M A S S A C H U S E T T S   I N S T I T U T E   O F   T E C H N O L O G Y

3.014 Materials Laboratory
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Laboratory 2: Module γ_2

Laser Diffraction from Macroscopic Planar Analogues

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Objectives
- Understand the fundamental diffraction process
- Become familiar with the Fourier transform formalism for treating far-field diffraction intensities.
- Develop an intuitive feel for real and reciprocal spaces, image and recoding planes, lens and lens-less diffraction experiments.
- Discover what information about an object is conveyed in diffraction patterns.
- Distinguish between Fraunhofer diffraction patterns of crystalline, quasicrystalline, and amorphous arrangements of repeated objects.
- Distinguish fundamental from derivative structures with superlattices by diffraction.

Summary of Tasks
1. Set up a laser diffractometer for recording object images and far-field (Fraunhofer) diffraction patterns.
2. Record Fraunhofer diffraction patterns from a series of two-dimensional objects and repeated objects with simple illustrative transmission functions.
3. Perform mathematically a Fourier transform of a simple object transmission function and compare to the experimentally recorded diffracted intensity.

Introduction

   Visible light is a convenient wave with which to observe diffraction effects because we can observe both objects and their diffraction effects directly with our eyes. The instrument generally used is the *laser diffractometer* (Fig. 11) that also doubles as a transmission light microscope. The wavelengths of visible light (λ = 300-700 nm) are sufficiently small that convenient macroscopic objects with millimeter dimensions d result in diffraction patterns at angles

$$\Theta \sim \lambda/d \sim 0.1° \tag{31}$$

conveniently small enough to record on flat-plate detectors, such as a fluorescent screen or digital camera CCD, and with a sufficiently *flat* Ewald sphere ($|\mathbf{k}| = \lambda^{-1}$ is large, Fig. 6b) that a whole plane of reciprocal space is sampled at once. Because diffraction phenomena scale with λ, a monochromatic source is necessary. Gas lasers of the 1-10 mW variety are a readily available source of intense, well-collimated, monochromatic
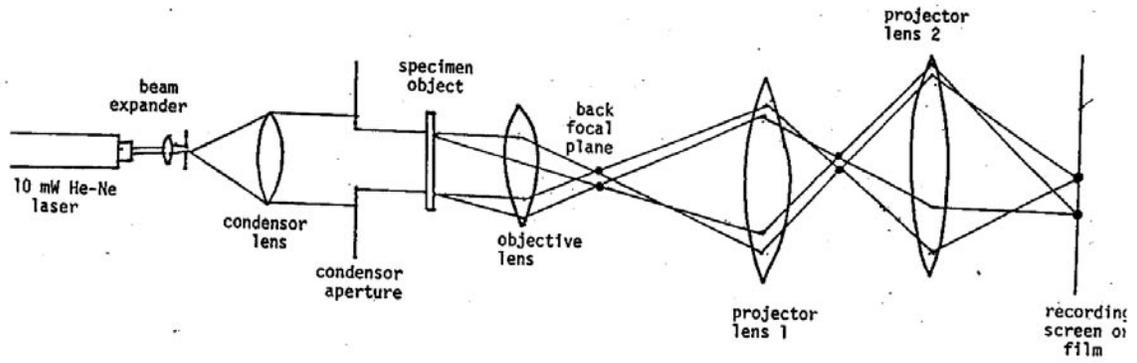
Fig. 11 LASER DIFFRACTOMETER

light. The He-Ne 3-mW laser used in this laser-diffractometer module has a wavelength $\lambda = 632.8$ nm and a beam diameter of about 0.8 mm.

The small beam diameter is inconvenient for observing readily objects of 5-10 mm dimensions and so must be expanded. This is easily done by focusing the laser beam onto a pinhole, which acts as a new source of spherical waves. If the pinhole is placed at the focal point of a convex lens ("condenser lens") with focal length $f = 200$ mm, the diverging source waves are converted to a parallel beam of much larger diameter (~20 mm). Illuminating the object with a parallel beam means we have satisfied one condition of Fraunhofer diffraction, that of an infinitely distant source: the condenser lens allows what would have to be an infinitely distant source to be situated at a finite distance from the object. Because the pinhole is not infinitely small, what is actually produced on the other side of the pinhole is an Airy function, the central disk of which is large enough to illuminate our objects.

The second criterion for Fraunhofer diffraction is that of an infinitely distant observer. While we could successfully demonstrate diffraction effects down the length of the Infinite Corridor, it is certainly more convenient to do so on a tabletop optical bench. We can analogously bring the distant observer up to a finite distance by employing a second lens ("objective lens") of similar focal length and situating the detector at the back focal plane of the lens. With this arrangement, the Fraunhofer diffraction pattern appears at the back focal plane of the objective lens. It turns out that it is at that point inconveniently small to observe by eye, so we employ a third lens ("projector lens") of much higher strength ($f \sim$ a few mm) to magnify the diffraction pattern appearing at the back focal plane, projecting the magnified image onto a translucent viewing screen, the back side of which is imaged by a digital camera coupled to computer imaging software. The projector lens is particularly convenient, because we can choose to focus either on the back focal plane of the object lens (the "diffraction pattern") or on the magnified image plane (the inverted "object image") a little further beyond the back focal plane. In the latter case, we have turned the optical bench into a microscope, making it easy to see what kind of object we have interposed (the "transmission function" of the object). This arrangement is *exactly* duplicated in a transmission electron microscope in which energetic electron beams ($\lambda \sim 3$ pm) are used instead of light, atomic-scale structures ($d \sim 300$ pm) diffract with small $\Theta \sim 1°$ diffraction angles (because of a ratio $\lambda/d \sim 100$ similar to the laser diffractometer), and image of the object transmission function can be obtained in the same way, with magnifications exceeding 10^6 possible because of small (pm) wavelength $\lambda << d$ does not impose a fundamental resolution limit at atomic dimensions.

Experimental Procedure

This experiment consists of a series of two-dimensional objects with amplitude transmission functions (no phase shifts) that have been reproduced on 35-mm film. These are to be sequentially interposed in the path of the expanded laser beam and their Fraunhofer diffraction patterns recorded. With each, start out by adjusting the projector lens so that a magnified image of the object is first produced on the screen (fine focusing can be accomplished using the objective lens). Then, move the projector lens towards the

objective lens until it focuses on the back focal plane, at which point the recording screen will display a magnified version of the Fraunhofer pattern. Save the screen images for each in the computer. If the intensity saturates the CCD in the central region of interest in the pattern, a neutral filter can be inserted between the final lens and the projection screen to reduce the laser intensity. A red filter over the camera lens reduces the contribution of stray white light from the ambient room lighting.

The objects to be examined (and their diffraction patterns explained using diffraction theory) are:

1. An edge (half black, half open). Try this both ways to see if the orientation makes a difference; the result demonstrates that diffraction introduces an inversion center even if none is present. Also note the orientation of the diffraction pattern with respect to the edge orientation, and rotate the edge to confirm that real space and reciprocal are rigidly attached to each other.

2. A vertical line aperture, and another of larger or smaller width. Note the effect of line width on the diffraction pattern.

3. A square aperture.

4. A rectangular aperture and a smaller rectangular aperture.

5. A circular aperture (this can be a difficult pattern to see, because the aberrations in the lenses are mostly rotationally symmetric and tend to confuse the pattern). The pinhole of the beam expander is another good example; you can observe that pattern by interposing a 3×5-inch file card between the pinhole and the condenser lens.

6. Two circular apertures. Vary the spacing of the apertures and their spacing.

7. Three circular apertures in a row, with equal spacing between them. This shows the effect of increasing the number of repeated objects.

8. An array of vertical line apertures (not all that expertly ruled, and the diffraction pattern picks this up!).

9. Four circular apertures arranged in a square pattern.

10. Four square apertures arranged in a square pattern. The result for both this and the four circular apertures shows how diffraction separates out the shape of a repeated object from the pattern of its repetition.

11. Four opaque squares arranged in a square pattern on a transparent background. This result demonstrates the principle of complementarity of transmission functions in diffraction. The diffraction patterns for this and its previous complement should be identical, except for a small region around the center of the pattern, where a δ-function should be evident for the transparent background. So it doesn't matter whether transmission functions are positive (radiation re-emitted) or negative (radiation absorbed), except near the origin.

12. The number "7" as an aperture. This is a classic asymmetrical object.

13. Two "7" apertures in a row.

14. A periodic array of "7" apertures. This should, again, show the separation of shape and repetition information.

15. An identical array of circular apertures. What stays the same compared to the previous object?

16. A two-dimensional "simple cubic" arrangement of identical circular apertures.

17. A corresponding two-dimensional "body-centered cubic" arrangement of identical circular apertures, with same lattice parameter as 16.

18. A two-dimensional "cesium chloride structure" arrangement of two different sizes of circular apertures with same lattice parameter as 17.

19. A periodic array of "atomic" apertures.

20. A corresponding periodic array of "diatomic molecule" apertures. This result shows the result of a zero in the structure factor $F(\boldsymbol{\kappa})$ for the unit cell canceling out intensity at reciprocal lattice points (or, in this example, a whole row of them), in this case when a feature in the unit cell (the "molecule" spacing) is a sub-multiple ($1/n$, n =2,3,4 *etc.*) of the real lattice period. Such "systematic absences" are used to deduce the internal structure of the unit cell, the size and shape of which will have been already deduced from the overall reciprocal lattice pattern.

21. A random array of circular apertures. This is a two-dimensional analogue of a glass or "amorphous" material. The apertures have been arranged so that they approach each other no closer than a fixed minimum distance (the "nearest neighbor" distance). Look for evidence of this minimum distance in the diffraction pattern. What is the symmetry of the pattern. Why?

22. A two-dimensional quasicrystal pattern. Quasicrystals are atom arrangements occurring in nature that show repeated (but not regularly repeated) structural features in which angular relationships are preserved, but with no positional periodicity. In two-dimensions, such a pattern can be constructed using as elements two rhombuses of different shape, as demonstrated by the Oxford University mathematician Roger Penrose. Quasicrystals can contain rotation axes that *cannot* exist in periodic crystalline arrangements, such as the 5- or 10-fold axes evident in this pattern and in its sharp, but non-periodic diffraction pattern (no periodic reciprocal lattice exists).

Derive *analytically* the diffraction pattern from a vertical line aperture, and an expression for the structure factor of the two-dimensional "CsCl" crystal structure analogue.