

MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
Department of Electrical Engineering  
And Computer Science

**6.977 Semiconductor Optoelectronics – Fall 2002**

**Problem Set 1 – Semiconductor electronics**

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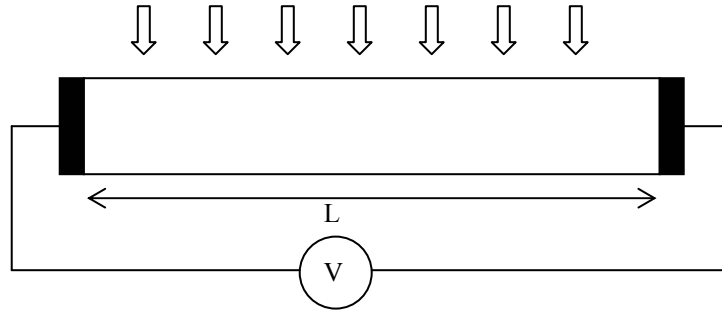
**Problem #1** *The purpose of this problem is to develop approximations to the Fermi integral for bulk semiconductors. Consider GaAs ( $N_V = 10^{18}$  at 300 K) with a donor concentration that varies from  $10^{15}$  to  $10^{19}$   $\text{cm}^{-3}$ .*

- Assuming Boltzmann statistics, calculate and plot the Fermi level as a function of the donor concentration.
- At high doping density the electron occupancy in the semiconductor is nearly metallic – the Fermi level is in the conduction band and the Fermi-Dirac function can be approximated to be at  $T=0$ . In this limit – known as the Sommerfeld approximation, show that the carrier density varies in proportion to

$$N \sim (E_F - E_C/kT)^{3/2}.$$

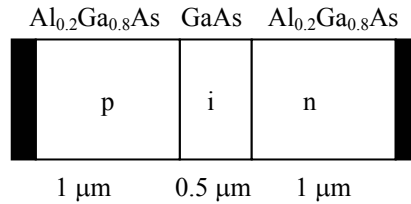
- Numerically estimate the Fermi-Dirac integral. Using numerical integration, determine the range of doping where the Fermi level calculation is accurate to 1% under the Boltzmann approximation and the doping range for 1% accuracy under the Sommerfeld approximation.
- A semiconductor is said to be degenerate when the Fermi level lies within either the conduction or valence bands. At what doping density is this n-type material degenerate?
- Determine the Fermi level for GaAs doped with acceptors instead of donors. Include the effects of the valence band degeneracy between light and heavy holes, but not the split-off band. At what doping density is the GaAs degenerate?

**Problem #2** The purpose of this problem is to familiarize you with the calculation of carrier concentrations out of equilibrium. A slab of GaAs is illuminated by a beam of light with a wavelength of 519 nm. At this wavelength, the absorption coefficient of GaAs is  $\alpha=10^4 \text{ cm}^{-1}$ . The excess carrier lifetimes are  $\tau_p=\tau_n=1 \text{ ns}$  and the slab is much thicker than  $1/\alpha$ .



- Determine the excess electron distribution ( $\Delta N(x)$ ) for incident optical powers of  $P=1, 10, \text{ and } 100 \text{ mW}$  in a beam area of  $50 \mu\text{m}^2$ . Neglect carrier diffusion and plot only for  $0\text{-}5 \mu\text{m}$  deep into the sample.
- Relate the excess carrier concentration to the electron quasi-Fermi level,  $E_{fc}(x)$ . Plot  $E_{fc}(x)$  for the various optical powers again from  $0\text{-}5 \mu\text{m}$ .
- Since the slab is much thicker than  $1/\alpha$ , the total photogenerated carrier concentration can be determined. If a DC electric field is applied along the length ( $L$ ) of the slab, determine the photocurrent that flows as a result of the applied field and the incident light.
- This type of photodetector is known as a photoconductive detector. From (c), it can be seen that the total current can be greater than  $qP/h\nu$  - which is the current if every photon generates one electron. Since there is excess photocurrent, there must be electrical gain. What is the physical origin of this gain?

**Problem #3** The purpose of this problem is to familiarize you with *SimWindows*. Consider a *p-i-n* diode consisting of doped  $\text{Al}_{0.2}\text{Ga}_{0.8}\text{As}$  layers, that are each  $1\ \mu\text{m}$  thick, surrounding an intrinsic (unintentionally doped) GaAs that is  $0.5\ \mu\text{m}$  thick. Consider that each of the  $\text{Al}_{0.2}\text{Ga}_{0.8}\text{As}$  regions are doped with acceptors/donors at  $5 \times 10^{17}\ \text{cm}^{-3}$ . Use *SimWindows* to generate all of the data for this exercise.



- a. Write out the Device File that describes this device.
- b. Plot the bandgap, electron effective mass, and dielectric constant as a function of position.
- c. Plot the bandedge diagrams and quasi-Fermi levels as a function of position for an applied bias of  $V=0$ ,  $V=+0.5$  Volts and  $V=-2$  Volts.
- d. On a log scale, plot the electron and hole density as a function of position for an applied bias of  $V=0$ ,  $V=+0.5$  Volts and  $V=-2$  Volts.
- e. Plot the I-V for this diode. Use a the maximum forward bias that corresponds to current densities of  $1\ \text{kA/cm}^2$  and a maximum reverse bias of  $V=-5$  Volts.
- f. For a forward biased current of  $1\ \text{kA/cm}^2$ , how much of the electron current that is injected into the GaAs from the n-type region recombines in the GaAs ?

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**MATLAB<sup>®</sup>\* Supplement for Problem Set 1**

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This document is intended as a MATLAB<sup>®</sup> tutorial for first time users and as a relevant example for more experienced MATLAB<sup>®</sup> users.

Below is an example of a MATLAB<sup>®</sup> implementation of the Fermi-Dirac integral and its inverse. The Fermi-Dirac integral is numerically evaluated using the 'quad8' function. Evaluating the inverse of the Fermi-Dirac integral is a bit tricky. This algorithm implements a search to look for the Fermi-level that corresponds to the appropriate integral.

```
function fd = fermi(y,v)
% fd = fermi(y,v);
%
% This is the integrand of the Fermi-Dirac integral from
% p. 416 of C&C.
%
% see also FERMIDIRACINT.M
%
% This is a simple example of a function in MATLAB. In the first line,
% function is the key word that lets matlab know that you are writing
% a function. The fd on the left hand side of the equal sign is
% the output argument, what is returned by the function. on the
% right hand side of the equal sign is the function name, fermi, with
% the input arguments in parameters. Note how fd, y, and v are used
% in the function below.
%
% To call this function, type:
% >> blah = fermi(1,3)
%
% blah will be equal to sqrt(1)./(1+exp(1-3)).

fd = sqrt(y)./(1+exp(y-v));

% note the dot preceding the divide. This indicates an
% element by element division for the vectors y and v.
%
% In general, a dot before an operator modifies it from a
% matrix operation to an element by element operation.
```

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\* The MathWorks Inc. MATLAB, Simulink, Stateflow, Handle Graphics, and Real-Time Workshop are registered trademarks, and TargetBox is a trademark of The MathWorks, Inc

```

function fdi = fermidiracint(v)
% fdi = fermidiracint(v);
%
% This function returns the Fermi-Dirac integral of order
% 1/2 givein in p. 416 of C&C
%
% see also FERMI.M

% relative and absolute tolerance

tol = [1e-3, 1e-4]; % pass [] to use defaults

%-----
% quad8 is some kind of numerical quadrature to approximate
% the integral. >>help quad8 OR >>type quad8 for more details
%
% help is the most valuable matlab command.
% type lets you see what the toolbox is doing, another way to
% learn matlab tricks.
%-----

warning off; % turns off the warnings

fdi = zeros(size(v)); % initializes fdi to a vector of zeros
% the same size as v

% step through each v and numerically integrate using a numerical
% quadrature. The first argument is the name of the function that
% returns the integrand. The second and third are the limits of
% integration. The third and fourth are the tolerance and the number
% of points for graphical output, the last is a parameter that is
% passed to fermi(integration_variable,parameter);

for I = 1:length(v)

    fdi(I) = quad8('fermi',0,max(40,v(I)*2),[],[],v(I));

    % index vectors and matrices with parentheses.

end

warning on; % turn on warnings.

% you will get warnings about reaching the recursion level limit
% if you leave warnings on. This is due to the sharp slope of the
% Fermi-Dirac integrand near y=0. compare the result with trapz.
% To check that it is correct.

```

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