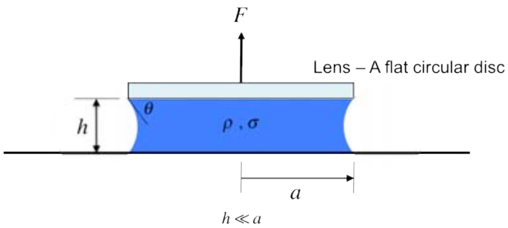
Supplementary note on the contact lens example

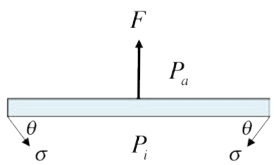
Why we need to slide, not to pull, contact lens.

Here, we assume a contact lens as a circular disc, as shown in the figure.



We are interested in calculating the force, F , which enables to hold the lens at its height, h , from an eye.

First, we consider the free body diagram for the lens, shown right.



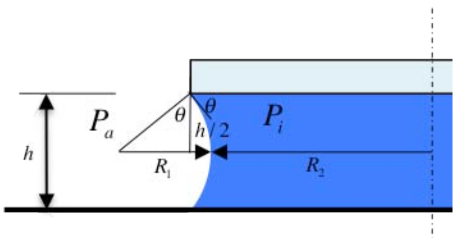
Force balance for the lens gives

$$F + (P_i - P_a)\pi a^2 = 2\pi a\sigma \sin\theta$$

or

$$F = (P_a - P_i)\pi a^2 + 2\pi a\sigma \sin\theta \qquad (1)$$

The pressure of the inside liquid P_i is determined by the Young-Laplace equation.



$$P_a - P_i = \sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$R_1 = \frac{h/2}{\cos\theta} \quad \& \quad R_2 = -a$$

Thus, $$P_a - P_i = \sigma\left(\frac{2\cos\theta}{h} - \frac{1}{a}\right) \qquad (2)$$

By combining the equation (1) and (2),

$$F = \sigma\left(\frac{2\cos\theta}{h} - \frac{1}{a}\right)\pi a^2 + 2\pi a\sigma \sin\theta$$
$$= \frac{2\pi a^2\sigma}{h}\left[\cos\theta - \frac{h}{a}\left(\frac{1}{2} - \sin\theta\right)\right]$$

For small h $\left(\frac{h}{a} \ll 1\right)$, $\quad F \approx \frac{2\pi a^2 \sigma \cos\theta}{h}$

How large is F ?

e.g., for $h = 1\ \mu\text{m}$ & $\theta = 0$

$$\frac{F}{\pi a^2} = \frac{2\sigma}{h} = 1.5 \text{ bar}$$

2.06 Fluid Dynamics
Spring 2013