1.022 Introduction to Network Models **Fall 2018**

Homework 7

7.1 Herding Experiment

In this problem, we will ask whether an information cascade can occur if each individual sees only the action of her immediate neighbor rather than the actions of all those who have chosen previously. Let's keep the same setup as in the "Simple Herding Experiment" part of the lecture, except that when individual i chooses, she observes only her own signal and the action of individual $i-1$.

Again, similar to the setup in the lecture, all individuals trust their own observations when there is a tie between two choices.

[5 point] a) Briefly explain why the decision problems faced by individuals 1 and 2 are unchanged by this modification to the information network.

[10 point] b) Individual 3 observes the action of individual 2, but not the action of individual 1. What can 3 infer about 2's signal from 2's action?

[10 point] c) Can 3 infer anything about 1's signal from 2's action? Explain.

[10 point] d) What should 3 do if she observes "Chinese" and she knows that 2 chose the Chinese restaurant? What if 3 's signal was "Indian" and 2 chose the Chinese restaurant?

[10 point] e) Do you think that a cascade can form in this world? Explain why or why not. A formal proof is not necessary; a brief argument is sufficient.

7.2 Influencers and the Wisdom of the Crowd

Consider a society of n agents and let $x(t) = [x_1(t), \dots, x_n(t)]^T$ represent their opinion on some *state of the world*. Suppose that people update their opinion following the DeGroot dynamics:

$$x(t+1) = Px(t),$$

where the transition matrix P is of the following form:

$$P = \begin{bmatrix} \frac{1}{n} & \frac{1}{n} & \frac{1}{n} & \cdots & \frac{1}{n} & \frac{1}{n} & \frac{1}{n} \\ \frac{1}{2} & \frac{1}{2} & 0 & \cdots & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \frac{1}{2} & 0 & 0 & \cdots & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \cdots & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & \cdots & 0 & 0 & \frac{1}{2} \end{bmatrix}.$$

[20 points] Draw the corresponding weighted graph for $n = 10$. Is this graph strongly connected? Without doing any computations, argue whether society will eventually reach a consensus on the matter (that is, $\lim_{t\to\infty} x_i(t) = x^*$ for all $i = 1, 2, \dots, n$). Check your findings numerically using python (see python notebook) for some initial opinion vector $x(0)$ of your choice with different elements.

[20 points] Let w^T be the left eigenvector of P corresponding to the eigenvalue 1, which is normalized such that $w^T e = 1$, where $e = [1, \ldots, 1]^T$. Express w^T as a function of n, and use it to find the consensus point x^* from the initial opinions $x(0)$.

[20 points] Assume $\theta = 1/n \sum_{i=1}^{n} x_i(0)$, where θ is the underlying state of the world. Use your result from the previous part to write x^* in terms of θ, $x_1(0)$, and n only. Show that in a very large society with $n \to \infty$, the consensus point $x^* \to \frac{1}{3}x_1(0) + \frac{2}{3}\theta$. Is this society wise? (i.e., do individuals learn the underlying state?) Explain.