therefore, the cycle does not violate the 1st law.

With the same choice of control volume, the 2nd law becomes:

$$0 = \frac{\dot{Q}}{T_r} + \dot{m}_{sea}(s_{in} - s_{out}) + \dot{S}_{gen} \quad \Rightarrow \qquad \dot{S}_{gen} = \dot{m}_{sea}(s_{out} - s_{in}) - \frac{\dot{Q}}{T_r} \tag{12}$$

where T_r=723 K (450°C) and $s_{out} - s_{in} = c_{sea} \ln \frac{T_{out}}{T_{in}}$. Then Eq. 12 yields $\dot{S}_{gen}$= 665 kW/K > 0, therefore the cycle does not violate the 2nd law either.