14.06 Lecture Notes

Intermediate Macroeconomics

George-Marios Angeletos

MIT Department of Economics

Spring 2004

Chapter 7

Endogenous Growth II: R&D and Technological Change

7.1 Expanding Product Variety: The Romer Model

- There are three production sectors in the economy: A final-good sector, an intermediate good sector, and an R&D sector.

- The final good sector is perfectly competitive and thus makes zero profits. Its output is used either for consumption or as input in each of the other two sector.

- The intermediate good sector is monopolistic. There is product differentiation. Each intermediate producer is a quasi-monopolist with respect to his own product and thus enjoys positive profits. To become an intermediate producer, however, you must first acquire a "blueprint" from the R&D sector. A "blueprint" is simply the technology or know-how for transforming final goods to differentiated intermediate inputs.

- The R&D sector is competitive. Researchers produce "blueprints", that is, the technology for producing an new variety of differentiated intermediate goods. Blueprints are protected by perpetual patents. Innovators auction their blueprints to a large number of potential buyers, thus absorbing all the profits of the intermediate good sector. But there is free entry in the R&D sector, which drive net profits in that sector to zero as well.

7.1.1 Technology

- The technology for final goods is given by a neoclassical production function of labor L and a composite factor Z:

$$Y_t = F(\mathcal{X}_t, L_t) = A(L_t)^{1-\alpha}(\mathcal{X}_t)^{\alpha}.$$

The composite factor is given by a CES aggregator of intermediate inputs:

$$\mathcal{X}_t = \left[\int_0^{N_t} (X_{t,j})^{\varepsilon} dj\right]^{1/\varepsilon},$$

where N_t denotes the number of different intermediate goods available in period t and $X_{t,j}$ denotes the quantity of intermediate input j employed in period t.

- In what follows, we will assume $\varepsilon = \alpha$, which implies

$$Y_t = A(L_t)^{1-\alpha} \int_0^{N_t} (X_{t,j})^{\alpha} dj.$$

Note that $\varepsilon = \alpha$ means the marginal product of each intermediate input is independent of the quantity of other intermediate inputs:

$$\frac{\partial Y_t}{\partial X_{t,j}} = \alpha A \left(\frac{L_t}{X_{t,j}}\right)^{1-\alpha}.$$

More generally, intermediate inputs could be either complements or substitutes, in the sense that the marginal product of input j could depend either positively or negatively on X_t.

- We will interpret intermediate inputs as capital goods and therefore let aggregate "capital" be given by the aggregate quantity of intermediate inputs:

$$K_t = \int_0^{N_t} X_{t,j} dj.$$

- Finally, note that if $X_{t,j} = X$ for all j and t, then

$$Y_t = AL_t^{1-\alpha} N_t X^{\alpha}$$

and

$$K_t = N_t X,$$

implying

$$Y_t = A(N_t L_t)^{1-\alpha}(K_t)^{\alpha}$$

or, in intensive form, $y_t = AN_t^{1-\alpha} k_t^{\alpha}$. Therefore, to the extent that all intermediate inputs are used in the same quantity, the technology is linear in knowledge N and capital K. Therefore, if both N and K grow at a constant rate, as we will show to be the case in equilibrium, the economy will exhibit long run growth like in an Ak model.

7.1.2 Final Good Sector

- The final good sector is perfectly competitive. Firms are price takers.

- Final good firms solve

$$\max \; Y_t - w_t L_t - \int_0^{N_t} (p_{t,j} X_{t,j}) dj$$

where w_t is the wage rate and $p_{t,j}$ is the price of intermediate good j.

- Profits in the final good sector are zero, due to CRS, and the demands for each input are given by the FOCs

$$w_t = \frac{\partial Y_t}{\partial L_t} = (1-\alpha)\frac{Y_t}{L_t}$$

and

$$p_{t,j} = \frac{\partial Y_t}{\partial X_{t,j}} = \alpha A \left(\frac{L_t}{X_{t,j}}\right)^{1-\alpha}$$

for all $j \in [0, N_t]$.

7.1.3 Intermediate Good Sector

- The intermediate good sector is monopolistic. Firms understand that they face a downward sloping demand for their output.

- The producer of intermediate good j solves

$$\max\ \Pi_{t,j} = p_{t,j}X_{t,j} - \kappa(X_{t,j})$$

subject to the demand curve

$$X_{t,j} = L_t\left(\frac{\alpha A}{p_{t,j}}\right)^{\frac{1}{1-\alpha}},$$

where $\kappa(X)$ represents the cost of producing X in terms of final-good units.

- We will let the cost function be linear:

$$\kappa(X) = X.$$

The implicit assumption behind this linear specification is that technology of producing intermediate goods is identical to the technology of producing final goods. Equivalently, you can think of intermediate good producers buying final goods and transforming them to intermediate inputs. What gives them the know-how for this transformation is precisely the blueprint they hold.

- The FOCs give

$$p_{t,j} = p \equiv \frac{1}{\alpha} > 1$$

for the optimal price, and

$$X_{t,j} = xL$$

for the optimal supply, where

$$x \equiv A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}}.$$

- Note that the price is higher than the marginal cost ($p = 1/\alpha > \kappa'(X) = 1$), the gap representing the mark-up that intermediate-good firms charge to their customers (the final good firms). Because there are no distortions in the economy other than monopolistic competition in the intermediate-good sector, the price that final-good firms are willing to pay represents the social product of that intermediate input and the cost that intermediate-good firms face represents the social cost of that intermediate input. Therefore, the mark-up $1/\alpha$ gives the gap between the social product and the social cost of intermediate inputs. (*Hint:* The social planner would like to correct for this distortion. How?)

- The resulting maximal profits are

$$\Pi_{t,j} = \pi L$$

where

$$\pi \equiv (p-1)x = \tfrac{1-\alpha}{\alpha} x = \tfrac{1-\alpha}{\alpha} A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}}.$$

7.1.4 The Innovation Sector

- The present value of profits of intermediate good j from period t and on is given by

$$V_{t,j} = \sum_{\tau=t} \frac{q_\tau}{q_t} \Pi_{\tau,j}$$

  or recursively

$$V_{t,j} = \Pi_{t,j} + \frac{V_{t+1,j}}{1 + R_{t+1}}$$

- We know that profits are stationary and identical across all intermediate goods: $\Pi_{t,j} = \pi L$ for all t, j. As long as the economy follows a balanced growth path, we expect the interest rate to be stationary as well: $R_t = R$ for all t. It follows that the present value of profits is stationary and identical across all intermediate goods:

$$V_{t,j} = V = \frac{\pi L}{R/(1+R)} \approx \frac{\pi L}{R}.$$

  Equivalently, $RV = \pi L$, which has a simple interpretation: The opportunity cost of holding an asset which has value V and happens to be a "blueprint", instead of investing in bonds, is RV; the dividend that this asset pays in each period is πL; arbitrage then requires the dividend to equal the opportunity cost of the asset, namely $RV = \pi L$.

- New blueprints are also produced using the same technology as final goods. In effect, innovators buy final goods and transform them to blueprints at a rate $1/\eta$.

- Producing an amount ΔN of new blueprints costs $\eta \cdot \Delta N$, where $\eta > 0$ measures the cost of R&D in units of output. On the other hand, the value of these new blueprints is $V \cdot \Delta N$, where $V = \pi L/R$. Net profits for a research firm are thus given by

$$(V - \eta) \cdot \Delta N$$

Free entry in the sector of producing blueprints then implies

$$V = \eta.$$

7.1.5 Households

- Households solve

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$
$$s.t. \quad c_t + a_{t+1} \leq w_t + (1 + R_t)a_t$$

- As usual, the Euler condition gives

$$\frac{u'(c_t)}{u'(c_{t+1})} = \beta(1 + R_{t+1}).$$

And assuming CEIS, this reduces to

$$\frac{c_{t+1}}{c_t} = [\beta(1 + R_{t+1})]^{\theta}.$$

7.1.6 Resource Constraint

- Final goods are used either for consumption by households, or for production of intermediate goods in the intermediate sector, or for production of new blueprints in the innovation sector. The resource constraint of the economy is given by

$$C_t + K_t + \eta \cdot \Delta N_t = Y_t,$$

where $C_t = c_t L$, $\Delta N_t = N_{t+1} - N_t$, and $K_t = \int_0^{N_t} X_{t,j} dj$.

7.1.7 General Equilibrium

- Combining the formula for the value of innovation with the free-entry condition, we infer $\pi L/R = V = \eta$. It follows that the equilibrium interest rate is

$$R = \frac{\pi L}{\eta} = \tfrac{1-\alpha}{\alpha} A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} L/\eta,$$

  which verifies our earlier claim that the interest rate is stationary.

- The resource constraint reduces to

$$\frac{C_t}{N_t} + \eta \cdot \left[\frac{N_{t+1}}{N_t} - 1\right] + X = \frac{Y_t}{N_t} = AL^{1-\alpha} X^{\alpha},$$

  where $X = xL = K_t/N_t$. It follows that C_t/N_t is constant along the balanced growth path, and therefore C_t, N_t, K_t, and Y_t all grow at the same rate, γ.

- Combining the Euler condition with the equilibrium condition for the real interest rate, we conclude that the equilibrium growth rate is given by

$$1 + \gamma = \beta^{\theta} \left[1 + R\right]^{\theta} = \beta^{\theta} \left[1 + \tfrac{1-\alpha}{\alpha} A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} L/\eta\right]^{\theta}$$

- Note that the equilibrium growth rate of the economy decreases with η, the cost of expanding product variety or producing new "knowledge".

- The growth rate is also increasing in L or any other factor that increases the "scale" of the economy and thereby raises the profits of intermediate inputs and the demand for innovation. This is the (in)famous "scale effect" that is present in many models of endogenous technological change. Discuss....

7.1.8 Pareto Allocations and Policy Implications

- Consider now the problem of the social planner. Obviously, due to symmetry in production, the social planner will choose the same quantity of intermediate goods for all varieties: $X_{t,j} = X_t = x_t L$ for all j. Using this, we can write the problem of the social planner simply as maximizing utility,

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t),$$

subject to the resource constraint

$$C_t + N_t \cdot X_t + \eta \cdot (N_{t+1} - N_t) = Y_t = AL^{1-\alpha} N_t X_t^{\alpha},$$

where $C_t = c_t L$.

- The FOC with respect to X_t gives

$$X_t = x^* L,$$

where

$$x^* = A^{\frac{1}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}}$$

represents the optimal level of production of intermediate inputs.

- The Euler condition, on the other hand, gives the optimal growth rate as

$$1 + \gamma^* = \beta^\theta \left[1 + R^*\right]^\theta = \beta^\theta \left[1 + \tfrac{1-\alpha}{\alpha} A^{\frac{1}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} L/\eta\right]^\theta,$$

where

$$R^* = \tfrac{1-\alpha}{\alpha} A^{\frac{1}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} L/\eta$$

represents that social return on savings.

- Note that

$$x^* = x \cdot \alpha^{-\frac{1}{1-\alpha}} > x$$

That is, the optimal level of production of intermediate goods is higher in the Pareto optimum than in the market equilibrium. This reflects simply the fact that, due to the monopolistic distortion, production of intermediate goods is inefficiently low in the market equilibrium. Naturally, the gap x^*/x is an increasing function of the mark-up $1/\alpha$.

- Similarly,

$$R^* = R \cdot \alpha^{-\frac{1}{1-\alpha}} > R.$$

That is, the market return on savings (R) falls short of the social return on savings (R^*), the gap again arising because of the monopolistic distortion in the intermediate good sector. It follows that

$$1 + \gamma^* > 1 + \gamma,$$

so that the equilibrium growth rate is too low as compared to the Pareto optimal growth rate.

- *Policy exercise:* Consider three different types of government intervention: A subsidy on the production of intermediate inputs; an subsidy on the production of final goods (or the demand for intermediate inputs); and a subsidy on R&D. Which of these policies could achieve an increase in the market return and the equilibrium growth rate? Which of these policies could achieve an increases in the output of the intermediate good sector? Which one, or which combination of these policies can implement the first best allocations as a market equilibrium?

7.1.9 Introducing Skilled Labor and Human Capital

notes to be completed

7.1.10 International Trade, Technology Diffusion, and other implications

notes to be completed

7.2 Improving Product Quality: A Simple Model

- Before analyzing the full-fledge Aghion-Howitt model, we consider a simplified version that delivers most of the insights.

- The economy is populated by a large number of finitely-lived households.

- Each producer in the economy is an "entrepreneur". He lives (be present in the market) for $1 + T$ periods, where T is random. In particular, conditional on being alive in the present period, there is probability $1 - n$ that the producer will be alive in the next period and a probability n that he will die (exit the market). n is constant over time and independent of the age of the producer.

- In each period, a mass n of existing producers dies, and a mass n of new producers is born. The population of producers is thus constant.

- In the first period of life, the producer is "endowed" with the aggregate level of knowledge in the economy. In the first period of life, he also has a "fresh mind" and may engage in R&D activity. In the later periods of life, he is too old for coming up with good ideas and therefore engages only in production, not innovation.

- Young producers engage in R&D in order to increase the profits of their own productive activities later in life. But individual innovation has spillover effects to the whole economy. When a mass of producers generate new ideas, the aggregate level of knowledge in the economy increases proportionally with the production of new ideas.

7.2.1 R&D Technology

- Let V_{t+1}^j denote the value of an innovation for individual j realized in period t and implemented in period $t+1$. Let z_t^j denote the amount of skilled labor that a potential innovator j employes in R&D and $q(z_t^j)$ the probability that such R&D activity will be successful. $q : \mathbb{R} \to [0,1]$ represents the technology of producing innovations and it is assumed to be strictly increasing and strictly concave and satisfy the relevant Inada conditions: $q(0) = 0$, $q' > 0 > q''$, $q'(0) = \infty$, $q'(\infty) = 0$.

- The potential researcher maximizes

$$q(z_t^j) \cdot V_{t+1}^j - w_t \cdot z_t^j.$$

It follows that the optimal level of R&D is given by

$$q'(z_t^j)V_{t+1}^j = w_t$$

or

$$z_t^j = g\left(\frac{V_{t+1}^j}{w_t}\right)$$

where the function $g(v) \equiv (q')^{-1}(1/v)$ satisfies $g(0) = 0$, $g' > 0$, $g(\infty) = \infty$. Note that the amount of labor devoted to R&D and the rate of innovation will be stationary only if both the value and the cost of innovation (V and w) grow at the same rate.

7.2.2 The Value of Innovation

- What determines the value of an innovation? For a start, let us assume a very simple structure. Let A_t^j represent the TFP of producer j in period t. The profits from his production are given by

$$\Pi_t^j = A_t^j \widehat{\pi}$$

where $\widehat{\pi}$ represents normalized profits. We can endogenize π, but we won't do it here for simplicity.

- When a producer is born, he automatically learns what is the contemporaneous aggregate level of technology. That is, $A_t^j = A_t$ for any producer born in period t. In the first period of life, and only in that period, a producer has the option to engage in R&D. If his R&D activity fails to produce an innovation, the his TFP remains the same for the rest of his life. If instead his R&D activity is successful, then his TFP increases permanently by a factor $1+\gamma$, for some $\gamma > 0$. That is, for any producer j born in period t, and for all periods $\tau \geq t+1$ in which he is alive,

$$A_\tau^j = \begin{cases} A_t & \text{if his R\&D fails} \\ (1+\gamma)A_t & \text{if his R\&D succeeds} \end{cases}$$

- It follows that a successful innovation increases profits also by a factor $1+\gamma$. That is, the innovation generates a stream of "dividends" equal to $\gamma A_t \widehat{\pi}$ per period that the producer is alive. Since the producer expects to survive with a probability $1-n$ in each period, the expected present value of the increase in profits is given by

$$V_{t+1} = \sum_{\tau=t+1}^{\infty} \left(\frac{1-n}{1+R}\right)^{\tau} (\gamma A_t \widehat{\pi}) = \gamma \widehat{v} A_t \tag{7.1}$$

where where R is the interest rate per period and

$$\widehat{v} \equiv \sum_{\tau=1}^{\infty} \left(\frac{1-n}{1+R} \right)^{\tau} \widehat{\pi} \approx \frac{\widehat{\pi}}{R+n}.$$

Note that the above would be an exact equality if time was continuous. Note also that $\widehat{v}$ is decreasing in both R and n.

- *Remark:* We see that the probability of "death" reduces the value of innovation, simply because it reduces the expected life of the innovation. Here we have taken n as exogenous for the economy. But later we will endogenize n. We will recognize that the probability of "death" simply the probability that the producer will be displaced by another competitor who manages to innovate and produce a better substitute product. For the time being, however, we treat n as exogenous.

7.2.3 The Cost of Innovation

- Suppose that skilled labor has an alternative employment, which a simple linear technology of producing final goods at the current level of aggregate TFP. That is, if l_t labor is used in production of final goods, output is given by $A_t l_t$. Since the cost of labor is w_t, in equilibrium it must be that

$$w_t = A_t. \tag{7.2}$$

7.2.4 General Equilibrium

- Combining (7.1) and (7.2), we infer that the value of innovation relative to the cost of R&D is given by

$$\frac{V_{t+1}}{w_t} = \gamma \widehat{v}$$

It follows that the level of R&D activity is the same across all new-born producers:

$$z_t^j = z_t = g\left(\gamma\widehat{v}\right).$$

- Note that the outcome of the R&D activity of the individual producer is stochastic. In every period, some researchers succeeds and some fail. By the law of large numbers, however, the aggregate outcome of R&D is deterministic. In particular, the aggregate rate of innovation in the economy is simply given by

$$\lambda_t = q(z_t) = \lambda\left(\gamma\widehat{v}\right)$$

where $\lambda\left(\gamma\widehat{v}\right) \equiv q\left(g\left(\gamma\widehat{v}\right)\right).$

- If each innovation results to a quality improvement in technology by a factor $1+\gamma > 1$, and a mass λ_t of R&D projects is successful, then the aggregate level of technology improves at a rate

$$\frac{A_{t+1}}{A_t} = 1 + \gamma\lambda_t.$$

This gives the equilibrium growth rate of the economy as

$$\frac{y_{t+1}}{y_t} = \frac{A_{t+1}}{A_t} = 1 + \gamma\lambda\left(\gamma\widehat{v}\right).$$

- An increase in $\widehat{v}$ increases the incentives for R&D in the individual level and therefore results to higher rates of innovation and growth in the aggregate level. An increase in γ has a double effect. Not only it increases the incentive for R&D, but it also increase the spill over effect from individual innovations to the aggregate level of technology.

7.2.5 Business Stealing

- Consider a particular market j, in which a producer j has monopoly power. Suppose now that there is an outside competitor who has the option to engage in R&D in an attempt to create a better product that is a close substitute for the product of producer j. Suppose further that, if successful, the innovation will be so "radical" that, not only it will increase productivity and reduce production costs, but it will also permit the outsider to totally displace the incumbent from the market.

- *Remark:* We will later discuss in more detail what is the market structure and how competition between the incumbent and an entrant is resolved. We will then see that the size of the innovation and the type of competition (e.g., Bertrand versus Cournot) will determine what is the fraction of monopoly profits that the entrant can grasp. For the time being, we assume for simplicity that a successful innovator simply becomes the new monopolist in the market.

- What is the value of the innovation for this outsider? Being an outsider, he has no share in the market of product j. If his R&D is successful, he expects to displace the incumbent and grasp the whole market of product j. That is, an innovation delivers a dividend equal to total market profits, $(1+\gamma)A_t\widehat{\pi}$, in each period of life. Assuming that the outsider also has a probability of death equal to n, the value of innovation for the outsider is given by

$$V_{t+1}^{out} = \sum_{\tau=t+1}^{\infty} \left(\frac{1-n}{1+R}\right)^{\tau} [(1+\gamma)A_t\widehat{\pi}] = (1+\gamma)\widehat{v}A_t$$

- Now suppose that the incumbent also has the option to innovate is later periods of life. If he does so, he will learn the contemporaneous aggregate level of productivity and improve upon it by a factor $1+\gamma$. The value of innovation in later periods of life is thus the same as in the first period of life:

$$V_{t+1}^{in} = \sum_{\tau=t+1}^{\infty} \left(\frac{1-n}{1+R}\right)^{\tau} [\gamma A_t\widehat{\pi}] = \gamma\widehat{v}A_t.$$

- Compare now the value of an innovation between the incumbent and the outsider. Obviously, $V_{t+1}^{out} > V_{t+1}^{in}$. That is because the incumbent values only the potential increase in productivity and profits, while the outsider values in addition the fact that he will be able to "steal the business" of the incumbent. This "business-stealing" effect implies that, ceteris paribus, that innovation will take place mostly in outsiders.

- *Remark:* Things could be reversed if the incumbent has a strong cost advantage in R&D, which could be the case if the incumbent has some private information about the either the technology of the product or the demand of the market.

- Using $V_{t+1}^{out}/w_t = (1+\gamma)\widehat{v}$, we infer that the optimal level of R&D for an outsider is given by

$$z_t^{out} = z_t = g\left((1+\gamma)\widehat{v}\right).$$

Assuming that only outsiders engage in R&D, we conclude that the aggregate level of innovation is

$$\lambda_t = q(z_t) = \lambda\left((1+\gamma)\widehat{v}\right)$$

and the growth rate of the economy is

$$\frac{y_{t+1}}{y_t} = \frac{A_{t+1}}{A_t} = 1 + \gamma\lambda\left((1+\gamma)\widehat{v}\right).$$

- Finally, we can now reinterpret the probability of "death" as simply the probability of being displaced by a successful outside innovator. Under this interpretation, we have

$$n = \lambda\left((1+\gamma)\widehat{v}\right)$$

and $\widehat{v}$ solves

$$\widehat{v} = \frac{\widehat{\pi}}{R + \lambda\left((1+\gamma)\widehat{v}\right)}$$

7.2.6 Pareto Allocations and Policy Implications

- Discuss the spillover effects of innovation... Both negative and positive...

- Discuss optimal patent protection... Trade-off between incentives and externalities...

7.3 Ramsey Meets Schumpeter: The Aghion-Howitt Model

notes to be completed