

## 12.520 Problem Set 2

Due 10/04/06

(Problem weights are 15%, 15%, 25%, 25%, 30%, respectively)

1) In Problem set 1 you considered the stress tensor

$$\sigma_{ij} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

and found the principal stresses and directions of  $\sigma_{ij}$  and the deviatoric stress tensor  $\sigma_{ij}^{dev}$

i) Construct the Mohr's circle for  $\sigma_{ij}$  in the  $\sigma_1 - \sigma_3$  plane.

ii) Construct the Mohr's circle for  $\sigma_{ij}^{dev}$  in the  $\sigma_1 - \sigma_3$  plane.

iii) What is the relation between the maximum shear for the two tensors? Explain why.

iv) Calculate the three invariants of both  $\sigma_{ij}$  and  $\sigma_{ij}^{dev}$ .

2) Give the stress tensors, principal stresses, Mohr's circles and maximum shear stresses for the following cases of tractions applied to a unit cube: [In all cases, the magnitude of the applied traction is  $\sigma$ .]

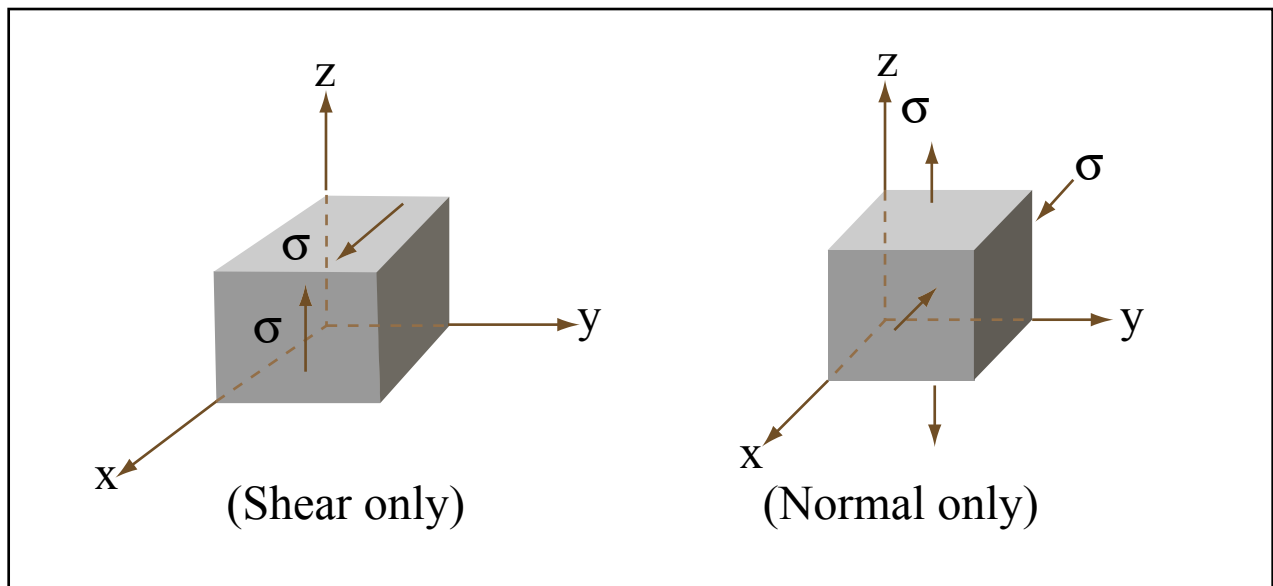


Figure by MIT OCW.

What can you say about the relationship between the stress tensors for the two examples shown?

3) In a laboratory test, samples of sandstone failed at the conditions:

a) confining “pressure” = 200 bars, uniaxial load stress = 1100 bars

b) confining “pressure” = 400 bars, uniaxial load stress = 1700 bars

If the strength of rock can be described by the Navier-Coulomb criterion, what are (i) the intrinsic strength  $S_0$ , (ii) the coefficient of internal friction  $\mu$ , (iii) the orientation of the fractures, (iv) the maximum shear stress (on any plane), (v) the shear traction on the failure plane in each test? Give the answers for dimensional quantities both in these units and in SI units.

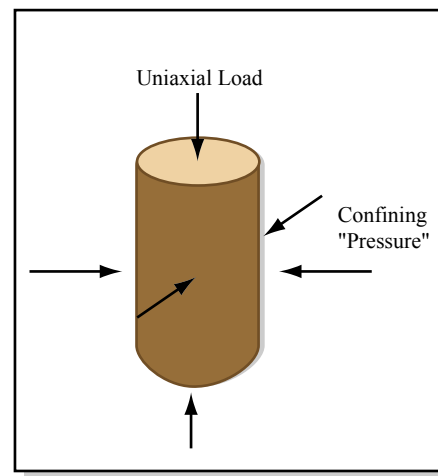


Figure by MIT OCW.

4) In a rock mechanics experiment, a cylindrical piece of rock is put into a press. The specimen is enclosed in a foil capsule to keep out fluids and a confining stress of  $-400$  MPa is applied by a fluid medium. An end load is applied, initially of  $-400$  MPa, then increasing in amplitude until the specimen fails.

Assume that the principal stress axes and principal stresses are as given in the figure. Assume that there is a crack in the sample oriented with its normal in the  $x_1$ - $x_3$  plane at  $60^\circ$  to  $x_1$ . The bulk of the sample is described by a Navier-Coulomb failure criterion:

$$S = S_0 + \mu N$$

with  $\mu = 0.6$  and  $S_0 = 4$  kb. The crack has the same coefficient of friction  $\mu$ , but since it is broken,  $S_0 = 0$ .

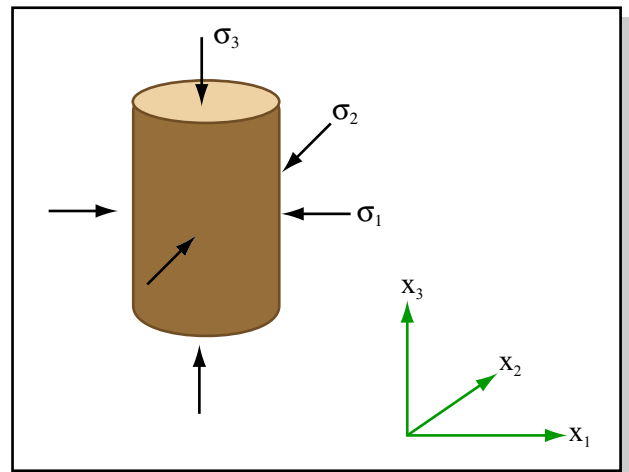


Figure by MIT OCW.

a) Draw the Mohr circles for  $\sigma_3 = -400, -600, \text{ and } -800$  MPa, along with the failure curves for the bulk sample and for the crack.

b) For these values of  $\sigma_3$ , what are the values of  $S$  and  $N$  on the crack?

c) Which fails first, the bulk sample or the preexisting crack? What is the value of  $\sigma_3$  required for failure? What are the values of  $S$  and  $N$  on the failure plane? What is the orientation of this plane?

d) Suppose that the sample leaks, so that fluid with pore pressure  $400$  MPa infiltrates the sample. Which fails first, the bulk sample or the preexisting crack? What is the value of  $\sigma_3$  required for failure? What are the values of  $S$  and  $N$  on the failure plane? What is the orientation of this plane?

5) In this problem you will get to know the World Stress Map, as well as other resources available for studying tectonics.

1. Obtain a stress map of a region of your choice using the on-line request tool (CASMO) in the World Stress Map web-site. This may be for example your home country, your field area, or just your favorite plate boundary. Your region should be large enough to include

at least 30 measurements. You can first look at the maps given on the website to get an impression of how the maps would look. Read the information given on the website under “Data Details”

2. Discuss the stress conditions shown in your map:
  - a. What are the main trends of stress directions?
  - b. What is the dominant regime of deformation?
  - c. Are the directions what you would expect given the location?
  - d. What techniques were used to obtain the measurements in your map?
  - e. What is the certainty of the measurements shown? What is the estimation of certainty based on?
  - f. How are the directions of stress correlating with other features, such as faults, topography, plate directions? Geologic maps of your region can be found online or in the geology labs, and plate motion directions can be calculated using the online tool on UNAVCO website.

Note: please see your TA if you encounter any technical difficulties.