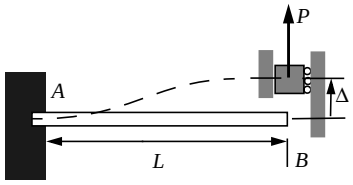
Problem Set #11 1.050 Solid Mechanics
Fall 2004
(Due Wednesday, 24 November)

Problem 10.1

A force *P* is applied to the end of a cantilever beam but the end, while free to move vertically, is restrained so that it can not rotate, i.e., the slope of the deflected curve is zero at *both* ends of the beam. We can write:

$P = K\,\Delta$



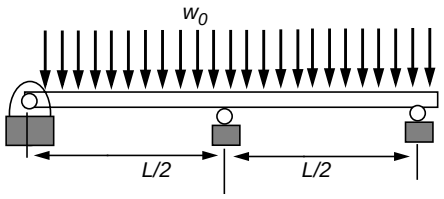
The beam is made from a material of Young's modulus *E* and its (symmetric) cross-section has bending moment of inertia *I*. Develop an expression for the stiffness *K* in terms of *E*, *I* and *L*, the length of the beam.

What are the reactive moments at the ends A and B in terms of Δ and the beam properties?

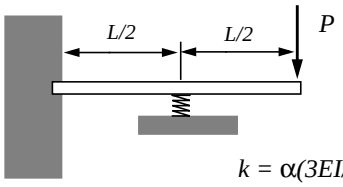
Problem 10.2

Determine the reactions at the three rollers of the redundantly supported beam which is uniformly loaded.

Sketch the shear force and bending moment distribution.



Problem 10.3



A cantilever beam is supported mid-span with a linear spring. The stiffness of the spring, *k*, is given in terms of the beam's stiffness as $k = \alpha(3EI/L^3)$

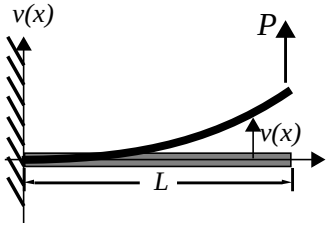
• Determine the reactions at the wall, and the way the shear force and bending moment vary along the beam.

• Compare the tip deflection with that of a cantilever without mid-span support.

• What if α gets very large? How do things change?

• What if α gets very small? How do things change?

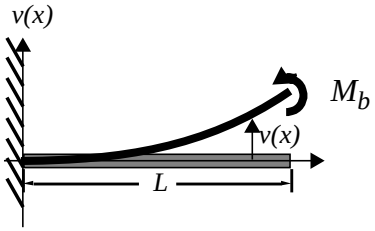
Displacements - **End-loaded Cantilever**



For $0<x<L$ $\quad v(x) = [\,PL^3/(6EI)][3(x/L)^2-(x/L)^3]$

$v|_{max} = PL^3/(3EI)$ $\quad$ *at* $x=L$

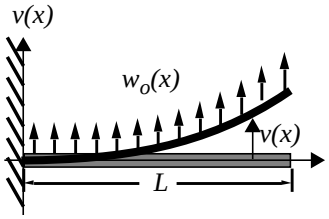
Couple, End-loaded Cantilever



For $0 < x < L$ $\quad v(x) =[\,ML^2/(2EI)]\,(x/L)^2$

$v|_{max} = ML^2/(2EI)$ $\quad$ *at* $x=L$

Uniformly Loaded Cantilever

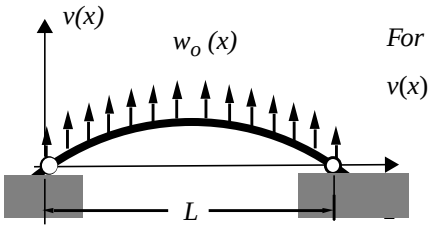


For $0 < x < L$

$v(x) = [\,w_o L^4/(24EI)]\,(x/L)^2[(x/L)^2 - 4(x/L) + 6]$

$v|_{max} = w_o L^4/(8EI)$ $\quad$ *at* $x=L$

Uniformly Loaded Simply-Supported Beam

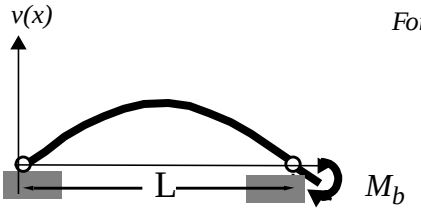


For $0 < x < L$

$v(x) = [w_0 L^4/(24EI)]\,(x/L)\,[1 - 2(x/L)^2 + (x/L)^3]$

$v|_{max} = [5w_o L^4/(384EI)]$ $\quad$ *at* $x=L/2$

Couple, End-loaded Simply-Supported Beam

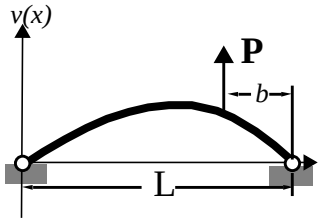


For $0 < x < L$

$v(x) = -\,[ML^2/(6EI)]\,(x/L)\,[1 - x^2/L^2]$

$v|_{max} = [\,ML^2/(9\sqrt{3}\,EI)]$ $\quad$ *at* $x=L/\sqrt{3}$

Point Load, Simply-Supported Beam



For $0<x<(L-b)$

$v(x) = [PL^3/(6EI)]\,(b/L)\,[\,-(x/L)^3 + (1-b^2/L^2)(x/L)\,]$

$v|_{max} = PL^3/[9\sqrt{3}\,EI)]\,(b/L)[1- b^2/L^2]^{3/2}$ $\quad$ *at* $x=(L/\sqrt{3})\sqrt{(1-b^2/L^2)}$

For $(L-b) <x <L$

$v(x) = [\,PL^3/(6EI)]\,(b/L)\,\{\,(L/b)\,[(x/L)- (1-b/L)]^3 - (x/L)^3+(1-b^2/L^2)(x/L)\}$