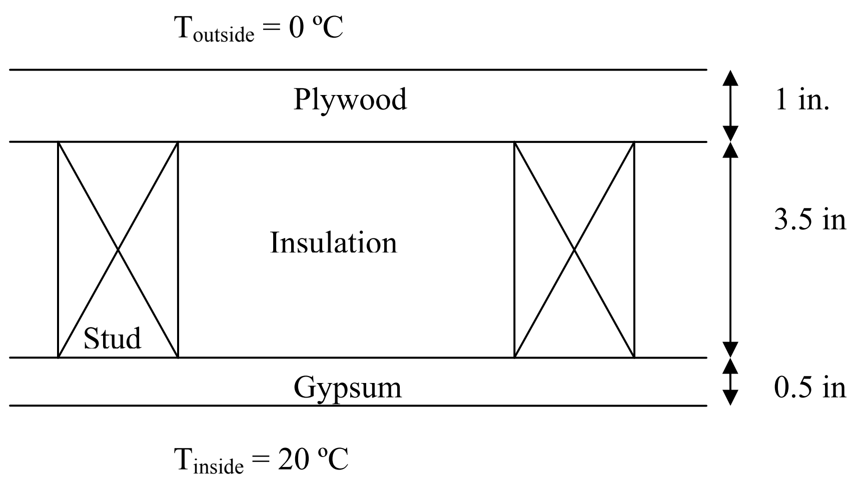
a.) Calculate the heat transfer in W/m^2 for the wall section shown in the figure below. The wood studs occupy 20% of the surface area of the wall and the insulation occupies 80%. The temperature of the air outside is 0ºC and the temperature of the air inside is 20ºC. Assume that there is no lateral heat transfer. Include convection and radiation heat transfer in your calculation. The outdoors and indoors can be considered black bodies at 0ºC and 20ºC respectively.

$k_{plywood}$ = 0.15 W/m-K
$k_{insulation}$ = 0.035 W/m-K
k_{gypsum} = 0.2 W/m-K
k_{wood} = 0.15 W/m-K



b.) A contractor you know suggests using steel studs instead of wood. He believes that because you need less steel than wood for structural stability, the increased insulation will decrease the total heat transfer across your wall section. Is he correct? Calculate the heat transfer in W/m^2 for a wall section with steel studs. Assume again that there is no lateral heat transfer. For this case, the insulation occupies 90% of the wall surface area and the steel occupies 10%. (k_{steel} = 43 W/m-K)

c.) Is the interior wall temperature the same over the insulated and stud sections? For case b, approximate the temperature of the interior wall over an insulated section. Compare this to the temperature of the interior wall over a steel stud.

Solution:

a.) Approximate $h_{out} \sim 20$ W/m^2-K, $h_{in} \sim 5$ W/m^2-K

$hr_{out} = 4\sigma T^3 = 4*5.67e\text{-}8*(273\ K)^3 = 4.6$ W/m^2-K

$hr_{in} = 4\sigma T^3 = 4*5.67e\text{-}8*(293\ K)^3 = 5.7$ W/m^2-K

Calculate total resistance for insulated section:

$R_{ins} = [1/(h_{out} + hr_{out})] + (L/k)ply + (L/k)ins + (L/k)gyp + [1/(h_{in} + hr_{in})]$

$R_{ins} = [1/(20 + 4.6)] + (0.0254/0.15) + (.09/.035) + (.0127/0.2) + [1/(5 + 5.7)]$

$R_{ins} = 2.94$ m^2-K/W

$q_{ins} = \Delta T/R = 20\ K/2.9$ m^2-K/W $= 6.8$ W/m^2

Calculate total resistance for wood stud section:

$R_{wood} = [1/(h_{out} + hr_{out})] + (L/k)ply + (L/k)wood + (L/k)gyp + [1/(h_{in} + hr_{in})]$

$R_{wood} = [1/(20 + 4.6)] + (0.0254/0.15) + (.09/.15) + (.0127/0.2) + [1/(5 + 5.7)]$

$R_{wood} = 0.97$ m^2-K/W

$q_{wood} = \Delta T/R = 20\ K/0.97$ m^2-K/W $= 20.7$ W/m^2

Calculate total heat transfer through wall:

$q_{total} = q_{ins} + q_{wood} = (0.8*6.8) + (0.2*20.7) =$ **9.6 W/m^2**

b.) The heat transfer through the insulation section is the same.

Calculate total resistance for steel stud section:

$R_{steel} = [1/(h_{out} + hr_{out})] + (L/k)ply + (L/k)wood + (L/k)gyp + [1/(h_{in} + hr_{in})]$

$R_{steel} = [1/(20 + 4.6)] + (0.0254/0.15) + (.09/43) + (.0127/0.2) + [1/(5 + 5.7)]$

$R_{steel} = 0.36$ m^2-K/W

$q_{steel} = \Delta T/R = 20\ K/0.36$ m^2-K/W $= 54.2$ W/m^2

Calculate total heat transfer through wall:

$q_{total} = q_{ins} + q_{steel} = (0.9*2.9) + (0.1*54.2) =$ **11.5 W/m^2**

The contractor was wrong. The steel stud wall allows more heat transfer than the wood stud wall, even though there is more insulation in the steel stud section.

c.) *Case 1 – Insulated section*:

$q_{ins} = \Delta T/R = \Delta T/[1/(h_{in} + hr_{in})]$

6.8 W/m^2 $= \Delta T/0.093 \rightarrow \Delta T = Tin – Twall = 0.6$ ºC

Twall = 20 – 0.6 = **19.4 ºC**

Case 2 – Steel stud section:

$q_{steel} = \Delta T/R = \Delta T/[1/(h_{in} + hr_{in})]$

54.2 W/m^2 $= \Delta T/0.093 \rightarrow \Delta T = Tin – Twall = 5$ ºC

Twall = 20 – 5 = **15 ºC**

4.42J / 1.044J / 2.45J Fundamentals of Energy in Buildings
Fall 2010