14.123 Microeconomics III—Problem Set 4

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Instructions. You are encouraged to work in groups, but everybody must write their own solutions. Each question is 33 points. Good Luck!

1. Ann is ambiguity-averse and has constant absolute risk aversion α. There is a stock that pays y per unit where $y \sim N(\mu, \sigma^2)$ where $\mu \in \left[\underline{\mu}, \bar{\mu}\right]$ is ambiguous. (She maximizes $\min_{\mu} E[u|\mu]$ where u is a CARA utility with α.) Ann can buy or sell any amount of stock.

   (a) Compute the demand of Ann for the stock as a function of price P of the stock.

   (b) Suppose there are n copies of Ann and Y total units of stock. Find the market clearing price.

   (c) Answer the above questions by assuming instead that Ann is an expected utility maximizer with known $\mu \in \left[\underline{\mu}, \bar{\mu}\right]$.

   (d) Briefly discuss your answers.

2. There is an asset that pays y where y has a continuous strictly increasing c.d.f. F on some interval $[a, b]$. Ann is a rank-dependent expected utility maximizer with linear utility (i.e. she is "risk-neutral") and with probability weighting function $w : [0,1] \to [0,1]$ where $w(x) = e^{-(-\log x)^{\alpha}}$ for some $\alpha \in (0,1)$. Let $P(\alpha)$ be the maximum amount Ann is willing to pay the asset. Compute $\lim_{\alpha \to 0} P(\alpha)$ and $\lim_{\alpha \to 1} P(\alpha)$.

3. Bob has just retired and has w_0 dollars. His utility from a consumption stream $(c_0, c_1, \ldots)$ is

$$\sum_{t=0}^{n} \delta^t u(c_t|x_t),$$

   where $u(c_t|x_t)$ is a reference dependent utility function

$$u(c_t|x_t) = v(c_t - x_t)$$

   with

$$v(y) = \begin{cases} y & \text{if } y \geq 0 \\ \lambda y & \text{otherwise} \end{cases}$$

   for some $\lambda \geq 1$ and x_t is the reference point at t. The initial reference point x_0 is 0.

   (a) Take $n = 1$, $x_t = 0$ for all t, and find the optimal consumption stream c^* with $c_0^* + c_1^* \leq w_0$.

   (b) Take $n = \infty$, $x_t = 0$ for all t, and find the optimal consumption stream c^* with $c_0^* + c_1^* + \cdots \leq w_0$.

   (c) Answer the above questions by taking $x_t = c_{t-1}$.

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