Thermal properties of foams

- closed cell foams widely used for thermal insulation
- only materials with lower conductivity are aerogels (tend to be brittle + weak) + vacuum insulation panels
- low thermal conductivity of foams arises from:
    - low volume fraction of solid
    - high volume fraction of gas with low λ
    - small cell size suppresses convection + radiation (through repeated absorption + reflection)

- applications: buildings, refrigerated vehicles, LNG tankers
- foams also have good thermal shock resistance since coeff. of thermal expansion of foam equal to that of the solid + modulus much lower ($\epsilon = \alpha \Delta T$ $\quad \sigma = E \alpha \Delta T = \sigma_f$)

  $\Rightarrow$ used as heat shields

- ceramic foams used as firebrick
  - ceramic has high T_m
  - foam - low λ - low heat loss
  - low heat capacity - lowers energy to heat furnace to temperature
  - good thermal shock resistance - resists spalling

Thermal conductivity, λ

- steady state conduction (T constant with time)

Fourier's law: $q = -\lambda \nabla T$

1D: $q = -\lambda \dfrac{dT}{dx}$

q = heat flux [J/(m²s)]

λ = thermal conductivity [W/mK]

∇T = temperature gradient

$= i \dfrac{\partial T}{\partial x} + j \dfrac{\partial T}{\partial y} + k \dfrac{\partial T}{\partial z}$

- non-steady heat conduction (T varies with time, τ)

$$\frac{\partial T}{\partial \tau} = a \frac{\partial^2 T}{\partial x^2}$$

a = thermal diffusivity $= \dfrac{\lambda}{\rho c_p}$
[m²/s]

ρ = density

c_p = specific heat = heat req'd to raise temp. of unit mass by 1°K

ρc_p = volumetric heat capacity [J/m³K]

- values for λ, a Table 7.1

Table 7.1 Thermal conductivities and diffusivities

Material	Thermal conductivity λ(W/m K)	Thermal diffusivity a (m^2/s)
Copper (solid)	384[a]	8.8×10^{-5} [a]
Aluminium (solid)	230[a]	8.9×10^{-5} [a]
Alumina (solid)	25.6[a]	8.2×10^{-6} [a]
Glass (solid)	1.1[a]	4.5×10^{-7} [a]
Polyethylene (solid)	0.35[a]	1.7×10^{-7} [a]
Polyurethane (solid)	0.25[c]	
Polystyrene (solid)	0.15[a]	1.0×10^{-7} [a]
Air	0.025[a]	–
Carbon dioxide	0.016[a]	–
Trichlorofluoromethane (CCl_3F)	0.008[a]	–
Oak ($\rho^*/\rho_s = 0.40$)	0.150[a]	–
White pine ($\rho^*/\rho_s = 0.34$)	0.112[a]	–
Balsa ($\rho^*/\rho_s = 0.09$)	0.055[a]	–
Cork ($\rho^*/\rho_s = 0.14$)	0.045[a]	–
Polystyrene foam ($\rho^*/\rho_s = 0.025$)	0.040[b]	1.1×10^{-6} [b]
Polyurethane foam ($\rho^*/\rho_s = 0.02$)	0.025[b]	9.0×10^{-7} [b]
Polystyrene foam ($\rho^*/\rho_s = 0.029$–0.057)	0.029–0.035[d]	
Polyisocyanurate foam, (CFC-11) ($\rho^* = 32\ \mathrm{kg/m^3}$)	0.020[d]	
Phenolic foam, (CFC-11, CFC-113) ($\rho^* = 48\ \mathrm{kg/m^3}$)	0.017[d]	
Glass foam ($\rho^*/\rho_s = 0.05$)	0.050[d]	
Glass wool ($\rho^*/\rho_s = 0.01$)	0.042[d]	
Mineral fibre ($\rho^*/\rho_s = 4.8$–$32\ \mathrm{kg/m^3}$)	0.046[d]	

All values for room temperature.

References

[a]Handbook of Chemistry and Physics, 66th edn (1985–6) Chemical Rubber Co. ed. R. C. Weast.

[b]Patten, G. A. and Skochdopole, R. E. (1962) *Mod. Plast.*, **39**, 149.

[c]Schuetz, M. A. and Glicksman, L. R. (1983) *Proc. SPI 6th International Technical/Marketing Conference*, pp. 332–40.

[d]Glicksman, L. R. (1994) Heat transfer in foams, in *Low Density Cellular Plastics* ed. Hilyard, N. C. and Cunningham, A. Chapman and Hall.

Data for thermal conductivity and thermal diffusivity

thermal diffusivity, a

- materials with a high value of a rapidly adjust their temp. to that of surroundings, because they conduct heat rapidly in comparison to their volumetric heat capacity; do not require much energy to reach thermal equilibrium

  e.g. Cu $a = 112 \times 10^{-6}\ m^2/s$

  nylon $a = 0.09 \times 10^{-6}\ m^2/s$

  wood $a = 0.082 \times 10^{-6}\ m^2/s$

Thermal conductivity of a foam, λ^*

λ^* - contributions from
- conduction through solid, λ_s^*
- " " gas, λ_g^*
- convection within cells, λ_c^*
- radiation through cell walls + across voids, λ_r^*

$$\lambda^* = \lambda_s^* + \lambda_g^* + \lambda_c^* + \lambda_r^*$$

- conduction through solid: $\lambda_s^* = \eta \lambda_s (\rho^*/\rho_s)$ η = efficiency factor ~ 2/3
- conduction through gas: $\lambda_g^* = \lambda_g (1 - \rho^*/\rho_s)$

For example, 2.5% dense closed-cell polystyrene foam $\lambda^* = 0.040$ W/mK

$\lambda_s = 0.15$ W/mK $\quad \lambda_g = 0.025$ W/mK (air)

$$\lambda_s^* + \lambda_g^* = \tfrac{2}{3}(0.15)(0.025) + (0.025)(0.975)$$
$$= 0.003 + 0.024$$
$$= 0.027 \text{ W/mK}$$

- most of conductivity comes from conduction through gas
- foams for insulation blown with low λ_g gases
- problem with aging - low λ_g gases diffuse out of foam over time, air diff. in, $\lambda_g^* \uparrow$

<u>convection within the cell</u>

- gas rises + falls due to density changes with temperature
- density changes - buoyancy forces
- also have viscous forces from drag of gas as it moves past cell wall

- convection important if Rayleigh number > 1000

$$Ra = \frac{\rho g \beta \Delta T_c l^3}{\mu a}$$

ρ = density of gas

g = grav. acc'n

β = volume expansion for a gas = $1/T$ (isobaric)

ΔT_c = temp. diff. across 1 cell

l = cell size

μ = dynamic viscosity of gas

a = thermal diffusivity

convection

for $Ra = 1000$ air $p = p_{atm}$ T = room temp $\beta = 1/T = 1/300$ $(°K^{-1})$

$\Delta T_c = 1°K$ $\mu_{air} = 2\times10^{-5}$ Pa·s $\rho_{air} = 1.2$ kg/m^3

$a_{air} = 2.0\times10^{-5}$ m^2/s

$\Rightarrow l = 20$ mm

- convection important if cell size > 20 mm
- most foams: cell size < 1 mm ⇒ convection negligible

---

radiation

- heat flux passing by radiation, $\dot{q}_r$, from a surface at temperature T_1 to one at a lower temp. T_0, with a vacuum between them, is:

$$\dot{q}_r = \beta_1 \sigma (T_1^4 - T_0^4) \qquad \text{Stefan's law}$$

σ = Stefan's constant $= 5.67\times10^{-8}$ W/m^{2}K^4

β_1 = constant (<1) describing emissivity of the surfaces

(emitted radiant flux per unit area of sample relative to black body radiator at same temp. + conditions; black body absorbs all energy; black body emissivity = 1) radiation

- if put foam between two surfaces, heat flux is reduced, since radiation is absorbed by the solid + reflected by cell walls

- attenuation $q_r = q_r^{\circ} \exp(-K^* t^*)$ Beer's law

  K^* = extinction coeff. for foam

  t^* = thickness of foam

- for optically thin walls + struts ($t < 10\mu m$) (transparent to radiation)

$$K^* = (\rho^*/\rho_s)\, K_s$$

- heat flux by radiation then

$$q_r = \lambda_r^* \frac{dT}{dx}$$

$$q_r = \beta_1 \sigma (T_1^4 - T_0^4) \exp[-(\rho^*/\rho_s) K_s t^*] = \lambda_r^* \frac{dT}{dx}$$

- obtain λ_r^* using some approximations:

approximations :

$$\frac{dT}{dx} \approx \frac{T_1 - T_0}{t^*} = \frac{\Delta T}{t^*}$$

$$T_1^4 - T_0^4 \approx 4\,\Delta T\,\bar{T}^3 \qquad \bar{T} = \left(\frac{T_1 + T_0}{2}\right)$$

$$q_r = \beta_1\,\sigma\,4\cancel{\Delta T}\,\bar{T}^3 \exp\left[-(\rho^*/\rho_s)\,k_s\,t^*\right] = \lambda_r^* \frac{\Delta T}{t^*}$$

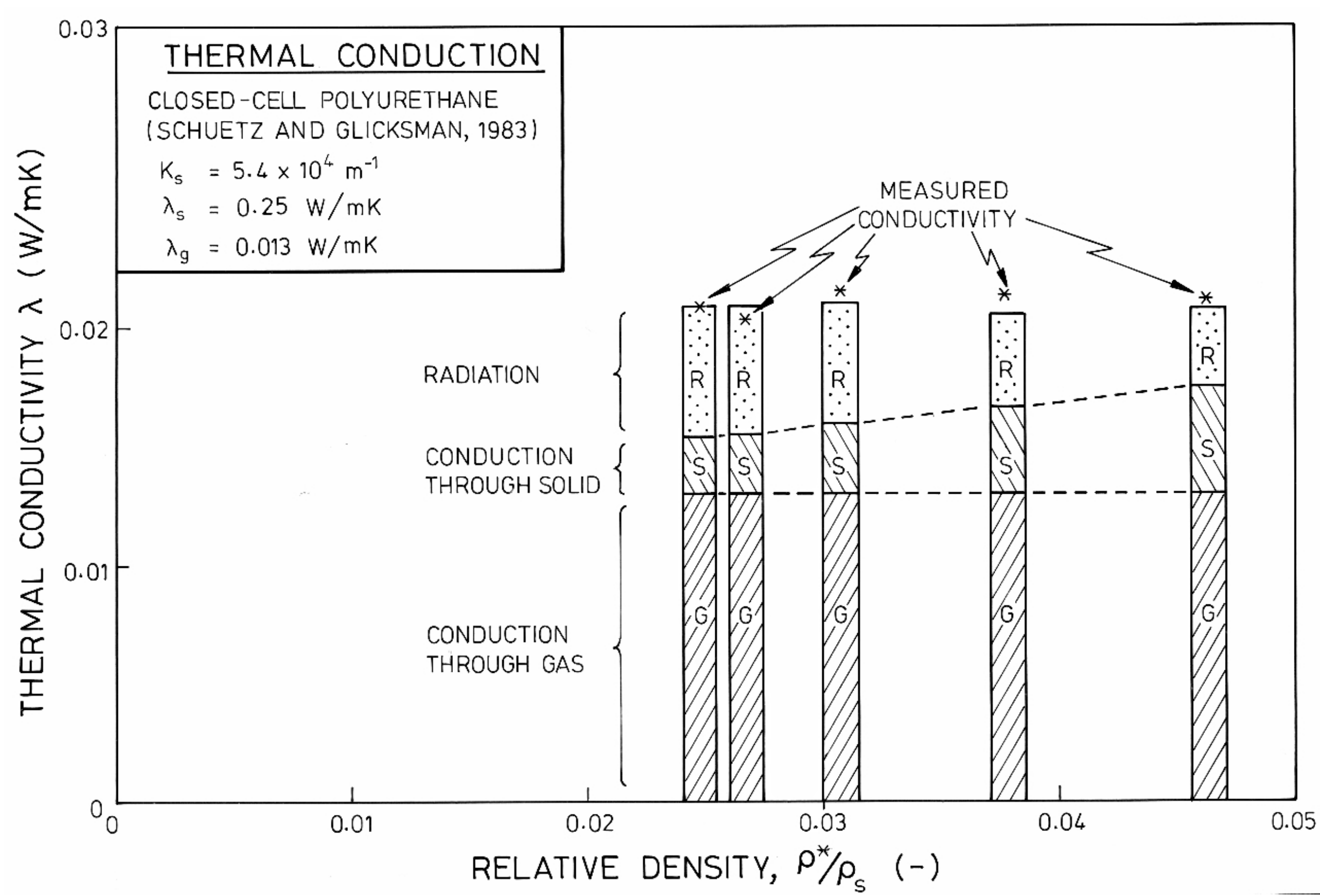
$$\lambda_r^* = 4\beta_1\,\sigma\,\bar{T}^3\,t^* \exp\left[-(\rho^*/\rho_s)\,k_s\,t^*\right]$$

as $\rho^*/\rho_s \downarrow$ $\lambda_r^* \uparrow$

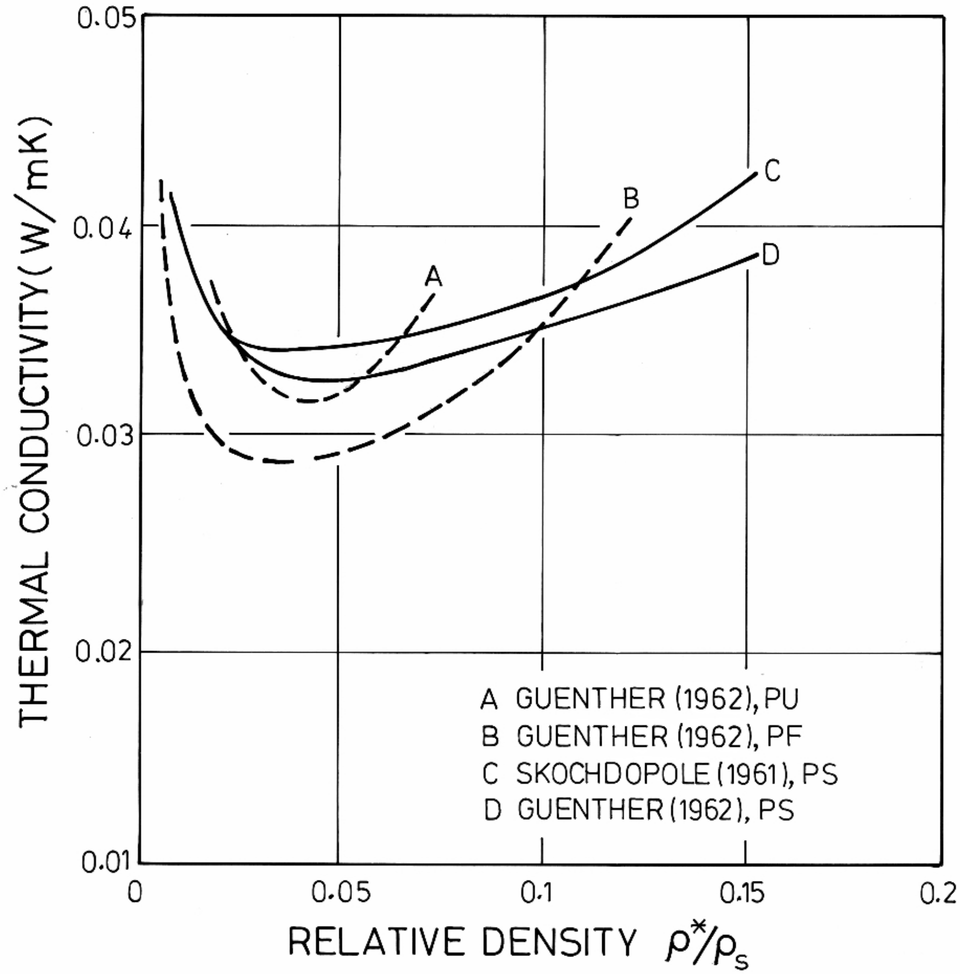
Thermal conductivity

- relative contributions of λ_s^*, λ_g^*, λ_r^* shown in Fig 7.1
  - largest contribution λ_g^*
- λ^* plotted against relative density Fig 7.2
  - minimum @ between ρ^*/ρ_s of 0.03 & 0.07
  - at which point λ^* only slightly larger than λ_g^*
  - at low ρ^*/ρ_s, λ^* increases - increasing transparency to radiation (also, walls may rupture)
  - tradeoff : as $\rho^*/\rho_s \downarrow$, $\lambda_s^* \downarrow$ but $\lambda_r^* \uparrow$

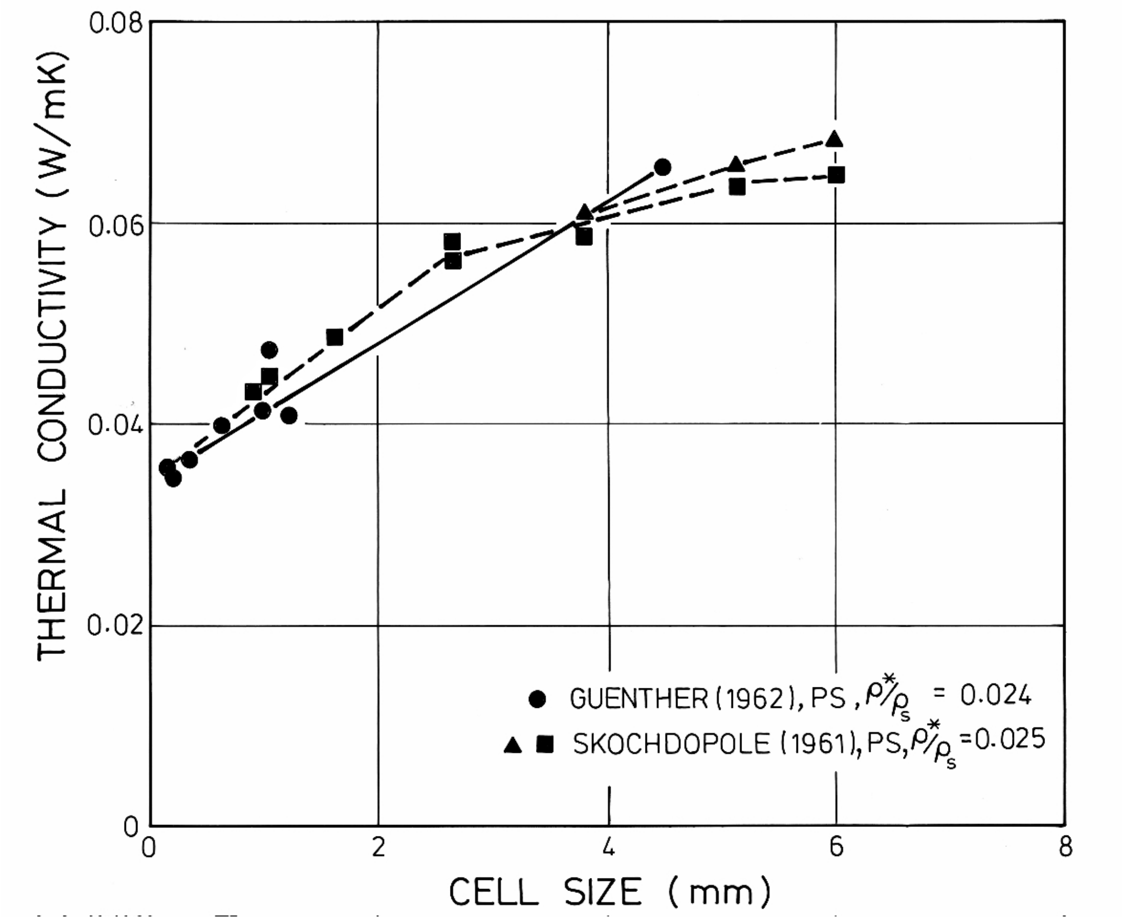
Thermal Conductivity



Cond. Vs. Relative Density



Cond. vs. Cell Size



Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

λ^* plotted against cell size Fig 7.3

- λ^* increases with cell size
- radiation reflected less often

Note: aerogels
- pore size < 100 nm
- mean free path of air at ambient pressure = 68 nm
  ↳ avg. distance molecules move before collision with another molecule
- aerogels - pore size < mean free path of air - reduces conduction through gas.

Specific heat C_p

- specific heat = energy req'd to raise temp. of unit mass by unit temp

$$C_p^* = C_{ps} \quad [J / kg\,°K]$$

Thermal expansion coefficient

$\alpha^* = \alpha_s$ (consider foam as framework)

(but if closed-cell foam cooled dramatically - gas can freeze, collapsing the cells; or if heated - gas expands, increasing the internal pressure + strains

Thermal shock resistance

- if material subjected to sudden change in surface temp. - induces thermal stresses at surface + cracking + spalling.
- consider material at T_1 dropped into water at T_2 ($T_1 > T_2$)
  - surface temp. drops to T_2, contracting surface layers
  - thermal strain $\epsilon_T = \alpha \Delta T$
- if surface bonded to underlying block of material - constrained to original dimensions

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$$\sigma = \frac{E \alpha \Delta T}{1-\nu} \quad \text{in the surface}$$

- cracking/spalling when $\sigma = \sigma_f$

$$\Delta T_c = \frac{\sigma_f (1-\nu)}{E \alpha} = \text{critical } \Delta T \text{ to just cause cracking}$$

- for foam: (open cells)

$$\Delta T_c^* = \frac{0.2\, \sigma_{fs} (\rho^*/\rho_s)^{3/2} (1-\nu^*)}{E_s (\rho^*/\rho_s)^2 \alpha_s} = \frac{0.2}{(\rho^*/\rho_s)^{1/2}} \frac{\sigma_{fs} (1-\nu)}{E_s \alpha_s} = \frac{0.2}{(\rho^*/\rho_s)^{1/2}} \Delta T_{cs}$$

- as foam density ↓ ΔT_c^* ↑   firebrick - porous ceramic

Case study: optimization of foam density for thermal insulation

- there is an optimal foam density for a given thermal insulation problem
- already saw λ^* has a minimum as a $f(\rho^*/\rho_s)$
- typically, have a constraint on the foam thickness, t^* $\quad t^* = \text{constant}$

$$\lambda^* = \frac{2}{3}(\rho^*/\rho_s)\lambda_s + (1-\rho^*/\rho_s)\lambda_g^* + 4\beta_1 \sigma \bar{T}^3 t^* \exp[-K_s(\rho^*/\rho_s)t^*]$$

- what is optimum ρ^*/ρ_s for a given t^*?

$$\frac{d\lambda^*}{d(\rho^*/\rho_s)} = 0 \Rightarrow (\rho^*/\rho_s)_{opt} = \frac{1}{K_s t^*} \ln\left[\frac{4K_s\beta_1\sigma\bar{T}^3 t^{*2}}{\frac{2}{3}\lambda_s - \lambda_g}\right]$$

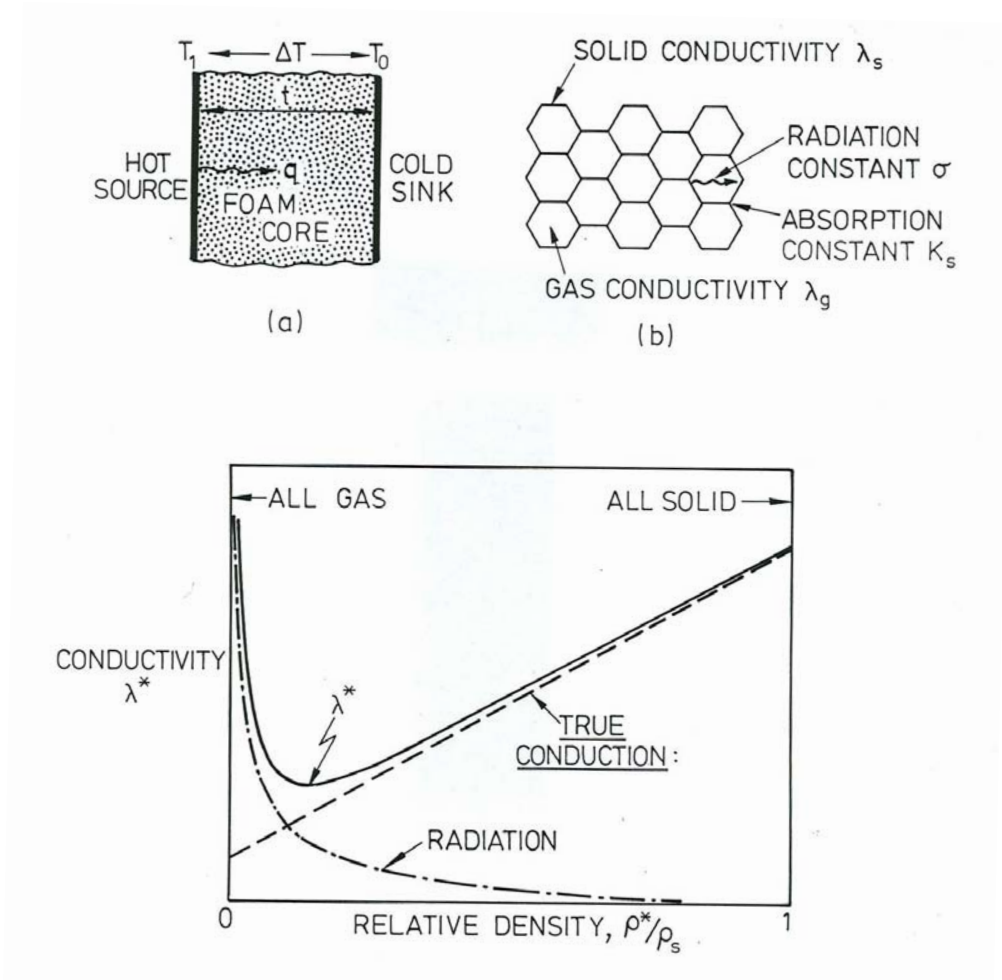
- as given thickness t^* increases, $(\rho^*/\rho_s)_{opt}$ decreases
- as $\bar{T}$ increases, $(\rho^*/\rho_s)_{opt}$ increases

e.g. coffee cup $t^* = 3\text{mm}$ $(\rho^*/\rho_s)_{opt} = 0.08$

refrigerator $t^* = 50\text{mm}$ $(\rho^*/\rho_s)_{opt} = 0.02$

(see PP slide Table 7.3 for data used in calculation).

Case Study: Optimization of Relative Density



Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figures courtesy of Lorna Gibson and Cambridge University Press.

Case Study: Optimum Relative Density

Table 7.3 Data for optimization case study

Extinction coefficient of solid polymer, K_s	$5.67 \times 10^4\ \mathrm{m}^{-1}$
Emissivity factor, β_1	0.5
Conductivity of solid polymer, λ_s	0.22 W/m K
Conductivity of gas, λ_g	0.02 W/m K
Mean temperature, $\bar{T}$	300°K
Stefan's constant, σ	$5.67 \times 10^{-8}\ \mathrm{W/m^2K^4}$

Case study: insulation for refrigerators

- insulation reduces energy cost, but has a cost itself
- total cost = cost of insulation + cost of energy lost by heat transfer through walls
- objective function: minimize total cost
- given: x = thickness of insulation $\quad$ C_M = cost of insulation/mass

  ΔT = temp. diff. across insulation $\quad$ C_E = " " energy/joule

  t_ℓ = design life of refrigerator $\quad$ C_T = total cost/area

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$$C_T = x \rho^* C_M + \lambda \frac{\Delta T}{x} t_\ell C_E \qquad \left(\text{heat flux } q = \lambda \frac{\Delta T}{x} \quad \frac{J}{m^2 s}\right)$$

define $M_1 = \frac{1}{\rho^* C_M}$ $\qquad M_2 = \frac{1}{\lambda}$

$$\frac{C_T}{x} = \frac{1}{M_1} + \left[\frac{\Delta T}{x^2} t_\ell C_E\right] \frac{1}{M_2}$$

- two terms are equal when:

$$M_2 = \left[\frac{\Delta T}{x^2}\, t_\ell\, C_E\right] M_1$$

↳ coupling constant

- family of parallel straight lines of constant value $\frac{\Delta T}{x^2} t_\ell C_E$
- Fig 13.11 $\Delta T = 20°$ $x = 10\,mm$ $C_E = \$0.01/MJ$

two lines for $t_\ell = 10$ years & $t_\ell = 1$ month

(note error in book $t_\ell = 10$ yrs line should be moved over)

- also plotted a set of curved contours - plots of G_T/x
  - as move up + to right of plot, the value of G_T/x ↓

- for $t_\ell = 10$ years ⇒ phenolic foam $\rho^* = 0.035\ Mg/m^3$

$t_\ell = 1$ month ⇒ EPS $\rho^* = 0.02\ Mg/m^3$

PP $\rho^* = 0.02\ Mg/m^3$

Case Study:
Insulation for Refrigerators