MIT 14.13 – Final Exam Spring 2020

May 19, 2020

- What materials can you use?
  - You can use slides and notes from lectures, recitations, and psets. You can also use a calculator.
  - You CANNOT receive help from others while taking the exam (online, in person, or any other way).
  - You CANNOT try to find answers to the questions online other than the Learning Modules website.
  - You CANNOT try to find questions or answers online other than looking at existing Piazza posts.
  - You can ask PRIVATE Piazza questions to clarify things if you think that is important and/or if you face technical difficulties, but you CANNOT ask public questions on Piazza.
  - You CANNOT watch lecture videos during the exam.
  - Support animals are fine!

- **Honor code:** We trust you to follow these rules. Part 4 asks you to type your name as an electronic signature confirming that you followed the rules given above for taking them exam.

- While taking this exam, always keep in mind that you are a wonderful person regardless of your answers in this exam. You will pass this class as long as you try your best.

- Good luck!

QUESTION 1: True, False, or Uncertain [30]

Please answer ALL of the the following five questions. State whether each of the following statements is true, false, or uncertain. Always explain your answer carefully and concisely. Your score is largely determined by the quality of your explanation. You only need to give the intuition for your answer, not a formal proof.

1. (6 points) Market solutions can help people with self-control problems but can also make things worse.

> **Solution:** True. If profit-seeking firms offer effective commitment devices (e.g., apps for tracking monthly spending totals), then sophisticated consumers with self-control problems will rationally pay for them and improve their future choices. But firms can also earn profits by exploiting naive consumers' self-control problems (e.g., offering credit cards with up-front spending bonuses and low introductory interest rates that increase over time).

2. (6 points) Consider a retail firm whose customers have reference-dependent preferences. A profitable strategy for the firm could be to advertise high prices and then give consumers discounts when they arrive at the store.

> **Solution:** False. As illustrated in problem set 3, a profit-maximizing firm should do the opposite. Low advertised prices will encourage customers to go to the store and update their reference points as they anticipate owning the advertised product. The shift in reference points increases consumers' willingness to pay, so they are willing to pay surprise add-on fees after they arrive at the store.

3. (6 points) The Chetty et al. (2009) paper on salience and taxation discussed in class finds that consumers are fully attentive to all forms of taxes.

> **Solution:** False. Chetty et al. study sales taxes on retail items and find a significant reduction in demand for treated products after tax-inclusive price tags are posted. They estimate that the salience parameter θ is about .75, which means that consumers react to sales tax changes only a quarter as much as they react to pre-tax price changes.

4. (6 points) Loewenstein (1987) finds that survey respondents i) prefer to kiss a movie star in three days rather than now, and ii) prefer to suffer a painful electric shock now rather than in one year. These responses can be explained by both discounting and anticipatory utility.

> **Solution:** False. Only anticipatory utility can explain these responses. Discounting would lead respondents to prefer undertaking pleasurable activities (like kissing a movie star) sooner and painful activities (suffering an electric shock) later. But respondents would be willing to delay the kiss if they derive anticipatory utility from thinking about it, and would be willing to suffer the shock now if they suffer anticipatory disutility from imagining the future pain.

5. (6 points) The evidence from Read and Van Leeuwen (1998) supports the idea that participation in commitment devices may depend on the state in which they are offered.

Solution: True. Read and Van Leeuwen find that when choosing future snacks, people are more likely to favor healthy options if they are not hungry when they make their choice. People are probably more likely to participate in a commitment device that encourages healthy snack food if they are offered the device when they are full.

QUESTION 2: Multiple Choice [30 points]

Please answer ALL of the below questions. Each of the following questions has a single correct answer option. Please select one answer option per question.

1. (6 points) Which of the following is an example of projection bias?
   (a) Believing that the 10th coin toss is likely to come up tails because the first 9 tosses have all come up heads
   (b) Deciding not to go running on a sunny day because it was rainy and miserable the last time you ran
   (c) Buying too many groceries (especially potato chips!) when you go shopping on an empty stomach
   (d) Planning to finish your problem set three days in advance, then procrastinating it until the last minute

Solution: (c). Projection bias occurs when we overweight current preferences (being hungry) in projecting future preferences (how many groceries I should buy for later consumption). Option (b) describes attribution bias: overweighting the state (rainy weather) in which you previously consumed a good (going running). Option (a) describes the gambler's fallacy and option (d) describes naive quasi-hyperbolic discounting.

2. (6 points) Reference-dependent preferences can explain why investors
   (a) exhibit delayed reactions to relevant market news
   (b) never save for retirement
   (c) hold stocks that have increased in value above the purchase price and sell stocks that have decreased in value below the purchase price
   (d) sell stocks that have increased in value above the purchase price and hold stocks that have decreased in value below the purchase price

Solution: (d). This is the disposition effect documented by Odean (1997). Option (c) describes the opposite of the disposition effect. Option (a) describes inattention and option (b) is best explained by present bias/procrastination.

3. (6 points) Madrian and Shea (2001) study 401(k) savings behavior among employees of a large company. They find that
   (a) Default enrollment in the 401(k) plan significantly increases 401(k) participation rates
   (b) Default enrollment in the 401(k) plan has no effect on 401(k) participation rates
   (c) Financial education significantly increases 401(k) participation rates
   (d) Increases in the employer's matching contribution rate significantly increase 401(k) participation rates

Solution: (a). Madrian and Shea find that participation increases significantly after default enrollment is instituted. They do not study the effects of financial education or changes in the employer match rate.

4. (6 points) An Expected Utility Maximizer with Constant Relative Risk Aversion (CRRA) preferences
   (a) is risk neutral
   (b) has preferences that are consistent with the finding in Kahneman and Tversky (1989) that people appear to be loss averse

(c) mispredicts utility from uncertain situations

(d) wants to invest a constant share of wealth in risky assets, irrespective of their level of wealth

Solution: (d). (a) is wrong because CRRA utility functions are strictly concave. (b) is wrong because an Expected Utility Maximizer does not have loss averse preferences. (c) is not relevant.

5. (6 points) Which of the following is an example of the Gambler's Fallacy?

(a) A woman rolls a fair die twice. It turns up six both times. She thinks the probability of a six on the next roll is $\frac{1}{6}$.

(b) A basketball player performed extremely well in the first half of the game. The crowd thinks "today is his day" and believes he's certain to perform well in the second half.

(c) A man sees four tosses of a fair coin all turn up heads. He concludes the fifth is almost certain to be a tail because it would be extremely unlikely for five out of five tosses to be heads.

(d) A test for a rare disease is 95% accurate. A woman gets a positive test result and concludes the probability that she has the disease is 95%.

Solution: (c). (b) is an example of the Hot-Hand Fallacy, and (d) is an example of Base-Rate Neglect. (a) is an example of correctly recognizing three rolls of a fair die are independent.

PART 3, QUESTION 1: Reference Dependence [30 Points]

Please make sure to explain your answers in this section carefully and concisely. Do not simply write an answer without an explanation of how you arrived at this answer. Answers without adequate explanation will not receive full credit.

Maddie has reference-dependent preferences over the number of cups of coffee she consumes in a day and the amount of money she has available for other food and drink for the day. Her daily utility over coffee and money takes the form

$$u(c_c - r_c) + v(c_m - r_m)$$

where c_c is the number of cups of coffee she consumes, c_m the amount of money she has for other food and drink, r_c her reference point for coffee, and r_m her reference point for money.

$u(x)$ takes the form

$$u(x) = \begin{cases} -10 & \text{if } x \leq -2 \\ -6 & \text{if } x = -1 \\ 0 & \text{if } x = 0 \\ 3 & \text{if } x = 1 \\ 4 & \text{if } x \geq 2 \end{cases}$$

and $v(x)$ takes the form

$$v(x) = \begin{cases} x & \text{if } x \geq 0 \\ 2x & \text{if } x < 0 \end{cases}$$

Maddie does not have equipment to make coffee at home so she buys coffee from a shop nearby, which charges \$2.50 per cup. Maddie does not like old coffee so she consumes any cup she purchases immediately and never saves it for later.

She has gotten into a routine of consuming two cups of coffee a day. She has a total of \$25 to spend on food and drink per day, so consuming two cups a day gives her \$20 to spend on other food and drink.

1. (5 points) First interpret Maddie's preferences. Does she exhibit loss aversion? Does she exhibit diminishing sensitivity? Why might we expect her to have $r_c = 2$ and $r_m = 20$?

> **Solution:** She exhibits loss aversion over both coffee and money. She exhibits diminishing sensitivity over coffee but not over money. We would expect $r_c = 2$ because her routine of drinking two cups of coffee a day would make that her reference point, for instance, if her reference point is given by expectations or status quo. This routine gives her \$20 for other food and drink so we would expect her reference point for spending on other food and drink to be 20.

2. (5 points) Assume $r_c = 2$ and $r_m = 20$. Suppose Maddie has had 2 cups of coffee one day. Alex is stopping by the coffee shop and asks Maddie if she would like him to pick up a coffee for her (for which she would need to reimburse him the price of \$2.50). Assuming Maddie considers only her utility of that particular day (i.e. she ignores any effects on her future utility), what is the maximum price Maddie is willing to pay for another cup of coffee? Will she accept Alex's offer?

> **Solution:** The maximum price she would pay for another cup is the price that makes her indifferent between consuming three cups that day and consuming two. Thus p_{max} solves
>
> $$u(3-2) + v(20 - p_{max} - 20) = u(2-2) + v(20-20)$$

Simplifying and rearranging,

$$u(1) + v(-p_{max}) = u(0) + v(0)$$

$$3 - 2p_{max} = 0$$

$$p_{max} = 1.5$$

so she would be willing to pay \$1.50. This is lower than the price (which is \$2.50) so she will not accept Alex's offer.

3. (5 points) On another day, Maddie buys her second cup just as the shop is closing for the day. On her way out, she runs into Will, who arrived at the shop to buy a coffee just a minute after it closed. Will decides to try to buy the cup Maddie just bought from her at price p. For what range of prices will Maddie accept Will's offer? (Continue to assume that $r_c = 2$ and $r_m = 20$; also assume there is no other way for Maddie to buy coffee on that day.) Explain why your answer is different from your answer to the previous question.

Solution: Maddie will accept Will's offer if doing so makes her better off than consuming two cups. She would accept any price p that satisfies:

$$u(1-2) + v(20 + p - 20) > u(2-2) + v(20 - 20)$$

Simplifying and rearranging,

$$u(-1) + v(p) > u(0) + v(0)$$

$$-6 + p > 0$$

$$p > 6$$

so she would accept any price over \$6.00.

4. (5 points) In the week before her dissertation was due (i.e. last week!), Maddie's need for coffee changed and she began drinking three cups a day. Explain why one might expect her reference points to adjust to $r_c = 3$ and $r_m = 17.50$ by the end of this week.

Solution: Three cups a day has become the new normal so it will be her reference point. At three cups a day, she would have \$17.50 to spend on other food and drink, so that is her new reference point for money.

5. (5 points) Assume we now have $r_c = 3$ and $r_m = 17.50$. Frank becomes concerned that Maddie is consuming too much coffee. He forbids her from going to the coffee shop but offers her the following deal: she can pay him \$2.50 to buy her a cup for the first two cups he buys, but she must pay him $\$q > \$2.50$ for the third cup and he will not let her have any more than three cups. What is the minimum value of q that Frank must set to ensure that Maddie consumes only two cups a day?

Solution: Maddie would choose two cups over three as long as

$$u(2-3) + v(20 - 17.5) > u(3-3) + v(20 - q - 17.5)$$

Simplifying and rearranging,

$$u(-1) + v(2.5) > u(0) + v(2.5 - q)$$

$$-6 + 2.5 > 5 - 2q$$

$$q > 4.25$$

so Frank must set a price over \$4.25. [ALSO NEED TO: show she would consume two and not one or zero cups.]

6. (5 points) Frank implements the policy, with a q in the range you identified in the previous part, for one month. After that, he removes the policy, allowing Maddie to buy as many cups a day at \$2.50 as she would like. Is is possible that when given free reign after a month of the policy, Maddie would choose to consume two rather than three cups a day?

Solution: Yes. We know that Maddie would consume two cups a day under Frank's policy. After one month of this, her reference points may have moved back to $r_c = 2$ and $r_m = 20$. We know from the earlier parts that if this is her reference point, she would consume two cups a day. [ALSO NEED TO: (a) check the last sentence is implied by the earlier parts, and (b) show that if Maddie's reference point hadn't changed, she would consume 3 cups.]

PART 3, QUESTION 2: The Better is the Enemy of the Good [30 points]

Please make sure to explain your answers in this section carefully and concisely. Do not simply write an answer without an explanation of how you arrived at this answer. Answers without adequate explanation will not receive full credit.

Jacqueline, Jared, and Drew are students in 14.13:

- The final exam is happening two days from now, at date $t = 3$. They can decide whether to prepare for the exam today ($t = 1$) OR tomorrow ($t = 2$). It is useless to prepare more than once.
- Today ($t = 1$) they can decide to work hard (instantaneous utility $u_1 = -c_2$), work moderately hard (instantaneous utility $u_1 = -c_1$), or not work at all (instantaneous utility $u_1 = 0$).
- Tomorrow ($t = 2$), if they have not worked yet, they will have the same menu but because of stress, instantaneous utility from working hard will be $u_2 = -c_2 - k_2$, from working moderately will be $u_2 = -c_1 - k_1$ and from not working $u_2 = 0$.
- Finally, on exam day, they will derive instantaneous utility $u_3 = Y_2$ if they worked hard in $t = 1$ or $t = 2$, $u_3 = Y_1$ if they worked moderately in either $t = 1$ or $t = 2$, and $u_3 = 0$ if they did not work at all.
- We assume that $c_2 > c_1$, $k_2 \geq k_1$, $Y_2 \geq Y_1$.

Each student's utility function is as follows:

$$U_t = u_t + \beta \sum_{\tau=t+1}^{3} \delta^{\tau-t} u_\tau \text{ for } 1 \leq t \leq 2 \tag{1}$$

1. (2 points) Briefly explain this utility function. What do the parameters β and δ measure? What do we typically assume about these parameters?

Solution:

The student's utility comes from the costs of working to prepare for the exam, and rewards of doing well on the exam.

β is the short-term discount factor and captures the student's degree of present bias. δ is the long-term discount factor. A large body of evidence suggests that people are more patient in tradeoffs involving only future periods than in tradeoffs that involve the present.

We usually assume both β and δ are between 0 and 1, with $\beta < 1$ for present-biased individuals, and $\beta = 1$ for exponential discounters. δ is usually assumed to be close to 1. Full credit for basic explanation and typical values. Half credit if discussion of either is missing.

In the rest of the problem, we always assume $\delta = 1$ and $\beta < 1$, so that the utility function becomes:

$$U_t = u_t + \beta \sum_{\tau=t+1}^{3} u_\tau \text{ for } 1 \leq t \leq 2 \tag{2}$$

2. (2 points) Briefly explain the assumptions $c_2 > c_1$, $k_2 \geq k_1$, $Y_2 \geq Y_1$.

Solution:

Working hard is strictly more costly than working moderately hard. The stress-related additional cost of working hard last minute is at least as big as the stress-related additional cost of working moderately last minute. Working hard leads to a reward on the exam that is at least as large as working moderately.

3. (3 points) Let's first consider Drew, who only cares about passing the exam (so that $Y_1 > 0$) but does not care at all about doing really well ($Y_2 = Y_1$). Given the latter assumption, show that Drew will never work hard or consider doing so.

Solution: For Drew, working hard in a given period is always dominated by working moderately hard. Both give the same continuation payoff of βY_1, but working hard generates a higher instantaneous utility cost (since $c_2 > c_1$ and $c_2 + k_2 > c_1 + k_1$). Therefore, we know that Drew will never put in more than moderate effort.

4. (3 points) Let's assume that Drew is fully sophisticated about his present bias ($\hat{\beta} = \beta$). Assume that $c_1 + k_1 < \beta Y_1$. Derive the threshold k_1^* (as a function of parameters c_1 and β) such that if $k_1 > k_1^*$, Drew will work today. Do not worry about the knife-edge case $k_1 = k_1^*$. When will he work (if at all) if $k_1 < k_1^*$?

Solution:

$k_1^* = \frac{1-\beta}{\beta} c_1$**. If** $k_1 < k_1^*$ **he will work at** $t = 2$**.**

First, since $c_1 + k_1 < \beta Y_1$, if Drew arrives tomorrow without having worked, he will work tomorrow (moderately hard) since this will bring him utility $U_2 = u_2 + \beta u_3 = \beta Y_1 - c_1 - k_1$ (as opposed to $U_2 = 0$ from not working).

Drew is sophisticated so he correctly predicts this today. Therefore, his decision today is between working (moderately) today or working (moderately) tomorrow. Working moderately today brings him:

$$U_1(\text{today}) = -c_1 + \beta Y_1$$

While working moderately tomorrow brings him (today):

$$U_1(\text{tomorrow}) = \beta(-c_1 - k_1 + Y_1)$$

He will work today if:

$$U_1(\text{today}) > U_1(\text{tomorrow}) \Leftrightarrow c_1 < \beta(c_1 + k_1) \Leftrightarrow k_1 > \frac{1-\beta}{\beta} c_1 =: k_1^*$$

If $k_1 < k_1^*$, Drew will work tomorrow.

5. (3 points) How would your answers to the previous question change if Drew was fully naive ($\hat{\beta} = 1$), but with the same true β?

Solution: The answers are unchanged.

If Drew was fully naive, he would still face the same choice conditional on getting to period 2 and would thus work at that point. In period 1, he would incorrectly predict his Period-2 utility to be $\hat{U}_2 = u_2 + u_3$ but he would correctly predict that he would work in period 2, because

$$\hat{U}_2(\text{working in period2}) - \hat{U}_2(\text{never working}) = Y_1 - c_1 - k_1 > \beta Y_1 - c_1 - k_1 > 0$$

As a result, he would face the exact same tradeoffs in period 1 as in the previous question, and the answers are unchanged.

6. (3 points) Jared is fully sophisticated and, just like Drew, does not care about his grades (he only wants to pass the class), i.e. $Y_2 = Y_1$. Jared's utility parameters satisfy the following assumptions:

$$c_1 > \beta Y_1 - k_1 \tag{3}$$
$$c_1 < Y_1 - k_1 \tag{4}$$
$$c_1 < \beta Y_1 \tag{5}$$
$$c_1 > \frac{\beta}{1-\beta} k_1 \tag{6}$$

What will be Jared's work decisions in period 1 and 2?

Solution:

Jared will work (moderately) in period 1.

We proceed by backward induction. If Jared has not worked in period 1, in period 2 he will face a decision between working, getting $U_2 = \beta Y_1 - c_1 - k_1$, and not working, getting $U_2 = 0$. From assumption (3), we know Jared would prefer not to work.

In period 1, Jared is sophisticated and thus understands that if he does not work today he won't work at all. Therefore, his utility from not working today is $U_1 = 0$ while his utility from working today is $U_1 = -c_1 + \beta Y_1$. We know from assumption (5) that he will thus prefer to work today.

7. (3 points) Now we assume that Jared is fully naive but maintain all the parametric assumptions on his utility from the previous question. How does your answer to the previous question change?

Solution:

Jared gets to the exam unprepared.

By the same token, we know that Jared would not work in period 2 if he gets there without having worked in period 1. However, because Jared is now fully naive, he fails to predict this. In period 1, he predicts his behavior in period 2 assuming that he will have utility $\hat{U}_2 = u_2 + u_3$. He thus believe that he will be choosing between working (getting predicted utility $\hat{U}_2 = -c_1 - k_1 + Y_1$) and not working (getting predicted utility $\hat{U}_2 = 0$). From assumption 4, this means that he predicts that he will work in period 2.

Jared in period 1 thus incorrectly believes that if he does not work today, he will work tomorrow and get period-1 utility $U_1 = \beta(-c_1 - k_1 + Y_1)$. On the other hand, working today still yields $U_1 = -c_1 + \beta Y_1$. Comparing these two quantities, and using assumption 6, we find that he prefers to not work today. Thus he procrastinates, thinking he will work tomorrow, and surprises himself by also not working tomorrow and getting to the exam unprepared.

8. (5 points) Now consider Jacqueline, who cares a lot about her grades (we relax the assumption that $Y_2 = Y_1$) and is fully naive about her present bias. We assume that Jacqueline's utility parameters verify the following

inequalities:

$$c_1 + k_1 < \beta Y_1 \tag{7}$$
$$c_1 < \beta(c_1 + k_1) \tag{8}$$
$$c_2 > \beta(c_2 + k_2) \tag{9}$$
$$c_2 + k_2 - c_1 - k_1 < Y_2 - Y_1 \tag{10}$$
$$c_2 + k_2 - c_1 - k_1 > \beta(Y_2 - Y_1) \tag{11}$$
$$c_2 - c_1 < \beta(Y_2 - Y_1) \tag{12}$$

What will be Jacqueline's level of work, and when will she be working, if at all?

Solution:

Jacqueline will wait and work moderately at $t = 2$.

We now have to consider that Jacqueline has five options: working hard today, working moderately today, working hard tomorrow, working moderately tomorrow, or not working at all.

Jacqueline is fully naive. To understand how she makes her decision today, let's first derive what she predicts that she will do tomorrow if she does not work today.

She believes that if she does not work today, tomorrow she will be maximizing $\hat{U}_2 = u_2 + u_3$. We have that:

$$\hat{U}_2(\text{working hard}) = Y_2 - c_2 - k_2$$

$$\hat{U}_2(\text{working moderately}) = Y_1 - c_1 - k_1$$

$$\hat{U}_2(\text{not working}) = 0$$

From assumption (10) we have that

$$\hat{U}_2(\text{working hard}) > \hat{U}_2(\text{working moderately})$$

And from assumption (7) (using that $\beta < 1$) we get that

$$\hat{U}_2(\text{working moderately}) > \hat{U}_2(\text{not working}) = 0$$

Therefore $\hat{U}_2$ is maximized by working hard. Jacqueline predicts that if she waits today, she will work hard tomorrow.

We can now derive her decision today. She is effectively deciding between working hard today, working moderately today, or waiting and working hard tomorrow (because she believes that she will do so). These three options correspond to the following utility levels:

$$U_1(\text{working hard today}) = -c_2 + \beta Y_2$$

$$U_1(\text{working moderately today}) = -c_1 + \beta Y_1$$

$$U_1(\text{waiting}) = \beta(-c_2 - k_2 + Y_2)$$

From assumption (12) we have that:

$$U_1(\text{working hard today}) > U_1(\text{working moderately today})$$

From assumption (9) we have that:

$$U_1(\text{waiting}) > U_1(\text{working hard today})$$

That is, Jacqueline decides to wait.

What does she actually do when tomorrow arrives? We can write down her real utility values from the three possible options, taking into account her present bias:

$$U_2(\text{working hard}) = \beta Y_2 - c_2 - k_2$$

$$U_2(\text{working moderately}) = \beta Y_1 - c_1 - k_1$$
$$U_2(\text{not working}) = 0$$

From assumption (7) we know that:

$$U_2(\text{working moderately}) > U_1(\text{not working})$$

And from assumption (11) we have that:

$$U_2(\text{working moderately}) > U_1(\text{working hard})$$

She will thus only work moderately, in period $t = 2$.

9. (3 points) What would be Jacqueline's decisions if she did not care about her grade ($Y_2 = Y_1$) but otherwise had the same preferences as in the previous question? i.e.:

$$c_1 + k_1 < \beta Y_1 \tag{13}$$
$$c_1 < \beta(c_1 + k_1) \tag{14}$$

Solution:

Jacqueline will work moderately in period 1.

In that case Jacqueline would never consider working hard.

Assumption (13) implies that $c_1 < \beta Y_1$, thus she would rather work moderately in period 1 rather than never working. In addition, (14) implies that she would rather work moderately in period 1 than in period 2. As a result, Jacqueline would just work moderately in period 1.

10. (3 points) We define Jacqueline's welfare for a given path of decisions as $W = u_1 + u_2 + u_3$.

Compare Jacqueline's welfare in the two previous questions. Comment. Is Jacqueline better or worse off when she cares about her grades than when she does not? Why?

Solution:

In the previous question, Jacqueline's welfare is $Y_1 - c_1$ while it was $Y_1 - c_1 - k_1$ when she cared about her grade. Jacqueline is worse off in this situation from caring about her grade, because it makes her consider a costlier effort level to reach a higher grade. However, she fails to realize that because of an increased stress from working hard last minute, she actually won't follow through with her plan, and will end up working moderately last minute, which ends up adding stress relative to working moderately in the first place.

PART 3, QUESTION 3: Beliefs and Studying Behavior [30 points]

Please make sure to explain your answers in this section carefully and concisely. Do not simply write an answer without an explanation of how you arrived at this answer. Answers without adequate explanation will not receive full credit.

Harry is studying for his last final exam, which takes place tomorrow. Today, he believes he will pass with probability p and fail with probability $1-p$. Harry's utility tomorrow if he passes is 10 and 0 otherwise. His long–term daily discount factor is $\delta \in (0,1)$ and his short-term daily discount factor is $\beta \in [0,1]$. That is, his expected utility today is

$$\beta\delta \cdot 10p.$$

1. (5 points) Suppose Harry can study for his exam. Studying costs $\gamma > 0$ today, and increases his chance of passing tomorrow to $(1+\theta)p$, where $\theta > 0$. Give a condition (in terms of β, δ, p, γ, and θ) that characterizes when Harry will study for his exam.

Solution: Harry's expected utility is $\beta\delta \cdot 10p$ if he does not study, and $-\gamma + \beta\delta \cdot 10p \cdot (1+\theta)$ if he studies. He studies if

$$\gamma < \beta\delta \cdot 10p \cdot \theta$$

or equivalently

$$\theta > \frac{\gamma}{\beta\delta \cdot 10p}.$$

2. (5 points) Suppose Harry does not study for his exam. Provide at least two economic reasons why he might not be studying (using the parameters from the previous questions).

Solution: We know Harry does not study if $\gamma > \beta\delta \cdot 10p \cdot \theta$, so, Harry will not study if

- studying is very costly (high γ)
- studying is not so effective (low θ)
- he was likely to fail anyway (low p)
- he is very impatient (low β)
- he considerably discounts future periods (low δ)

For the rest of this problem, suppose Harry is anxious about his performance. Specifically, his quality of sleep tonight depends on his beliefs about how well he will do tomorrow on his exam (but sleep does not directly affect his performance). His (expected) utility is now

$$f(p) + \beta\delta \cdot 10p.$$

The better Harry thinks he will do, the better he sleeps, so $f(p)$ is strictly increasing in p. In particular, $f(p) = \ln p$.

3. (5 points) Show that if Harry studied for the exam in part 1, then he will also study now.

Solution: The condition for Harry to study is now

$$\beta\delta \cdot 10p \cdot \theta + f(p(1+\theta)) - f(p) > \gamma$$

If he studied in part 1, $\beta\delta \cdot 10p \cdot \theta > \gamma$. We also know $f(p)$ is increasing, which means $f(p(1+\theta)) - f(p) > 0$. Thus if Harry studied in part 1, the new condition for studying will be met.

Another approach is to simplify using $f(p) = \ln p$ to determine that

$$\beta\delta \cdot 10p \cdot \theta + f(p(1+\theta)) - f(p) = \beta\delta \cdot 10p \cdot \theta + \ln(1+\theta) > \beta\delta \cdot 10p \cdot \theta > \gamma$$

This follows from $\theta > 0$ and $\beta\delta \cdot 10p \cdot \theta > \gamma$, and implies the condition is met.

Intuitively, Harry's anxiety creates additional benefits of studying; studying now reduces his anxiety in addition to making him more likely to pass. Therefore, if he studied in part 1, he will also study now.

4. (5 points) We now further explain the origins of p. The exam is "hard," with probability $1-q$, or "easy" with probability q. Harry knows he will pass the easy exam (i.e., he will pass the easy exam with probability 1), but will have more trouble passing the hard exam (he passes with probability $\hat{p} \in (0, p)$). In particular,

$$p = q \cdot 1 + (1-q) \cdot \hat{p}.$$

Suppose Harry has lost his books and cannot study no matter what. Harry has a friend, Hermione, who knows (don't ask how) whether the exam is hard or easy.

Hermione is going to text Harry today with news of whether tomorrow's exam is hard or easy. Will this text message make Harry better off (in terms of expected utility)?

Solution: The condition for the information making Harry better off when Harry cannot study simplifies to

$$qf(1) + (1-q)f(\hat{p}) > f(q \cdot 1 + (1-q)\hat{p}).$$

Since $f(p) = \ln p$ is concave, we know that this is not satisfied (Harry is information-averse). The future arrival of Hermione's text therefore reduces Harry's expected utility.

5. (5 points) A mysterious stranger has found and returned Harry's books. Studying still costs γ. It has no benefit when the exam is easy, but changes Harry's probability of passing when the exam is hard from $\hat{p}$ to $(1+\hat{\theta})\hat{p}$, where $\hat{\theta} > \theta$ satisfies $(1+\theta)p = q + (1-q)\hat{p}(1+\hat{\theta})$.

Suppose that if Harry did *not* know Hermione, and therefore wouldn't receive a text from her, he would study for the exam.

When does Hermione's text to Harry increase Harry's expected utility? (I.e., for which parameter values?) When does the text reduce Harry's expected utility?

Solution:

With the news, Harry suffers from a night's sleep he expects to be worse by

$$qf(1) + (1-q)f((1+\hat{\theta})\hat{p}) - f((1+\theta)p).$$

However, he benefits by *not* having to study when he learns the exam is easy, i.e., benefits in expectation by

$$q\gamma,$$

since the probability the exam is easy is q and the cost of studying is γ. Therefore he is better off from Hermione's text message, in terms of expected utility, whenever his expected avoided effort exceeds his expected increase in anxiety:

$$q\gamma > f((1+\theta)p) - qf(1) - (1-q)f(\hat{p}(1+\hat{\theta})).$$

(*Not needed for your answer, but note that Harry will study if he learns the exam is hard: that's because we know he plans to study, so*

$$\gamma < \beta\delta \cdot 10p \cdot \theta + \ln(1+\theta). \tag{15}$$

In addition, $\hat{\theta} > \theta$ *(as defined above) and* $\hat{p}\hat{\theta} > p\theta$ *(since* $\theta p = (1-q)\hat{p}\hat{\theta}$*). Therefore (15) implies that* $\gamma < \beta\delta \cdot 10\hat{p}\hat{\theta} + \ln(1+\hat{\theta})$.)

6. (5 points) Suppose the setup is as in part 5, except that Harry would NOT study if he didn't receive a text from Hermione. When does Hermione's text to Harry increase Harry's expected utility? When does the text reduce Harry's expected utility?

Solution: If Harry learns the exam is easy, he will still not study. If Harry learns the exam is hard, he will study if

$$\gamma < \beta\delta \cdot 10\hat{p} \cdot \hat{\theta}.$$

If the information that the exam is hard does not change Harry's decision to study, he is worse off, because of his dislike of information (his night's sleep is, in expectation, worse).

However, if the information that the exam is hard does change Harry's decision to study, then Harry is better off from receiving the text if the additional expected net benefit from studying when the exam is hard,

$$(1-q) \cdot \left[\beta\delta \cdot 10\hat{p} \cdot \hat{\theta} - \gamma\right],$$

exceeds the direct utility cost from learning the news,

$$qf(1) + (1-q)f(\hat{p}(1+\hat{\theta})) - f(p).$$

PART 3, QUESTION 4: Are Your TAs Altruistic? [30 points]

Please make sure to explain your answers in this section carefully and concisely. Do not simply write an answer without an explanation of how you arrived at this answer. Answers without adequate explanation will not receive full credit.

After taking 14.13, you become interested in understanding the extent to which your team of TAs has social preferences. Casual observation suggests the TAs are all very nice. However, you decide to study this more rigorously, by having your TAs play a series of lab experiment games to determine the extent to which each of them are altruistic.

In all of the lab experiment games, the TAs split 1kg of apples according to various rules.

You know that Aaron and Alex have preferences with the following form:

$$u_s(x_s, x_o) = \begin{cases} \rho_s x_s^{1/2} + (1-\rho_s)x_o^{1/2} & \text{if } x_s \geq x_o \\ \sigma_s x_s^{1/2} + (1-\sigma_s)x_o^{1/2} & \text{if } x_s < x_o \end{cases}$$

where x_s is the kilograms of apples that Aaron or Alex get for themselves and x_o is the kilograms of apples that the other TA in the game gets.

1. (3 points) Suppose that, for Aaron, $\rho_s = \sigma_s = .5$. Describe Aaron's preferences. How does Aaron's marginal utility of apples change in the kilograms of apples that Aaron gets?

> **Solution:** Aaron is altruistic and cares equally about his own payoff and his fellow TAs payoffs; this is true regardless of whether or not his payoff is greater than the other TA's payoff.
> Aaron's marginal utility of apples is decreasing in the kilograms of apples that he gets.

2. (3 points) Suppose that Aaron and Alex play a game where Aaron chooses how much each person gets and Alex makes no choice. Solve for Aaron's choices of (distributing 1kg of apples between) x_s and x_o.

> **Solution:** Aaron solves
> $$\max_{.5x_s}\{x_s^{1/2} + .5(1-x_s)^{1/2}\}.$$
> Aaron chooses $x_s^* = .5$

3. (3 points) Suppose now that Alex chooses how much each person gets and Aaron makes no choice. Suppose that, for Alex, $\rho_s = \sigma_s = 1$. Describe Alex's preferences and solve for Alex's choices of x_s and x_o.

> **Solution:** Alex is narrowly self interested and only cares about his own payoff; this is true regardless of whether or not his payoff is greater than the other TA's payoff.
>
> Alex solves
> $$\max_{x_s}\{x_s^{1/2}\} \quad \text{s.t. } x_s \leq 1$$
> Alex chooses $x_s^* = 1$

4. (3 points) Suppose that the game is changed. First, Alex makes Aaron an offer under which Alex will get x_s and Aaron will get x_o. Then Aaron chooses to accept or reject the offer. If Aaron accepts, then Alex gets x_s and Aaron gets x_o. If Aaron rejects, then Alex and Aaron both get nothing.

   Suppose that Alex believes that Aaron will reject any offer with $x_o < 0.5$. What will Alex choose as his offer for x_s and x_o? What is the intuition for how Alex makes his choice?

Solution: Alex believes that if he chooses $x_s > .5$, then his utility will be 0. Thus Alex solves

$$\max_{x_s}\{x_s^{1/2}\} \quad \text{s.t. } x_s \leq .5$$

Alex chooses $x_s^* = .5$

Alex is selfish and so he chooses to keep as many apples to himself as he thinks Aaron will allow.

5. (3 points) Suppose that the format of the game is the same as in Part 4, but Aaron makes the offer and Alex accepts or rejects.

   Also suppose that Aaron believes that Alex will reject any offer lower than \$0.5. What will Aaron choose as his offer for x_s and x_o?

Solution: Aaron's optimal offer when Alex had no choice was $x_s^* = x_o^* = .5$. Since Aaron believes that Alex will accept this offer, he still chooses $x_s^* = x_o^* = .5$.

6. (4 points) Do Aaron and Alex make different choices when making offers in Part 4 and Part 5? Could we distinguish Aaron and Alex's preferences using the games from Part 4 and Part 5 only? Would having data from the games from Parts 2 and 3 help us distinguish Aaron and Alex's preferences? Why?

Solution: Aaron and Alex make the same choices in Part 3 and 4, so we cannot distinguish their preferences from these games alone. Aaron and Alex make different choices in Parts 2 and 3, so that helps us distinguish their preferences. The reason that having data from Parts 2 and 3 helps is because beliefs about the other player's decision to accept or reject the offer no longer result in Aaron and Alex making the same choice.

7. (4 points) Suppose that Will has preferences with the following form:

$$u_W(x_W, x_A, x_a, x_M, x_P) = \begin{cases} \rho_W x_W^{1/2} + (1-\rho_W)\sum_{j\in\{A,a,M,P\}} x_j^{1/2} & \text{if } x_W \geq \sum_{j\neq W} x_j \\ \sigma_W x_W^{1/2} + (1-\sigma_W)\sum_{j\in\{A,a,M,P\}} x_j^{1/2} & \text{if } x_W < \sum_{j\neq W} x_j \end{cases}$$

   where x_W is the kilograms of apples that Will gets and x_j $(j \neq W)$ is the kilograms of apples that TA j gets.

   Suppose that Will and Maddie play a game where Will chooses how much to keep for himself x_W and how much to give to Maddie x_M and Maddie makes no choice.

   After the game, Will can give some apples to Aaron x_A, Alex x_a, or Pierre–Luc x_P (with $x_A + x_a + x_P \leq x_W$). Will keeps $x_{\tilde{W}} = x_W - x_A - x_a - x_P$.

   Suppose Will has $\rho_W = \sigma_W = .5$. What will Will choose for x_W and x_M in the game? What will Will keep $x_{\tilde{W}}$ at the end of the day?

Solution: Will knows that he can give money to Aaron, Alex, and Pierre–Luc after the game, so he solves

$$\max_{x_W, x_M, x_A, x_a, x_P}\left\{.5x_W^{1/2} + .5\sum_{j\neq W} x_j^{1/2}\right\} \quad \text{s.t. } \sum_j x_j = 1$$

Will's marginal utility from giving money to each TA is $.25x^{-1/2}$. Will equates the marginal utility of giving money to each TA, so he chooses $x_j^* = .2$.

Thus Will chooses $x_W = .8$ and $x_M = .2$ in the game.

At the end of the day, Will keeps $x_{\tilde{W}} = .2$.

8. (4 points) Suppose we incorrectly assumed that Will had preferences with the following form:

$$u_W(x_W, x_A, x_a, x_M, x_P) = \begin{cases} \rho_W x_W^{1/2} + (1-\rho_W)x_M^{1/2} & \text{if } x_W \geq x_M \\ \sigma_W x_W^{1/2} + (1-\sigma_W)x_M^{1/2} & \text{if } x_W < x_M \end{cases}$$

Suppose that we only see how much Will gave to Maddie (and that we assume that Will gives no apples to Aaron, Alex, or Pierre-Luc). Show that we would infer $\rho_W = 2/3$ from Will's choice (of x_W and x_M) that you solved for in Part 7.

Solution: Under this assumption, Will would solve

$$\max_{x_W} \{\rho_W x_W^{1/2} + (1-\rho_W)(1-x_W)^{1/2}$$

The FOC is

$$.5\rho_W x_W^{-1/2} - .5(1-\rho_W)(1-x_W)^{-1/2} = 0.$$

Rearranging for ρ_W gives

$$\rho_W = \frac{\sqrt{x}}{\sqrt{1-x} + \sqrt{x}}.$$

Substituting the solution from Part 6, gives

$$\rho_W = \frac{\sqrt{.8}}{\sqrt{.2} + \sqrt{.8}} = 2/3.$$

9. (3 points) Describe, in words, what it means when to infer that Will has $\rho_W = 2/3$ as opposed to $\rho_W = 1/2$. Explain why the conclusions from Part 8 make sense if we only see how much Will gave to Maddie (and assume he gives nothing to the other TAs).

Solution: It means that we are inferring that Will cares more about his own preferences and less about the other TAs preferences. Will chooses $x_W = .8$ and $x_M = .2$. Because he keeps most of the apples and gives few away, we infer that he cares more about his own preferences than the other TAs. However, in fact, this is not the case.