6.006 Introduction to Algorithms
Spring 2008

6.006 Recitation

Build 2008.38

6.006 Proudly Presents

- Warmup: Maxing out sums
- Fun: Tetris pwnage
- Bonus:
  - Pwn Mario v2: mushrooms, monsters

Max. Sum Sub-array

i	0	1	2	3	4	5	6	7	8	9
a[i]	31	-41	59	26	-53	58	97	-93	-23	84

- a is a list of real numbers
- want i, j so that $\sum a[i:j]$ is as large as possible
- want to compute this as fast as possible
- answer for this case
  - i = 2
  - j = 6
  - sum = 187

Max. Sum Sub-array: Naive Solution

i	0	1	2	3	4	5	6	7	8	9
a[i]	31	-41	59	26	-53	58	97	-93	-23	84

- max_sum, max_i, max_j = 0, 0, 0
- for i in 0:len(a)
  - for j in i:len(a)
    - if max_sum < $\sum$a[i:j]
      - max_sum, max_i, max_j = $\sum$a[i:j], i, j

Running Time for Naive Solution

- i, j go through all possible intervals a[i:j]
  - $O(N^2)$ intervals
- evaluating $\sum$a[i:j] at each interval
  - $O(N)$ work per interval
- $O(N^3)$ total

Max. Sub-Array: Smarter Solution A

- Notice that $\sum a[i:j] = \sum a[i:j-1] + a[j]$
- Rewrite inner block to eliminate computing $\sum a[i:j]$, replace with a running sum
- Running time: work per interval drops to $O(1)$, total work drops to $O(N^2)$

Max. Sub-Array: Smarter Solution B

- Hints
  - we're using a 'fancy' data structure
    - $s[i] = \sum a[0:i]$
  - again, we're trying to cut the work per interval

Max. Sub-Array: Smarter Solution B

- Notice that $\sum a[i:j] = \sum a[0:j] - \sum a[0:i-1]$
- Pre-compute $\sum a[0:i]$ into s[i]
- Rewrite the inner block of the naive algorithm to compute $\sum a[i:j]$ in O(1)
- Running time: again $O(N^2)$

Max. Sub-Array: Uber-Pro Solution Hint

- Hint: we will go through the motions of DP, but arrive at a very interesting conclusion
- Hint II: so start thinking of the optimal sub-structure

Max. Sub-Array: Uber-Pro Solution 1

- Problem: the max. sum sub-array in a
- Sub-problem
  s[i] = max. sum sub-array ending at a[i]
- Optimal sub-structure: if the max. sub-array includes a[i], it starts with the max. sum sub-array ending at a[i]

Max. Sub-Array: Uber-Pro Solution II

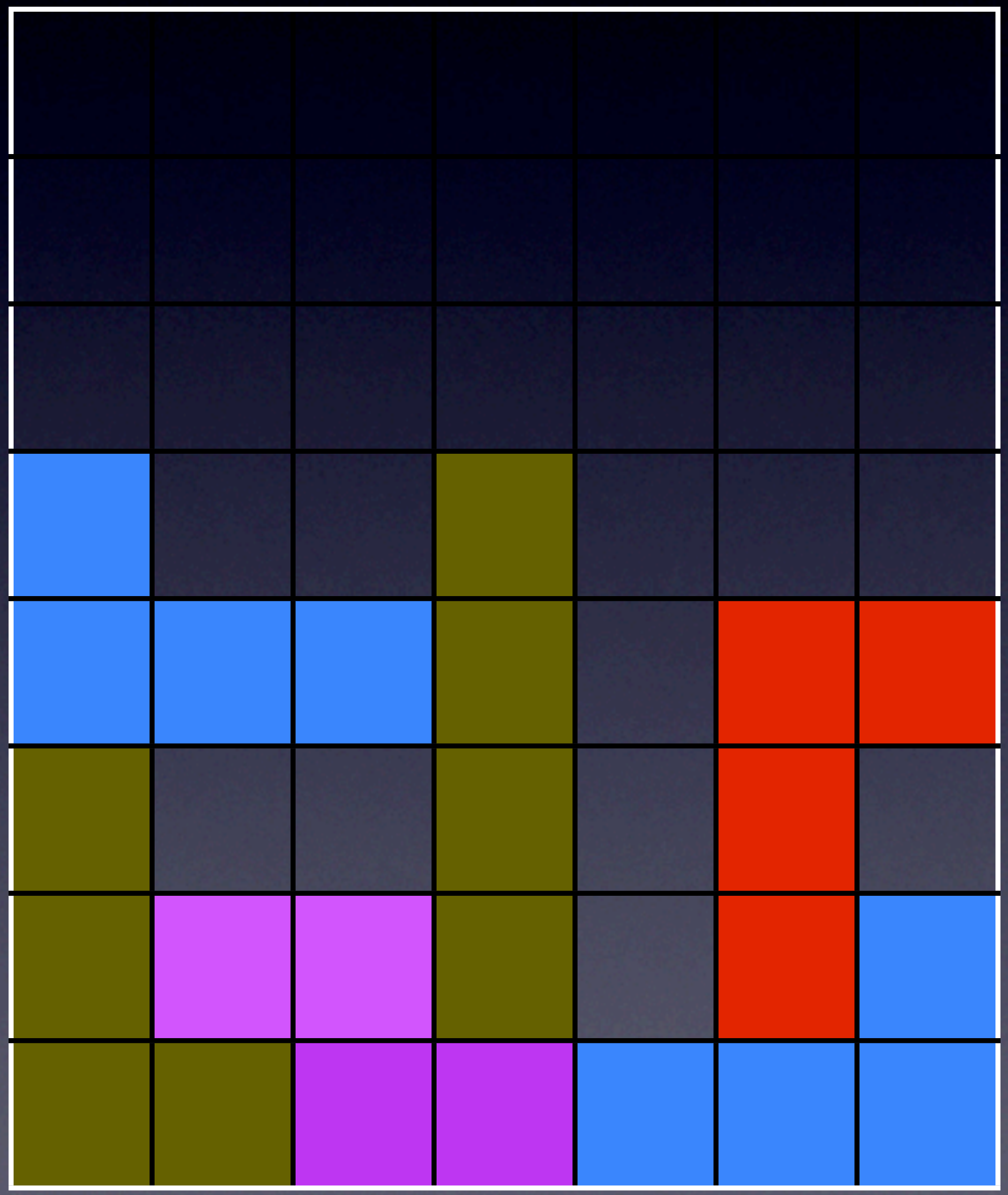
- s[i] = max(s[i - 1] + a[i], a[i])
- So we keep adding to the current sub-array until the sub-array sum becomes negative
- Discussion: bottom-up implementation, constant-space implementation

Tetris Pwnage: This is How Pros Do It

- For each piece
  1. Instantly rotate and move the piece
  2. Let the piece drop
- Don't care about making lines disappear; if you pwn it, they will come
- Last for as many pieces as possible

Tetris Pwnage: Formal Problem

- Board of width N
- K pieces, each of its own shape
- Must fit as many pieces as possible
- For each piece, must return rotation and position where it falls from



Tetris Pwnage: The Vision

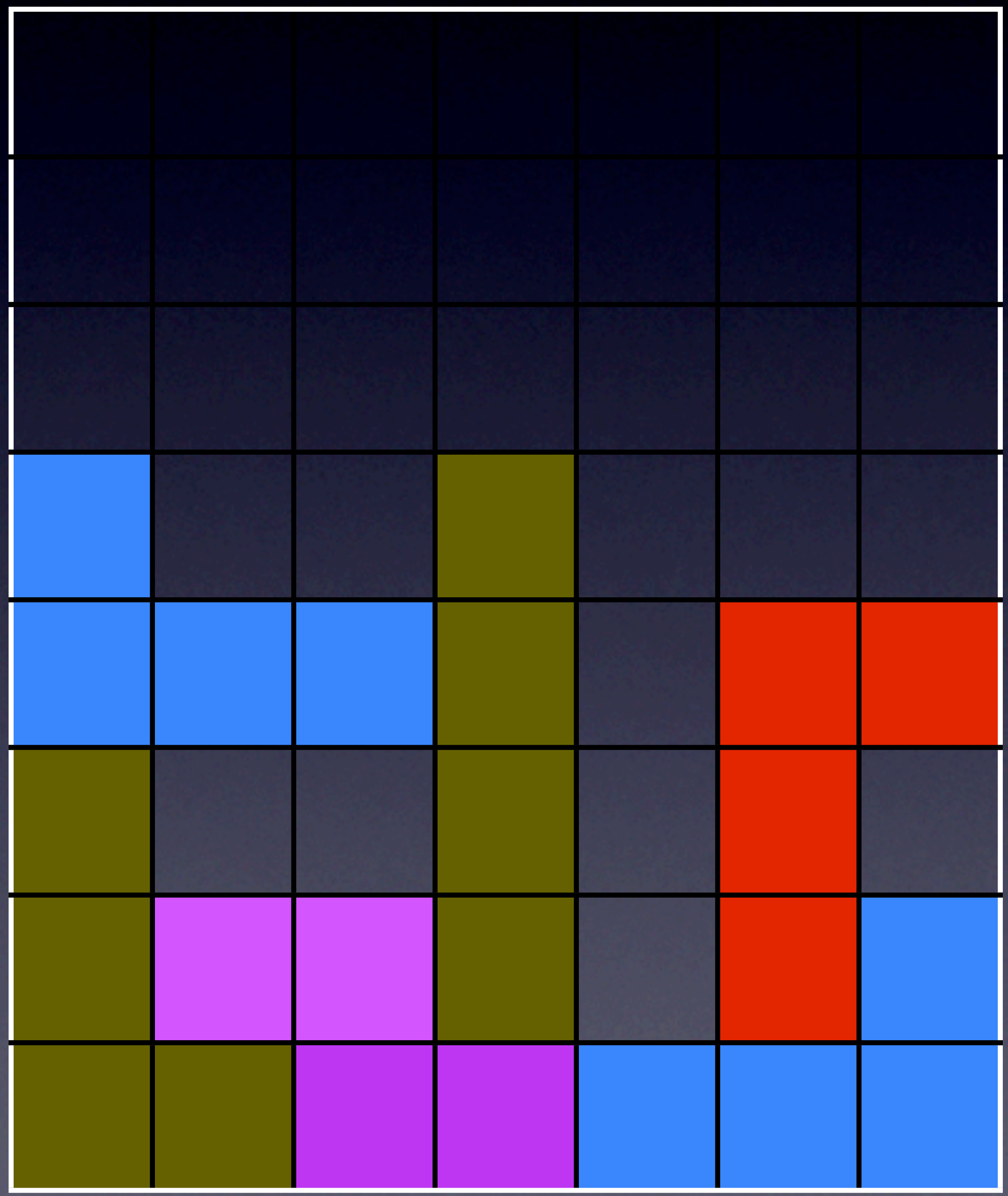
This is a game. Act accordingly.

Tetris Pwnage: The Approach

1. Find all the variables that make a position
2. Reduce the position representation
3. Use BFS
4. Figure out a way to do this bottom-up

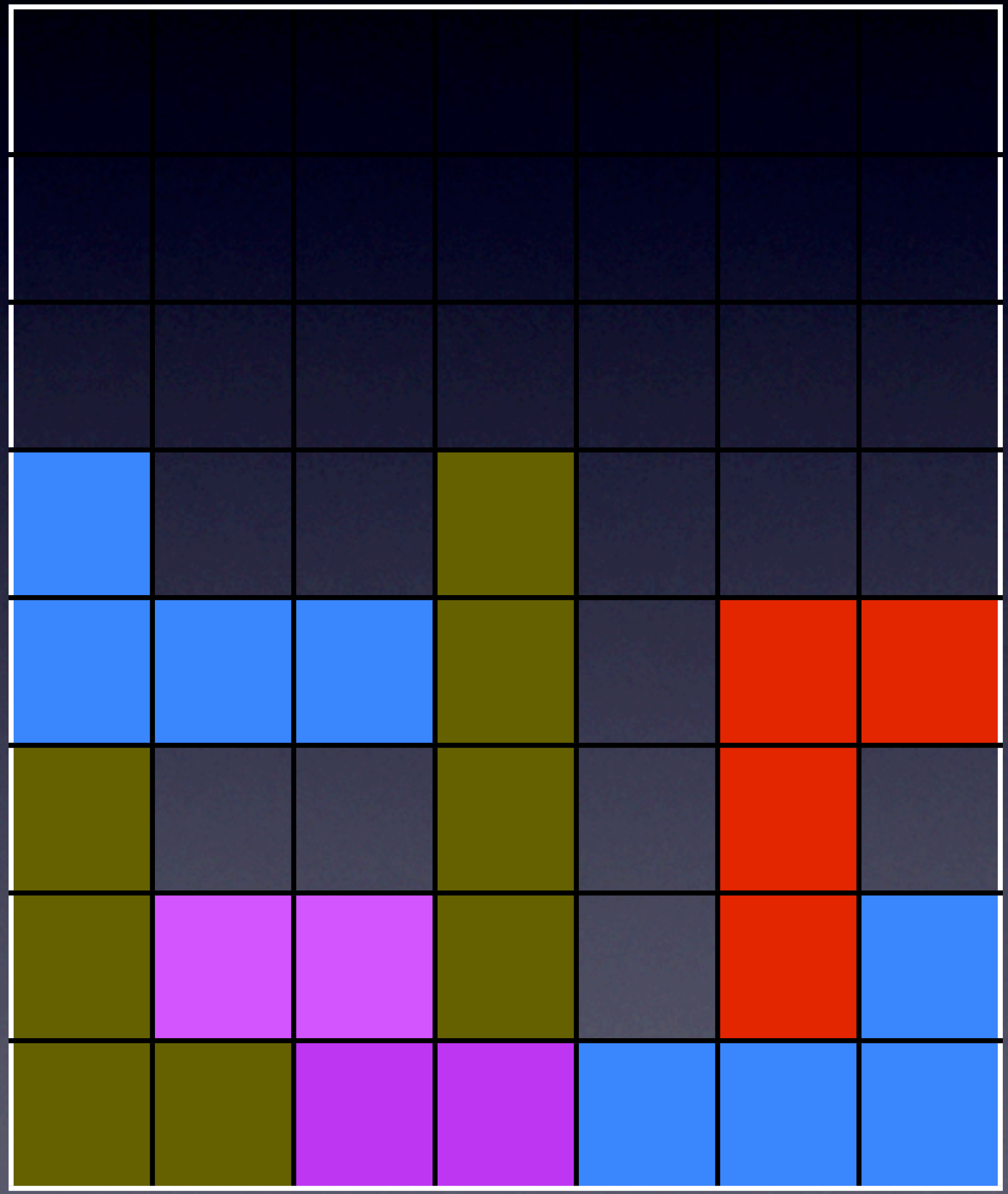
Tetris Pwnage: The Solution 1

- A configuration is the # of pieces on the board and the "skyline"
- Pieces can't go through other pieces, so it doesn't matter what's under the "skyline"
- Example at the right: 6 pieces, (5 4 4 5 1 4 4)



Tetris Pwnage: The Solution II

- Bottom-line solution: configurations of P pieces only depend on configurations of P-1 pieces
- d[p][skyline] = 1 if can use p pieces to achieve the given skyline



Bonus Discussion: Mario v2

- Monsters 1...m patrol platforms
  - moster i moves between platforms m[i][0], m[i][1]...m[i][mp_i], $1 \leq mp_i \leq 4$
- Special platforms contain mushrooms
  - mushroom state is an extra life - lost when in the same position as a monster