16.333 Handout #3
Prof. J. P. How Oct 12, 2004
**Due:** Oct 26, 2004

# 16.333 Homework Assignment #3

Please include all code used to solve these problems.

1. What is the wing surface area of the "Peacemaker" B-36?

2. For the F-4C aircraft we get the data on the following page from the tables. There are three flight conditions. For each flight condition:

   (a) Find the phugoid and short period frequencies.

   (b) Comment on how these mode frequencies change with the flight condition, and how do the numbers compare with the B747 analyzed in class?

   (c) Find the Spiral, Dutch roll, and Roll modes frequencies. Comment on how these mode frequencies change with these flight conditions. How do the numbers compare with the B747 analyzed in class?

3. The longitudinal model approximations are thought to be significantly better than those developed for the lateral dynamics.

   (a) The Dutch roll approximate model is obtained by looking at sideslip and yawing motions, neglecting the rolling motion. Show that the resulting model is of the form:

   $$\begin{bmatrix} \dot{v} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} Y_v/m & -U_o \\ (I'_{zx}L_v + N_v/I'_{zz}) & (I'_{zx}L_r + N_r/I'_{zz}) \end{bmatrix} \begin{bmatrix} v \\ r \end{bmatrix} + \begin{bmatrix} Y_{\delta_r}/m \\ (I'_{zx}L_{\delta_r} + N_{\delta_r}/I'_{zz}) \end{bmatrix} \delta_r$$

   For the B747, examine this conjecture by plotting the following transfer functions for the actual and approximate models given below:

   - $G_{v\delta_r}(s)$ - actual and Dutch roll approximate models
   - $G_{r\delta_r}(s)$ - actual and Dutch roll approximate models

   (b) Compare the accuracy of the approximate model in the frequency range near the mode that it approximates. Do your results support the conjecture given above? How well does this model approximate the actual dynamics in the other (higher and lower) frequencies ranges?

   (c) An approximate model for the spiral mode is obtained by looking at changes in the bank and heading angles. Sideslip is usually small (can ignore the side force equation), but cannot ignore $\beta$ completely. Not much roll motion, so set $p = \dot{p} = \phi = \theta_0 = 0$. The result is of the form:

$$\dot{r} + \lambda_r r = 0$$

What is $\lambda_r$, and how well does this model agree with the full dynamics?

(d) An approximate roll model is given by the equation

$$\dot{p} + \lambda_p p = 0$$

What is $\lambda_p$, and how well does this model agree with the full dynamics?

4. Using classical techniques (PID or Lead/Lag), design a pitch attitude autopilot for the B747 Jet using pitch angle and/or rate feedback. Use the short period approximation of vehicle dynamics. The goal is to put the short period roots in the vicinity of $w_{sp} =$ 4 rad/sec and $\zeta_{sp} = 0.4$. Assume there is an actuator servo that can be modeled as a first order lag with a time constant of 0.1 sec and has a DC gain of 1.

   (a) Show how you arrived at your design (show a root locus or Bode plot).

   (b) Plot the time response to a step $\theta_c$ command.

   (c) Check your pole locations on the full set of longitudinal dynamics. Is the response stable?

   (d) Use the short period model and design the controller using *state space* techniques. Develop a full state feedback controller that puts the *regulator* poles where required. Compare the time response to a step $\theta_c$ command to your classical controller.

5. The attached figures show the planform of the monocoupe that we have just started flying autonomously. Here are some key additional parameters:

   - $U_0 = 20\text{m/s}$
   - Mass=9.55Kg,
   - $\eta = 0.9$
   - $i_t \approx 0,\ i_w \approx 0$
   - Tail section is (NACA 0009), so that $C_{l_\alpha} \approx 6.25/\text{rad}$
   - $C_{D\,\min} \approx 0.017$,
   - Can assume CG at 1/4 chord point of the wing
   - $C_{L_{\alpha w}} = 4.4/\text{rad}$.

   Given this information, estimate the four main longitudinal derivatives $X_u$, $Z_u$, $M_w$, and $M_q$ and use them to predict the frequency and damping of the Phugoid and short period modes.

   Some basic questions:

   (a) What is your estimate of the trim angle of attack. Recall that:

$$C_{L_{\alpha_T}} = C_{L_{\alpha w}} + \eta \frac{S_t}{S} C_{L_{\alpha_t}} \left(1 - \frac{d\epsilon_0}{d\alpha}\right)$$

   (b) What are $C_{L_0}$ and $C_{D_0}$?

   (c) What is $C_{m_{cg}}$?

$$\begin{aligned} C_{m_{cg}} &= C_{m_0} + C_{m_\alpha}\alpha \\ C_{m_0} &= C_{m_{ac_w}} + \eta V_H C_{L_{\alpha_t}}(\epsilon_0 + i_w - i_t) \\ C_{m_\alpha} &= C_{L_{\alpha w}}(h - h_{\bar{n}}) - \eta V_H C_{L_{\alpha_t}}\left(1 - \frac{d\epsilon}{d\alpha}\right) \end{aligned}$$

   (d) Can you estimate the derivatives needed to form the more accurate estimates of the frequencies and damping (e.g., using the full approximations)?

$\mathbf{F-4C}$ $\quad S = 530\ \text{ft}^2, \quad b = 38.67\text{ft} \quad \bar{c} = 16\ \text{ft} \quad m = 1210\text{slugs}$

| Condition | 1 | 2 | 3 |
|---|---|---|---|
| $U_0$ ft/sec | 1.4520e+3 | 9.5200e+2 | 5.8400e+2 |
| $h$ ft | 4.5000e+4 | 1.5000e+4 | 3.5000e+4 |
| $I_{xx}$ | 2.5040e+4 | 2.4970e+4 | 2.7360e+4 |
| $I_{yy}$ | 1.2219e+5 | 1.2219e+5 | 1.2219e+5 |
| $I_{zz}$ | 1.3973e+5 | 1.3980e+5 | 1.3741e+5 |
| $I_{xz}$ | -3.0330e+3 | 1.1750e+3 | -1.6432e+4 |
| $\Theta_0$ | 0 | 0 | 0 |
| $X_u$ | -8.7120e+0 | -2.6015e+1 | -2.1296e+1 |
| $X_w$ | -2.4983e+1 | -7.5371e+0 | -5.0244e+1 |
| $Z_u$ | -1.4520e+1 | -1.7424e+2 | -1.4036e+2 |
| $Z_w$ | -5.9725e+2 | -1.4022e+3 | -3.3607e+2 |
| $Z_{\dot{w}}$ | -4.3250e-1 | -2.5420e+0 | -1.2245e+0 |
| $Z_q$ | -2.7104e+3 | -7.2600e+3 | -2.2022e+3 |
| $M_u$ | 3.0548e+2 | -5.3764e+2 | -1.2219e+1 |
| $M_w$ | -2.4354e+3 | -2.1820e+3 | -4.0319e+2 |
| $M_{\dot{w}}$ | -1.0267e+1 | -5.8656e+1 | -3.0129e+1 |
| $M_q$ | -5.9629e+4 | -1.2133e+5 | -3.7512e+4 |
| $Y_v$ | -1.4275e+2 | -2.6018e+2 | -6.8477e+1 |
| $L_v$ | -1.7206e+2 | -7.3048e+2 | -3.8660e+2 |
| $L_p$ | -2.4614e+4 | -5.6532e+4 | -1.8796e+4 |
| $L_r$ | 7.9878e+3 | 1.6930e+4 | 7.6882e+3 |
| $N_v$ | 9.7965e+2 | 1.7578e+3 | 5.0705e+2 |
| $N_p$ | -1.1178e+3 | 1.1184e+3 | -8.2446e+2 |
| $N_r$ | -4.4294e+4 | -7.5632e+4 | -2.0474e+4 |
| $X_{de}$ | 0 | 0 | 0 |
| $Z_{de}$ | -8.5511e+4 | -1.2947e+5 | -2.5386e+4 |
| $M_{de}$ | -1.9550e+6 | -3.0548e+6 | -5.9873e+5 |
| $Y_{dr}$ | 1.7364e+4 | 3.2368e+4 | 7.9860e+3 |
| $L_{dr}$ | 4.0064e+4 | 1.3199e+5 | -9.4666e+3 |
| $N_{dr}$ | -2.9008e+5 | -7.9211e+5 | -1.9279e+5 |
| $Y_{da}$ | -3.4969e+3 | -5.7487e+3 | -1.0672e+3 |
| $L_{da}$ | 1.7022e+5 | 4.3648e+5 | 1.1609e+5 |
| $N_{da}$ | 3.0042e+4 | 6.2491e+4 | -1.7039e+4 |

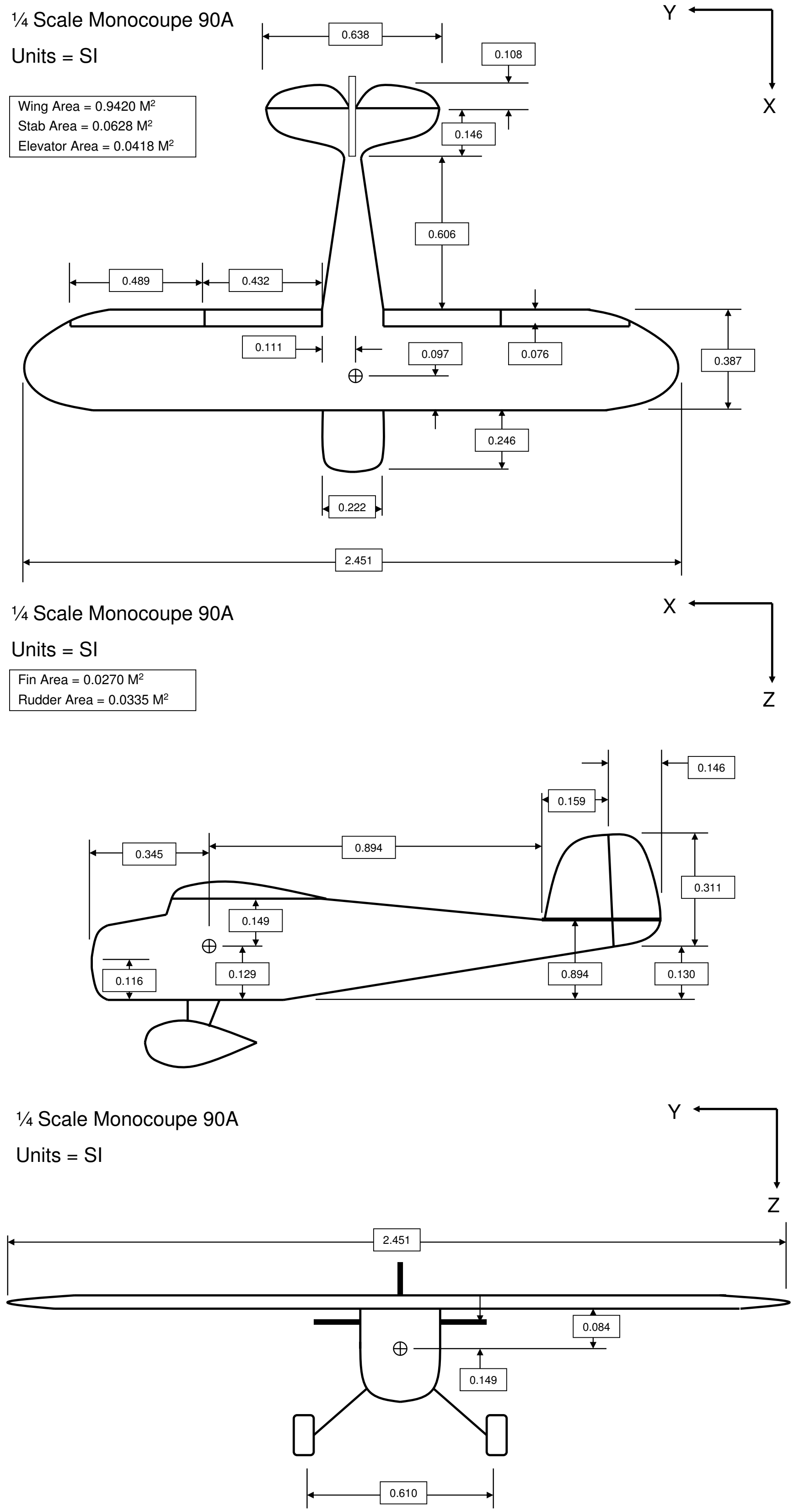


Figure 1: Monocoupe Planform