2.003 Engineering Dynamics
Problem Set 2

Problem 1: Tangent and normal unit vectors: A small box of negligible size slides down a curved path defined by the parabola $y = 0.4x^2$. When it is at point A(x_A=2m, y_A=1.6m), the speed of the box is $V_{box} = 8$m/s and the tangential acceleration due to gravity is $dV_{box}/dt = 4m/s^2$.

A. Determine the normal component and the total magnitude of the acceleration of the box at this instant.

B. Draw a free body diagram and determine if the tangential acceleration is consistent with the forces acting on the body. In particular, can you determine if the friction coefficient is greater than zero, and if it is, what is it?

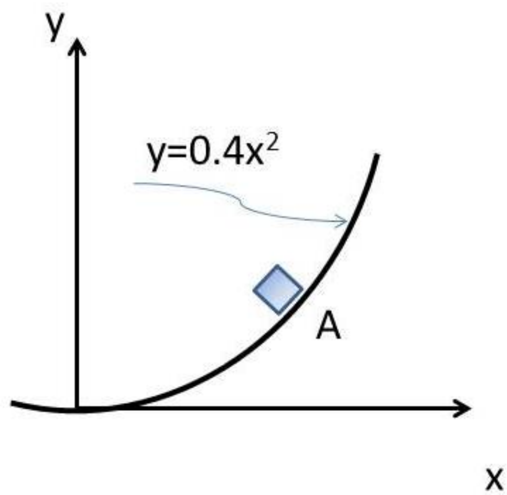


Figure 1

Concept question: When a particle travels along a curved path it experiences an acceleration given by the speed squared divided by the radius of curvature. In what direction is the acceleration?

A. Tangent to the path

B. Towards the concave side of the curved path (i.e. the inside of the curve, for example toward the center of a circular path)

C. Towards the convex side of the path

Problem 2: Polar coordinate challenge: The 0.5-lb ball is guided along a circular path, which lies in a vertical plane. The length of the arm OA is given by equal to $r_{P/O} = 2r_C \cos\theta$ using the arm OA. If the arm has an angular velocity $\dot{\theta} = 0.4\text{rad/s}$ and an angular acceleration $\ddot{\theta} = 0.8\text{rad/s}^2$ at the instant $\theta = 30^o$, determine the force of the arm on the ball. Neglect friction and the size of the ball. Set $r_C = 0.4\text{ft}$.

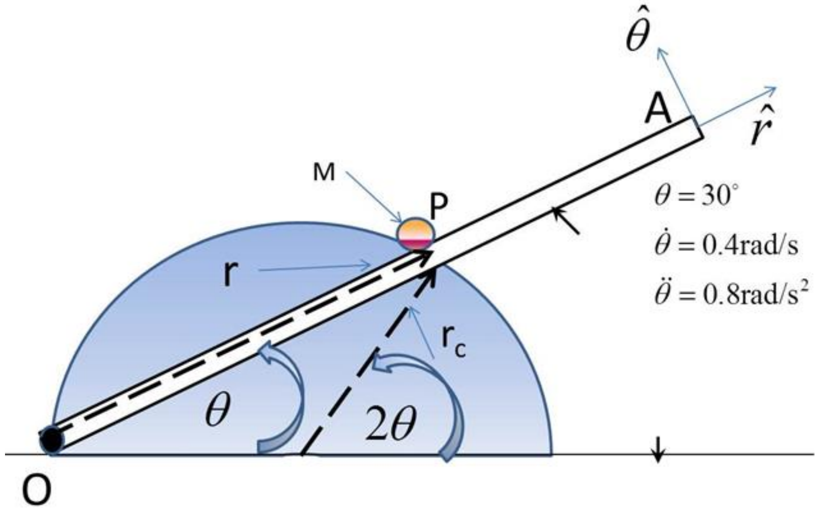


Figure 2

Concept question: In what direction is the force of the arm on the ball?

A. Parallel to the arm

B. Perpendicular to the arm

C. Parallel to the radius of the circular arc, with radius r_C

Problem 3: Helicopter Rotor: The rotor blades of a helicopter are of radius 5.2m and rotating at a constant angular velocity of 5rev/s. The helicopter is flying horizontally in a straight line at a speed of 5.0 m/s and an acceleration of 0.5m/s^2, as sketched in the figure. Determine the velocity and acceleration of point A, which is the outermost tip of one of the blades (with respect to ground), when θ is 90^o.

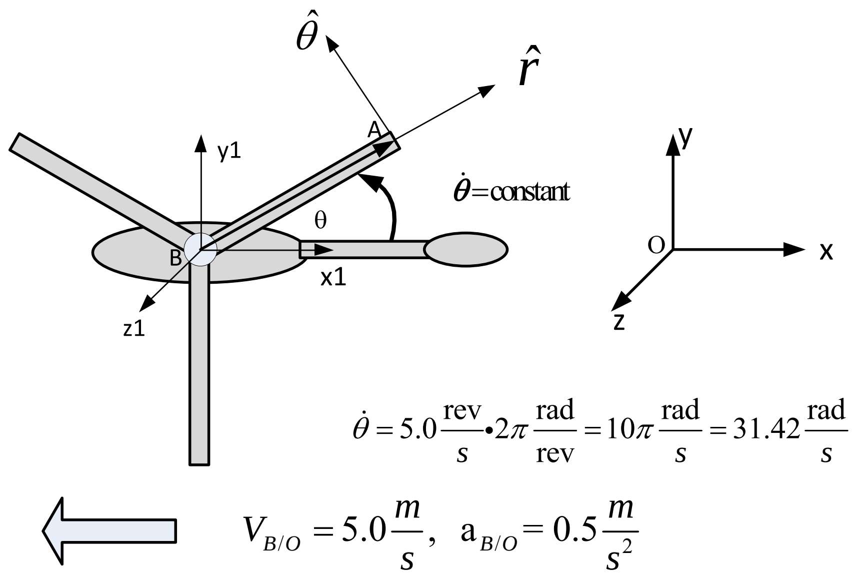
The reference frames defined in Figure 3 (below) are as follows: O_{xyz} is fixed to ground; B_{x1y1z1} is fixed to the translating helicopter.

Concept question: When is the magnitude of the acceleration at Point A the greatest?

A. $\theta = 0^\circ$

B. $\theta = 90^\circ$

C. $\theta = 180^\circ$



$$\dot{\theta} = 5.0\frac{\text{rev}}{s} \bullet 2\pi\frac{\text{rad}}{\text{rev}} = 10\pi\frac{\text{rad}}{s} = 31.42\frac{\text{rad}}{s}$$

$$V_{B/O} = 5.0\frac{m}{s},\ \ a_{B/O} = 0.5\frac{m}{s^2}$$

$$r_{A/B} = 5.2m$$

Figure 3

Problem 4: An amusement park ride: A ride at an amusement park consists of a rotating horizontal platform with an arm connected at point B to the outside edge. The arm rotates at 10 rev/min with respect to the platform. The platform rotates at 6 rev/min with respect to the ground. The axes of rotation for both are in the z direction, out of the plane of the page as shown in the drawing. Gravity acts in the z direction. A seat is located at C at the end of arm BC, which is 2m long. The radius from A to B is 6 m long. The mass of the passenger is 75kg. Find the velocities and accelerations of the passenger.

The reference frames defined in Figure 4 are as follows: O_{xyz} is fixed to ground; A_{x1y1z1} is fixed to the platform at the axis of rotation. $B_{x2\ y2\ z2}$ is fixed to the arm BC at B and rotates with the arm.

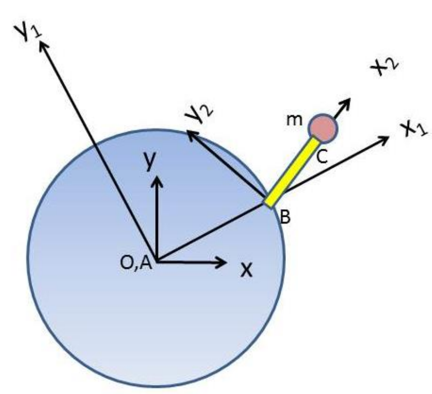


Figure 4

Concept question: What is the rotation rate experienced by the passenger with respect to the fixed coordinate system attached to the ground?

A. 4 rev/min

B. 10 rev/min
C. 16 rev/min

Problem 5: Cart and rotating mass: A block of mass M_b is constrained to horizontal motion by rollers. Let the acceleration of the block in the inertial frame O_{xyz} be designated as $\vec{a}_{A/O} = \ddot{x}\hat{i}$ which is assumed to be known. Point A is fixed to the block. A massless rod rotates about point A with an angular rate of $\dot{\vartheta}$. A point mass, m, is attached to the end of the rotating rod at point B. This problem will appear several times over the span of this subject. It introduces one of the most important and most common problems in mechanical engineering—the unbalanced rotor. You can watch an extreme illustration of this effect in this video.

A. Find an expression for the total acceleration of the mass, m, in the fixed inertial frame O_{xyz}. That is find $\mathbf{a}_{B/O}$. Express the result in terms of x and y components in the O_{xyz} frame. In other words use the unit vectors associated with the O_{xyz} frame. Remember to include the contribution from $\vec{a}_{A/O} = \ddot{x}\hat{i}$ in your answer.

B. What is the magnitude and direction of the force that the rod applies to the mass, m? What is the force and direction that the rod places on the pivot point at A?

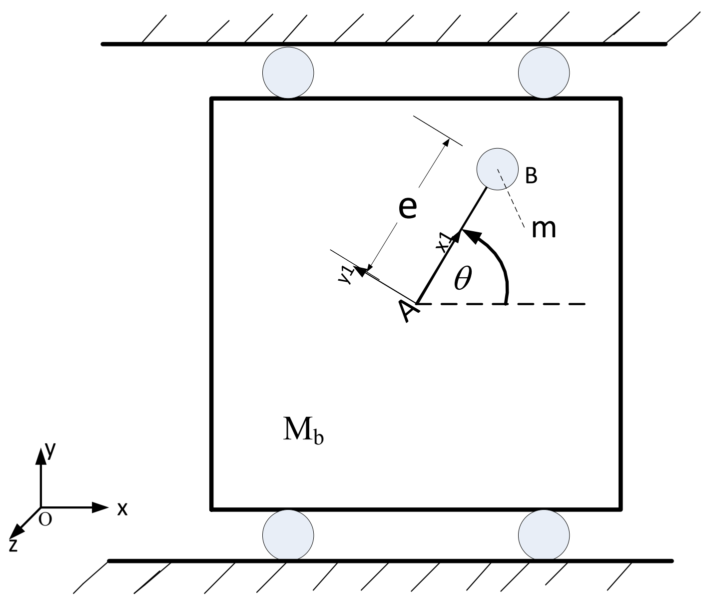


Figure 5

Concept question: When considered as a two particle system, the block and the rotating mass have a center of mass which one could calculate as a function of the position of the rod as it rotates. When viewed from a fixed inertial frame external to the block, the center of mass of the system

A. Moves left when the ball is in the left half of the circular track
B. Moves right when the ball is in the left half of the circular track
C. Does not move left or right