6 CAPTIVE MEASUREMENTS

Before making the decision to measure hydrodynamic derivatives, a preliminary search of the literature may turn up useful estimates. For example, test results for many hull-forms have already been published, and the basic lifting surface models are not difficult. The available computational approaches should be considered as well; these are very good for predicting added mass in particular. Finally, modern sensors and computer control systems make possible the estimation of certain coefficients based on open-water tests of a model or a full-scale design.
In model tests, the Froude number $Fr = \frac{U}{\sqrt{gL}}$, which scales the influence of surface waves, must be maintained between model and full-scale surface vessels. Reynolds number $Re = \frac{UL}{\nu}$, which scales the effect of viscosity, need not be matched as long as the scale model attains turbulent flow (supercritical Re). One can use turbulence stimulators near the bow if necessary. Since the control surface(s) and propeller(s) affect the coefficients, they should both be implemented in model testing.

6.1 Towtank

In a towtank, tow the vehicle at different angles of attack, measuring sway force and yaw moment. The slope of the curve at zero angle determines Y_v and N_v respectively; higher-order terms can be generated if the points deviate from a straight line. Rudder derivatives can be computed also by towing with various rudder angles.

6.2 Rotating Arm Device

On a rotating arm device, the vessel is fixed on an arm of length R, rotating at constant rate r: the vessel forward speed is $U = rR$. The idea is to measure the crossbody force and yaw moment as a function of r, giving the coefficients Y_r and N_r. Note that the lateral force also contains the component $(m - Y_{\dot{v}})r^2R$. The coefficients Y_v and N_v can also be obtained by running with a fixed angle of attack. Finally, the measurement is made over one rotation only, so that the vessel does not re-enter its own wake.

6.3 Planar-Motion Mechanism

With a planar motion mechanism, the vessel is towed at constant forward speed U, but is held by two posts, one forward and one aft, which can each impose independent sway motions, therefore producing variable yaw. The model moves in pure sway if $y_a(t) = y_b(t)$, in a pure yaw motion about the mid-length point if $y_a(t) = -y_b(t)$, or in a combination sway and yaw motion. The connection points are a distance l forward and aft from the vessel origin.
Usually a sinusoidal motion is imposed:

$$\begin{aligned} y_a(t) &= a \cos \omega t \\ y_b(t) &= b \cos(\omega t + \psi), \end{aligned} \tag{95}$$

and the transverse forces on the posts are measured and approximated as

$$\begin{aligned} Y_a(t) &= F_a \cos(\omega t + \theta_a) \\ Y_b(t) &= F_b \cos(\omega t + \theta_b). \end{aligned} \quad (96)$$

If linearity holds, then

$$\begin{aligned} (m - Y_{\dot{v}})\dot{v} - Y_v v + (mU - Y_r)r + (mx_G - Y_{\dot{r}})\dot{r} &= Y_a + Y_b \\ (I_{zz} - N_{\dot{r}})\dot{r} + (mx_G U - N_r)r + (mx_G - N_{\dot{v}})\dot{v} - N_v v &= (Y_b - Y_a)l. \end{aligned} \quad (97)$$

We have $v = (\dot{y}_a + \dot{y}_a)/2$ and $r = (\dot{y}_b - \dot{y}_a)/2l$. When $a = b$, these become

$$\begin{aligned} v &= -\frac{a\omega}{2}\left(\sin \omega t(1 + \cos \psi) + \cos \omega t \sin \psi\right) \\ \dot{v} &= -\frac{a\omega^2}{2}\left(\cos \omega t(1 + \cos \psi) - sin\omega t \sin \psi\right) \\ r &= -\frac{a\omega}{2l}\left(\sin \omega t(\cos \psi - 1) + \cos \omega t \sin \psi\right) \\ \dot{r} &= -\frac{a\omega^2}{2l}\left(\cos \omega t(\cos \psi - 1) - \sin \omega t \sin \psi\right). \end{aligned} \quad (98)$$

Equating the sine terms and then the cosine terms, we obtain four independent equations:

$$\begin{aligned} (m - Y_{\dot{v}})\left(-\frac{a\omega^2}{2}\right)(1 + \cos \psi) - & \\ Y_v\left(-\frac{a\omega}{2}\right)\sin \psi + & \\ (mU - Y_r)\left(-\frac{a\omega}{2l}\right)\sin \psi + & \\ (mx_G - Y_{\dot{r}})\left(-\frac{a\omega^2}{2l}\right)(\cos \psi - 1) &= F_a \cos \theta_a + F_b \cos \theta_b \end{aligned} \quad (99)$$

$$\begin{aligned} (m - Y_{\dot{v}})\left(-\frac{a\omega^2}{2}\right)(-\sin \psi) - & \\ Y_v\left(-\frac{a\omega}{2}\right)(1 + \cos \psi) + & \\ (mU - Y_r)\left(-\frac{a\omega}{2l}\right)(\cos \psi - 1) + & \\ (mx_G - Y_{\dot{r}})\left(-\frac{a\omega^2}{2l}\right)(-\sin \psi) &= -F_a \sin \theta_a - F_b \sin \theta_b \end{aligned} \quad (100)$$

$$(I_{zz} - N_{\dot{r}})\left(-\frac{a\omega^2}{2l}\right)(\cos \psi - 1) + \quad (101)$$

$$\begin{aligned}
(mx_G U - N_r)\left(-\frac{a\omega}{2l}\right)\sin\psi + & \\
(mx_G - N_{\dot{v}})\left(-\frac{a\omega^2}{2}\right)(1+\cos\psi) - & \\
N_v\left(-\frac{a\omega}{2}\right)\sin\psi & = l(F_b\cos\theta_b - F_a\cos\theta_a)
\end{aligned}$$

$$\begin{aligned}
(I_{zz} - N_{\dot{r}})\left(-\frac{a\omega^2}{2l}\right)(-\sin\psi) + & \\
(mx_G U - N_r)\left(-\frac{a\omega}{2l}\right)(\cos\psi - 1) + & \\
(mx_G - N_{\dot{v}})\left(-\frac{a\omega^2}{2}\right)(-\sin\psi) - & \\
N_v\left(-\frac{a\omega}{2}\right)(1+\cos\psi) & = l(-F_b\sin\theta_b + F_a\sin\theta_a)
\end{aligned} \tag{102}$$

In this set of four equations, we know from the imposed motion the values $[U, \psi, a, \omega]$. From the experiment, we obtain $[F_a, F_b, \theta_a, \theta_b]$, and from the rigid-body model we have $[m, I_{zz}, x_G]$. It turns out that the two cases of $\psi = 0$ (pure sway motion) and $\psi = 180^o$ (pure yaw motion) yield a total of eight independent equations, exactly what is required to find the eight coefficients $[Y_{\dot{v}}, Y_v, Y_{\dot{r}}, Y_r, N_{\dot{v}}, N_v, N_{\dot{r}}, N_r]$. Remarkably, we can write the eight solutions directly: For $\psi = 0$,

$$\begin{aligned}
(m - Y_{\dot{v}})\left(-\frac{a\omega^2}{2}\right)(2) & = F_a\cos\theta_a + F_b\cos\theta_b \\
-Y_v\left(-\frac{a\omega}{2}\right)(2) & = -F_a\sin\theta_a - F_b\sin\theta_b \\
(mx_G - N_{\dot{v}})\left(-\frac{a\omega^2}{2}\right)(2) & = l(F_b\cos\theta_b - F_a\cos\theta_a) \\
-N_v\left(-\frac{a\omega}{2}\right)(2) & = l(-F_b\sin\theta_b + F_a\sin\theta_a),
\end{aligned} \tag{103}$$

to be solved respectively for $[Y_{\dot{v}}, Y_v, N_{\dot{v}}, N_v]$. For $\psi = 180^o$, we have

$$\begin{aligned}
(mx_G - Y_{\dot{r}})\left(-\frac{a\omega^2}{2l}\right)(-2) & = F_a\cos\theta_a + F_b\cos\theta_b \\
(mU - Y_r)\left(-\frac{a\omega}{2l}\right)(-2) & = -F_a\sin\theta_a - F_b\sin\theta_b \\
(I_{zz} - N_{\dot{r}})\left(-\frac{a\omega^2}{2l}\right)(-2) & = l(F_b\cos\theta_b - F_a\cos\theta_a) \\
(mx_G U - N_r)\left(-\frac{a\omega}{2l}\right)(-2) & = l(-F_b\sin\theta_b + F_a\sin\theta_a),
\end{aligned} \tag{104}$$

to be solved for $[Y_{\dot{r}}, Y_r, N_{\dot{r}}, N_r]$. Thus, the eight linear coefficients for a surface vessel maneuvering, for a given speed, can be deduced from two tests with a planar motion mechanism. We note that the nonlinear terms will play a significant role if the motions are too large, and that some curve fitting will be needed in any event. The PMM can be driven with more complex trajectories which will target specific nonlinear terms.