We can solve the equations for homogeneous, incompressible, two–dimensional flow

$$\frac{d}{dt}\mathbf{u} + f\hat{\mathbf{k}} \times \mathbf{u} = -\nabla\frac{p}{\rho} + \nu\nabla^2\mathbf{u}$$

$$\nabla \cdot \mathbf{u} = 0 \qquad \text{or} \quad u = -\frac{\partial}{\partial y}\psi \ , \quad v = \frac{\partial}{\partial x}\psi$$

using a streamfunction

$$\mathbf{u} = -\nabla \times \hat{\mathbf{k}}\psi(x, y, t)$$

From the divergence of the momentum equations, we find

$$\nabla^2\frac{p}{\rho} = \nabla \cdot f\nabla\psi - \frac{\partial}{\partial x_i}u_j\frac{\partial}{\partial x_j}u_i$$

so, given ψ, we can find the velocities and the pressure p/ρ. From the curl of the momentum equations, we find the vorticity equation

$$\frac{d}{dt}q = \nu\nabla^2 q$$

with

$$q = f + \hat{\mathbf{k}} \cdot \nabla \times \mathbf{u} = f + \frac{\partial}{\partial x}v - \frac{\partial}{\partial y}u = f + \nabla^2\psi$$

Given ψ, we can find q and then evaluate the advection and diffusion terms to step q forward in time. Inverting the Laplace operator allows us to calculate ψ at the new time.

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