

# 16.485: VNAV - Visual Navigation for Autonomous Vehicles

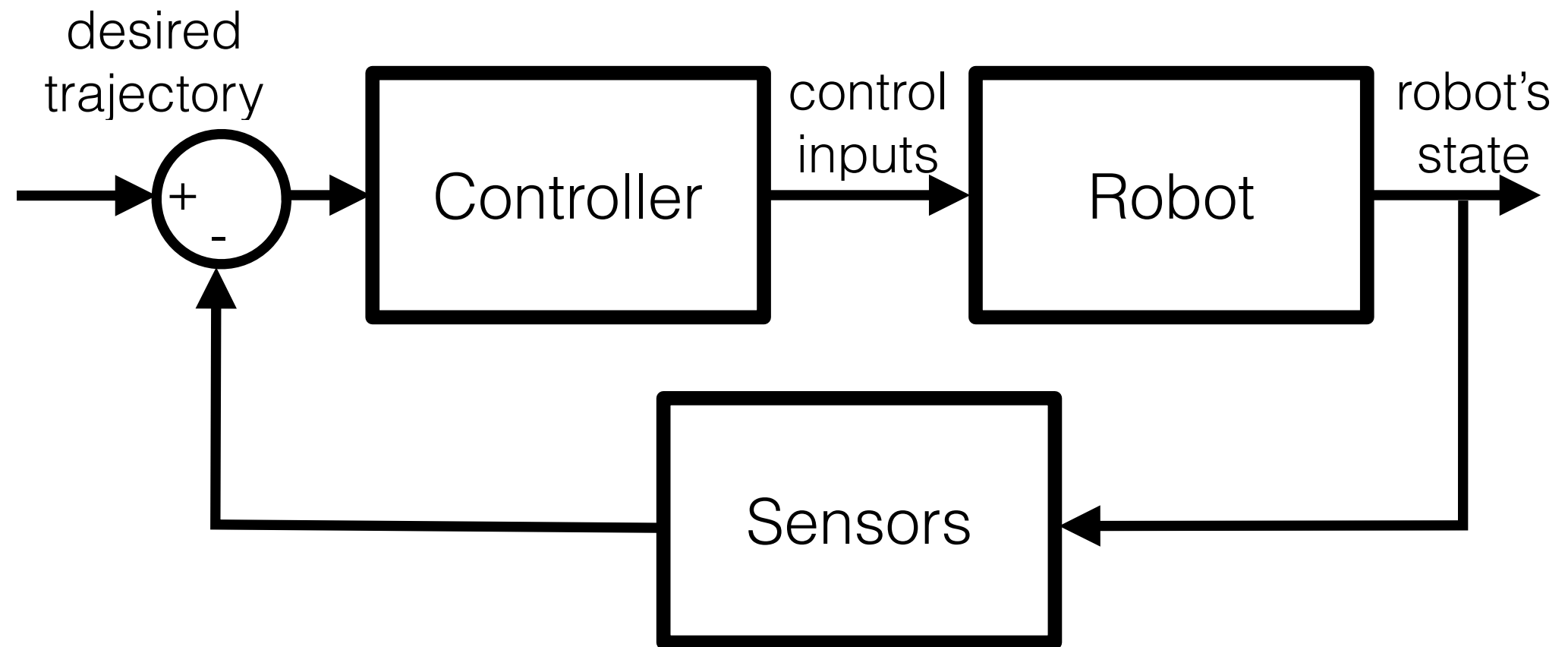
Lecture 8: Trajectory Optimization

**Luca Carlone**



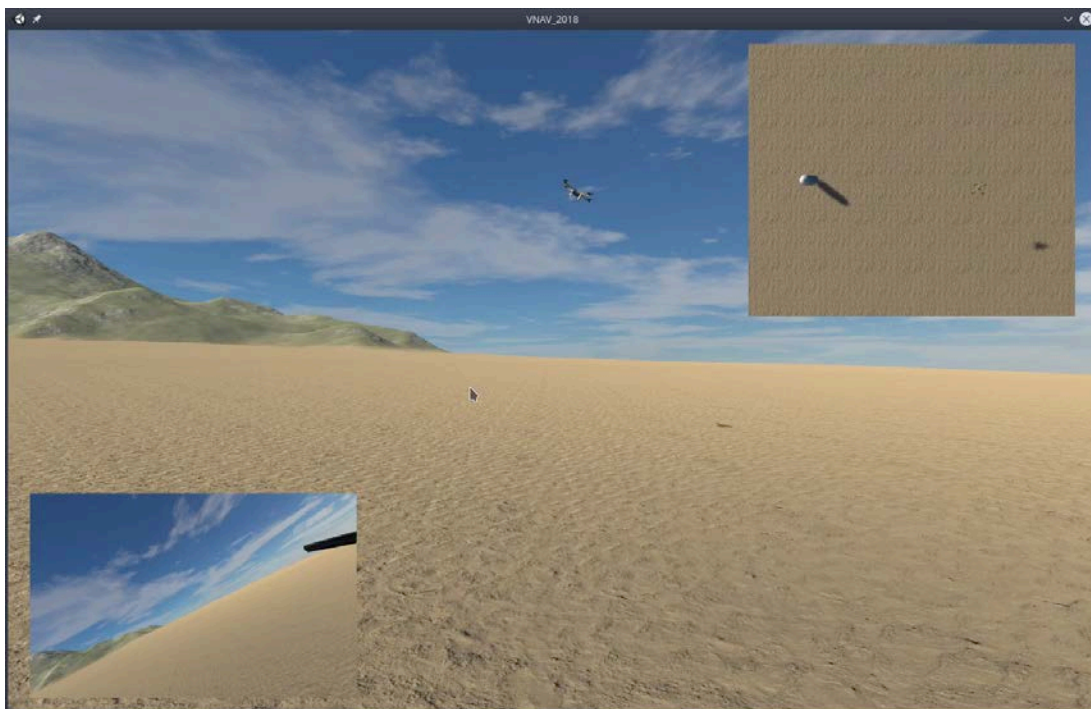
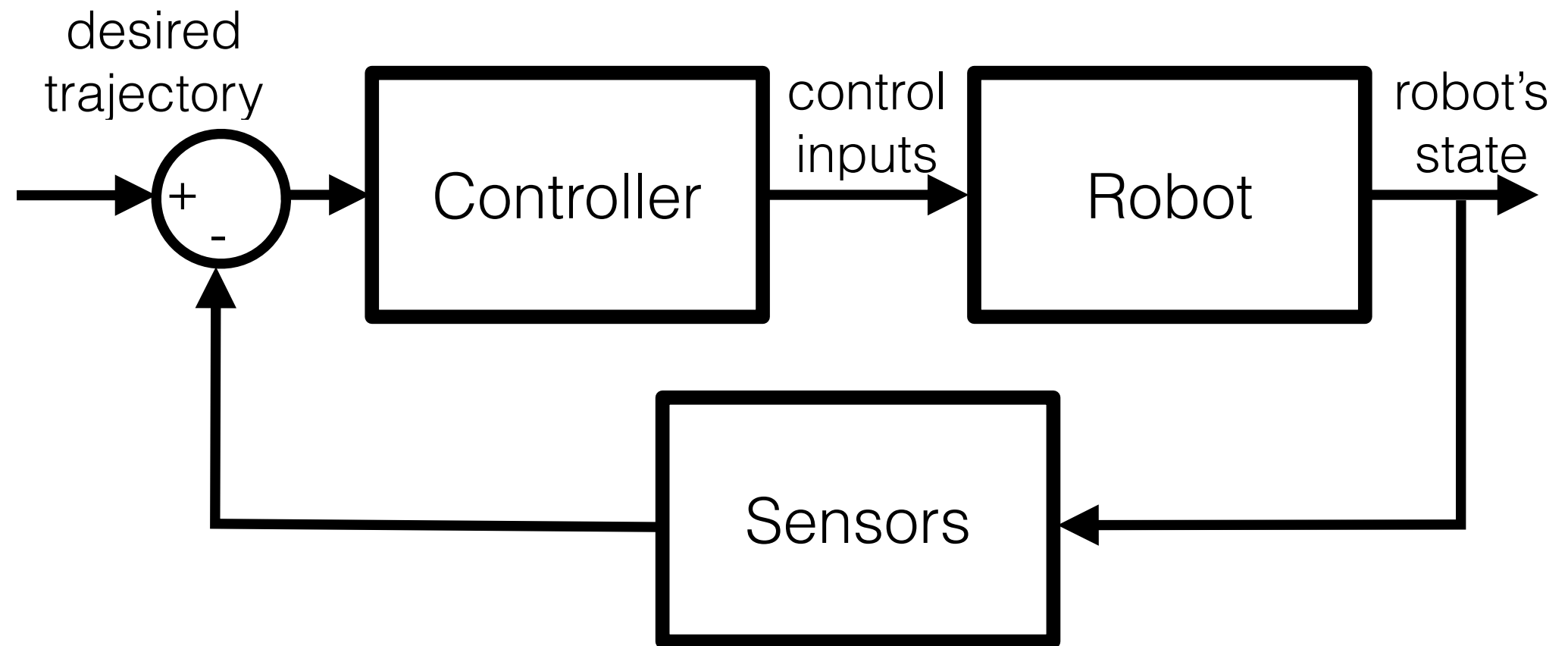
# Planning vs. Control

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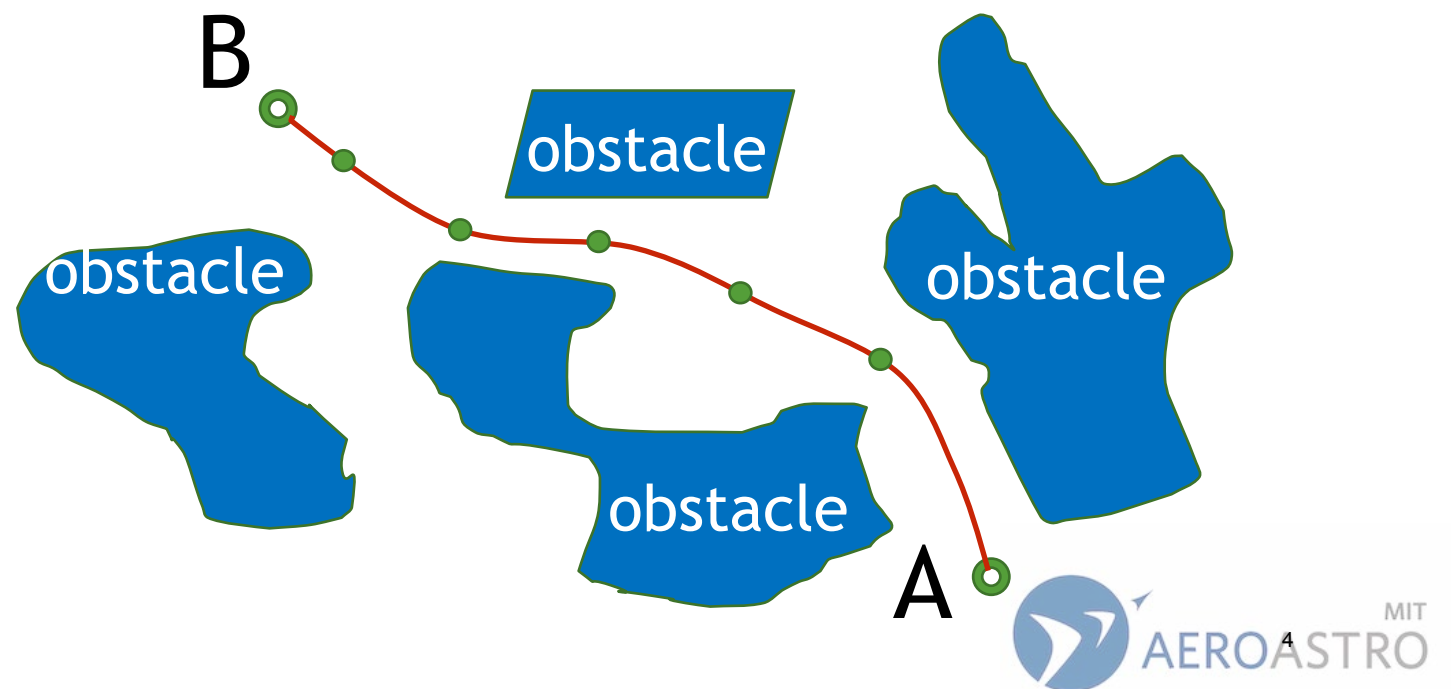
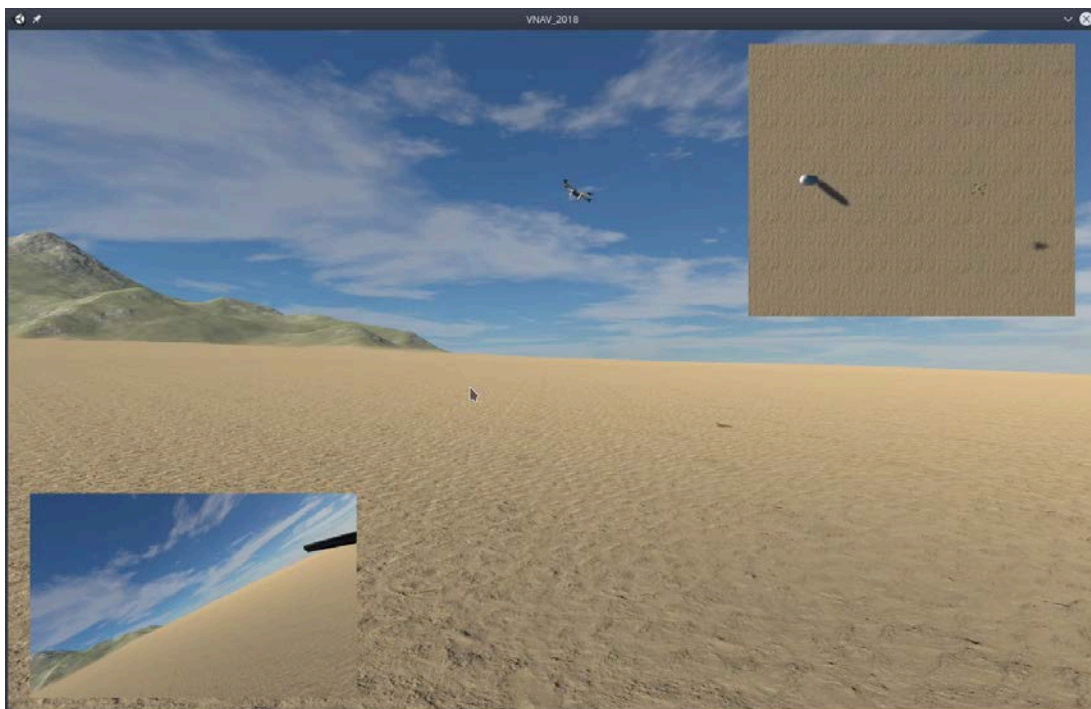
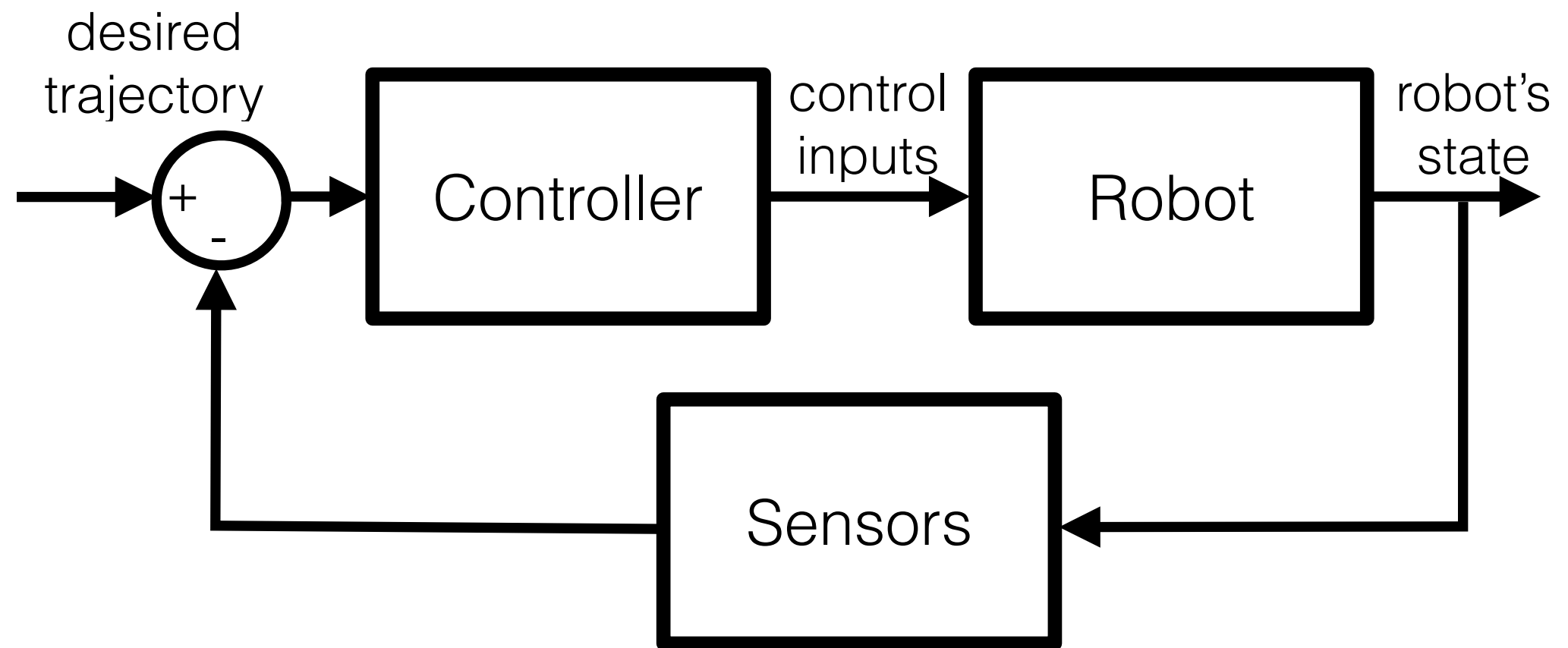


# Planning vs. Control

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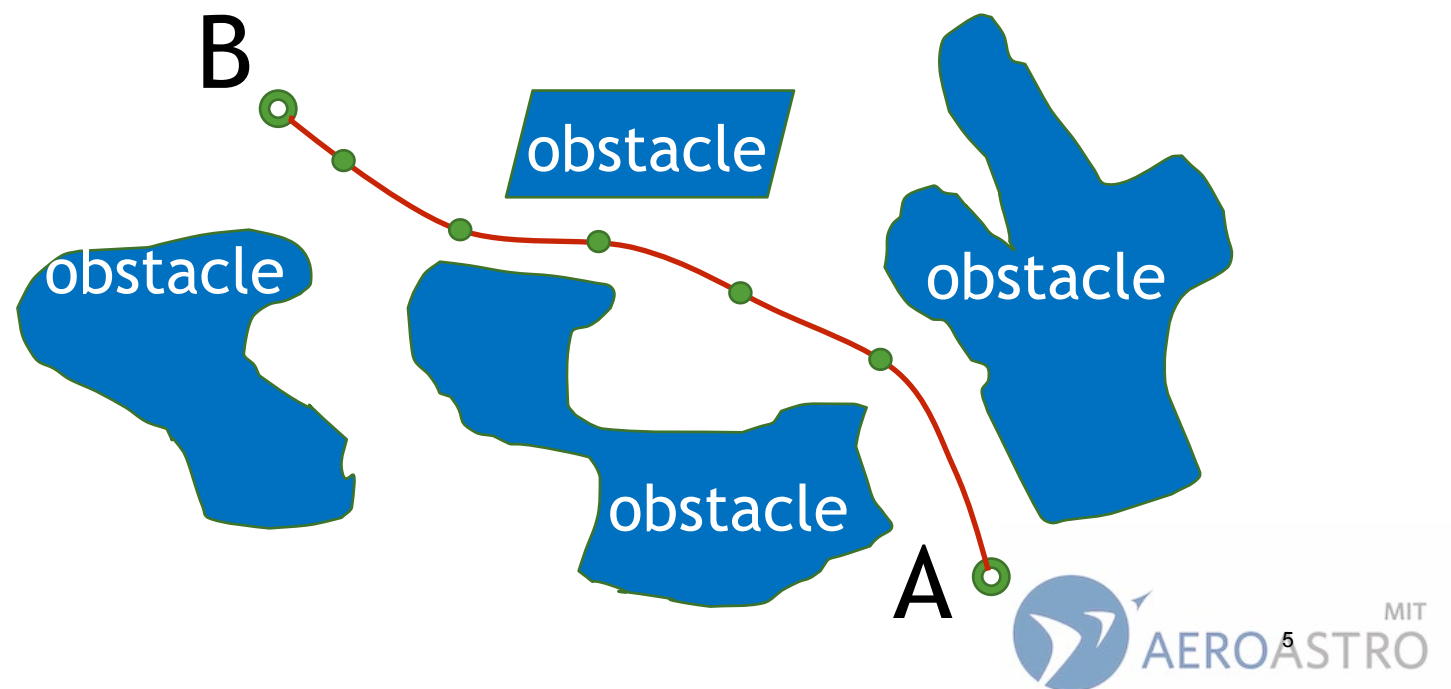
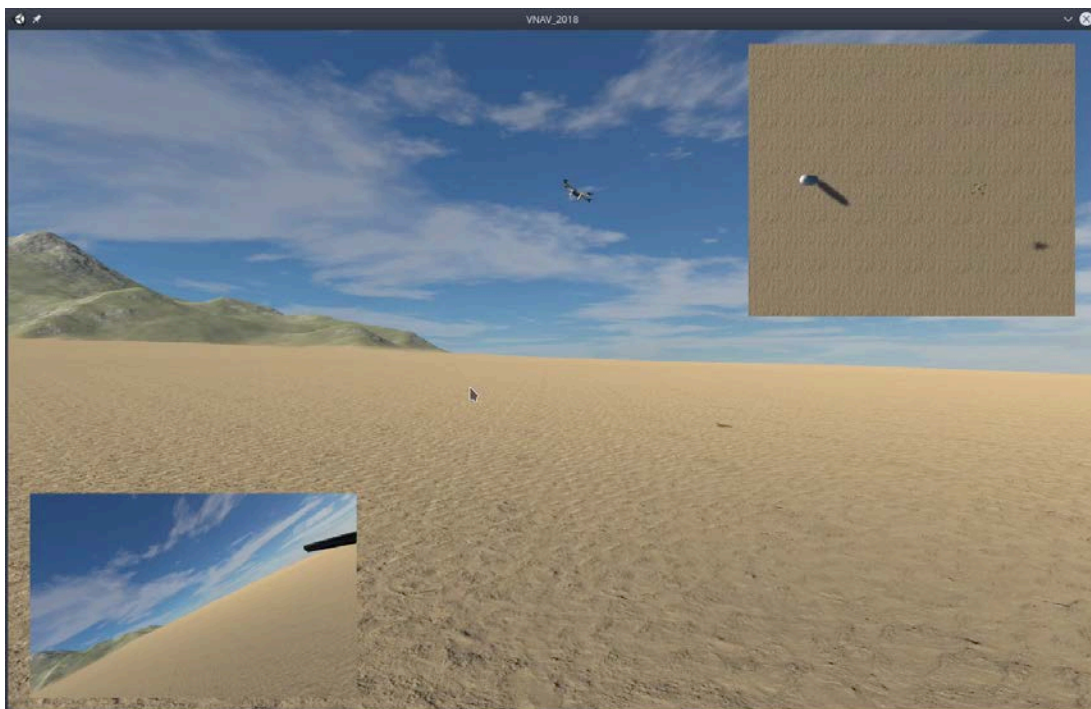
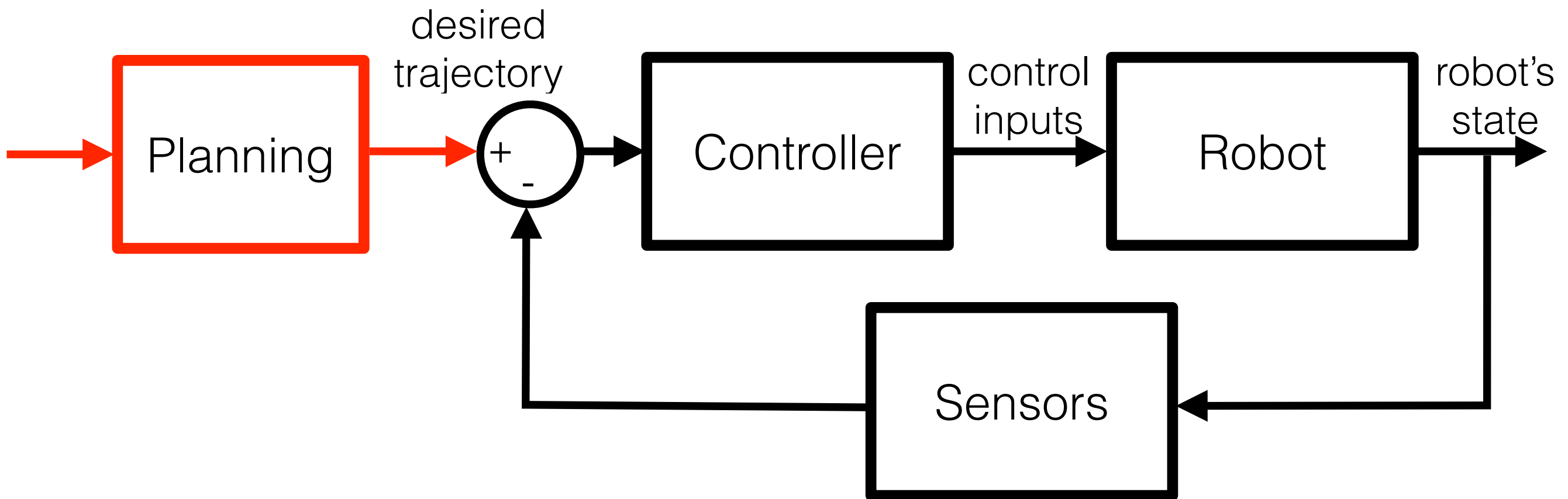


# Planning vs. Control





# Planning vs. Control

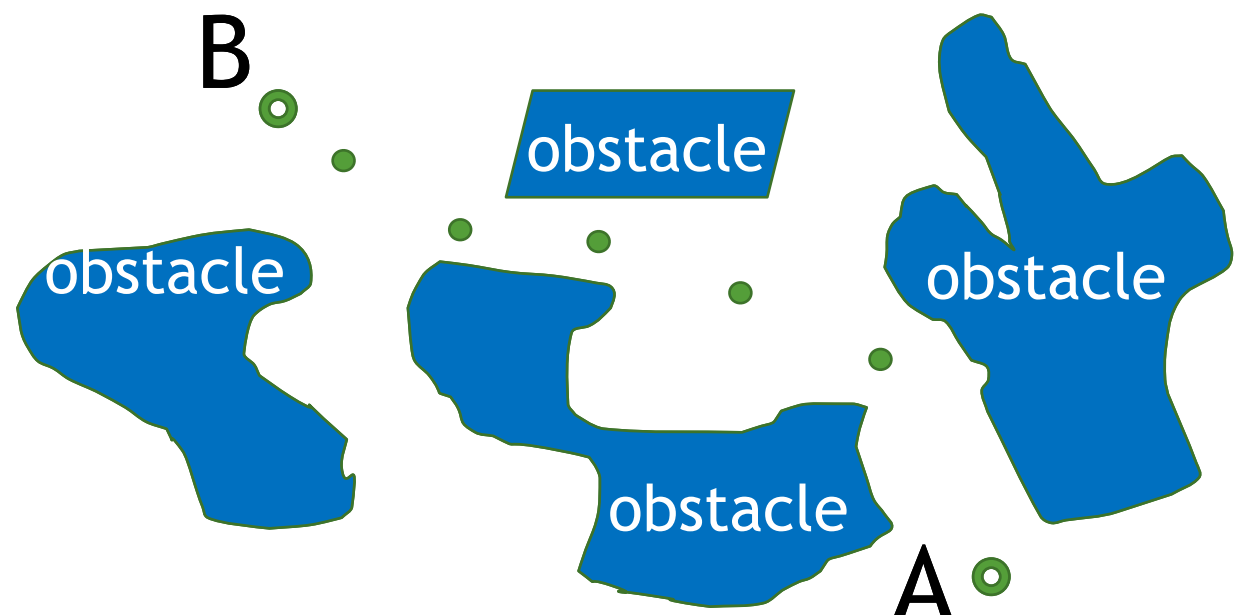


# Path Planning vs. Trajectory Planning

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- **Path** is a sequence of waypoints (in the obstacle-free space), without *time labels* or *information about velocity or higher order of derivatives*.

**DOES NOT ACCOUNT FOR DYNAMICS**



# Path Planning vs. Trajectory Planning

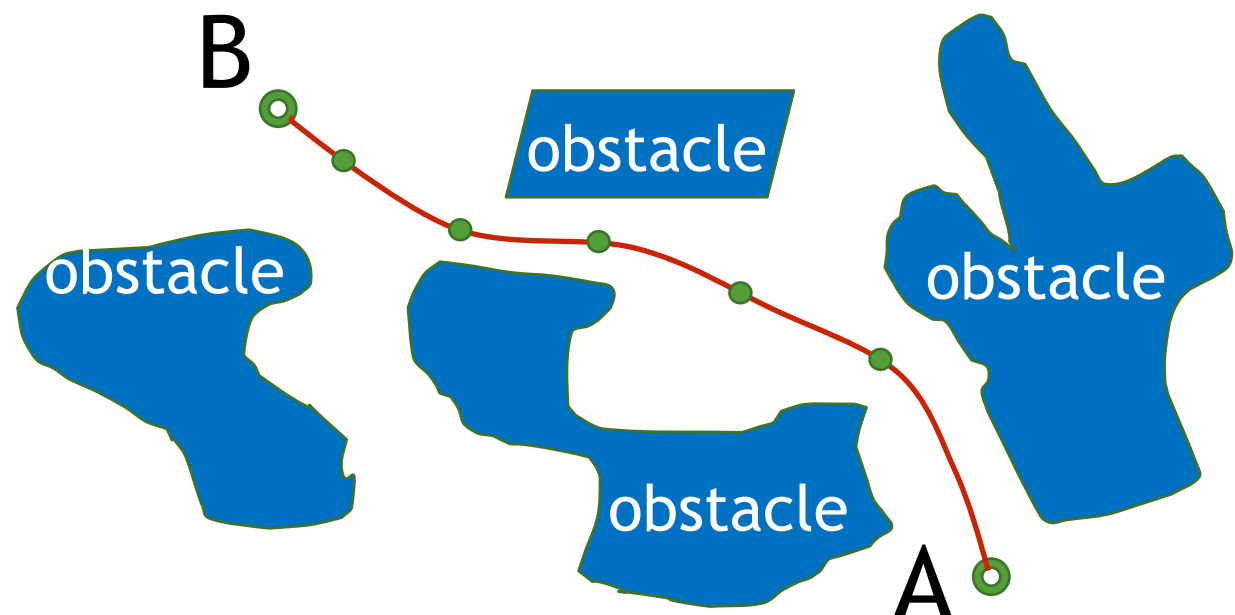
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**DOES NOT ACCOUNT FOR DYNAMICS**

- **Trajectory** is the sequence of movements the robot should make.

**ACCOUNTS FOR DYNAMICS**



# Path Planning vs. Trajectory Planning

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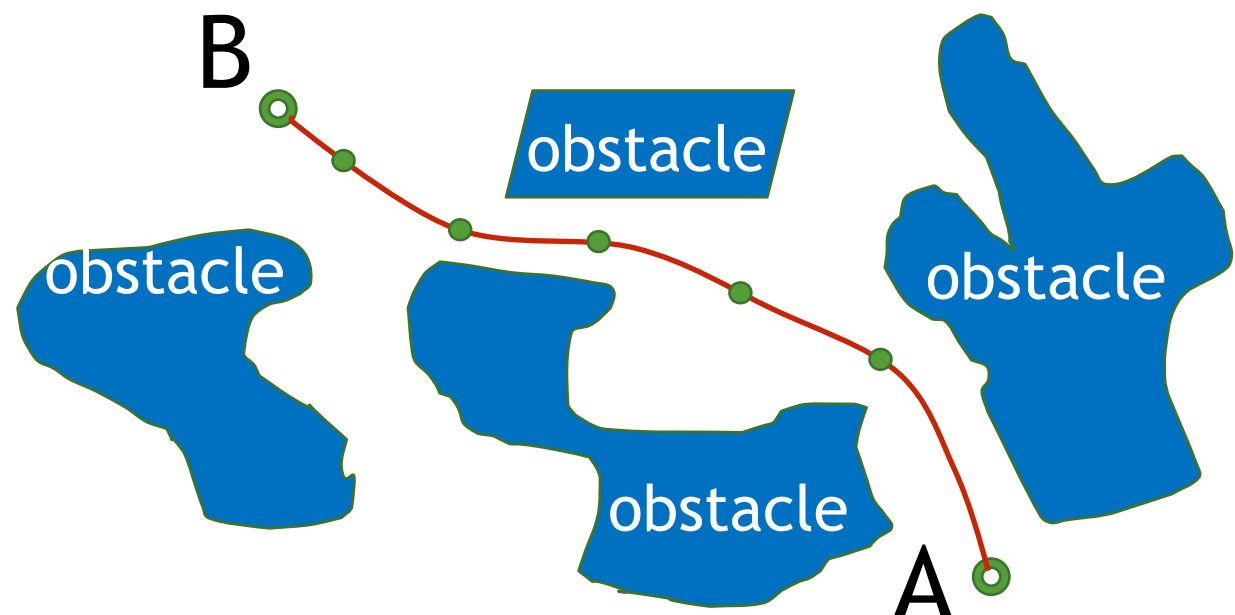
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**ACCOUNTS FOR DYNAMICS**

**MUST BE COLLISION FREE**





# Path Planning vs. Trajectory Planning

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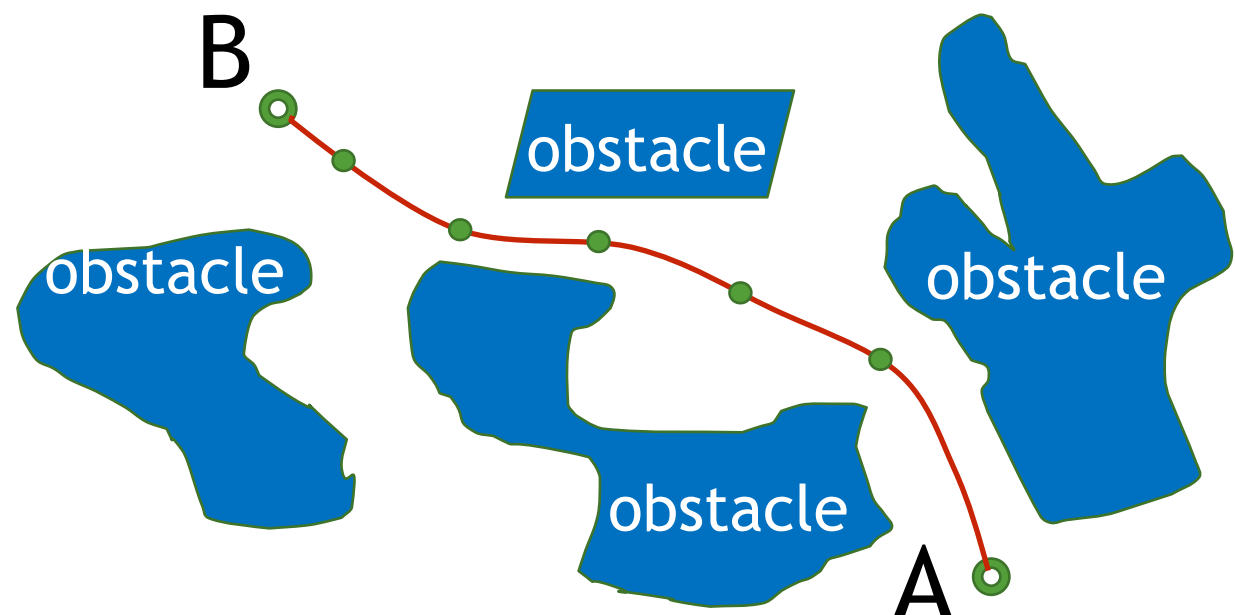
**DOES NOT ACCOUNT FOR DYNAMICS\***

\*Can account for dynamics but can be slow (Bry et al., IJRR '15)

- **Trajectory** is the sequence of movements the robot should make.

**ACCOUNTS FOR DYNAMICS**

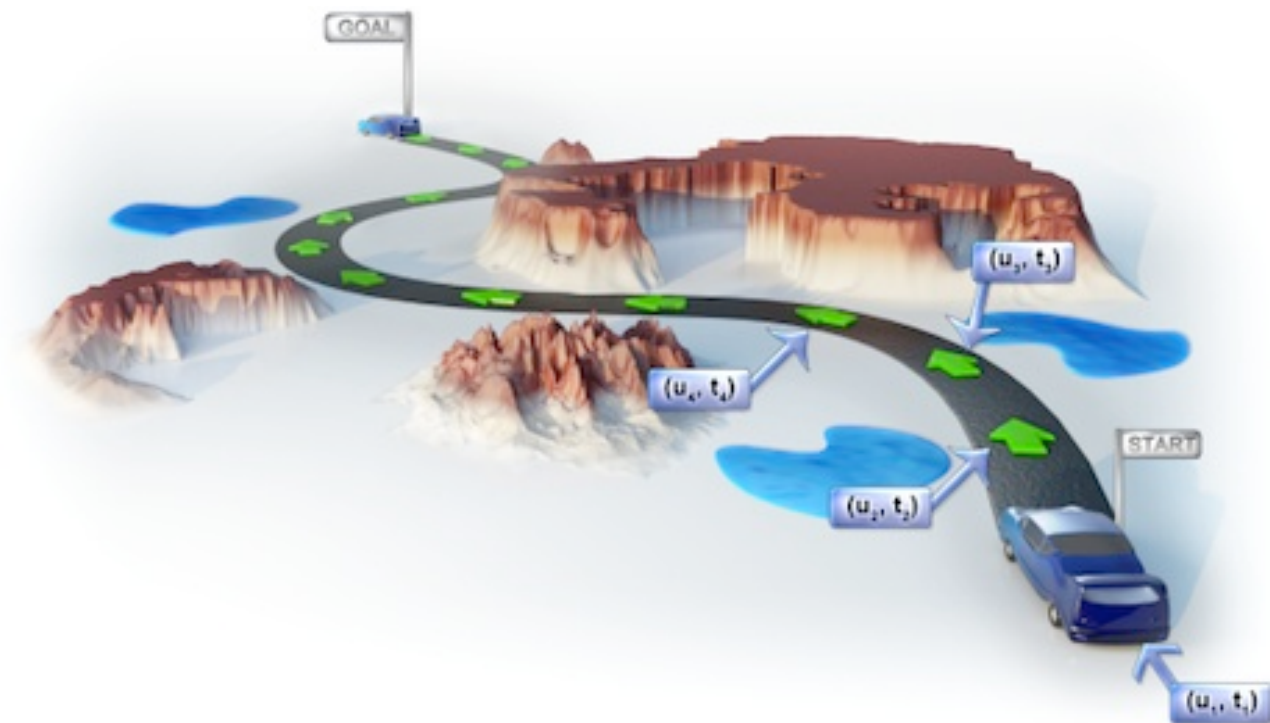
**MUST BE COLLISION FREE**



# Planning algorithms

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- Open-source libraries
  - Open Motion Planning Library (OMPL)
    - <http://ompl.kavrakilab.org/>
  - Motion Strategy Library (MSL)
    - <http://msl.cs.uiuc.edu/msl/>
  - RRT\* Library
  - Sampling Based Planning Library

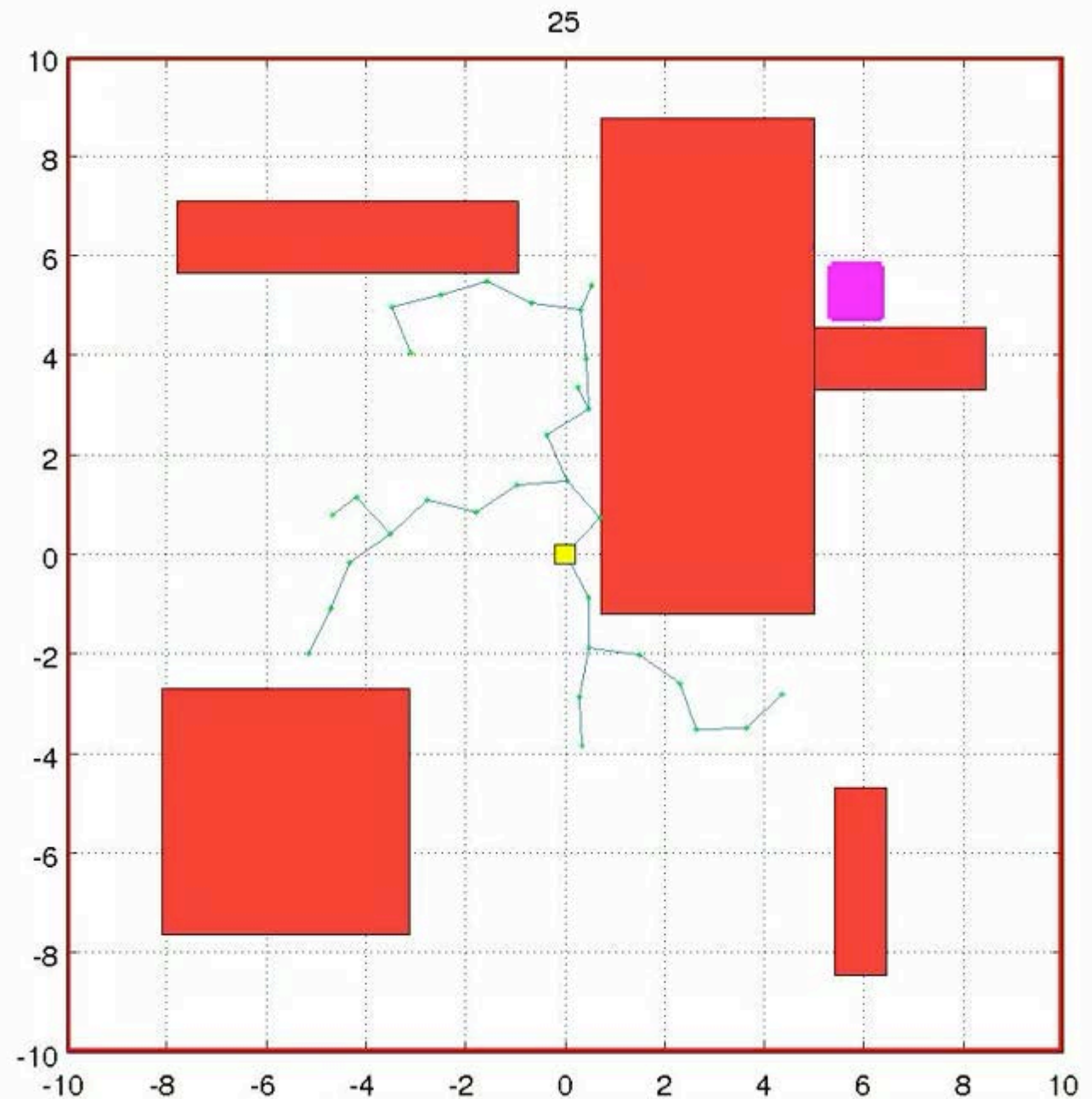


## References

- Howie Choset et al., “Principles of Robot Motion,” MIT press, 2005.
- Steven Lavalle, “Planning Algorithms,” Cambridge University Press, 2006.

# Path Planning Example

## RRT\*: Rapidly exploring Random Trees





# Path Planning Example

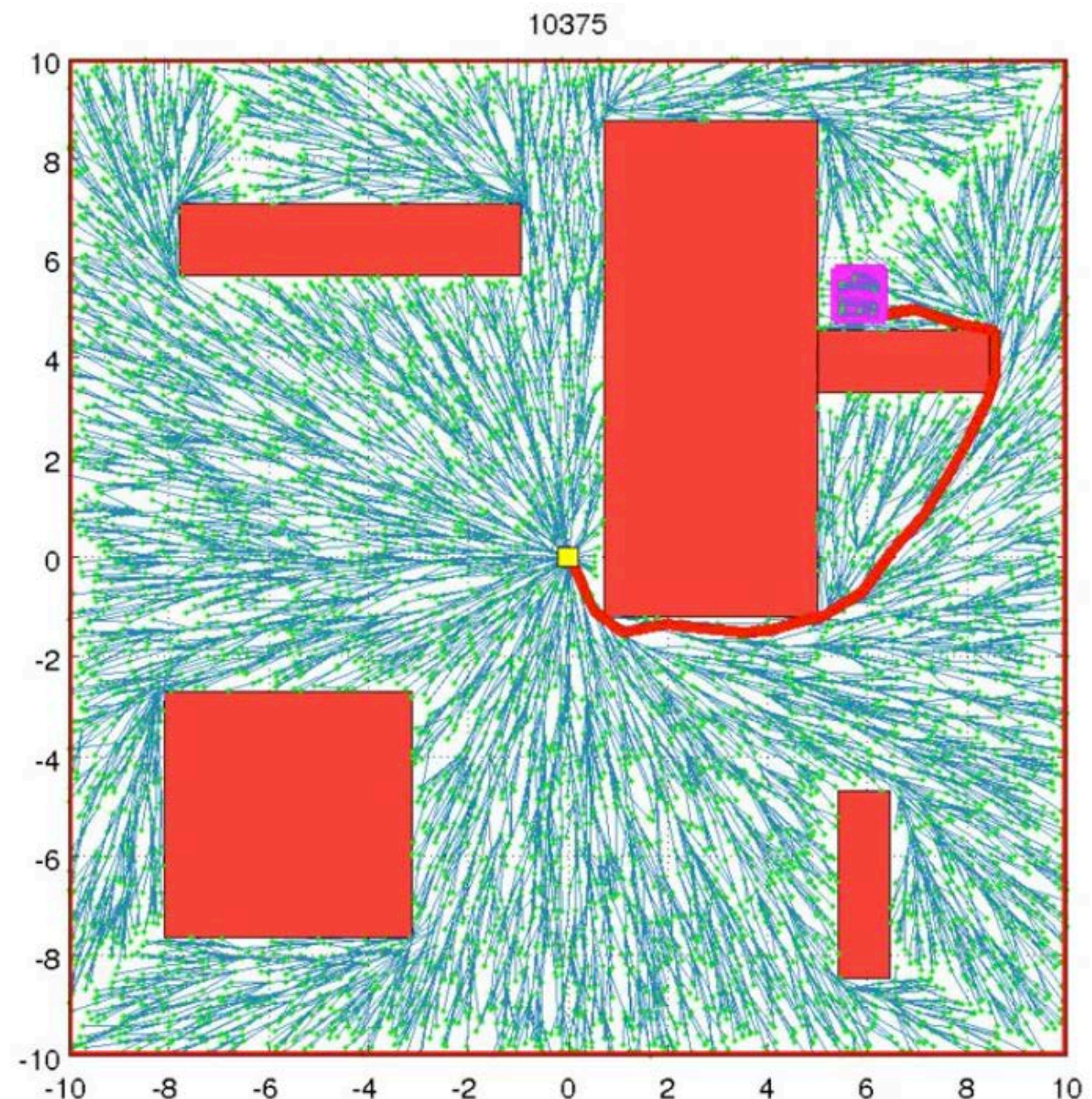
## RRT\*: Rapidly exploring Random Trees

### Pros

- Finds optimal path (if one exists)

### Cons

- Impractical running time *if asked for path with smooth trajectory* (Bry et al., IJRR '15)
- Jagged path *otherwise*



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# Path Planning Example

## RRT\*: Rapidly exploring Random Trees

### Pros

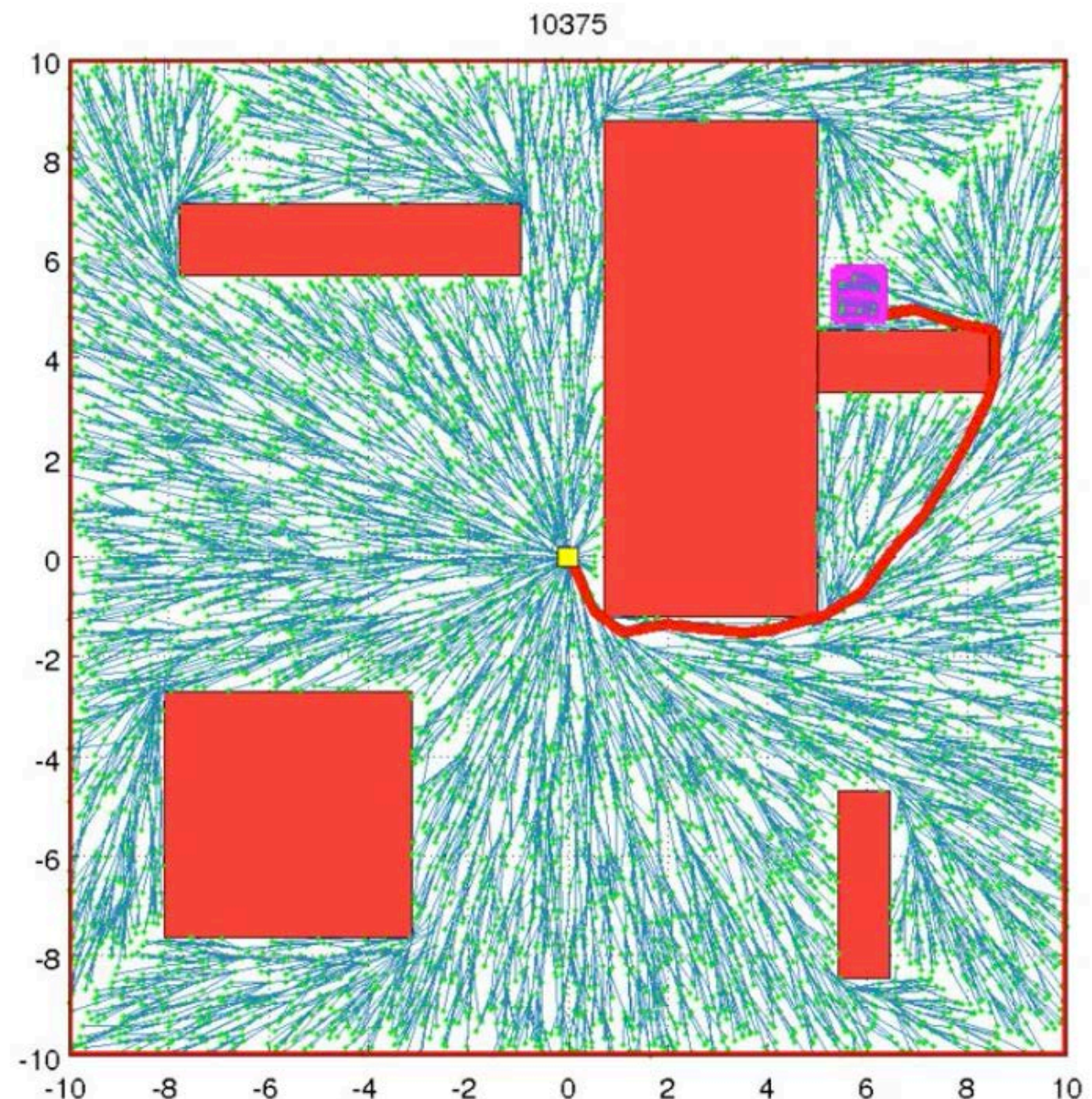
- Finds optimal path (if one exists)

### Cons

- Impractical running time *if asked for path with smooth trajectory* (Bry et al., IJRR '15)

Difficult to apply for online planning in unknown/dynamic environments

- Jagged path *otherwise*





# Path Planning Example

## RRT\*: Rapidly exploring Random Trees

### Pros

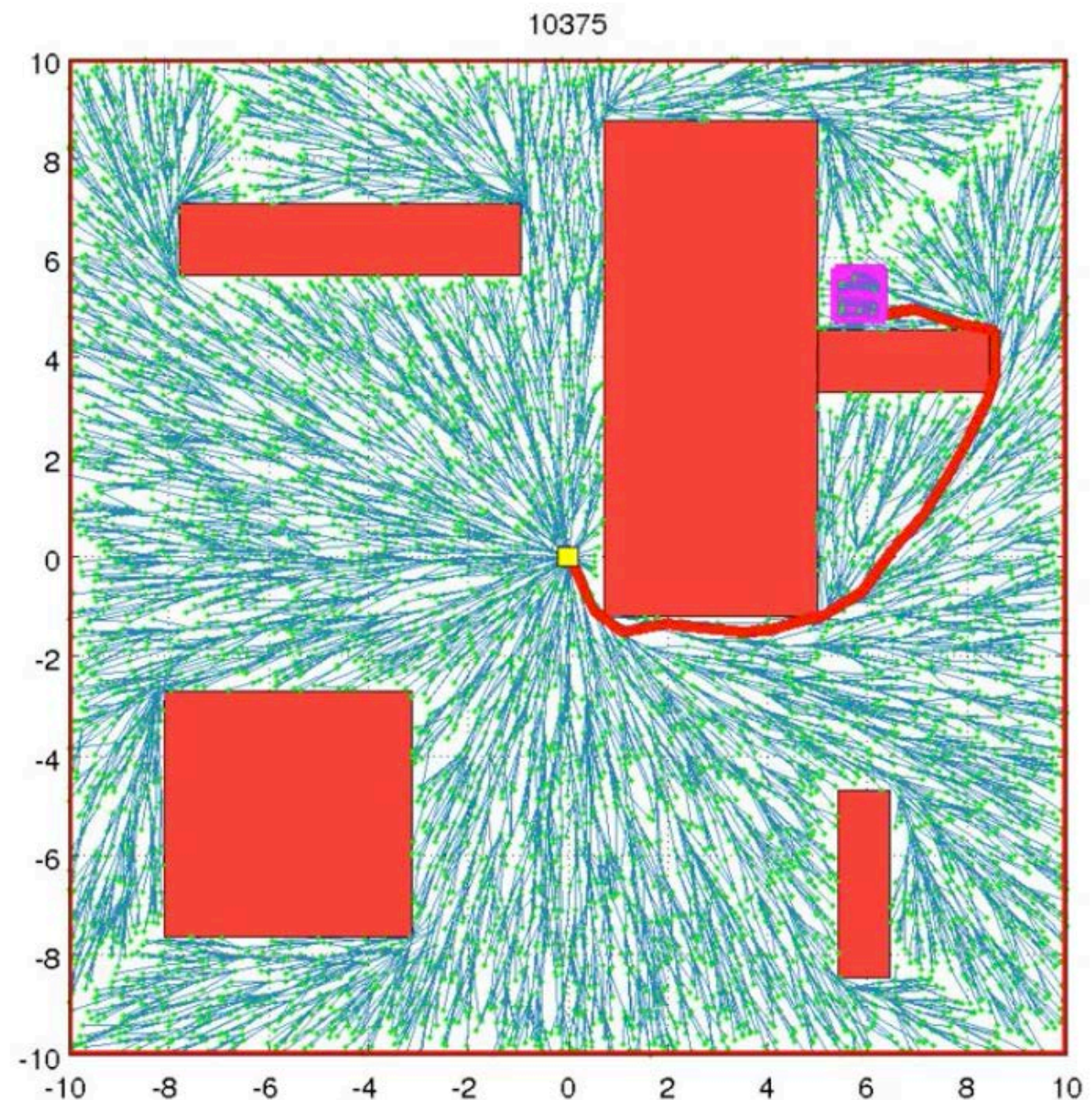
- Finds optimal path (if one exists)

### Cons

- Impractical running time *if asked for path with smooth trajectory*\* (Bry et al., IJRR '15)

\*the simultaneous path+trajectory planning is called **direct trajectory planning**

- Jagged path *otherwise*

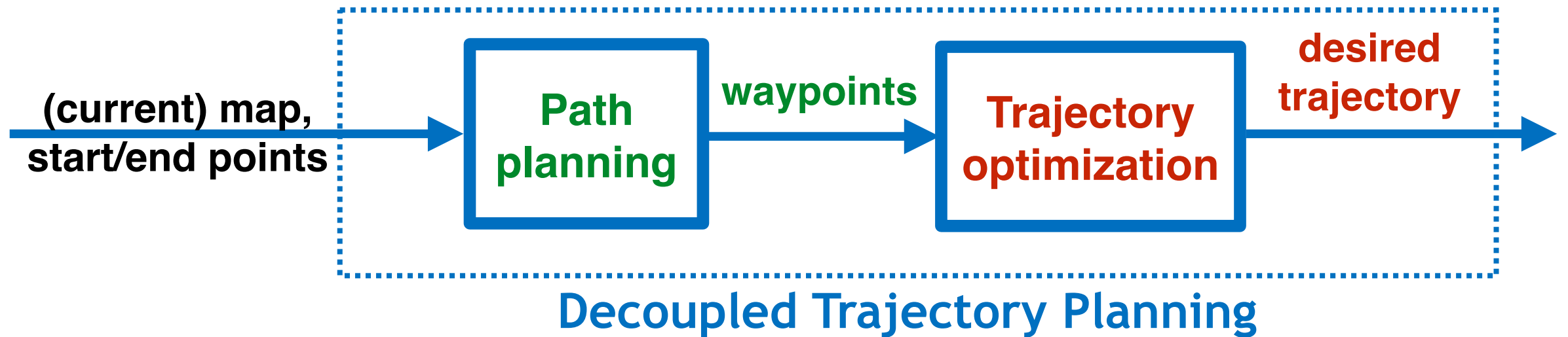


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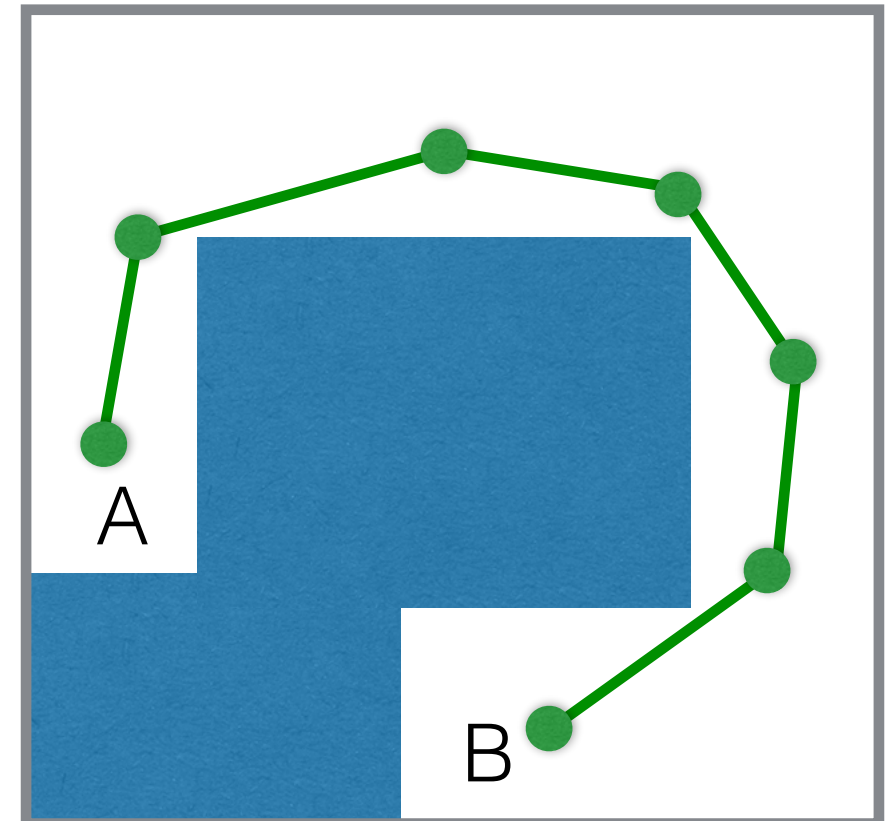
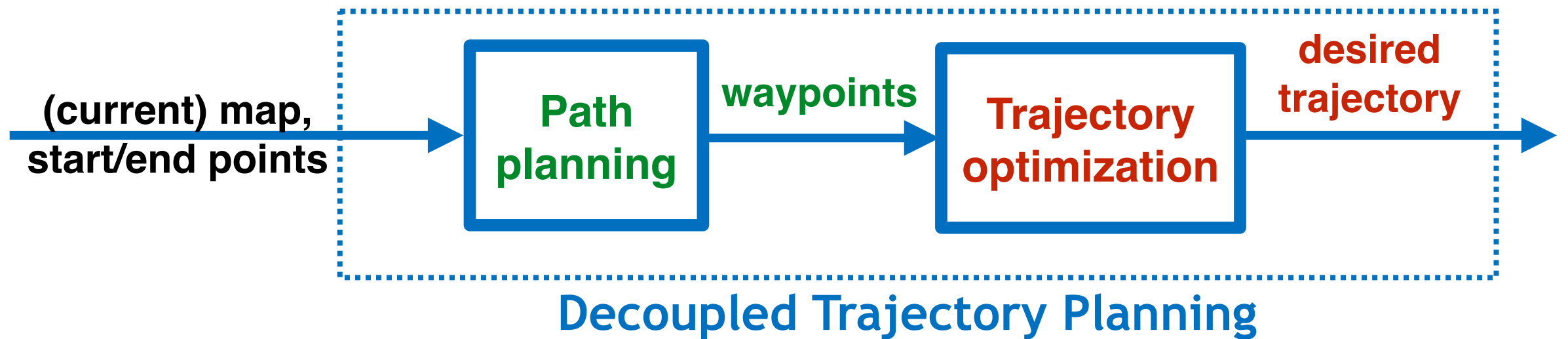
[courtesy of Prof. Sertac Karaman]

# Decoupled Trajectory Planning

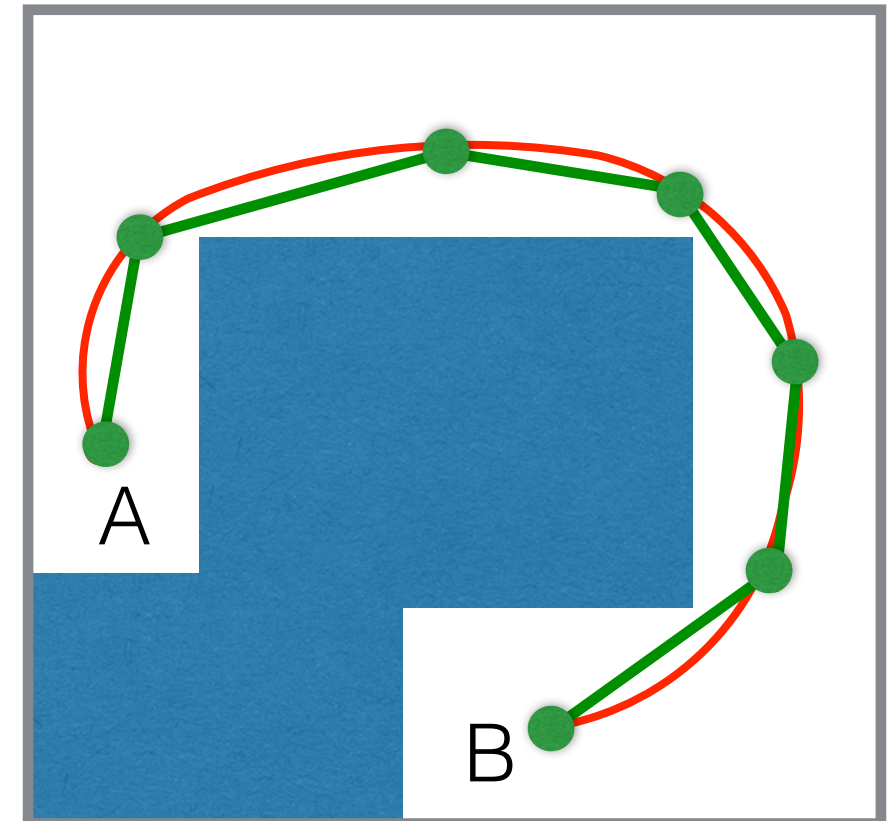
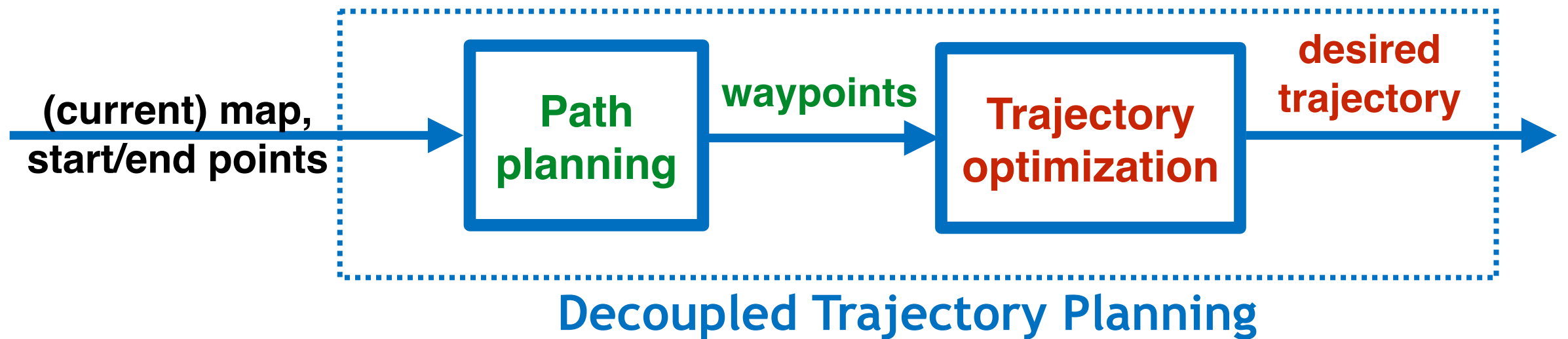
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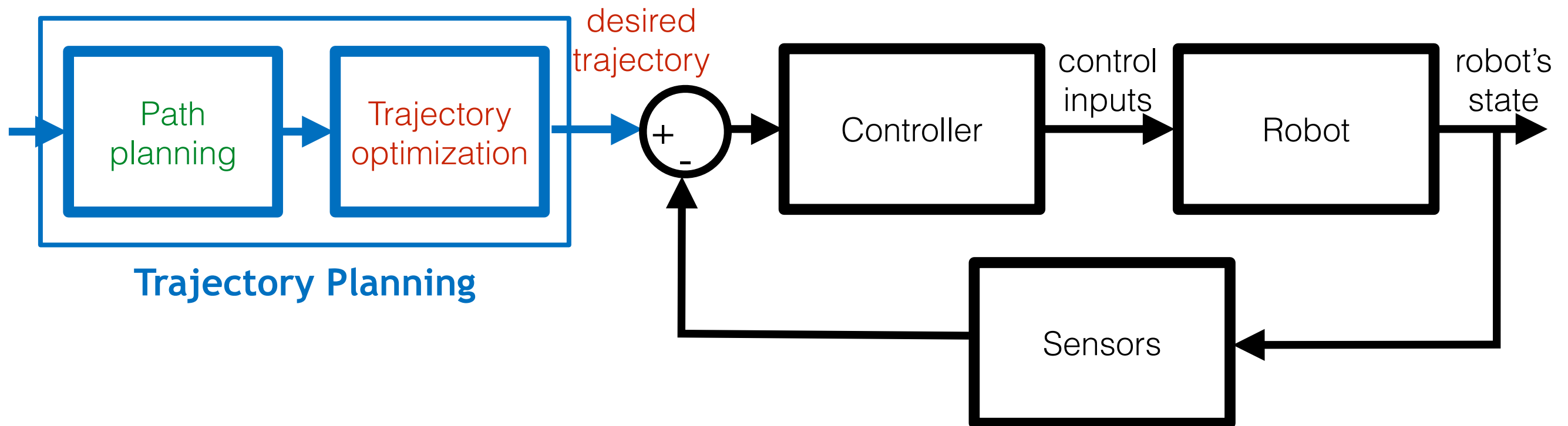
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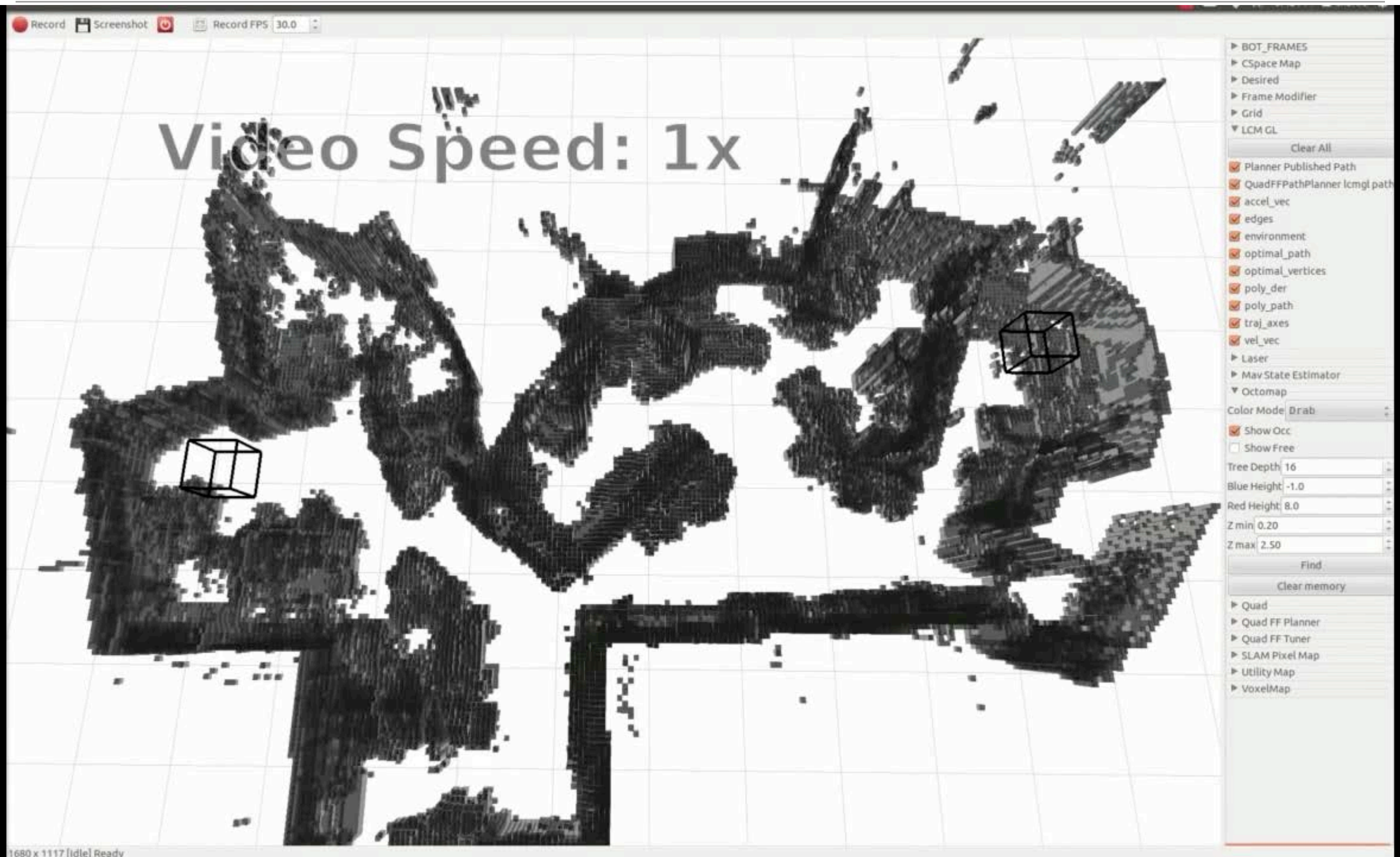


# Decoupled Trajectory Planning





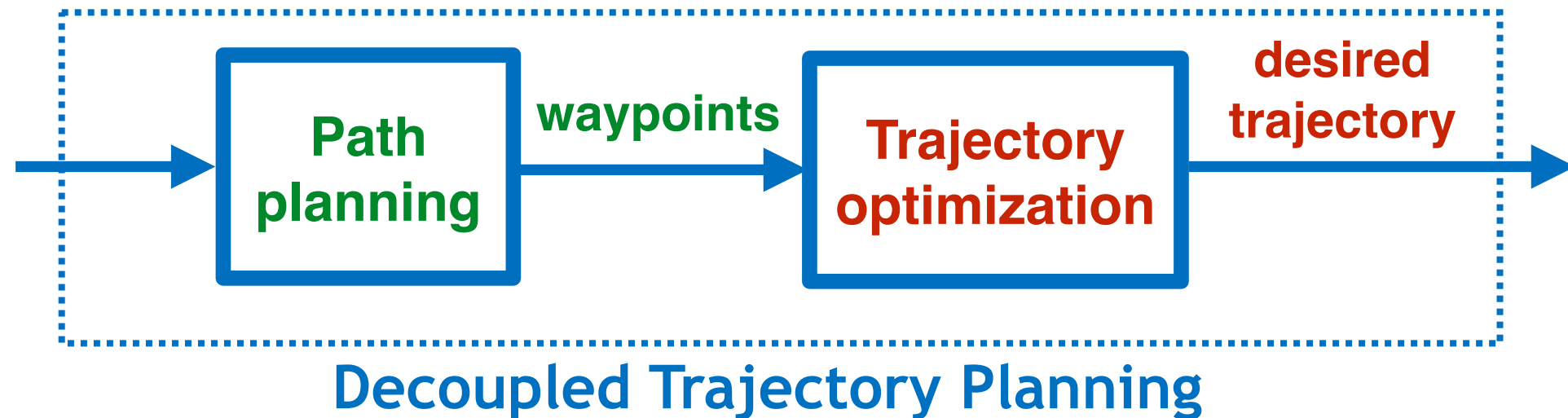
# Trajectory Optimization



2013

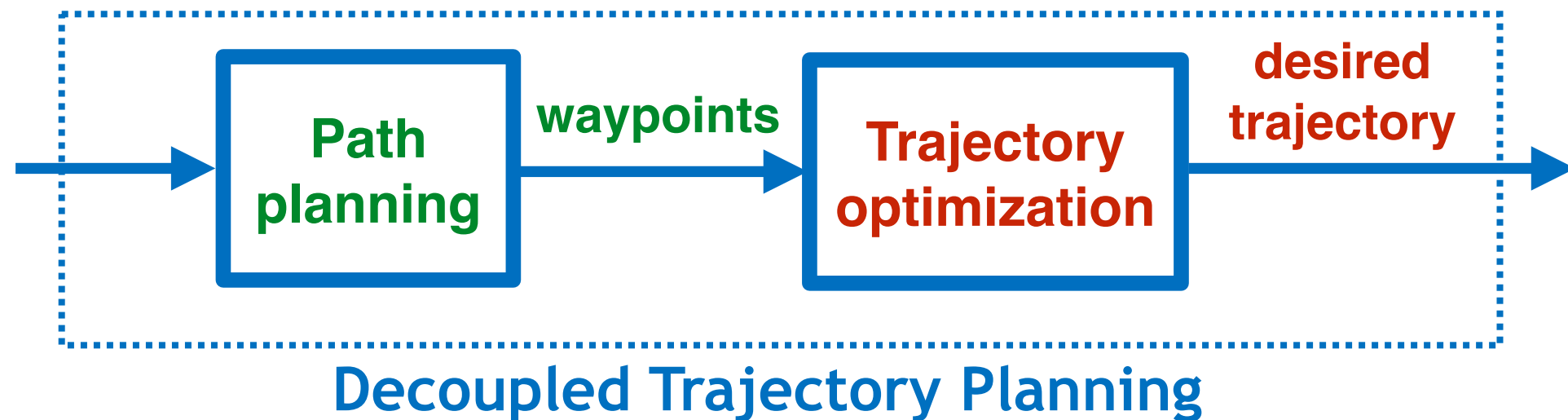
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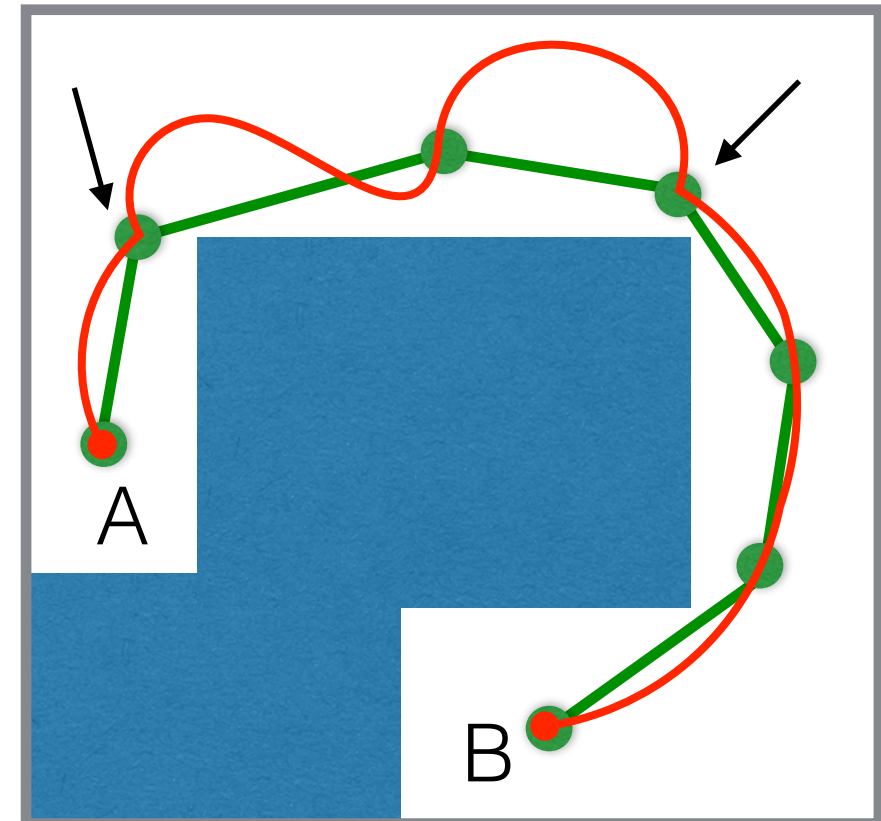


- Need to enforce “**continuity**” between segments for smooth trajectory

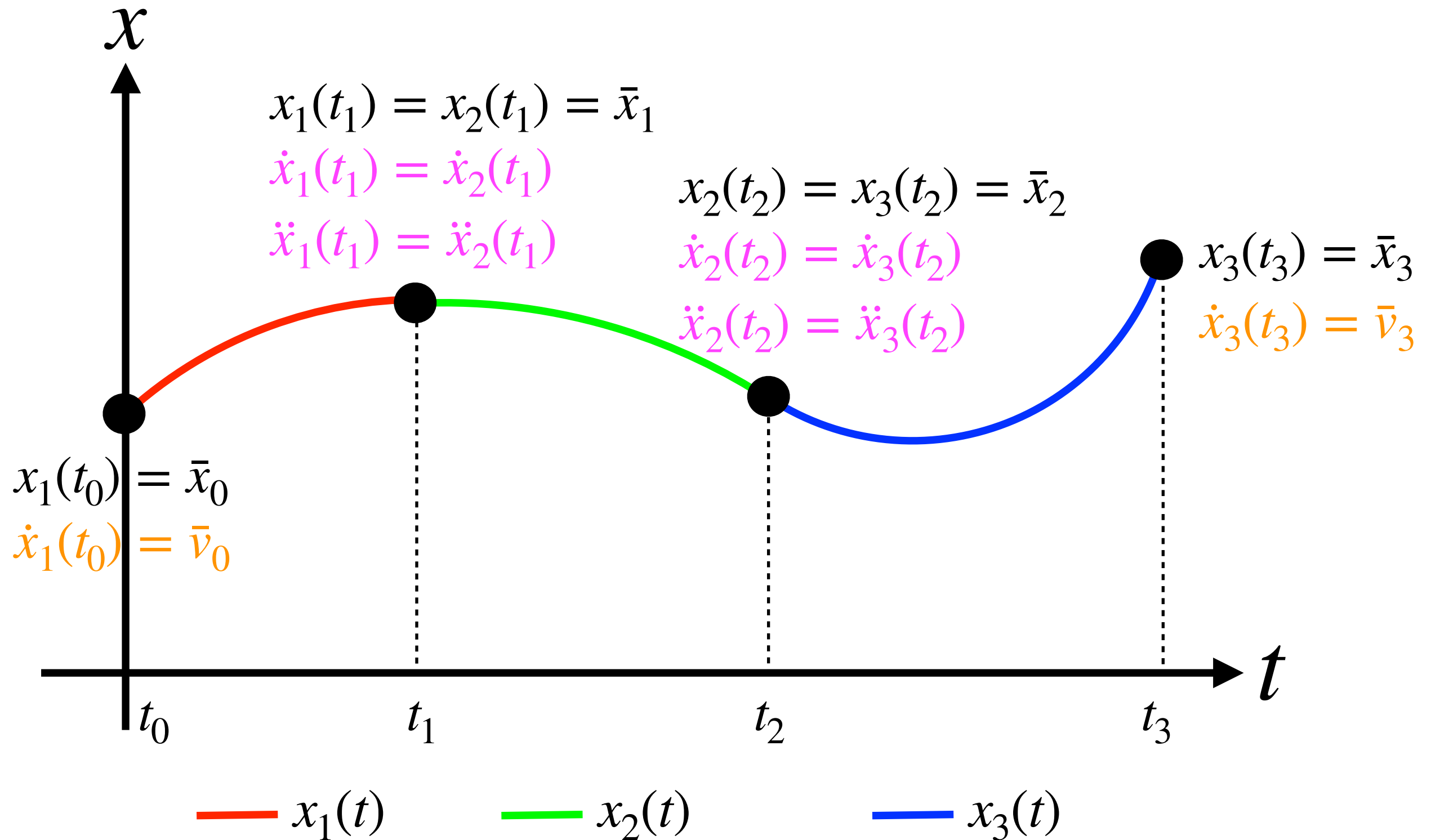
# Decoupled Trajectory Planning



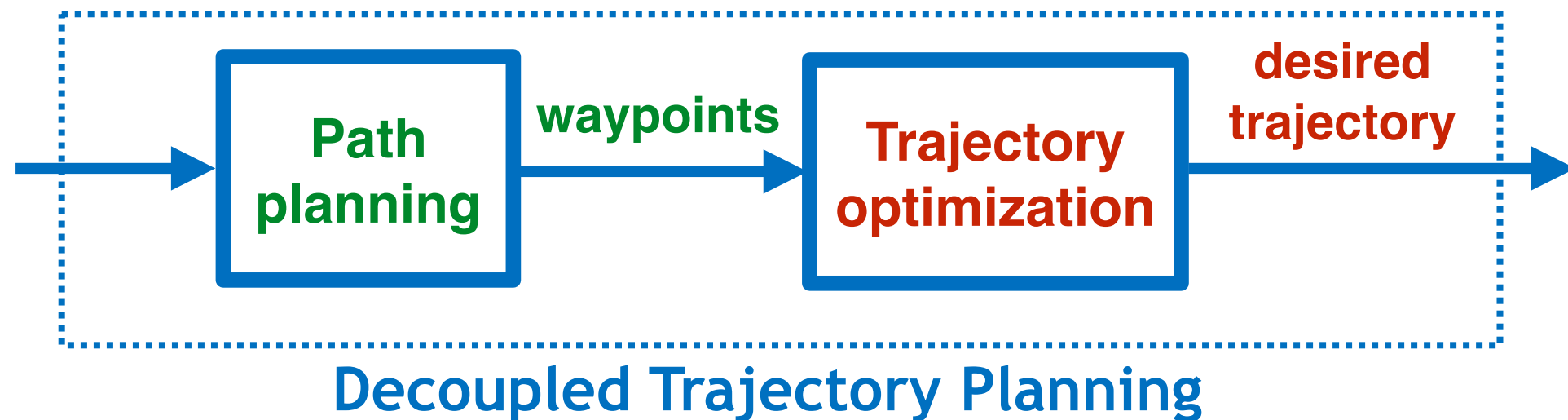
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# Trajectory Optimization

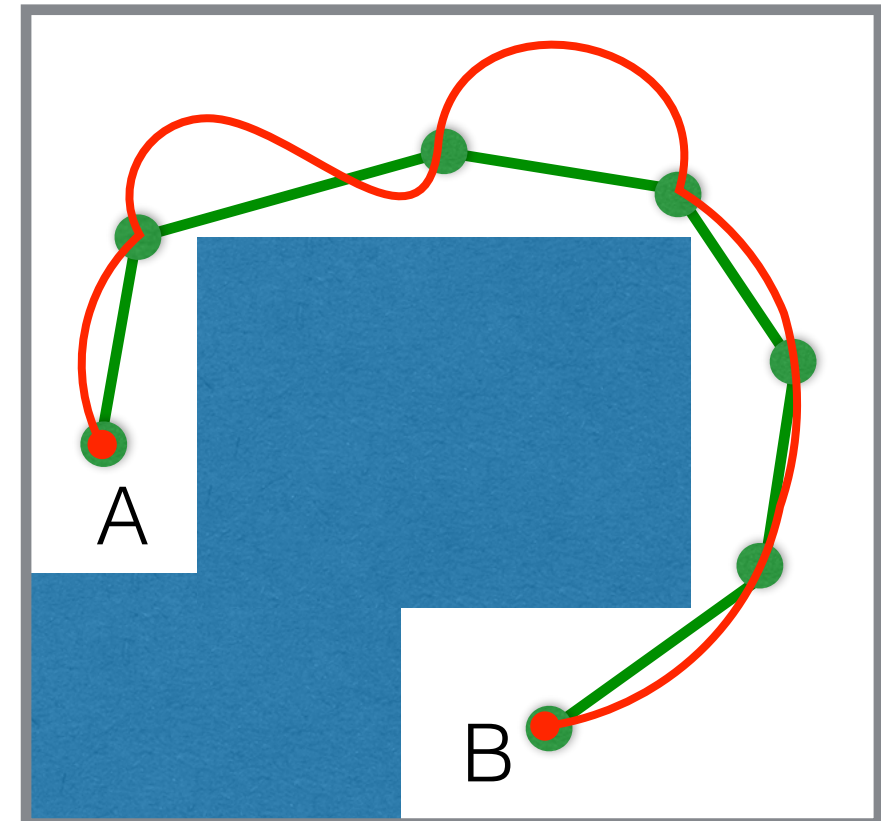


# Decoupled Trajectory Planning



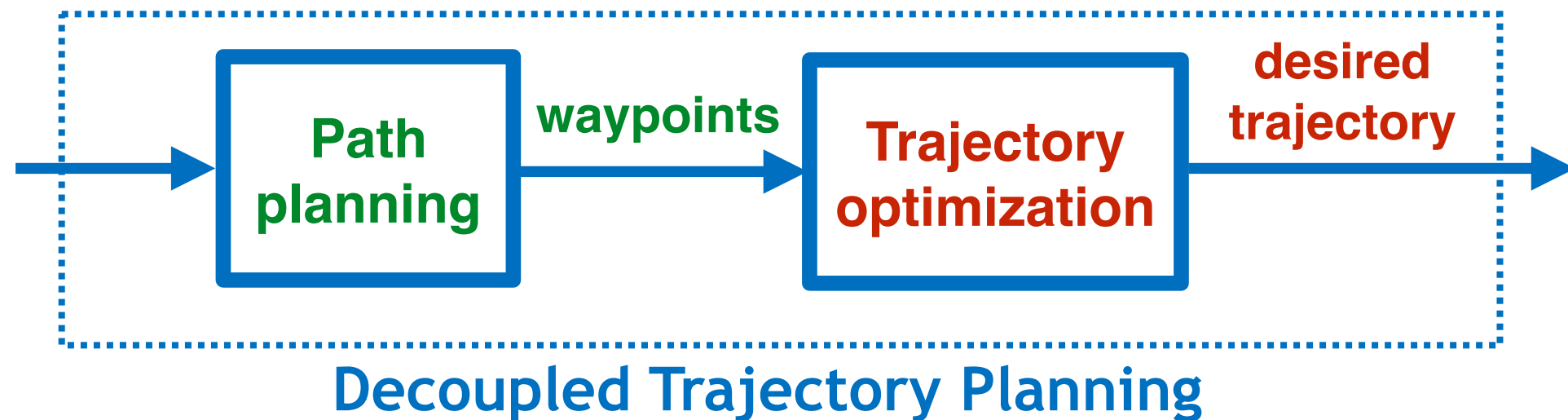
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- Need to ensure “**minimal**” motion



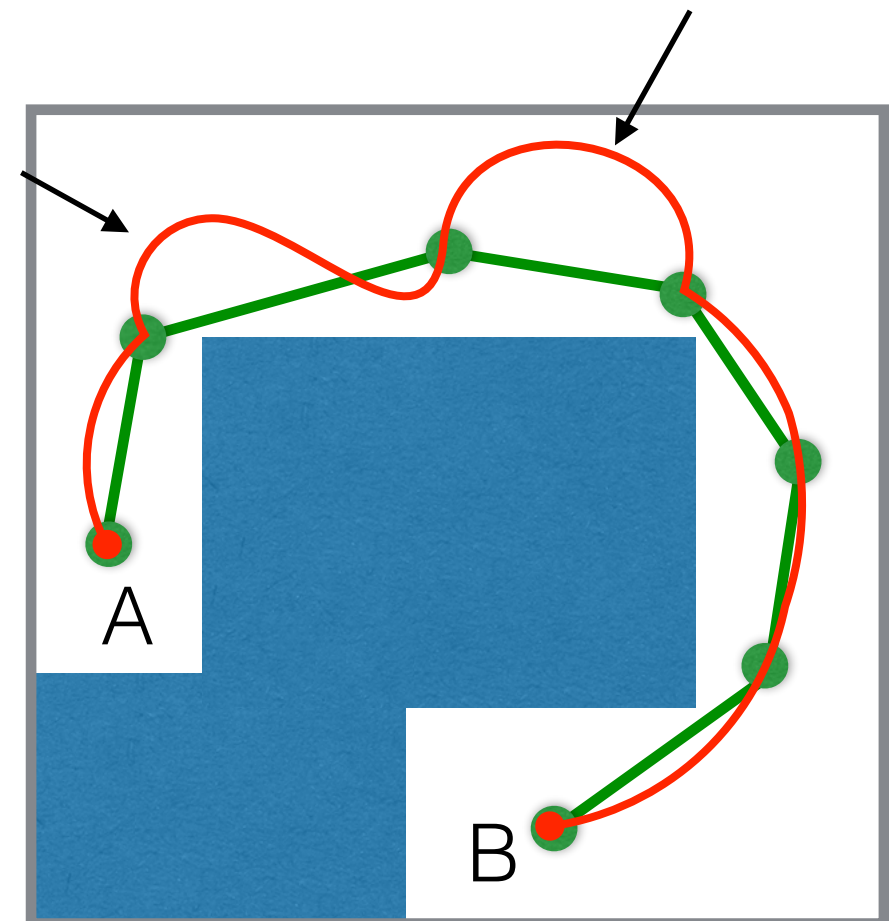


# Decoupled Trajectory Planning

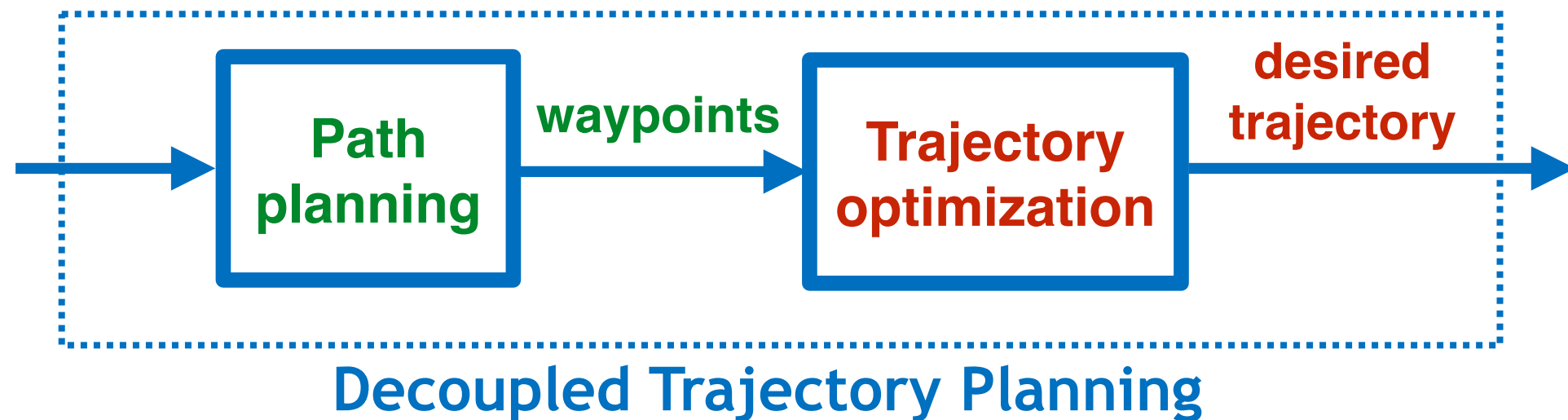


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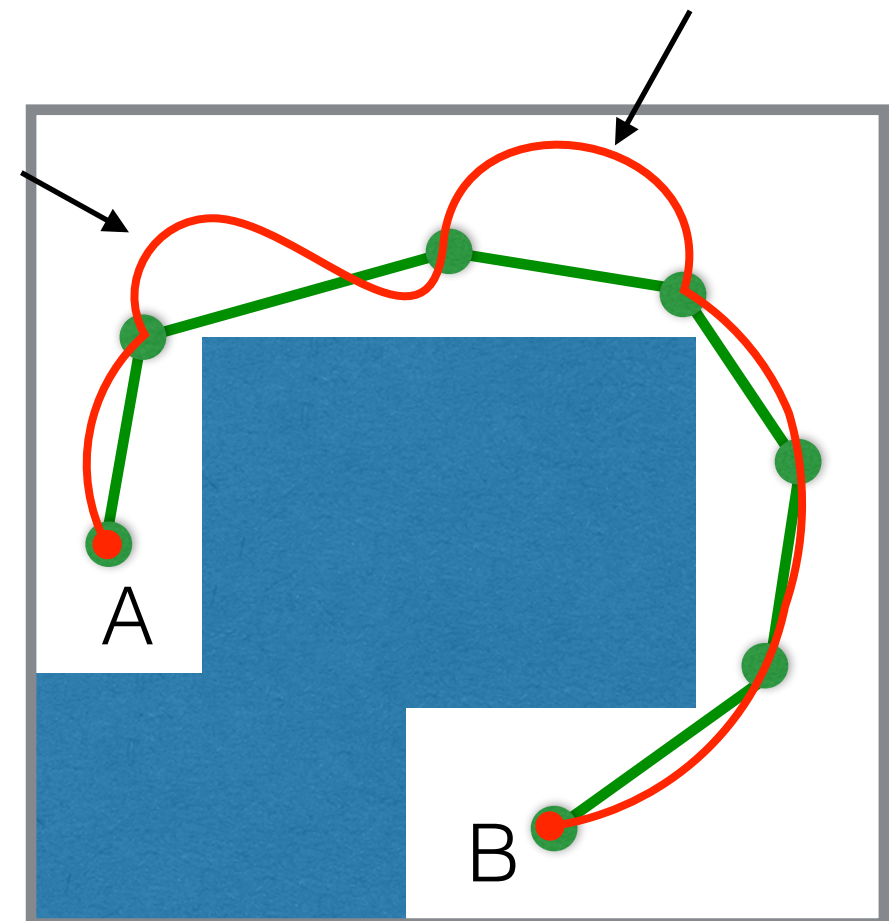
# Decoupled Trajectory Planning



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$$\min_{x(t), u(t)} J(t_A, t_B, x(t), u(t))$$



# Trajectory Optimization

## Estimation, Control and Planning for Aggressive Flight with a Small Quadrotor with a Single Camera and IMU

Giuseppe Loianno  
Vijay Kumar

Chris Brunner  
Gary McGrath

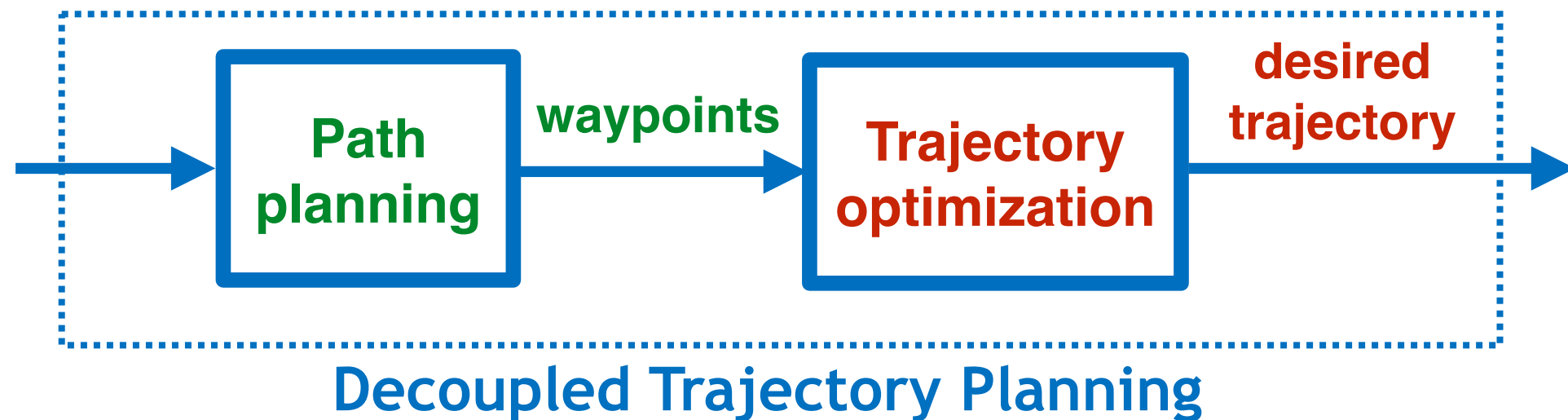


Qualcomm Technologies Inc.  
Qualcomm Research is a division of Qualcomm Technologies Inc.

[www.kumarrobotics.org](http://www.kumarrobotics.org)

2016/2017

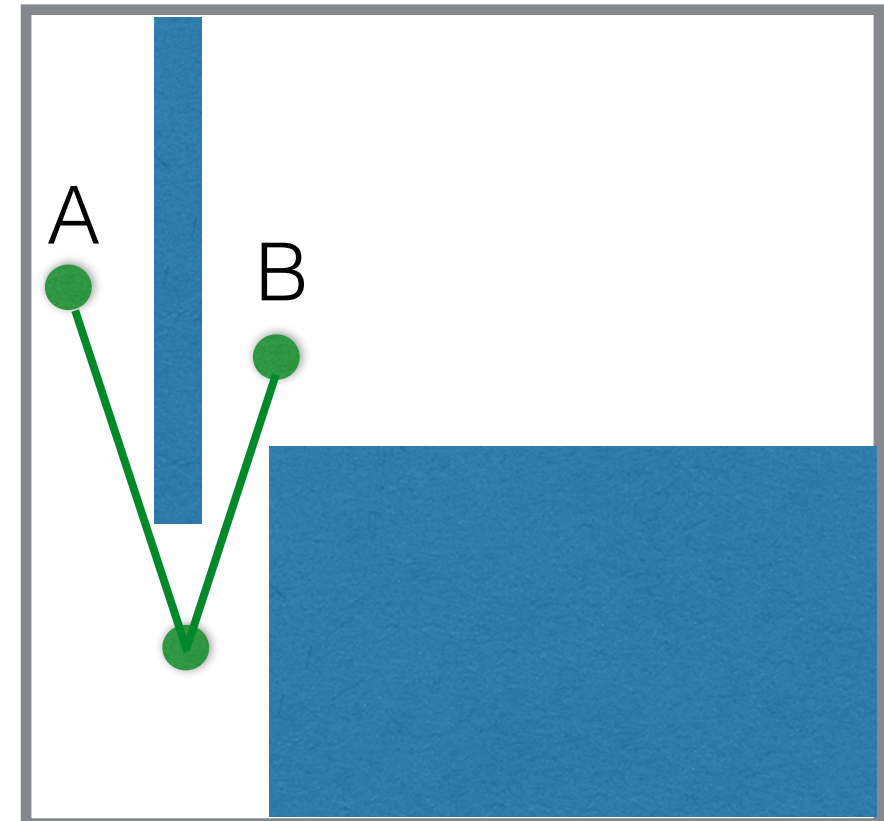
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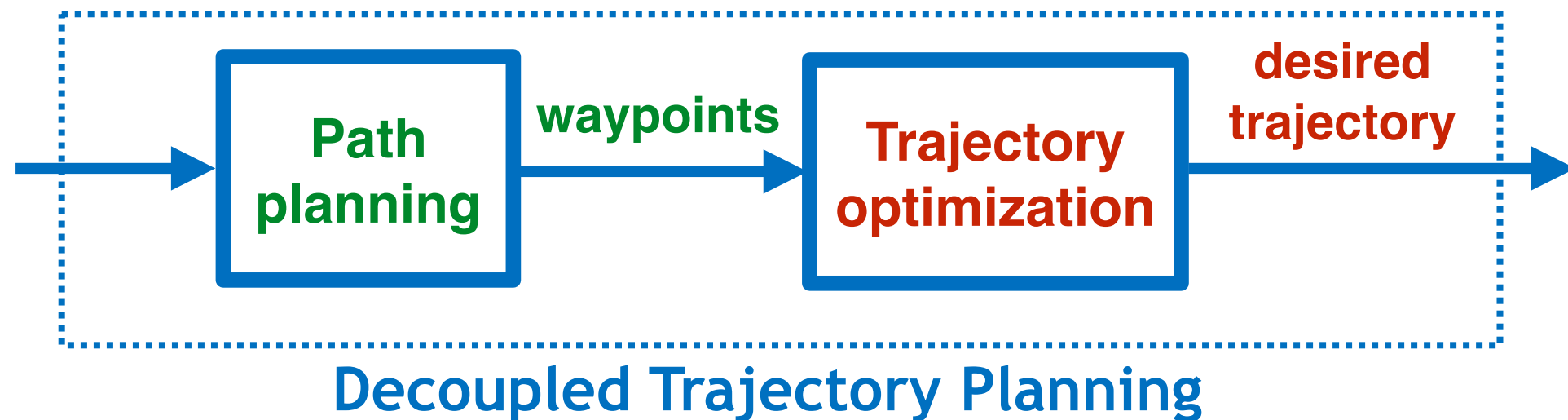
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- Need to ensure **feasibility**; e.g.:
  - Hit no obstacles
  - Don't saturate your controller



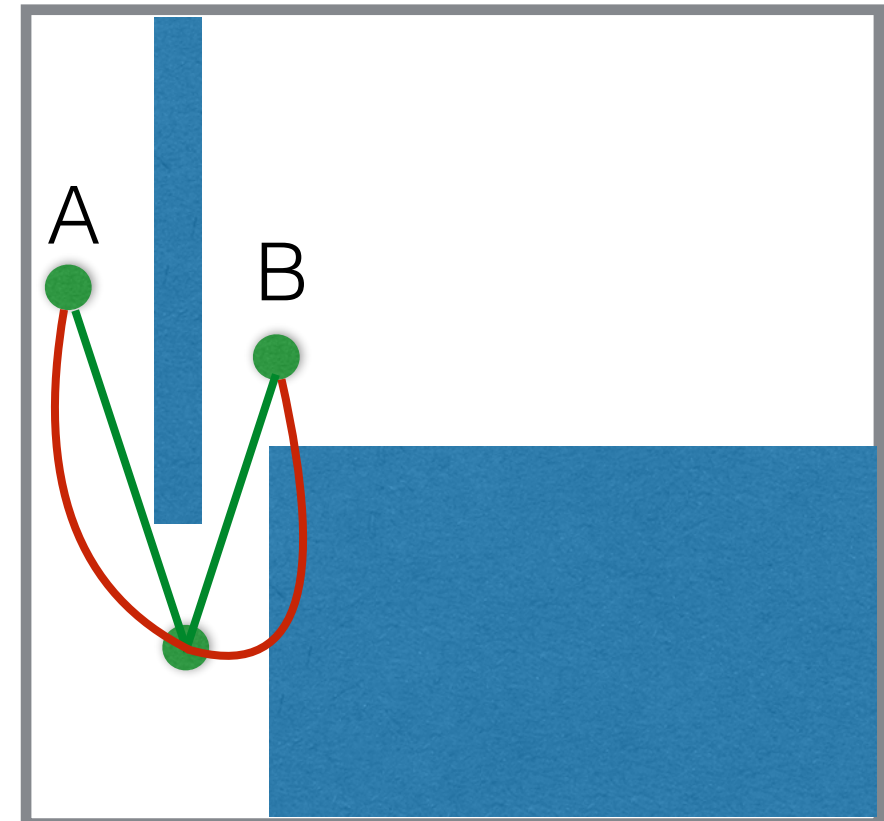
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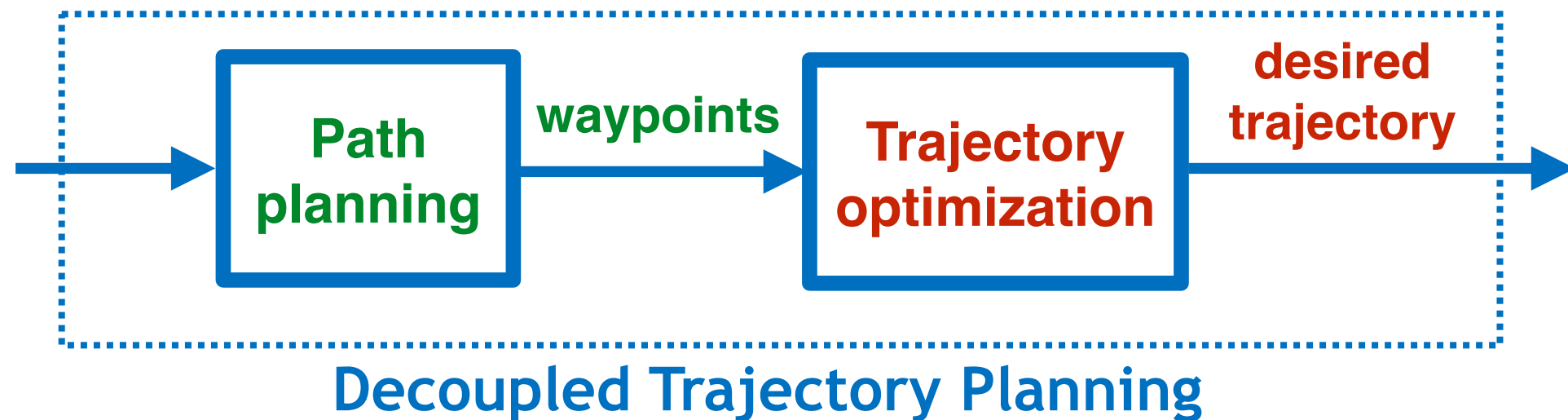
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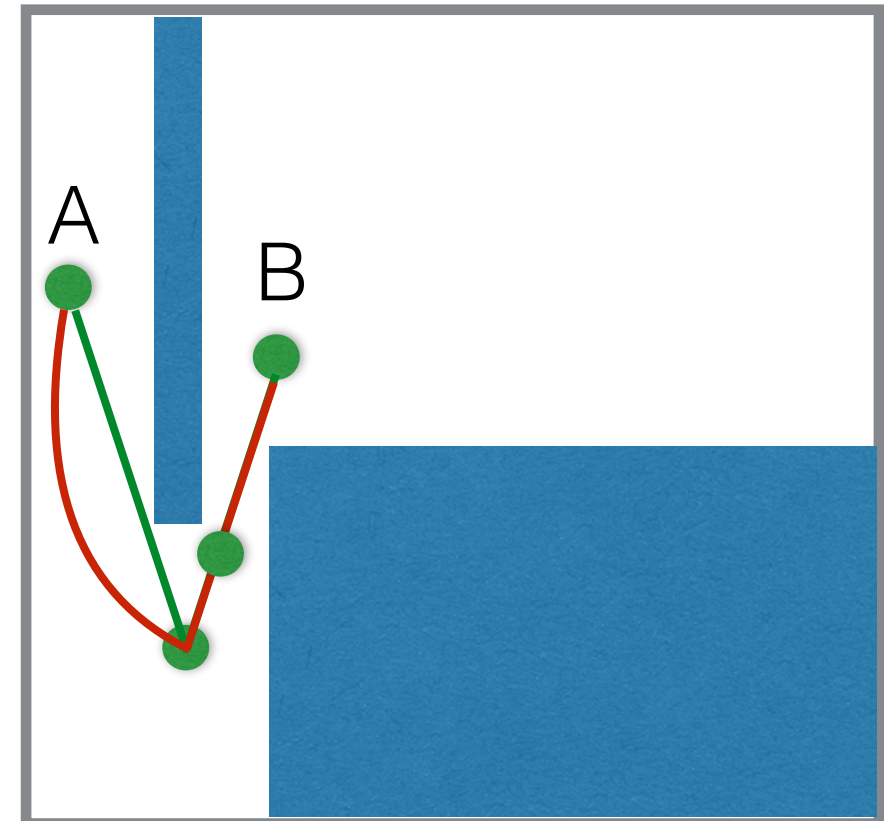
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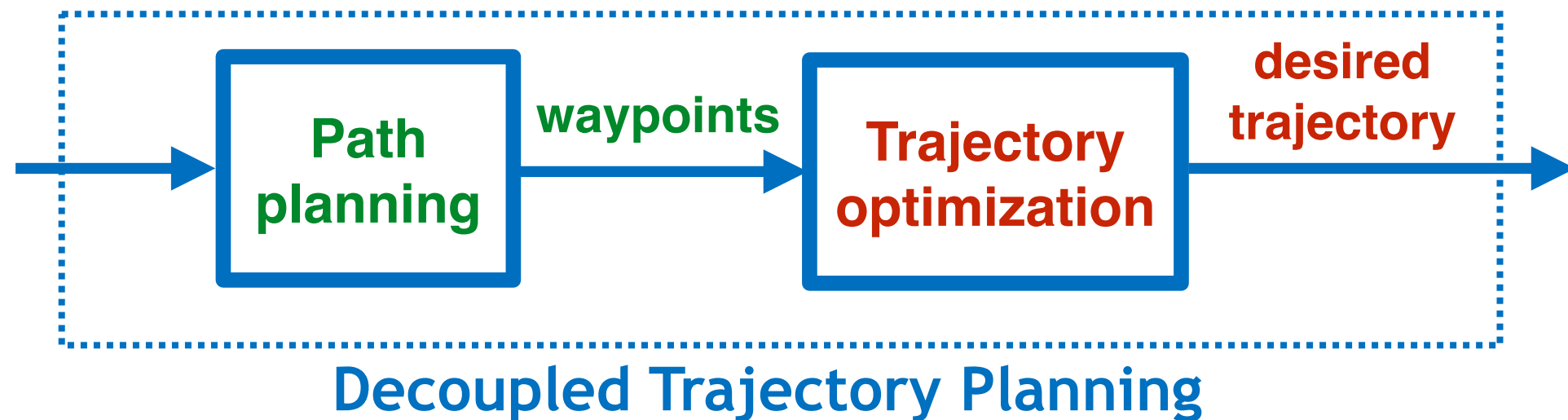
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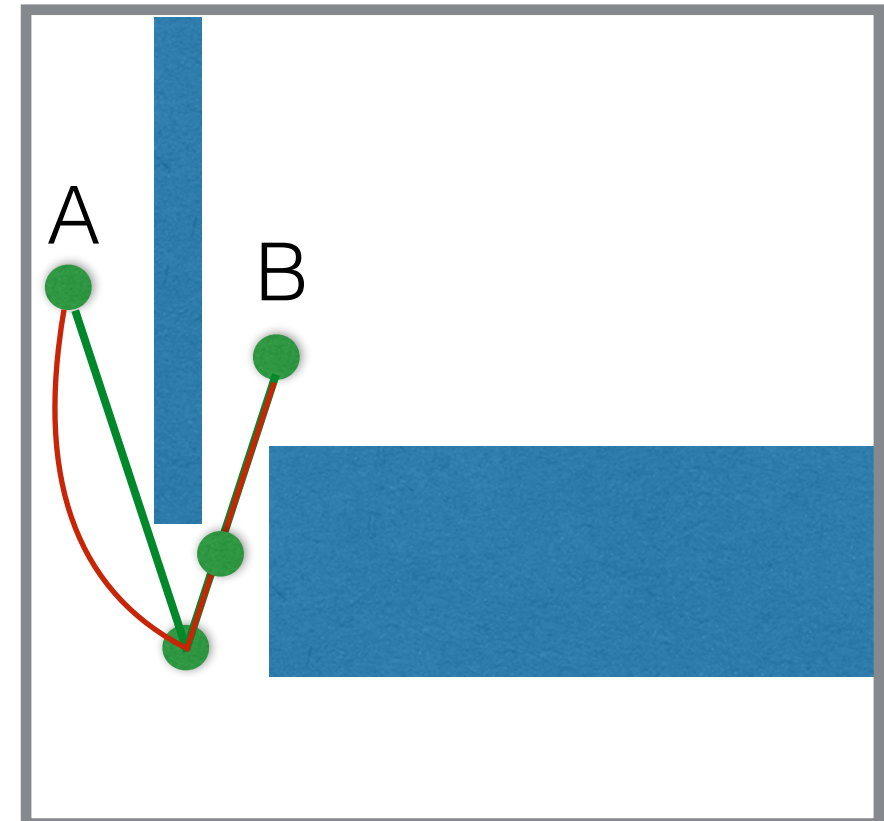
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16.485 Visual Navigation for Autonomous Vehicles (VNAV)  
Fall 2020

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