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Lecture 16 - Auction Types & Settings

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Independent private values

- N bidders, each has valuation for good, v_i, and independent draw from $U[0, 1]$

$$\text{bidder utility is} \begin{cases} V_i - P & \text{if wins} \\ 0 & \text{if loses} \end{cases}$$

- V_i known only to bidder
- what situation is this a good model for?
  - good worth different amount to each bidder
  - amounts are independent
  - suggests the good will not be resold, like a bottle of wine, or job that will be performed, like a paving contract

English auction

- bid b increases from 0 to 1 in continuous time, each bidder keeps hand up until no longer is willing to bit, last remaining bidder wins at $P = b$
- equilibrium strategy: bidder i stays in until $b = V_i$, then drops out
- expected revenue:

$$\begin{aligned}
&= \int_0^1 V\{\underbrace{N(N-1)F(V)^{N-2}f(v)(1-F(v)}_{\substack{\text{distribution of 2nd order statistic} \\ \text{from distribution } f}}\}\,\mathrm{d}V \\
&= N(N-1)\int_0^1 (V^{N-1} - V^N)\,\mathrm{d}V \text{ (for valuations distributed uniformly on unit interval)} \\
&= N(N-1)(\frac{1}{N} - \frac{1}{N+1}) \\
&= \frac{N-1}{N+1}
\end{aligned}$$

Second price sealed bid

- not so common, but has nice properties
- players submit secret bids, winner is one with highest bid but pays second highest bid
- unique equilibrium strategy: bidder i bids $b_i = V_i$
  - note that not all auctions have this property of truthful revelation

- proof:
  - if switch to $b_i' > V_i$, payoff changes only if they lose at V_i and win at b_i', but that's bad because then you win and pay more than your valuation
  - if switch to $b_i' < V_i$, payoff changes only if you win at V_i and lose at b_i', but that's bad because you would have been willing to pay more to win
- expected revenue: $\frac{N-1}{N+1}$

First price sealed bid auction

- very common for government auctions
- same as second price sealed bid, but bidders pay their bid –do you think the equilibrium bidding strategy will be the same?
- can't be, really, since what you pay if you win is different
- unique equilibrium strategy: bidder i bids $B_i = \frac{N-1}{N} V_i$
- no proof but intuition: bidders shade bids down, trading off bidder surplus if they win vs chance of losing, makes sense that shading would be function of N
- expected revenue: $\frac{N-1}{N+1}$
- note: bidders shading down ends up exactly canceling out effect of paying top bid instead of second highest

Dutch auction

- like English auction, but with b decreasing from 1 to 0
- bidders raise their hand as soon as they're willing to bid and the auction ends when a hand is raised
- equilibrium strategy and expected revenue: same as in first price sealed bid

Revenue equivalence

- N-bidder IPV setting with evaluations V_i F on $[\underline{v}, \bar{v}]$
- if object must be sold (no reserve) and all bidders must receive non-negative surplus (participation constraint), then any auction satisfying the following properties maximizes the seller's expected revenue:
  - in equilibrium, the winning bidder is the highest valuation bidder
  - the $\underline{\mathrm{v}}$ type gets zero expected surplus
  - this rules out crazy or random auctions
- first price, second price, English, and Dutch auctions are all revenue equivalent

Exceptions to revenue equivalence

- sellers can often do better by setting and announcing a reservation price
- if bidders are risk-adverse, first price auction is better for the seller than second price
  - intuition: revenue from second price is unchanged and still $b_i = V_i$
  - people shade bids down less in first price than they normally would because they're afraid of losing (increasing bid by and reduces surplus in states where utility is less steep and increases in states where utility is steeper)

Common values

- N bidders, good has value V to winner of auction
- no one observes V but every bidder receives some information S_i ("signal") about V

  e.g., $S_i = V + \epsilon_i$ where know $V \sim N(0,1)$ and $\epsilon_i \sim N(0,\sigma)$

- using Bayes rule, every bidder can calculate $E(V|S_i) = W_i = \frac{1}{1+\sigma^2} S_i$

Second price sealed bid auction

- do not want to bid W_i
- why? remember everyone has independent information about the value of the good –if you win by bidding your best guess of its value based upon your information, the fact that you won is bad news about the value
- this is what's known as the "winner's curse" (could be more accurate translation of caveat emptor than "buyer beware")
- note: in equilibrium, bidders should shade bids down to take winner's curse into account
- other auction mechanisms? shade value to account for adverse selection.

In reality

- most auctions have some private value and some common value character to them:
  - e.g., Van Gogh painting –I will enjoy seeing it on my wall but I also care about the prestige associated with owning it and the resale value if I want to eventually sell it.
  - e.g., I'm bidding on a highway paving contract –there is relevant information for me in my rivals' bids because we're all just estimating how much it's going to cost, but I have a cost advantage because I'm currently working on a job close by.
  - e.g., timber tract –impossible to count exact number and types of trees, but can "cruise" tract to estimate (knowing the market value of milled timber). however, my mill is currently running at capacity and my loggers are busy so I wouldn't be able to log the tract for several months.

Empirical studies of auctions

- lab & field experiments
  - e.g., auction off identical items repeated withed posted reserve, secret reserve, no reserve and see how you do
  - lab: pay undergrads to sit in special labs and bid on made-up goods
  - field: set up actual auctions to conduct experiments
  - what if we change auction mechanisms –do we make more money? should I set a reserve price?
  - of more academic value, testing validity of theory
- "reduced form"
  - suggests a theory, derives equilibrium predictions in that theory, sees whether predictions are borne out in reality.

  - e.g., revenue for identical items should be greater in English than in second price if valuations are positively correlated because information revelation leads to more intense competition.

- "structural"
  - assumes a theory, uses that theory to back out primitives that are not observable with primitives
  - can run "counterfactuals", such as what auction mechanism maximizes revenue but relies on theory being correct
  - e.g., 1st price sealed bid auction where you know N –if you have data on all bids B_i can back out valuations V_i and then run counterfactuals

Edelman, Ostrovsky, Schwartz

- first paper to study the type of auction that Google runs for sponsored search –called it a "position auction"
- even though it's a generalized second price auction, they showed that it is not a "truth-telling mechanism"
- in other words, it is not optimal for bidders to bid their valuations (this is because Google is auctioning off several positions at once)
  - e.g., 3 bidders and 2 positions
  - valuations are $V_1 = 10$, $V_2 = 9$, and $V_3 = 2$
  - positions have the following clicks: $P_1 = 200$ and $P_2 = 199$
  - truthtelling: $\pi_1 = 200(10 - 9) = 200$
  - if 1 bids below 2: $\pi_1 = 199(10 - 2) = 1592$
  - still achieves efficient allocation, max revenue
- there have been additional papers studying this auction design
- sidenote: Bing, Yahoo, Google have recently changed the way they label sponsored search, more often using the term "ad" or "advertising" or "paid advertising", partly due to research from Ben Edelman showing that consumers didn't know what sponsored search was.