6.302 Feedback Systems

Recitation 14: Bode Obstacle Course

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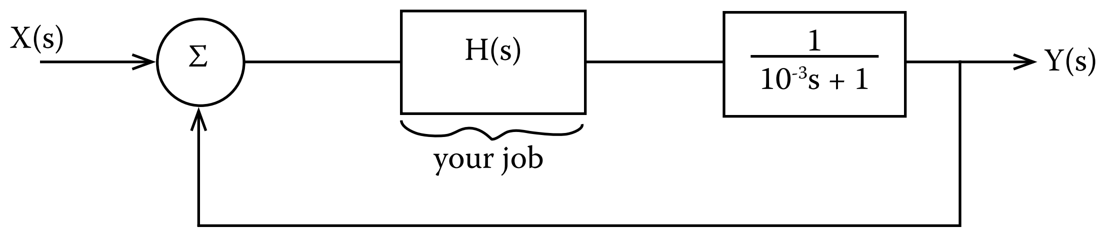
Now we're moving into the part of the course that for many of you will be the most fun: design of feedback systems.

Problem: How do we translate closed-loop specifications into specifications on our loop transmission?

CLASS EXERCISE:

Let's say that you're asked to control a plant as shown:

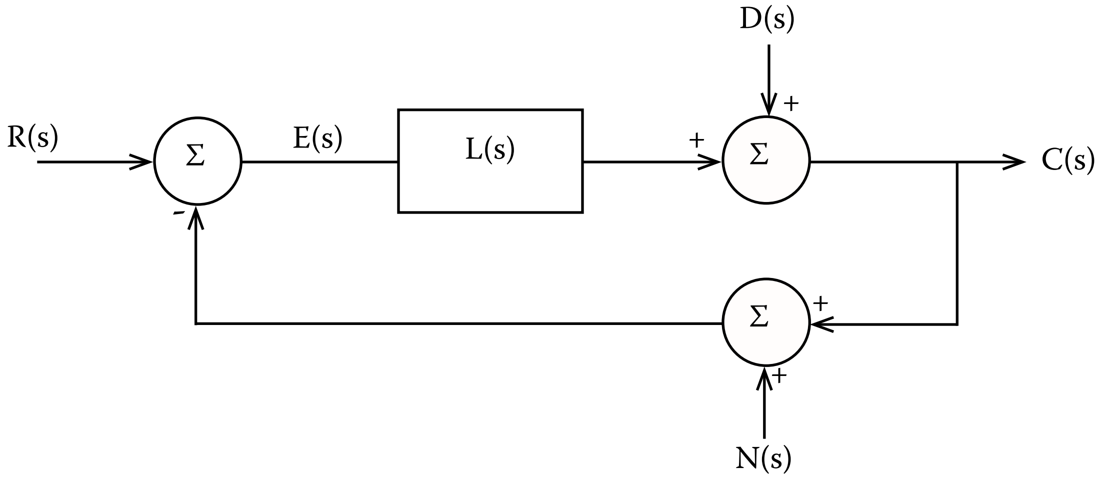


Requirements:

1. Zero steady-state error in response to a step
2. Loop crossover ω_C no higher than 100 rps.

→Design a compensator H(s) that meets these specifications.

Specs that we care about in feedback systems



1) Command following ⇒ the extent to which

$$\frac{C(s)}{R(s)} = \frac{L(s)}{1 + L(s)} \approx 1$$

Good command following requires that $|L(j\omega)| \gg 1$ over the frequency range of interest.

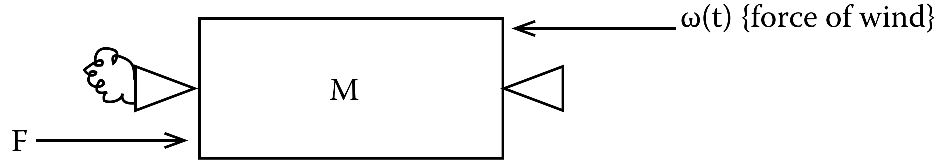
2) Small steady-state error/small dynamic tracking error

⇒ extent to which $\dfrac{E(s)}{R(s)} = \dfrac{1}{1 + L(s)} \ll 1$

requires that $|L(j\omega)| \gg 1$ over frequency range of interest.

3) Disturbance Rejection

EXAMPLE: Rocket sled buffeted by wind.



The wind in this case is a disturbance that we would like to reject.

Returning to general case, we want $\dfrac{C(s)}{D(s)}$ to be small, so disturbance rejection is the extent to which

$$\frac{C(s)}{D(s)} = \frac{1}{1 + L(s)} \ll 1 \longrightarrow \text{want } |L(j\omega)| \gg 1 \text{ over freq. range of interest.}$$

4) Noise Rejection

Our sensors aren't perfect: they give us the data that we want, plus some noise. Noise is always specified as a spectral density, e.g. the noise voltage associated with a resistor is $4kTR\Delta f$. The larger the bandwidth of your system, the larger the RMS noise voltage you're going to see at your output.

→This is one reason why extra bandwidth is bad←

since $\frac{C(s)}{N(s)} = \frac{L(s)}{1 + L(s)}$, we want $|L(j\omega)| << 1$ <u>outside</u> the frequency band of interest.

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Command-following and disturbance-rejection requirements produce inequalities like:

$$|L(j\omega)| > A, \text{ for } \omega < \omega_1$$

Noise rejection specs might be given as

$$|L(j\omega)| < A_2, \text{ for } \omega > \omega_2$$

We can draw a Bode Obstacle course as follows (see next page):

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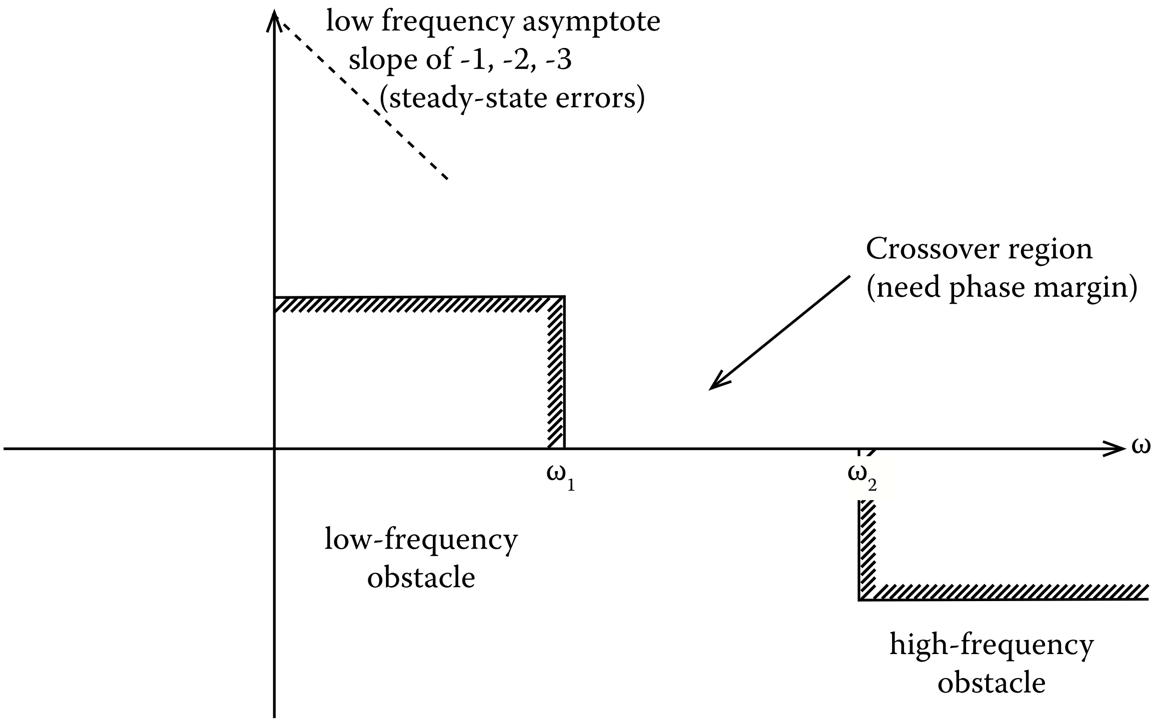


DESIGN EXAMPLE:

Design in acceptable L(s) that results in the following closed-loop performance specs:

1. Steady-state error in response to a ramp less than 10^{-2}
2. Disturbance rejection better than 10:1 for frequencies below 10 rps
3. Closed-loop bandwidth > 50 rps
4. Magnitude peaking Mp<1.4
5. Noise rejection better than 40dB above 1000 rps

How does this guide our design?

1. Steady-state error in response to a ramp is <u>bounded</u>, but not zero. This implies a pole @ origin.

$$\lim_{s \to 0} \cancel{s}\left(\frac{1}{s^{\cancel{2}}}\right) \frac{1}{1 + \frac{k}{s}F(s)} = \lim_{s \to 0} \frac{1}{\cancel{s}} \frac{\cancel{s}}{1 + kF(s)}$$

$$= \frac{1}{1 + kF(0)}$$

For F(s), if there are no singularities at the origin, we are free to assume F(0) = 1. As we fill in the details of F(s), we do so in the following fashion.

$$F(s) = \frac{(\tau_{z1}s+1)\,(\tau_{z2}s+1)\,\cdots\,(\tau_{zN}s+1)}{(\tau_{p1}s+1)\,(\tau_{p2}s+1)\,\cdots\,(\tau_{pm}s+1)} = \frac{\prod_{i=1}^{N}(\tau_{zi}s+1)}{\prod_{j=1}^{M}(\tau_{pj}s+1)}$$

This way, F(0)=1.

1) $\rightarrow \frac{1}{1+k} < 0.01 \rightarrow k > 100$

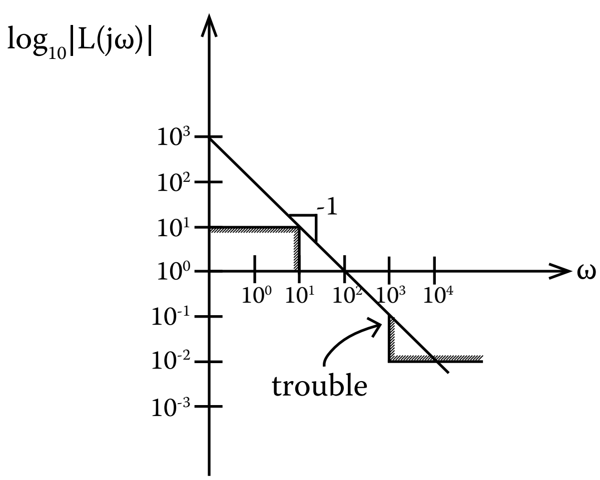
2) $\rightarrow |L(j\omega)| > 10$ for $\omega < 10$ rps

3) $\rightarrow \omega_C > 50$ rps

4) $\rightarrow \phi_m > 45^\circ$

5) $\rightarrow |L(j\omega)| < 0.01$ for $\omega > 10^3$ rps

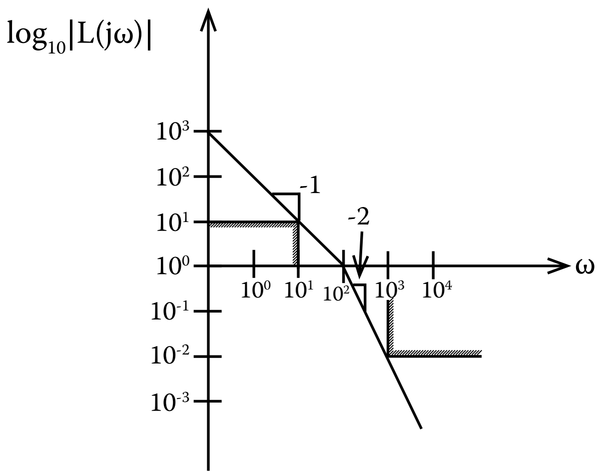
As a first stab, let's try L(s) = 100/s

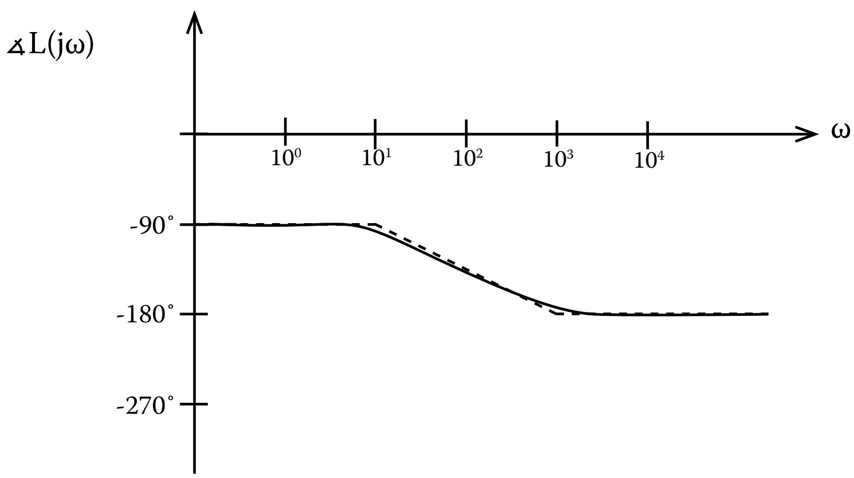


What to do? We need another pole somewhere in order to meet our high-frequency spec. But if we put the pole too low (in frequency), we'll lower ω_C and our phase margin.

What about a pole right at 100 rps? Using aymptotes on the Bode Plot, that would fix ω_C right at 100 rps, and the phase margin would be 45˚....

$$\text{Try } L(s) = \frac{100}{s\,(0.01s+1)}$$





Actual numbers: $\omega_C \approx 80$ rps, $\phi_m \approx 50$ rps