6.003 (Fall 2011)

Quiz #1 *October 5, 2011*

Name:

Kerberos Username:

Please circle your section number:

Section	*Time*
2	11 am
3	1 pm
4	2 pm

Grades will be determined by the correctness of your answers (explanations are not required).

Partial credit will be given for ANSWERS that demonstrate some but not all of the important conceptual issues.

You have **two hours.**

Please put your initials on all subsequent sheets.

Enter your answers in the boxes.

This quiz is closed book, but you may use one 8.5×11 sheet of paper (two sides).

No calculators, computers, cell phones, music players, or other aids.

1	/25
2	/25
3	/25
4	/25
Total	/100

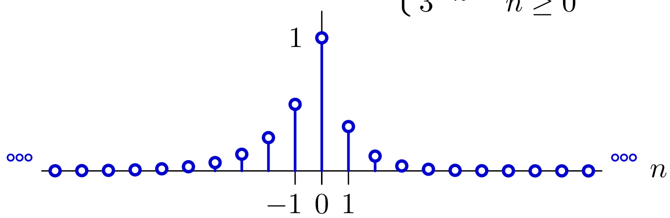
1. Find the coefficients *[25 points]*

Find constants c_1, c_2, and c_3 so that the solution to the difference equation

$$c_1 y[n-1] + c_2 y[n] + c_3 y[n+1] = \delta[n]$$

is equal to the signal $y[n]$ shown below.

$$y[n] = \begin{cases} 2^n & n < 0 \\ 3^{-n} & n \geq 0 \end{cases}$$



Enter numerical values (or closed-form numerical expressions) for the constants below. You need only show one valid answer, even if multiple answers exist. If no solution exists, enter **none**, and briefly explain why no solution exists.

$c_1 = \boxed{-\frac{2}{5}}$ $c_2 = \boxed{\frac{7}{5}}$ $c_3 = \boxed{-\frac{3}{5}}$

If $n < 0$ then $y[n] = 2^n$ and $c_1 2^{n-1} + c_2 2^n + c_3 2^{n+1} = 0$, which is true iff $c_1 + 2c_2 + 4c_3 = 0$.

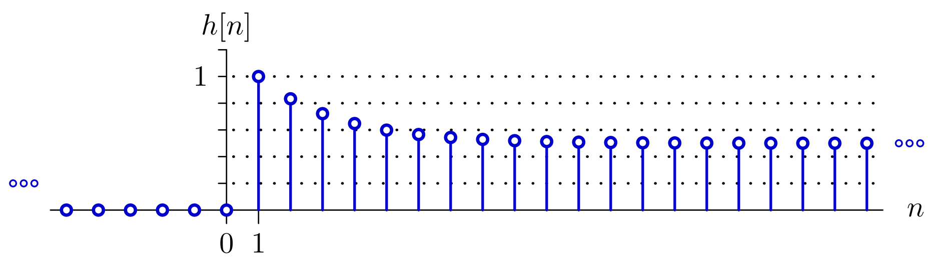
If $n > 0$ then $y[n] = 3^{-n}$ and $c_1 3^{-n-1} + c_2 3^{-n} + c_3 3^{-n+1} = 0$, which is true iff $9c_1 + 3c_2 + c_3 = 0$.

If $n = 0$ then $\frac{1}{2}c_1 + c_2 + \frac{1}{3}c_3 = 1$.

Solving these equations yields a single unique solution, $c_1 = -2/5$, $c_2 = 7/5$, and $c_3 = -3/5$.

2. Unit-sample response *[25 points]*

The unit-sample response of a system that contains just a small number of adders, gains, and delays (and no other types of elements) is shown in the following plot.



Notice: $h[n] = 0$ for $n \le 0$ and $h[n]$ approaches $\frac{1}{2}$ for $n > 20$.

Find the poles and zeros of this system.

You need only show one valid answer, even if multiple answers exist.

Enter the number of poles and zeros and list their approximate values below. If a pole or zero is repeated k times, then enter that value k times. If there are more than 5 poles or zeros, enter just 5 of them. If there are fewer than 5 poles or zeros, write **none** in the remaining boxes.

# of poles:	2				
poles:	1	$\frac{2}{3}$	none	none	none
# of zeros:	1				
zeros:	$\frac{5}{6}$	none	none	none	none

The unit-sample response has the form

$$h[n] = \frac{1}{2}u[n-1] + \frac{1}{2}\alpha^{n-1}u[n-1]$$

where $\alpha \approx \frac{2}{3}$. The corresponding operator form

$$\mathcal{H}(\mathcal{R}) = \frac{1}{2}\left(\frac{\mathcal{R}}{1-\mathcal{R}}\right) + \frac{1}{2}\left(\frac{\mathcal{R}}{1-\frac{2}{3}\mathcal{R}}\right)$$

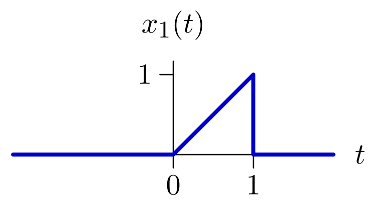
corresponds to a system function

$$H(z) = \frac{1}{2}\left(\frac{1}{z-1}\right) + \frac{1}{2}\left(\frac{1}{z-\frac{2}{3}}\right) = \frac{1}{2}\frac{2z-\frac{5}{3}}{(z-1)(z-\frac{2}{3})} = \frac{z-\frac{5}{6}}{(z-1)(z-\frac{2}{3})}.$$

Thus, there is a zero at $z = \frac{5}{6}$ and poles at $z = 1$ and $z = \frac{2}{3}$.

3. Laplace transforms *[25 points]*

Part a. Determine the Laplace transform of $x_1(t)$ shown below; $x_1(t)$ is zero outside the indicated range.



Enter a closed-form expression for the Laplace transform and the region of convergence (ROC) for this expression in the boxes below.

Laplace transform = $\frac{1-e^{-s}-se^{-s}}{s^2}$

ROC = all s

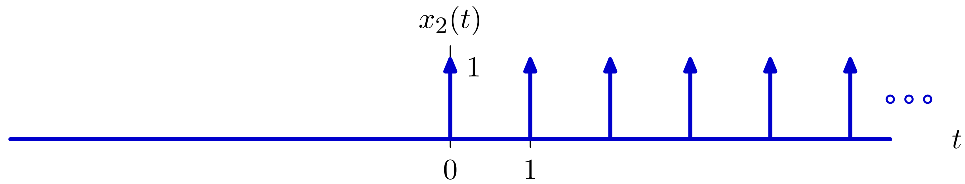
$$X_1(s)=\int_0^1 \underbrace{t}_{u}\,\underbrace{e^{-st}}_{dv}\,dt=\underbrace{t}_{u}\,\underbrace{\frac{e^{-st}}{-s}}_{v}\Bigg|_0^1-\int_0^1 \underbrace{1}_{du}\,\underbrace{\frac{e^{-st}}{-s}}_{v}\,dt$$

$$=-\frac{e^{-s}}{s}-\frac{e^{-st}}{s^2}\Bigg|_0^1=-\frac{e^{-s}}{s}-\frac{1}{s^2}(1-e^{-s})$$

$$=\frac{1-e^{-s}-se^{-s}}{s^2}$$

Alternatively, the signal is a unit ramp minus a delayed unit ramp - a delayed unit step. The transform is the sum of the transforms of these signals.

Part b. Determine the Laplace transform of the signal $x_2(t)$, which is a train of unit-impulse functions that starts at $t = 0$ and extends to positive infinity, with each impulse separated from the previous one by 1 unit of time.



Enter a closed-form expression for the Laplace transform and the region of convergence (ROC) for this expression in the boxes below.

Laplace transform = $\boxed{\dfrac{1}{1-e^{-s}}}$

ROC = $\boxed{\text{Re}(s) > 0}$

$$X_2(s) = \int_{-\infty}^{\infty} \sum_{n=0}^{\infty} \delta(t-n)e^{-st}dt = \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \delta(t-n)e^{-st}dt = \sum_{n=0}^{\infty} e^{-sn} = \frac{1}{1-e^{-s}}$$

provided $|e^{-s}| < 1$, i.e., $\text{Re}(s) > 0$.

4. Coupled oscillator *[25 points]*

A system with input $x(t)$ and output $y(t)$ can be described by

$$\dot{w}(t) = y(t) + x(t)$$
$$\dot{y}(t) = -w(t)$$

where $w(t)$ is an internal variable.

The system function $H(s) = \dfrac{Y(s)}{X(s)}$ for this system can be written in the form

$$H(s) = \frac{c_1}{s - p_1} + \frac{c_2}{s - p_2}\,.$$

Determine numerical values (or closed-form numerical expressions) for the constants c_1, c_2, p_1, and p_2.

$c_1 =$	$-\dfrac{j}{2}$	$p_1 =$	$-j$
$c_2 =$	$\dfrac{j}{2}$	$p_2 =$	j

$$H(s) = -\frac{1}{s^2+1}$$

$$h(t) = -(\sin t)u(t)$$