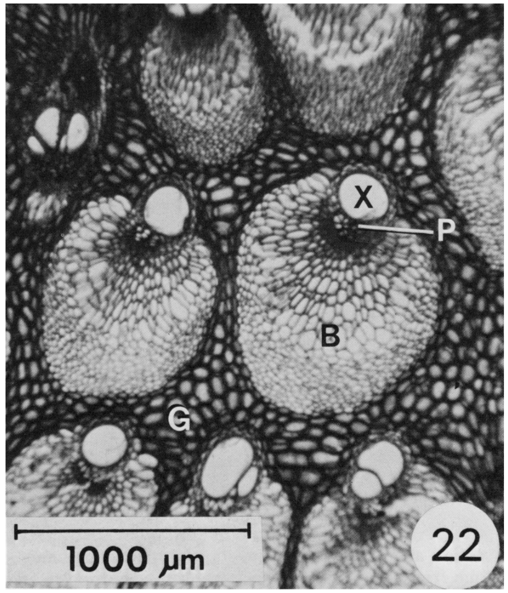
Plant Stems with Radial Density Gradients

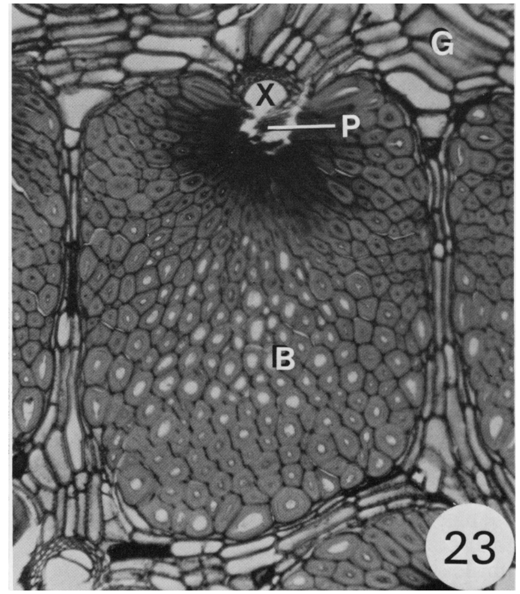
Coconut Palm
http://en.wikipedia.org/wiki/
Image:Palmtree_Curacao.jpg

Palm: Density Gradient

Vascular bundles: Honeycomb

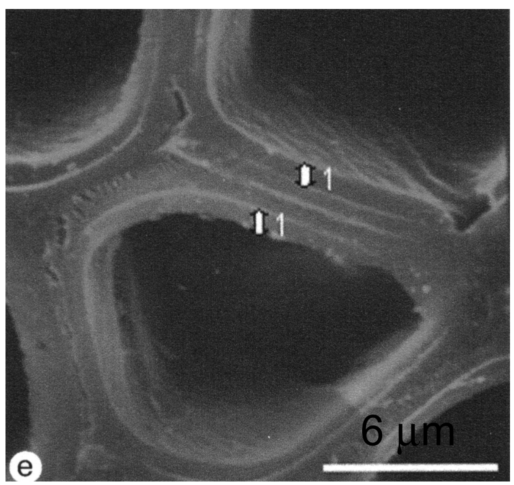
Ground tissue (Parenchyma): Foam



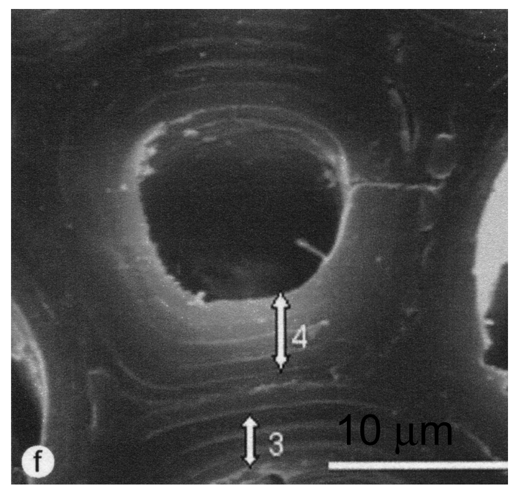


Peripheral Stem Tissue

Rich, 1987



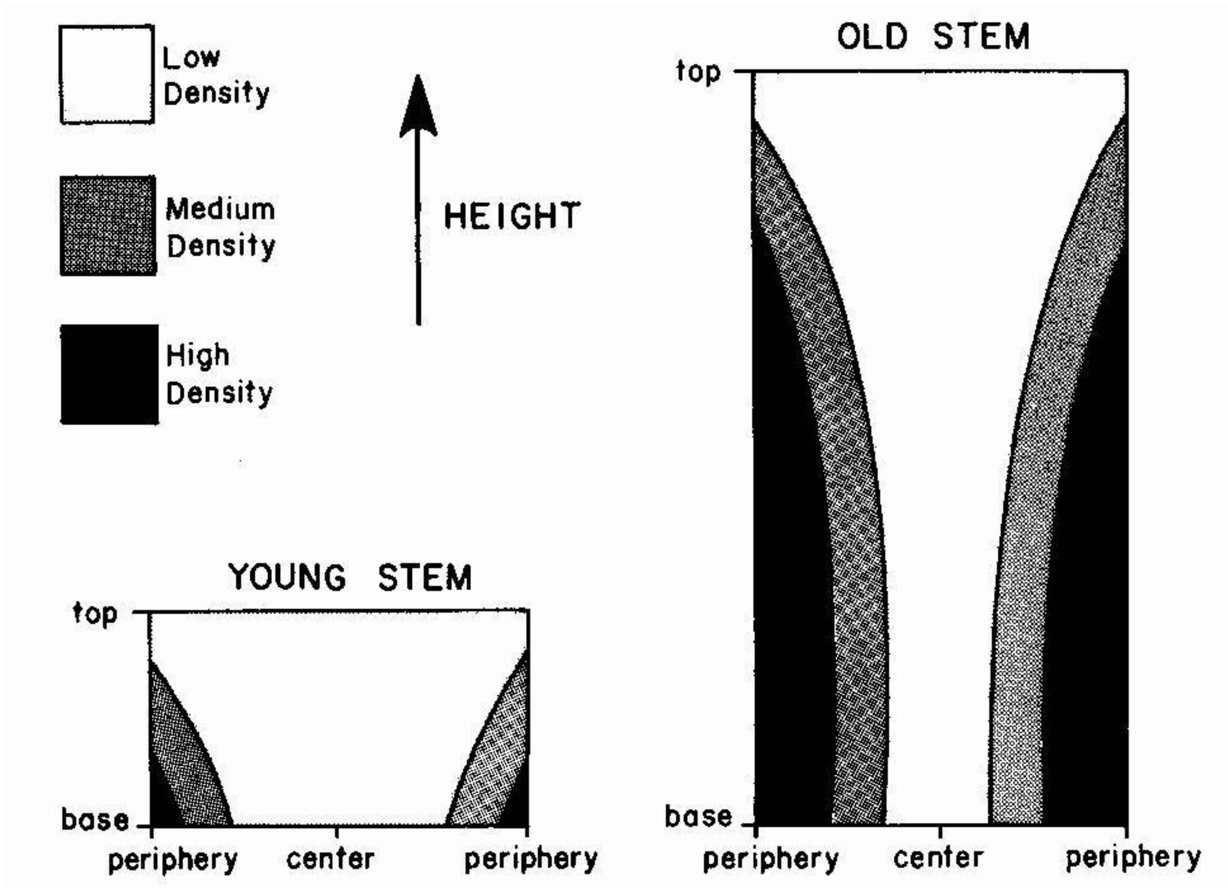
Young



Old

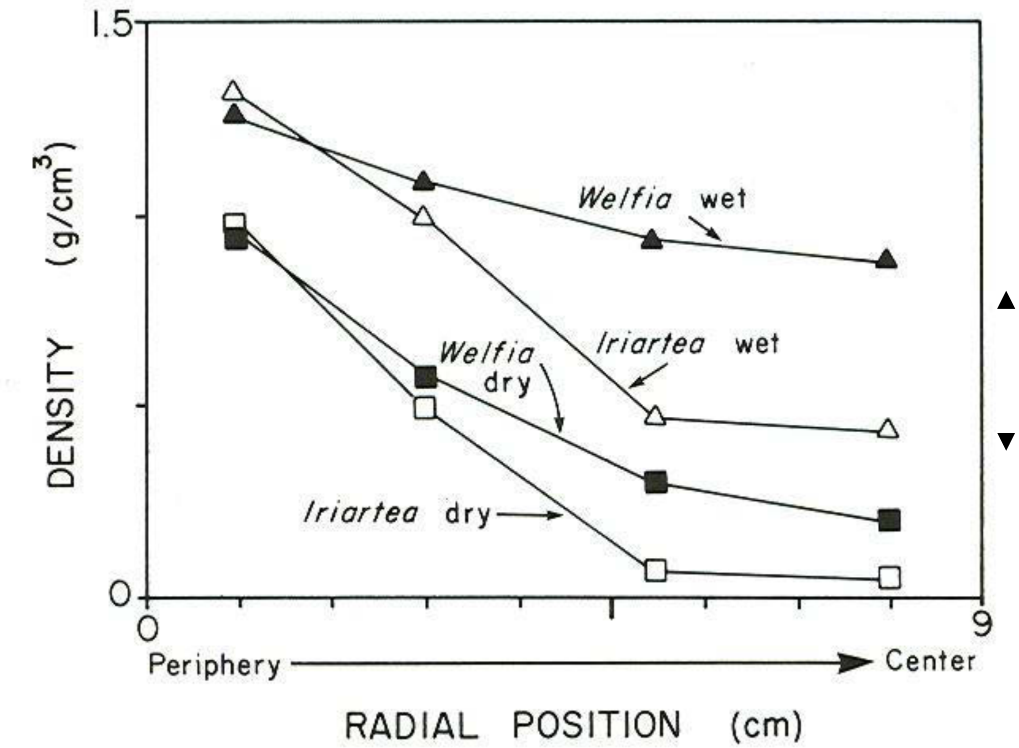
Kuo-Huang et al., 2004

Palm Stem: Density Gradient



Rich, PM (1987) Bot.Gazette 148, 42-50.

Palm Stem: Density at Breast Height



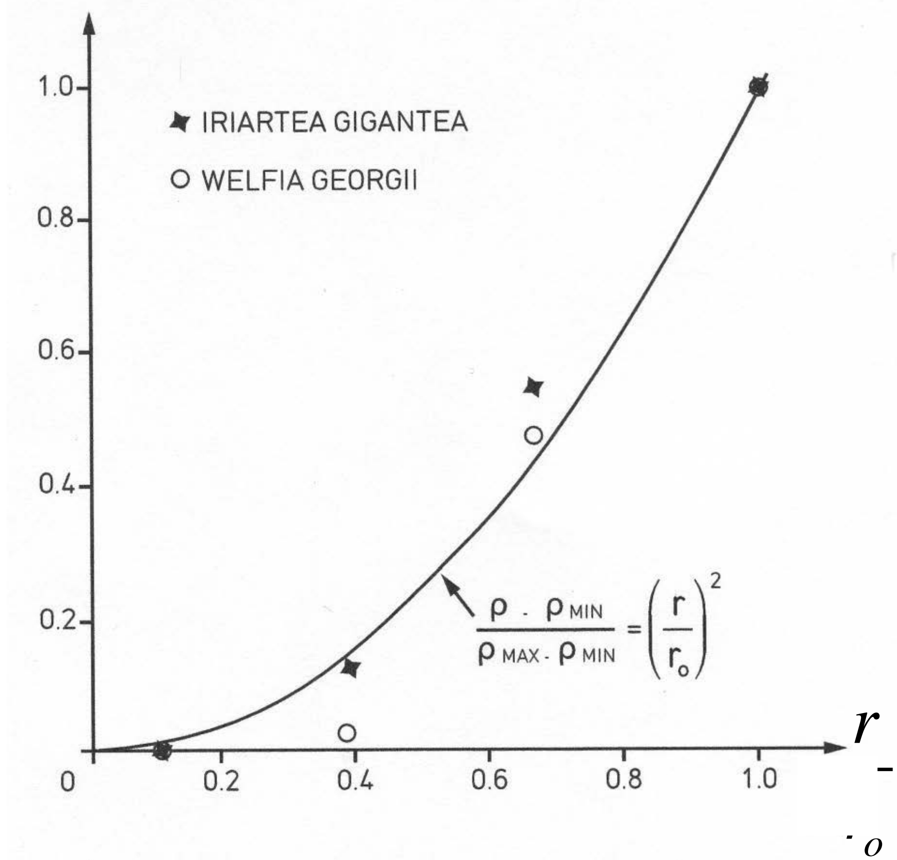
Densities of common woods

A single mature palm has a similar range of density as nearly all species of wood combined

Rich, PM (1987) Bot.Gazette 148, 42-50.

Palm Stem: Density Gradient

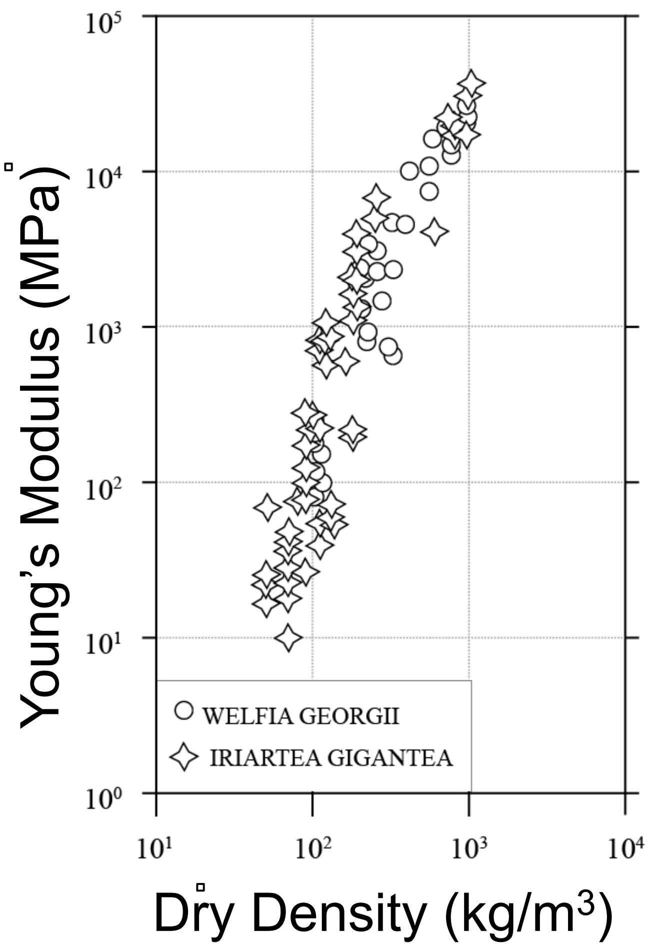
$$\frac{\rho - \rho_{min}}{\rho_{max} - \rho_{min}}$$

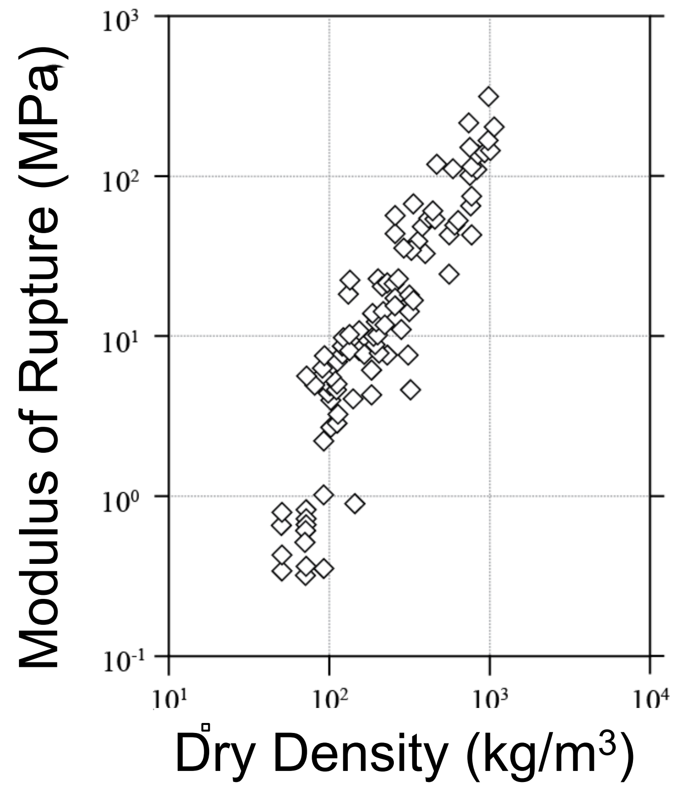


Iriartea gigantea:
$\rho_{min} \approx 0$

$$\frac{\rho^*}{\rho_{max}} = \left(\frac{r}{r_o}\right)^2$$

r_o is the outer radius





Along Grain

Iriartea gigantea

$$E^* = C\left(\frac{\rho^*}{\rho_{\max}}\right)^{2.5}$$

$$\sigma^* = C\left(\frac{\rho^*}{\rho_{\max}}\right)^{2}$$

Rich, PM (1987) Bot.Gazette <u>148</u>, 42-50.

Palm Properties

- Prismatic cells in palm deform axially (like wood loaded along the grain)
- If E_s was constant, would expect: $E^* = E_s\left(\rho^*/\rho_s\right)$
- But measure: $E^* = C\left(\rho^*/\rho_{\max}\right)^{2.5}$
- Similarly with strength

Palm Properties

- E_s = 0.1-3.0 GPa in low density palm tissue from *Washingtonia robusta* (Rueggeberg et al., 2008)
- Estimate in dense tissue (E^* = 30 GPa; ρ^*= 1000 kg/m^3) E_s = 45 GPa
- Large variation in E_s due to additional secondary layers in cell walls of denser tissue and increased alignment of cellulose microfibrils in those layers

Palm: Mechanical Efficiency Bending Stiffness

$$\rho = \left(\frac{r}{r_o} \right)^n \rho_{\max}$$

$$\left(EI \right)_{gradient} = \frac{C \pi r_o^4}{mn + 4}$$

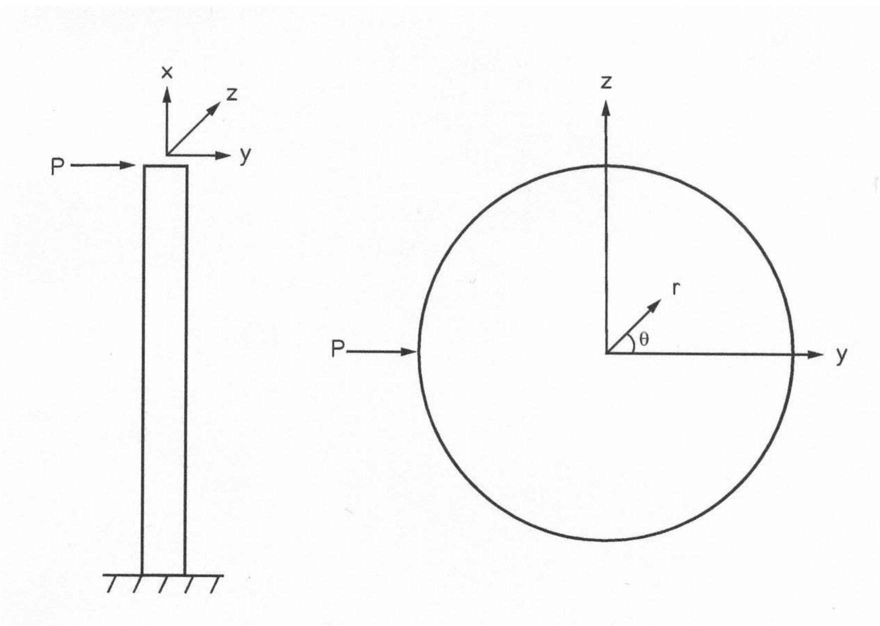
$$E = C \left(\frac{\rho}{\rho_{\max}} \right)^m = C \left(\frac{r}{r_o} \right)^{mn}$$

$$\frac{\left(EI \right)_{gradient}}{\left(EI \right)_{uniform}} = \frac{4}{mn + 4} \left(\frac{n + 2}{2} \right)^m$$

Iriartea gigantea: n = 2, m = 2.5

$(EI)_{gradient}/(EI)_{uniform} = 2.5$

Palm: Mechanical Efficiency Bending *Stress* Distribution



$$\sigma(y) = E\varepsilon = E\kappa y$$

$$\sigma(r,\theta) = C\left(\frac{r}{r_o}\right)^{mn} \kappa r \cos\theta \propto r^{mn+1}$$

I. gigantea: n =2, m = 2.5

$$\sigma \propto r^6$$

Palm: Mechanical Efficiency Bending *Strength* Distribution

$$\sigma^* \propto \left(\frac{\rho}{\rho_{\max}} \right)^q \propto \left(\frac{r}{r_o} \right)^{nq}$$

Iriartea gigantea: n = 2, q = 2

$$\sigma^* \propto r^4$$

Palm bending stress, strength

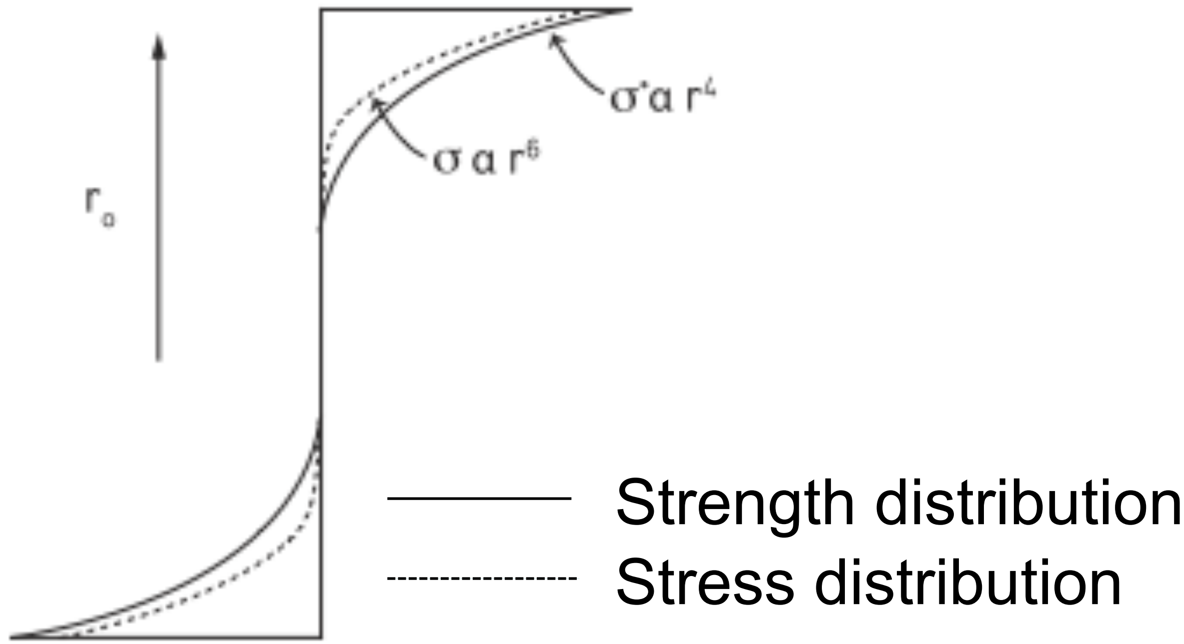


Figure sources

Sources for all figures in:
Cellular Materials in Nature and Medicine (2010)

Circular sections with radial density gradients: Palm stems

- palms can grow up to 20-40 m – largest stresses from hurricane winds
- unlike trees, palms do not have a cambium layer at the periphery, with dividing cells to allow increase in diameter as palm grows in height
- instead, diameter of palm roughly constant as it grows in height
- increasing stress resisted by cell walls increasing in thickness
- add additional layers of secondary cell wall

- produces radial density gradient
  - density higher at periphery + at base of stem
  - a single stem can have densities from 100 - 1000 kg/m^3, nearly spanning the density range of all woods (balsa ~200 kg/m^3 → lignum vitae 1300 kg/m^3)
- specimens of palm taken from different radial positions tested in bending (Paul Rich, 1980s)
- found $E^*_{axial} = C' \rho^{*2.46}$
- might expect $E^*_{axial} \propto \rho$ – vascular bundles honeycomb-like

- but additional cell wall layers change E_s : data $E_s = 0.1 - 3$ GPa
- also: lower density palm has more ground tissue (parenchyma) with
  $E \propto \rho$ if at high turgor, but $E \propto \rho^2$ if at low turgor. (bending specimens dry)
- modulus of rupture $\sigma^* = C'' \rho^{*\,2.05}$
- radial density gradient increases flexural rigidity
- compare (EI) with density gradient to (EI) of section of same mass + radius
  but uniform density

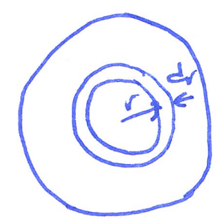
- for Iriartea gigantea:

$$\left(\frac{\rho^*}{\rho_{max}}\right) = \left(\frac{r}{r_o}\right)^n \qquad r_o = \text{outer radius}, \quad n = 2$$

$$E = C\left(\frac{\rho}{\rho_{max}}\right)^m = C\left(\frac{r}{r_o}\right)^{mn}$$

$$(EI)_{gradient} = \int_0^{r_o} C\left(\frac{\rho}{\rho_{max}}\right)^m \frac{2\pi r\, r^2\, dr}{2}$$

$$= \int_0^{r_o} C\left(\frac{r}{r_o}\right)^{mn} \pi r^3\, dr$$



$$\int r^2\, 2\pi r\, dr = J = 2I$$

$$(EI)_{gradient} = \frac{C\pi}{r_0^{mn}} \int_0^{r_0} r^{mn+3}\, dr$$

$$= \frac{C\pi}{r_0^{mn}} \frac{r_0^{mn+4}}{mn+4}$$

$$= \frac{C\pi r_0^4}{mn+4}$$

Equivalent mass, r_0, uniform density $\bar{\rho}$:

$$\frac{\bar{\rho}}{\rho_{max}} = \frac{1}{\pi r_0^2} \int_0^{r_0} \left(\frac{r}{r_0}\right)^n 2\pi r\, dr = \frac{1}{\pi r_0^2} \frac{2\pi}{r_0^n} \frac{r_0^{n+2}}{n+2} = \frac{2}{n+2}$$

$$(EI)_{\substack{uniform\\density}} = C\left(\frac{\bar{\rho}}{\rho_{max}}\right)^m \frac{\pi r_0^4}{4}$$

$$= C\left(\frac{2}{n+2}\right)^m \frac{\pi r_0^4}{4}$$

$$\frac{(EI)_{gradient}}{(EI)_{uniform}} = \frac{C\pi r_0^4}{mn+4} \frac{4}{C\pi r_0^4} \left(\frac{n+2}{2}\right)^m = \frac{4}{mn+4}\left(\frac{n+2}{2}\right)^m$$

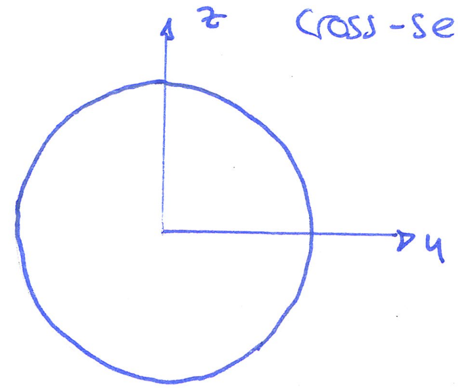
I. gigantea $m = 2.5$ $n = 2$

$$\frac{(EI)_{gradient}}{(EI)_{uniform}} = 2.5$$

Stress + Strength distribution

$$\frac{\rho}{\rho_{max}} = \left(\frac{r}{r_0}\right)^n$$

cross-section



κ = curvature at the cross-section

y = distance from neutral axis

$E = C\left(\frac{\rho}{\rho_{max}}\right)^m$

Stress

$$\sigma(y) = E\epsilon = E\kappa y$$

$$= C\left(\frac{\rho}{\rho_{max}}\right)^m \kappa y$$

$$= C\left(\frac{r}{r_0}\right)^{mn} \kappa r$$

$$\sigma(r) \propto r^{mn+1}$$

I. gigantea $m = 2.5$ $n = 2$ $\sigma(r) \propto r^6$

Strength

$$\sigma^*(r) = C'\left(\frac{\rho}{\rho_{max}}\right)^q = C\left(\frac{r}{r_0}\right)^{nq}$$

$$\sigma^*(r) \propto r^{nq}$$

I gigantea $q = 2$ $n = 2$ $\sigma^*(r) \propto r^4$

Figure: if max normal stress @ $r = r_0$ is $\sigma = \sigma^*$ then bending stress distribution closely follows strength distribution!