

24.949

Language Acquisition

Class 10
Numerals

Number and numerals

- Natural numbers (**N**):
 - ▶ the set of all positive integers;
 - ▶ used for counting and ordering;
 - ▶ abstract objects defined in purely mathematical terms
- Numerals:
 - ▶ linguistic objects that include some notion of cardinality in their denotation

Number and numerals

- Clearly the two are connected
 - mathematical language is expressible using natural language numerals
 - numerals encode a mathematical notion in their meanings
- connection exploited in developmental research
 - children's knowledge of natural number is diagnosed by their successes and failures to appreciate the meanings of numerals

Number and numerals

- Not the same
 - ▶ number knowledge and numeral knowledge may diverge in both directions
 - ▶ there are acquisition tasks that are proprietary to the linguistic domain

Today

- Acquisition of the semantics of numerals, and its connection with the acquisition of natural number

Early cognitive resources

- Two evolutionarily “ancient” domains of numerical ability:
 - i) Approximate Number System (ANS)
 - ii) Parallel Individual System

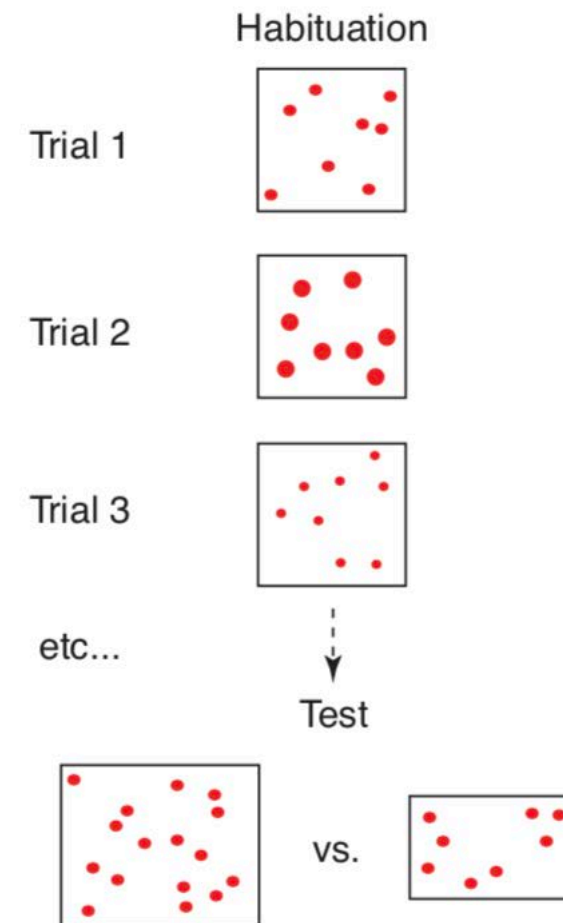
ANS

- Many species, including preverbal infants, have cognitive resources to estimate and compare collections of objects (and portions of stuff) in terms of their quantity
- Representations of quantity in this system are approximate (noisy) and the noisiness is a function of the quantity to be represented: the larger the quantity, the larger the variability

ANS

- Resolution increases with age
 - ▶ 6-month-olds succeed with 8 vs. 16 and 16 vs. 32, fail with 8 vs. 12 and 16 vs. 24
 - ▶ 10-month-olds succeed with 2:3 ratios as well
 - ▶ adults can do well up to 7:8
- Amodal: reproduce same effects with sounds

(a) Habituation experiments

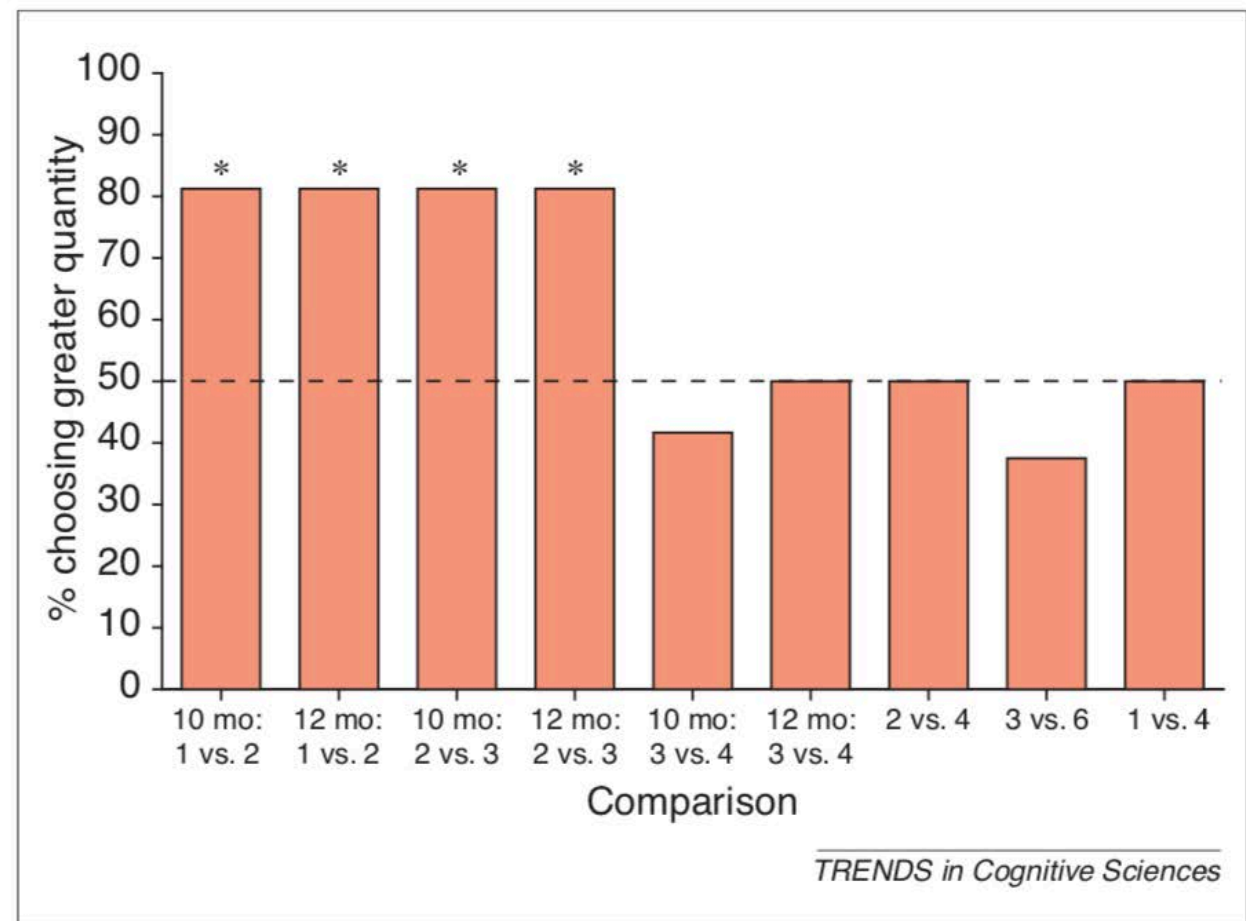
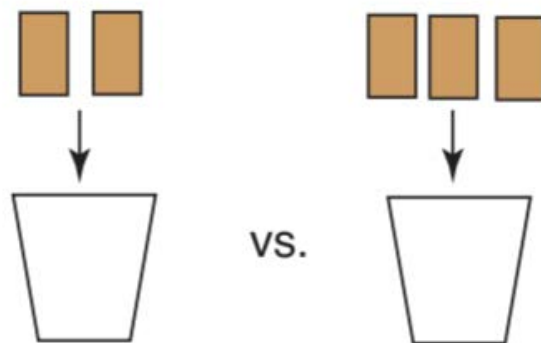


Parallel Individual System

- Many species, including preverbal infants, have cognitive resources to track objects wrt location, physical properties, etc.
- This system is capped at 3 or 4, but supports mathematical reasoning within that range

Parallel Individuation System

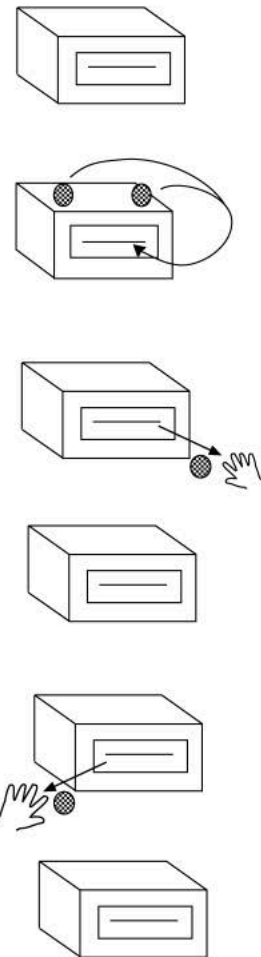
(c) Cracker choice experiments



Parallel Individuation System

Feigenson & Carey 2003

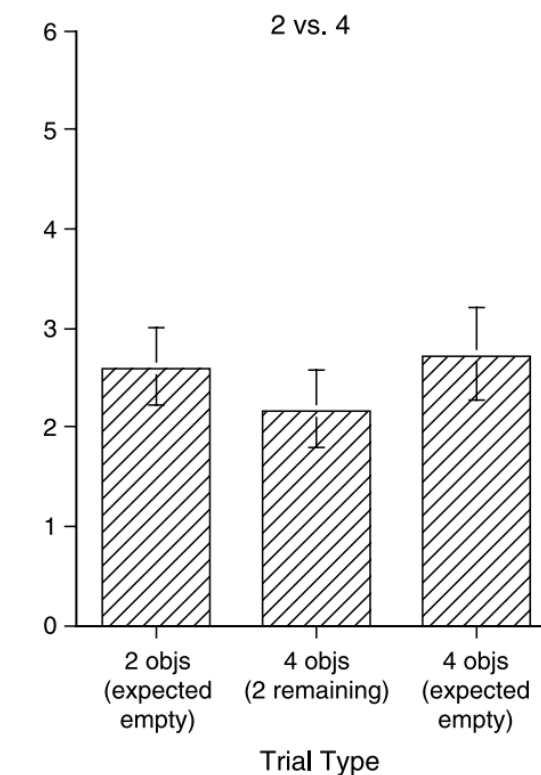
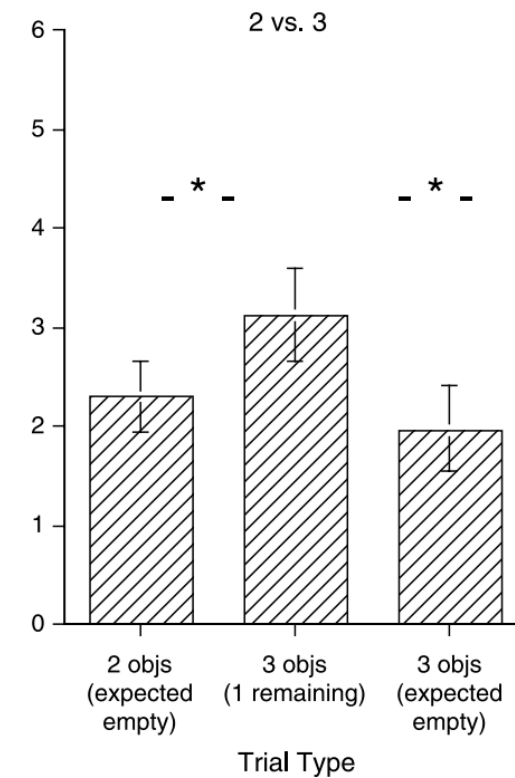
- 1) Box is placed on table.
- 2) Experimenter places 2 balls on box, then hides them inside.
- 3) Infant allowed to retrieve 1 ball. Experimenter surreptitiously removes 2nd ball.
- 4) 2-Objects (1 remaining) trial: Infant's searching is measured. 1 ball expected inside.
- 5) Experimenter 'finds' 2nd ball.
- 6) 2-Objects (expected empty) trial: Infant's searching is measured. Box expected empty.



*measurement period

*measurement period

Figure 2 2-Objects (1 remaining) trial and 2-Objects (expected empty) trial in Experiment 1.



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Natural number

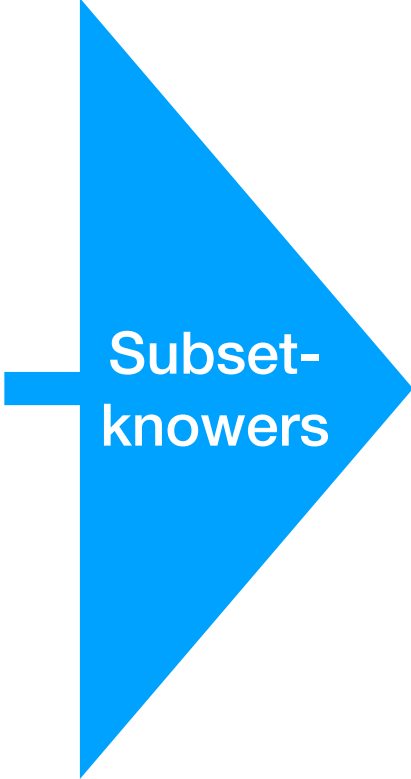
- Neither gives you the power to distinguish 6 from 7, which at some point, human beings can because we have natural numbers

Natural number

- But acquiring a full understanding of the natural numbers is a surprisingly difficult task.
 - ▶ Only humans succeed at it
 - ▶ It seems to requires some form of instruction
 - ▶ Initially, for the first three or four numbers (1,2,3,4), it is strictly incremental
 - ▶ Each stage takes a significant amount of time

Natural number

- Wynn (1990, 1992): Give N Task
 - “One-knowers”: succeed with 1, do not seem to distinguish 2, 3, *many*
 - “Two-knowers”: succeed with 1, 2, do not distinguish 3,4, *many*
 - “Three-knowers”: succeed with 1,2,3, do not distinguish 4, 5, *many*
 - “Four-knowers”: succeed with 1,2,3,4... begin to generalize



Subset-knowers



CP-knowers

Subset vs. CP-knowers

Qualitative shift at learning 4 (becoming “CP-knowers”)

- know the **Cardinality Principle**, which specifies that the last number word used in the counting process indicates the total number of items in a collection
- understand the **succession principle**, that for all $n \in \mathbb{N}$, $S(n) \in \mathbb{N}$
 - Sarnecka and Carey (2008): only CP-knowers know that adding objects to a set corresponds to a forward shift on the number line; removing objects corresponds to a backward shift
- make an induction to numbers even beyond their count list
 - Lipton and Spelke (2006): when a set beyond the child’s counting range has an item removed and replaced with a different item, CP-knowers understand that it retains its cardinality and thus that the same number word applies

Questions

- What is the contribution of language to the development of N?
- How does acquiring the meanings of number-related expressions of language interact with the development of N?

Inventory of number-related words

- i. Bare simplex numerals/cardinals: *one, two, three...*
 - ii. Complex cardinals: *twenty three, one hundred and seventy*
 - iii. Modified numerals: *at least two, more than three, exactly three, between one and three, about ten...*
 - iv. Proportional quantifiers: *more than half, less than a third, $n\%$*
 - v. Frequency quantifiers: *every other, two out of three...*
- Nearly all of the acquisition work has focused on bare numerals

Bare numerals

- What do bare numerals mean?
 - ▶ special type of entity: d or n
 - ▶ predicative or modifier: $\langle d, t \rangle$ or $\langle e, t \rangle$ or $\langle dt, dt \rangle$ or $\langle et, et \rangle$
 - ▶ generalized quantifier: $\langle dt, t \rangle$, $\langle et, t \rangle$

Bare numerals

- Different types in different environments?
 - ▶ we probably need a type d meaning for numerals in measure phrases (e.g. John is 2 inches taller than Bill) and for names of numbers
 - (1) Two plus two is/*are four.
 - (2) Two and two apples make/*makes four apples.

Bare numerals

- Different types in different environments?
 - ▶ we probably need the quantifier type to explain scope-taking behavior
- (1) He hasn't read two books. not > 2; 2 > not

Bare numerals

- Different types in different environments?
 - ▶ we probably need the modifier type to explain their ability to combine with determiner quantifiers (all three boys) and to appear in predicative positions
- (1) The Coen Brothers are two famous directors from the midwest.

Bare numerals

- at least, exactly, at most readings
 - (1) How many children do you have?
I have three. #In fact, four.
 - (2) Do you have three children? (For tax purposes)
I have three. In fact, four.
- The effect of modals
 - (3) You are required to read 10 books.
 - (4) You are allowed to read 10 books.
- NB: These readings are about *sentences*, not about numerals per se. You can have an exact number value in the denotation and still derive *at least* readings (cf. category mistake in Huang, Spelke and Snedeker)

Bare numerals

- How do we derive these readings?
 - Basic weak meaning (numerals/DPs containing numerals have existential semantics) + scalar implicature (Horn 1972, Fox 2006)

(1) John read 10 books = $\exists x[\text{books}(x) \ \& \ |x|=10 \ \& \ \text{John read } x]$

- Strong meaning (numerals inherently maximized) + scope of MAX

(2) John read 10 books = $\text{MAX}\{d: \exists x[\text{books}(x) \ \& \ |x| = d \ \& \ \text{John read } x]\} = 10$

Early use of numerals

- Primarily, if not exclusively, in count sequences

*CHI: one two three four five six seven eight nine
ten eleven twelve sixty forty eight nine ten eleven
twelve thirty six forty four eight nine ten eleven
twelve six seven eight nine ten eleven twelve thirty
six forty four six nine eight ten eleven twelve .

*MOT: oh

[Anne, 2;9]

Early use of numerals

- Primarily, if not exclusively, in count sequences

*MOT: how many have you got ?

*CHI: one three .

*MOT: all right .

*MOT: how many have you got ?

*CHI: bang .

*CHI: one two three four five .

*MOT: no .

[Aran, 2;4]

Early use of numerals

- Primarily, if not exclusively, in count sequences

*MOT: how many spoons have you got there , Becky ?

*CHI: one two five four .

*MOT: no .

*MOT: well .

[Becky, 2;3]

Early use of numerals

- Primarily, if not exclusively, in count sequences

*CHI: Daddy put some more batterys in later .
*MOT: he willn't . because he's not coming home tonight .
*CHI: Daddy put it on thirty (1)four .
*MOT: thirty four ?
*CHI: yes .
*CHI: my Daddy put it in thirty four [/] four .
*MOT: Daddy put it in what ?
*CHI: thirty (1)four I said .
*MOT: I heard the I said .
*MOT: I don't understand the thirty four bit .
*CHI: watch .
*CHI: put [/] put it in thirty four .
*MOT: okay .
*MOT: I don't know what you're on about but okay .

[Dominic, 2;7]

Early use of numerals

- Gelman & Gallistel (1978): if young toddlers understand what they're doing when they count, they must have the capacity to represent number
- But early on, they don't seem to understand what they are doing. There's a clear disconnect between counting ability and number knowledge.
- Wynn's "one-knowers" are often happily counting up to ten

Early representations of numerals

- How do children represent the numerals in their count sequence *before* they have the requisite number knowledge?
- How do subset-knowers represent the numerals for which they *do* have the requisite number knowledge?

Early representations of numerals

- Given the qualitative shift that takes place between being a subset-knower and a CP-knower, one possibility might be that:
 - ▶ subset-knowers' representations of numerals they use correctly is nevertheless non-adult
 - ▶ E.g. a subset knower's representation of *three sheep*:
(1) $\exists x \exists y \exists z [x \text{ is a sheep} \ \& \ y \text{ is a sheep} \ \& \ z \text{ is a sheep}]$
- Moreover, they might simply not have any idea about the rest of the numerals in their counting sequence

Condry and Spelke 2008

- Probably not the case:
 - ▶ subset knowers understand numerals they “use” but don't fully “know” as being the same kind of beast as those they do know

Condry and Spelke 2008

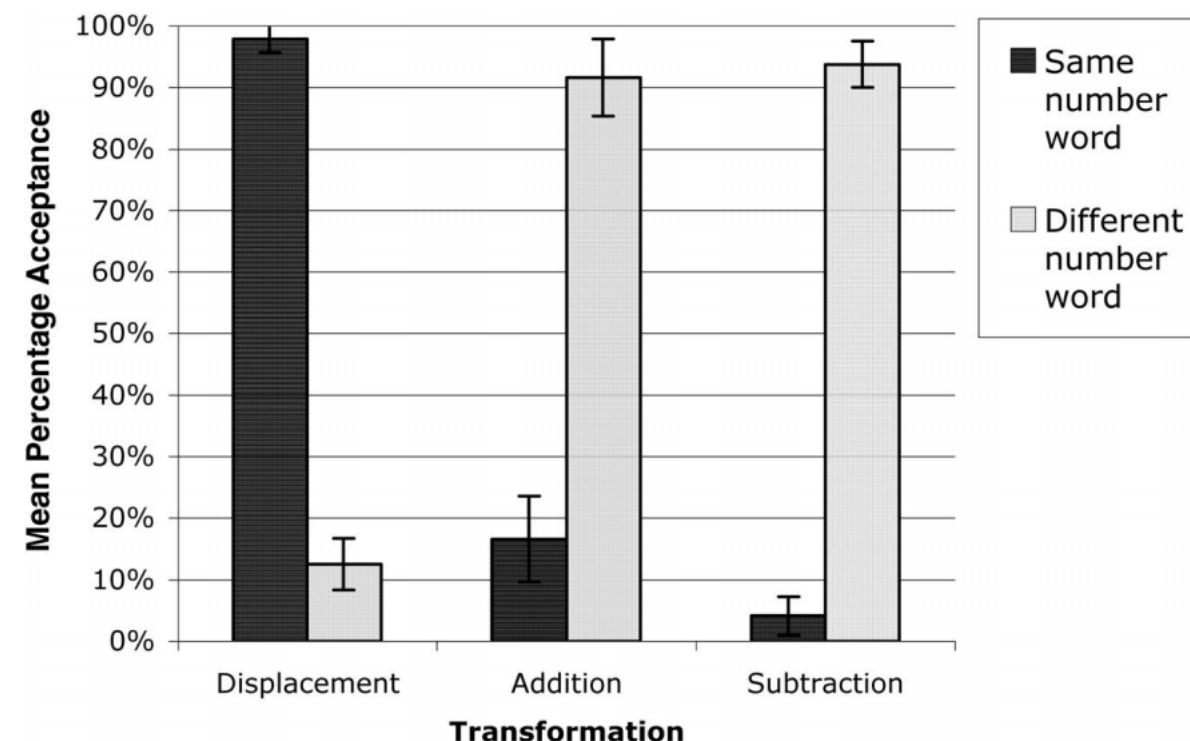
- Two pre-tests
 - Counting: only include children who can count to 10
 - Give-N: only include subset knowers

Condry and Spelke 2008

- Baseline (CS Exp 2): subset-knowers are really subset-knowers
 - ▶ 16 3yos
 - ▶ Children were presented with two arrays of objects, and asked to point to the one with n objects (half the time the number that would pick out the larger array)
 - ▶ In the known n trials, at least one of the arrays presented a cardinal value named by a word that the child had fully mastered; in the unknown n trials, neither array represented a number the child knew.
 - ▶ Children were well above chance (86%) on known n trials, but at chance (49%) on unknown n trials

Condry and Spelke 2008

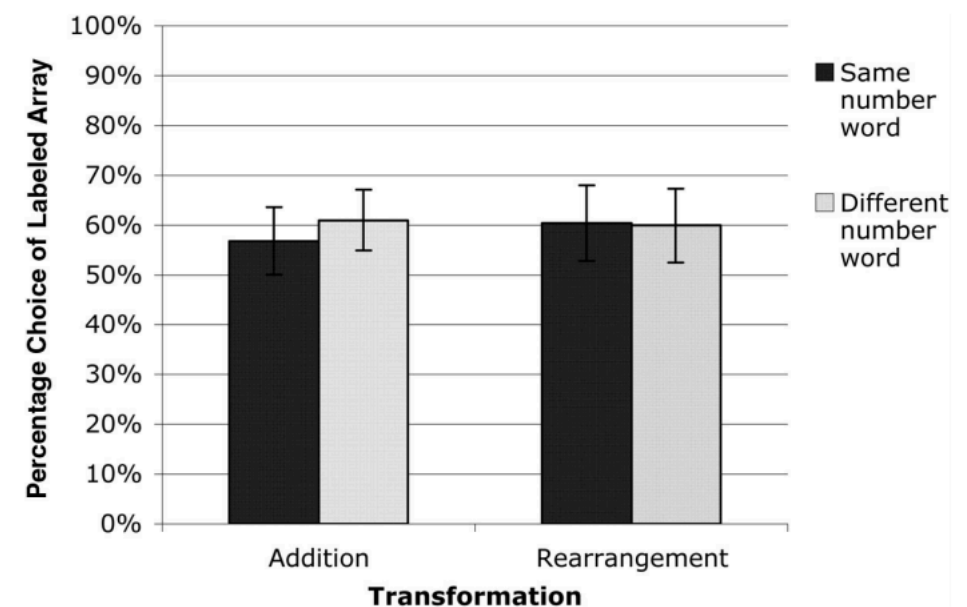
- Baseline 2 (CS Exp 4): subset-knowers can do “arithmetic” with known numbers
 - ▶ N=24
 - ▶ Identification: “Look there are **three** sheep here!” + Transformation: Displacement (move objects to another box), Addition (add more during displacement), Subtraction (remove one during displacement)
 - ▶ “Now are there [same] or [different] sheep here?”



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Condry and Spelke 2008

- Baseline 3 (CS Exp 4): subset knowers can't perform "arithmetic" with unknown numbers
 - ▶ N=16
 - ▶ Two sets of objects; one set labeled, "Look! There are *five* sheep here!" and then transformed (add new object or rearranged ordered)
 - ▶ "Now can you point to the one with *five/six* sheep?"

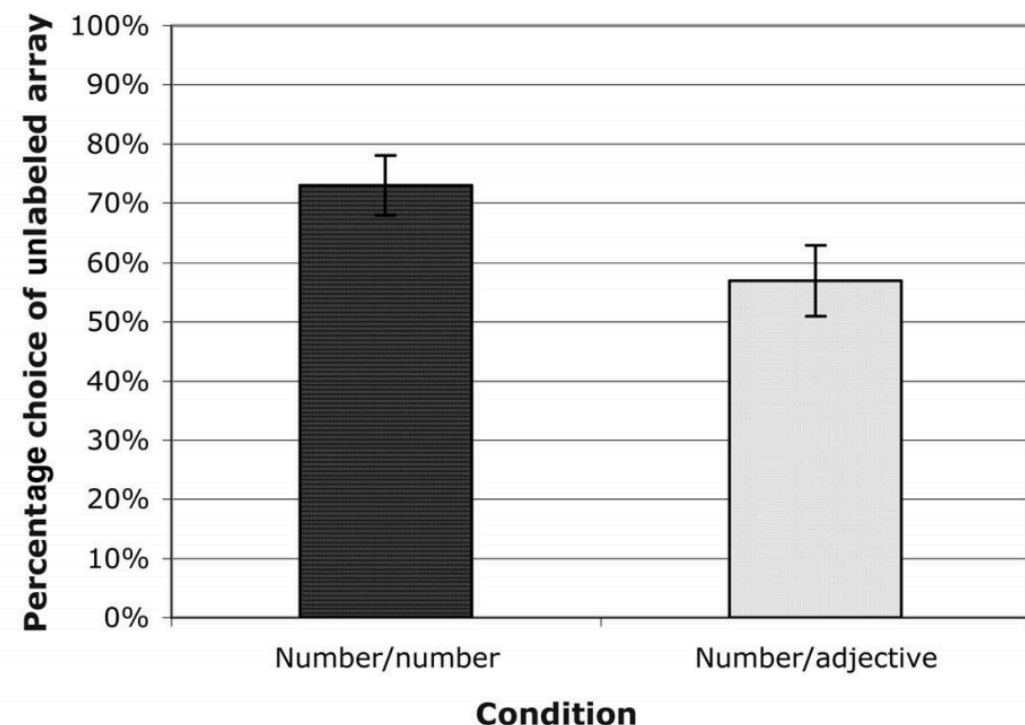
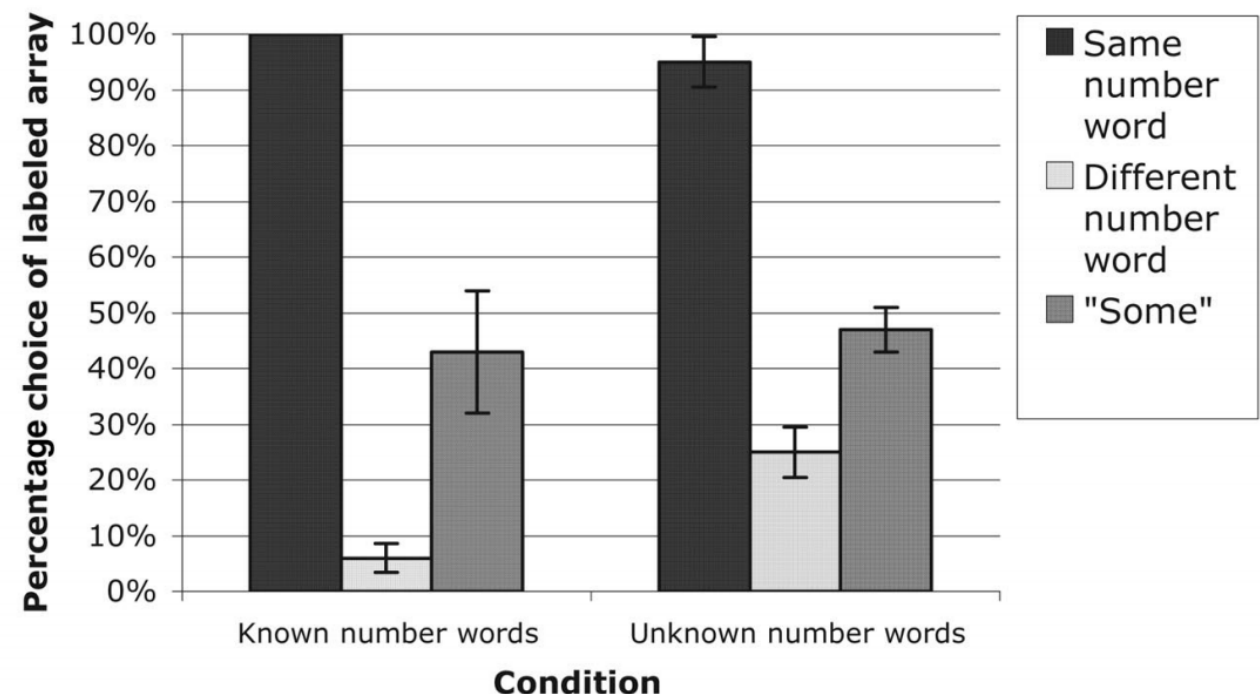


Condry and Spelke 2008

- Direction on the scale (Exp 3): subset-knowers know something about unknown numbers
 - ▶ 16 3yos
 - ▶ “Which one is more?” task, where they have to compare whether e.g. in an apple-picking context, whether a person who picked 4 apples or 7 apples picked more apples.
 - Known n trials: one of the words represented an n that the child knew
 - Unknown n trials: neither words represented an n that the child knew
 - ▶ Children successfully identified the larger number (even though they didn’t know its cardinal value) as *more* when contrasted with a known number (79%), but failed when neither word represented a cardinal value that was known (49%)

Condry and Spelke 2008

- Linguistic contrast (Exp 1): subset-knowers know that numerals contrast with each other, even when they don't know the exact cardinal value of those numerals
 - ▶ 32 3yos, 16 per group
 - ▶ Numeral contrast group: “There are *ten* sheep on this one. This one has sheep too. Can you show me a card with *six* sheep?”
 - ▶ Adjectival contrast group: “There are *ten* sheep on this one. This one has sheep too. Can you show me a card with *happy* sheep?”
 - ▶ Existential control (both groups): “Can you show me a card with some sheep?”



Condry and Spelke 2008

- Even when they do not know their cardinal values, numerals on the child's count list contrast with other numerals (and not other types of expressions)

Barner and Bachrach 2009

- Main idea:
 - numerals/DPs containing numerals have weak meanings
 - “N-knowers” know the meaning of N, and also know something about N+1
 - e.g. *one*-knowers might not know what number *two* is, but know that it is the successor of *one* on some number-related scale
- They compute a scalar implicature based on the fact that N+1 wasn't used to arrive at an exact meaning for N

Barner and Bachrach 2009

- Supporting evidence:
 - ▶ *one* vs. *a*
 - ▶ When presented a context in which there were two objects in a circle (e.g., two bananas), 2- to 5-year-old children replied “yes” when asked “Is there a banana in the circle?” but said “no” when asked “Is there one banana in the circle?”

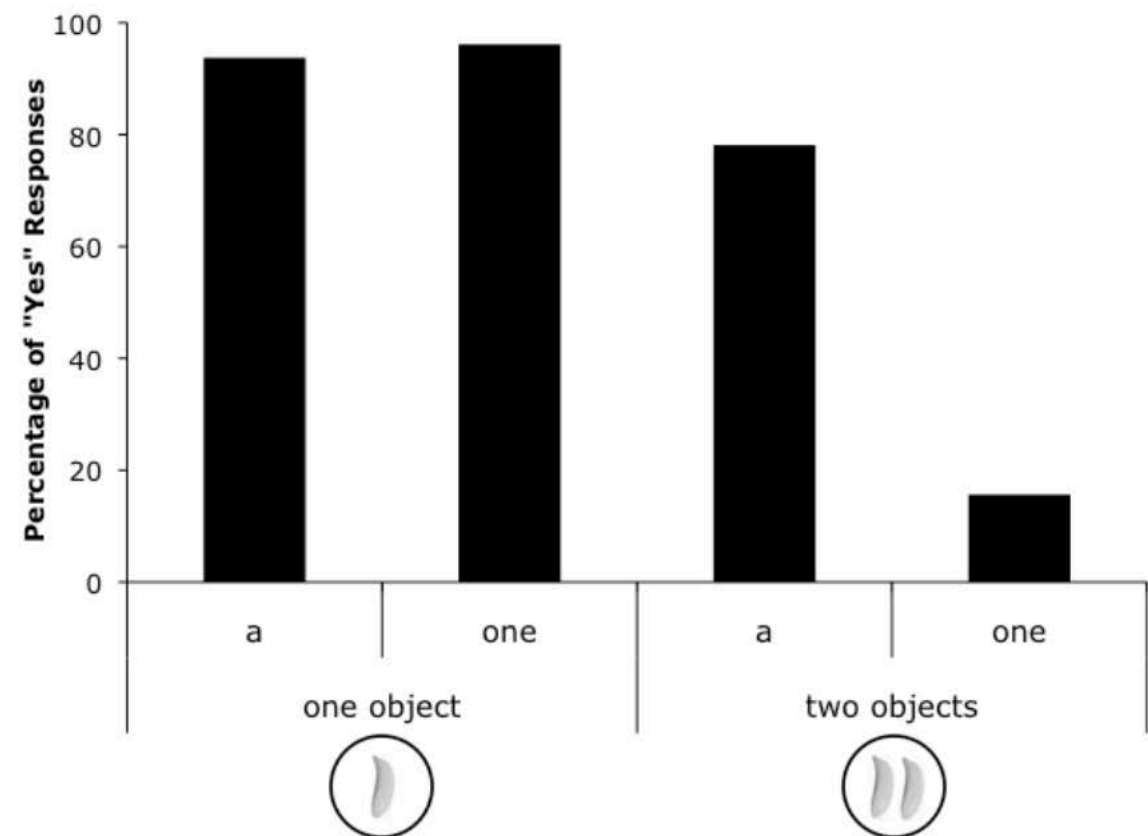


Figure 2. The percentage of yes responses for *a* and *one* for one or two objects, when 2-year-olds were asked – e.g., *Is there a banana in the circle?* or *Is there one banana in the circle?*

Barner and Bachrach 2009

- Supporting evidence:
 - ▶ N-knowers respond correctly to N+1 at higher-than-chance rates

Table 1

Average percentage of trials on which N-knowers gave N + 1 when asked for N + 1.

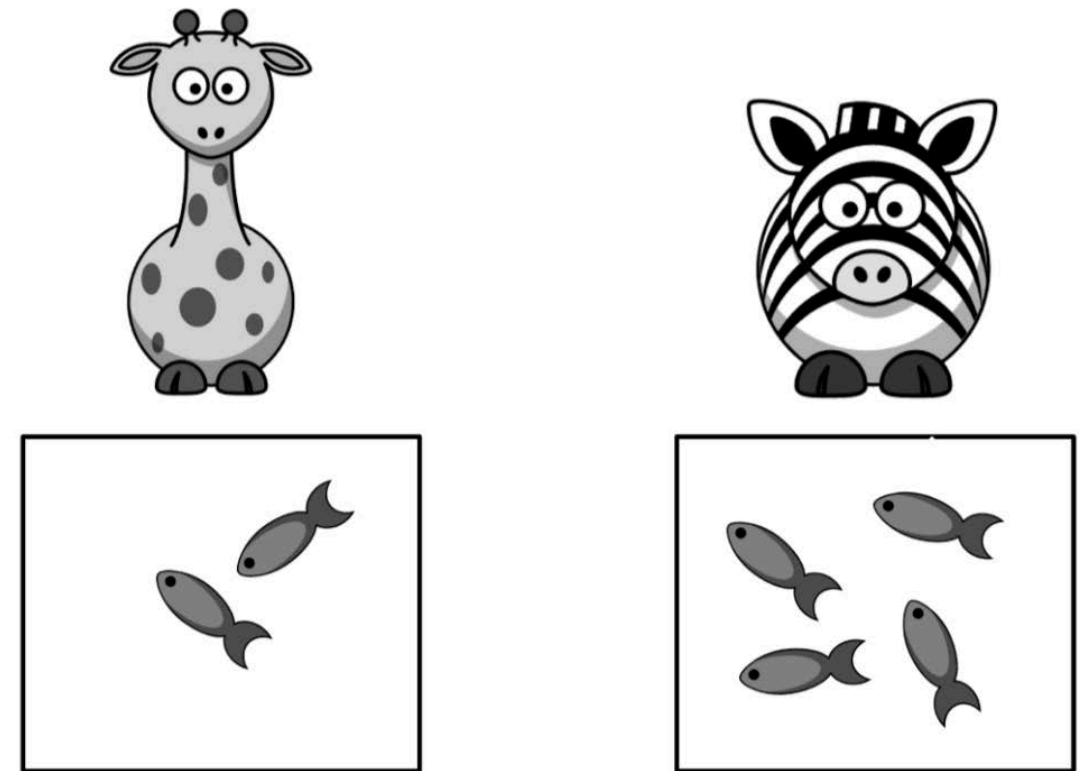
Data source	Language	Non-knower	One-knower	Two-knower	Overall
LeCorre et al.	English	62	47	27	43
Sarnecka et al.	English	75	73	49	66
	Japanese	70	63	53	65
	Russian	100	67	45	60
Barner et al.	English	23	37	24	29
	Japanese	38	54	53	44
Average		51	60	41	53

Feiman et al. 2019

- Subset knowers treat known and unknown numerals as related by entailment

Feiman et al. 2019

- Participants (Exp2): 133 English-speaking children, ages 2;8–5;0; 72 subset-knowers
- “Split” condition, where the exact number requested (e.g. 3) was not represented by either choice, but there was a larger set.



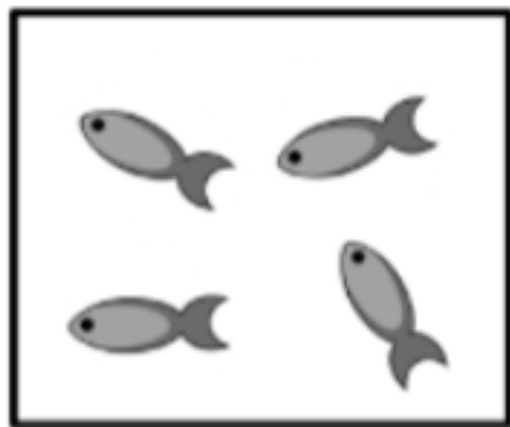
Can you find the one that has *three* fish?
Can you find the one that doesn't have *three* fish?

Feiman et al. 2019

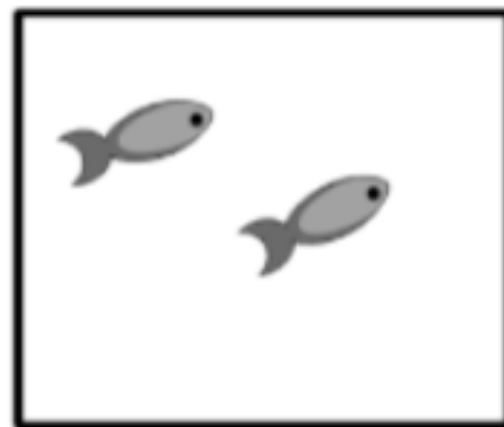
N= the max number that the child knows, here 3

Can you find the one that has 3 fish?

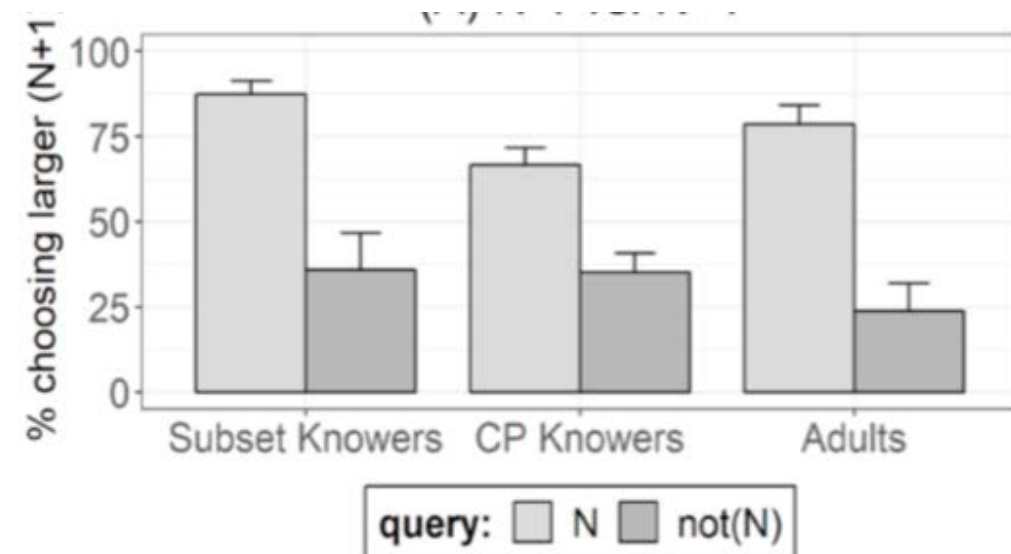
Can you find the one that doesn't have 3 fish?



4



2

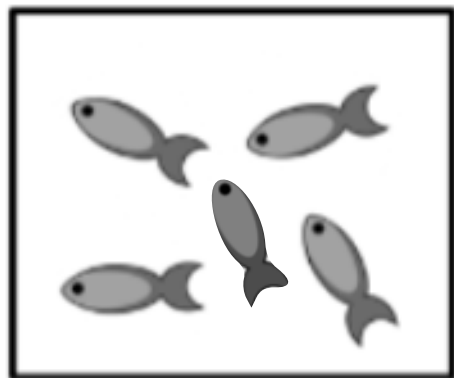


Feiman et al. 2019

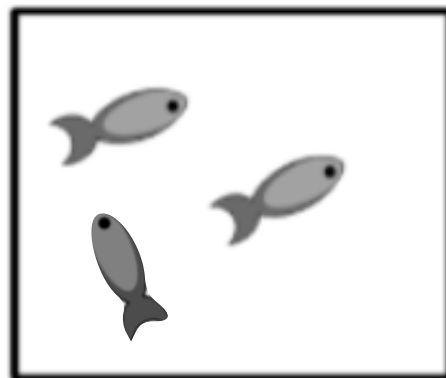
N= the max number that the child knows, here 3

Can you find the one that has 4 fish?

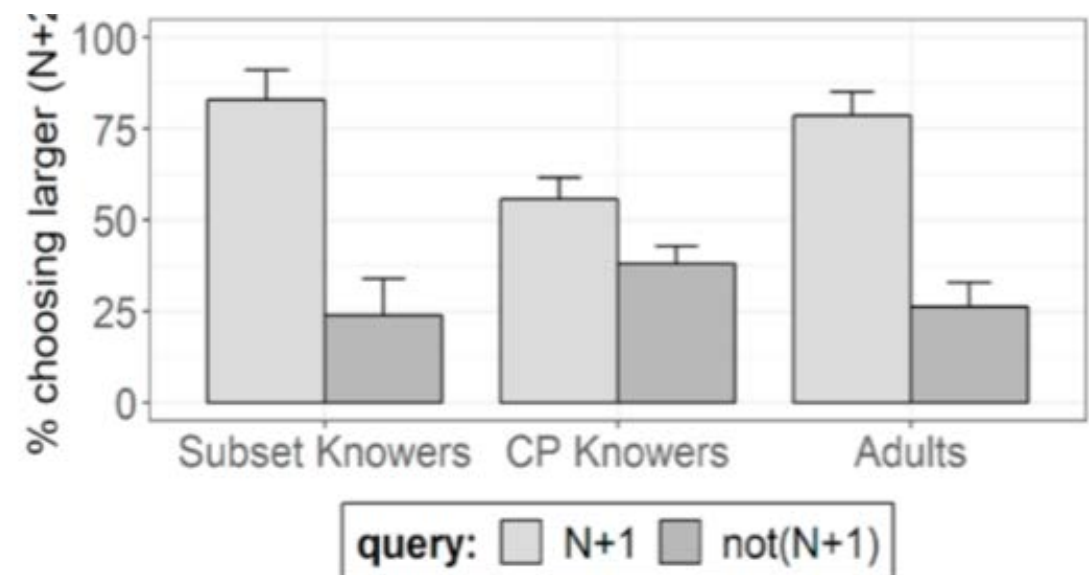
Can you find the one that doesn't have 4 fish?



5



3

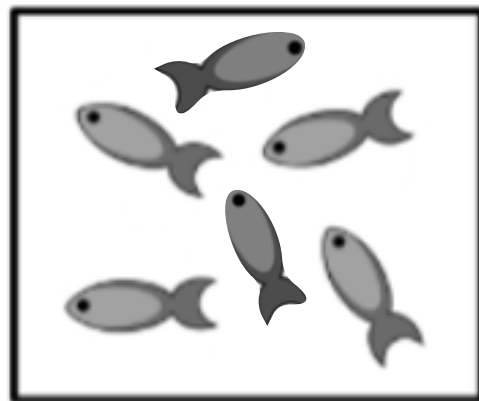


Feiman et al. 2019

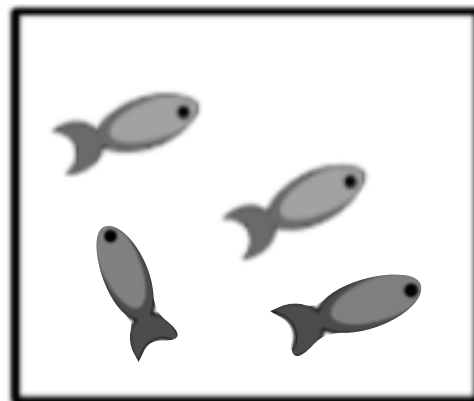
N = the max number that the child knows, here 3

Can you find the one that doesn't have 5 fish?

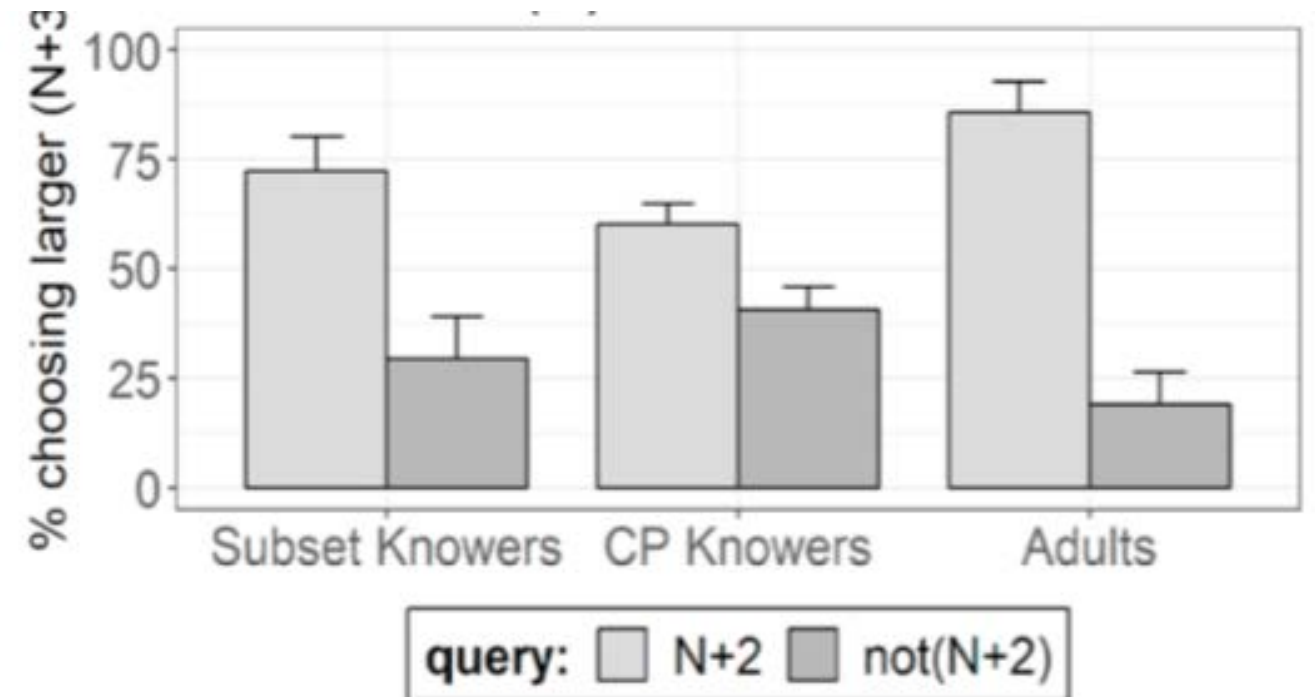
Can you find the one that has 5 fish?



6



4



Later numeral knowledge

- What do CP knowers know about numerals?

Davidson et al. 2012

- **Participants:** 84 CP-knowers
- Split by counting ability:
 - Low counters (up to 19)
 - Medium counters (up to 29)
 - High counters (30+)

Davidson et al. 2012

- **Unit Task:** Do CP-knowers know that n is *one more* than $n-1$?
- Experimenter puts some number of beads in a box (small $n = 4$ or 5 , medium $n = 13, 14, 15$, large $n = 24, 25$) and says how many. Then they added either one or two beads to the box, and asked: How many beads are there now, $n+1$ or $n+2$?

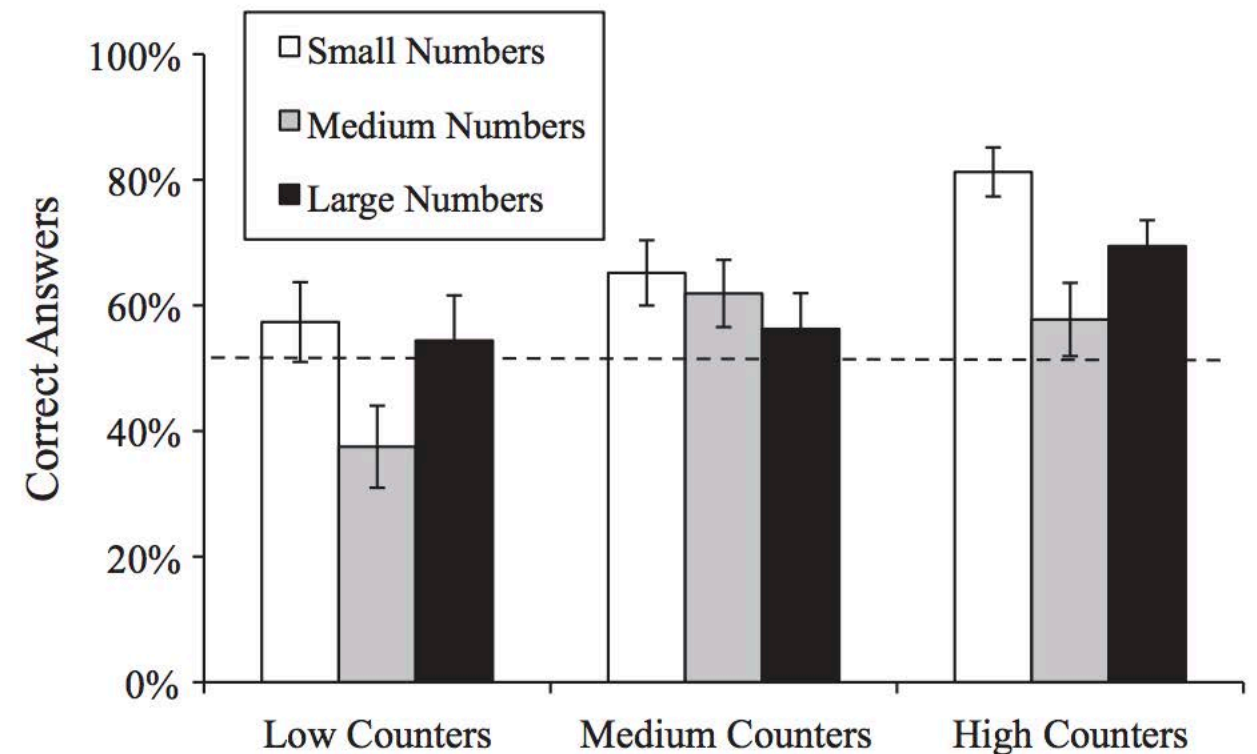
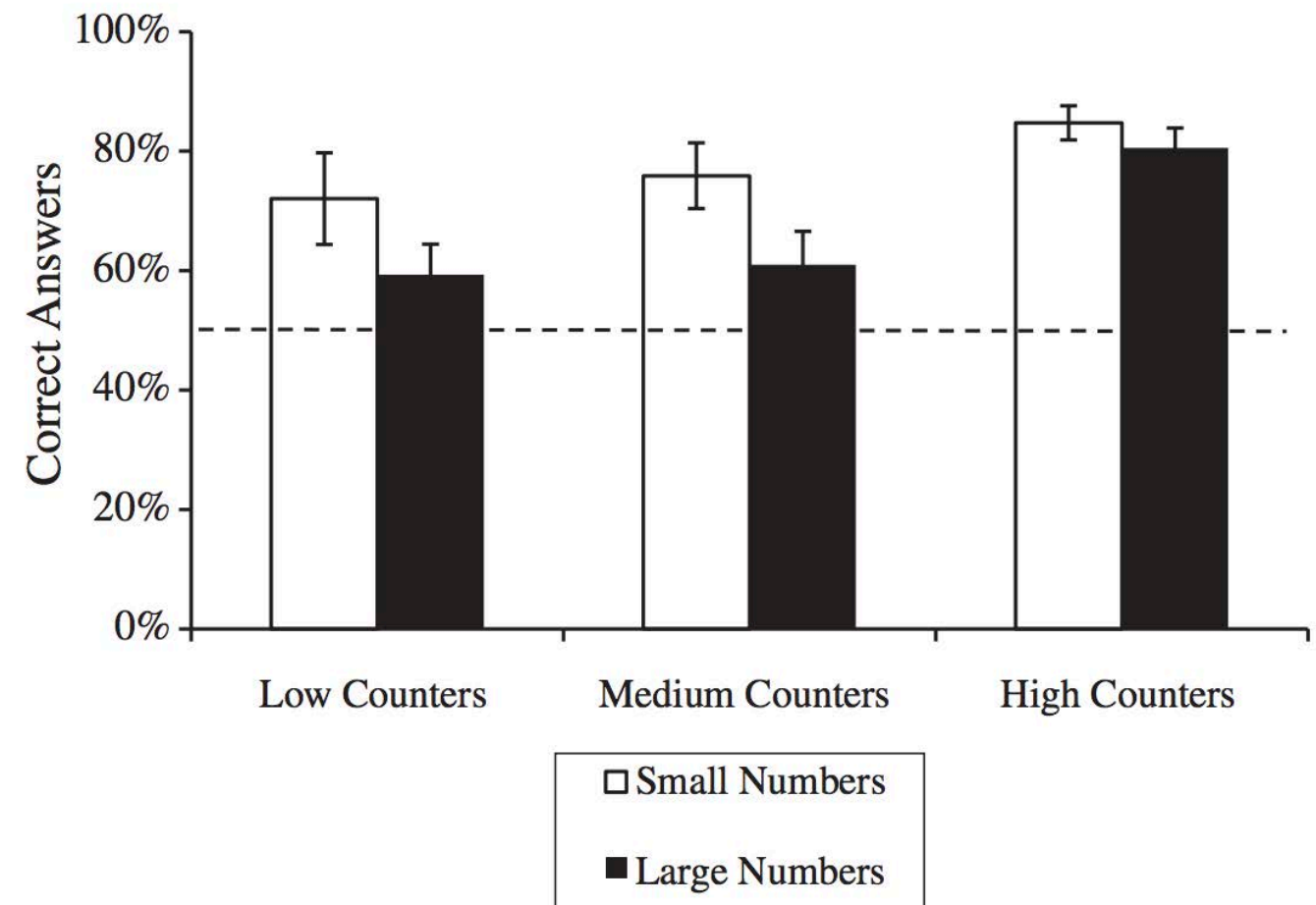


Fig. 2. Average percentage of correct answers by count group on the Unit task with small, medium, and large numbers.

Davidson et al. 2012

- **More Task:** Do CP knowers know the later-greater principle?
- Two of the boxes were placed in front of the child. Then, the experimenter said, “This box has N stickers and this box has M stickers. Which box has more stickers?”
- Small quantifies (< 10) and large quantifies (> 20) were tested



What's the issue?

- Davidson et al.'s interpretation: CP-knowers don't yet have the successor principle
 - ▶ wholesale, it's implausible: children do have access to recursive functions
 - ▶ perhaps they lack knowledge of the quantitative distance between any two adjacent elements in the sequence

What's the issue?

- “Large numbers” may introduce difficulties of their own
- Cardinal numbers are syntactically and semantically complex (Hurford 1975, Ionin and Matushansky 2006 et seq.)

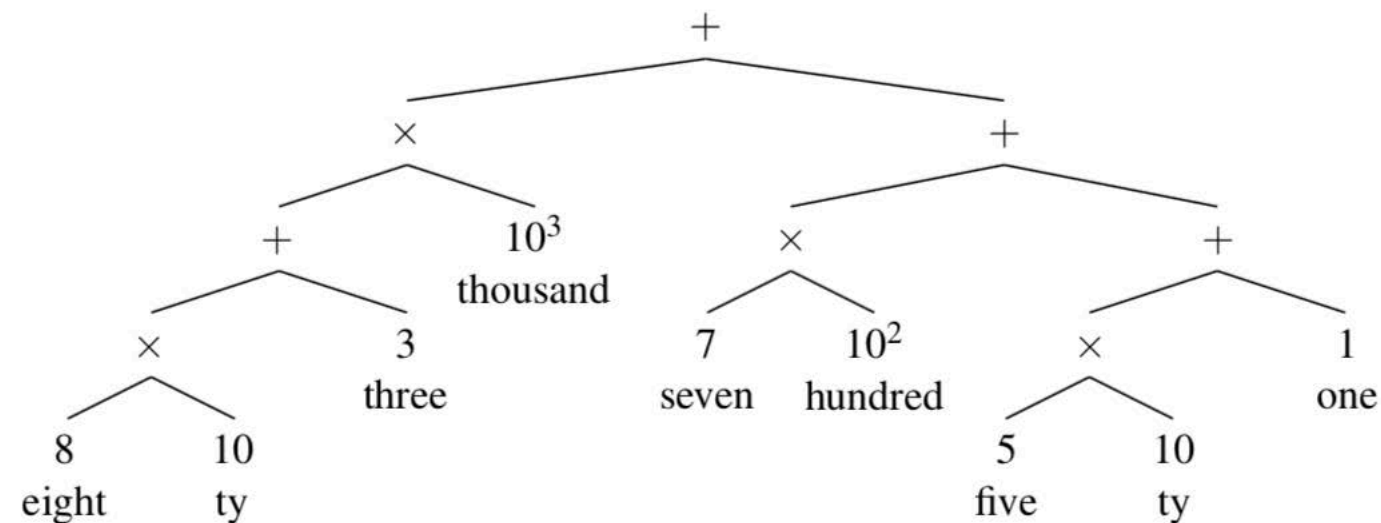
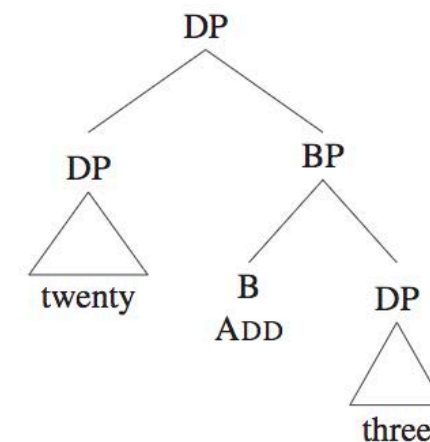


Figure 1 Structure of number 83751 (Hurford 1975, 1987)

What's the issue?

- They compose via operations like *addition* and *multiplication*, carried out by appropriate heads in the syntactic structure (see below)
- On this view, figuring out what number the numeral *twenty-three* makes reference to involves carrying out some arithmetic aided by the linguistic structure (plus figuring out what the base is etc.)

(27) Structure of an additive numeral (*twenty-three*):



a. $\llbracket \text{ADD} \rrbracket = \lambda D'. \lambda D''. \lambda d. \exists d' d'' [d = d' + d'' \wedge D'(d') \wedge D''(d'')]$

b. $\llbracket \text{MUL} \rrbracket = \lambda D'. \lambda D''. \lambda d. \exists d' d'' [d = d' \times d'' \wedge D'(d') \wedge D''(d'')]$

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$$\llbracket \text{twenty ADD three} \rrbracket = \lambda d \exists d', d'' [d = d' + d'' \wedge \llbracket \text{three} \rrbracket (d') \wedge \llbracket \text{twenty} \rrbracket (d'')]$$

Interaction of numerals and modals

- The salient readings of numerals is a lower-bounded one under universal modals, and an upper-bounded one under existential modals:
 - (1) You have to get three answers right in order to pass. [= 4 is a pass]
 - (2) You can get three answers wrong and pass. [= 4 is a fail]

Musolino 2004

- 20 children (M=5;0) in a TVJT
- Two between conditions (N=10 per group):
 - ▶ “at least” condition
 - ▶ “at most” condition

Musolino 2004

Table 1

The puppet's statements/questions on test stories, 'at least' condition

Test story 1	Goofy said that the Troll had to put two hoops on the pole in order to win the coin. Does the Troll win the coin?
Test story 2	Pink Panther said that if the horse made it over two obstacles he would win. Does the horse win the blue ribbon?
Test story 3	Mickey said that if Cookie Monster got two basketballs in the net he would get the donut. Does Cookie Monster get Mickey's donut?
Test story 4	Smurfette said that if Carpenter Smurf knocked down two pins she would give him her flower. Does Carpenter Smurf get Smurfette's flower?

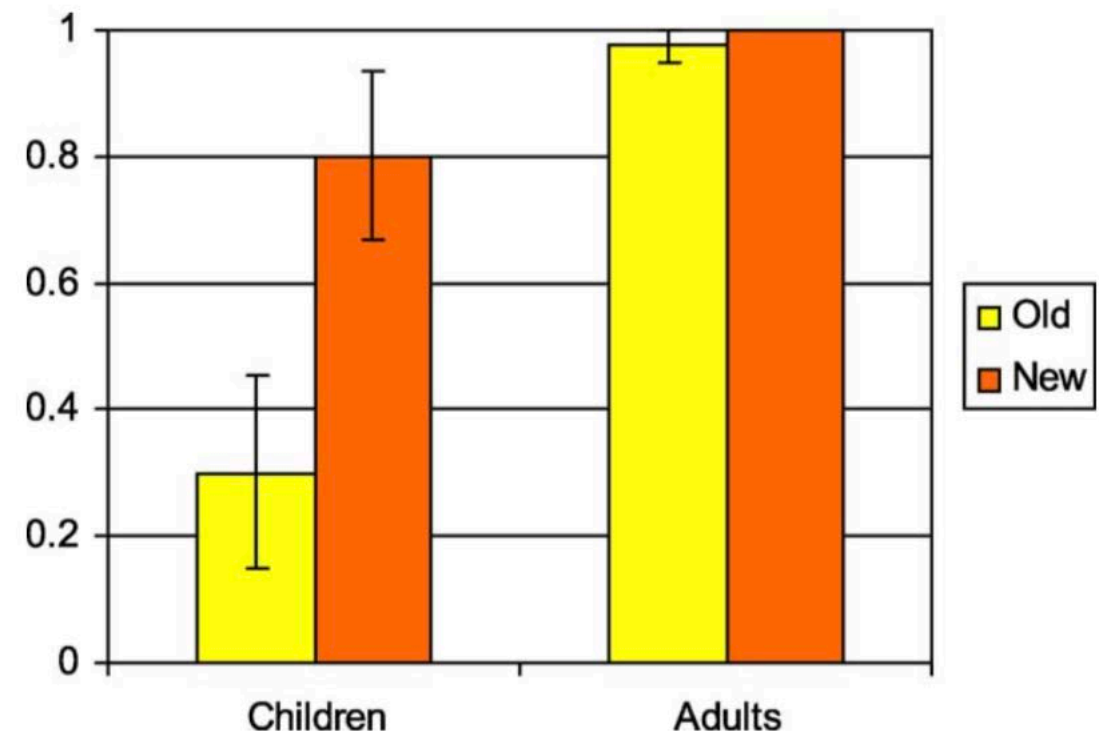
Table 2

The puppet's statements on test stories, 'at most' condition

Test story 1	Goofy said the Troll could miss two hoops and still win the coin. Does the Troll win the coin?
Test story 2	Pink Panther said the horse could knock down two obstacles and still win the blue ribbon. Does the horse win the blue ribbon?
Test story 3	Mickey said that Cookie Monster could miss two shots and still get the donut. Does Cookie Monster get Mickey's donut?
Test story 4	Smurfette said that Carpenter Smurf could miss two pins and still get her flower. Does he get Smurfette's flower?

Musolino 2004

- Children seem to have overly strict criteria for success in games and could have been biased towards an exact interpretation in the at least condition
- Exp 2: fix the “at least” condition
- 20 children (M=5;0) in a TVJT; 10 per group (replication vs. fixed)



**beyond simplex bare numerals
in DP-internal position**

Modified numerals

- Musolino 2004, Exp.3
 - ▶ Picky Puppet Task: “Children were then told that the puppet only likes to keep cards with *exactly/at least/at most two Ns* on them and they were asked, for each card, to tell the experimenter whether the puppet would want to keep that card.”
 - ▶ 16 children (M=5;2), 8 in the *at least* condition; 8 in the *at most* condition
 - ▶ Imperfect control: half the sample received a *more than two Ns* control

Modified numerals

Children were adult-like with *exactly two* and *more than two*

8/8 kids showed the *exactly two* pattern for “at least two”

5/8 kids showed the *exactly two* pattern for “at most two”; some showed *more than two* pattern

Measure phrase numerals

- Numerals can serve as differential expressions in comparative constructions, where they seem to contribute an arithmetic meaning:

(1) John is three inches taller than Bill

(2) John has three more cookies than Bill.

- These measure phrase numerals indicate the *difference* between two extents, e.g.

(3) **max**(λd_1 . tall(d_1)(John)) – **max**(λd_2 . tall(d_2)(Bill)) \geq 3-inches

(4) **max**(λn_1 . John has n_1 -many cookies) – **max**(λn_2 . Bill has n_2 -many cookies) ≥ 3

Measure phrase numerals

- Arai et al. 2016 (English & Japanese data, focusing on English here, but patterns are identical)
- 16 children (M=4;9) in a modified TVJT

Arii et al.

- All trials involved comparing quantities or extents
- Two blocks:
 - ▶ Comparatives w/ GAs, e.g. *two meters taller*
 - ▶ Amount comparatives w/ nominals, e.g. *more than two oranges*

Arii et al.

- Comparatives w/ GAs involved a novel unit of measure, *chipanis*

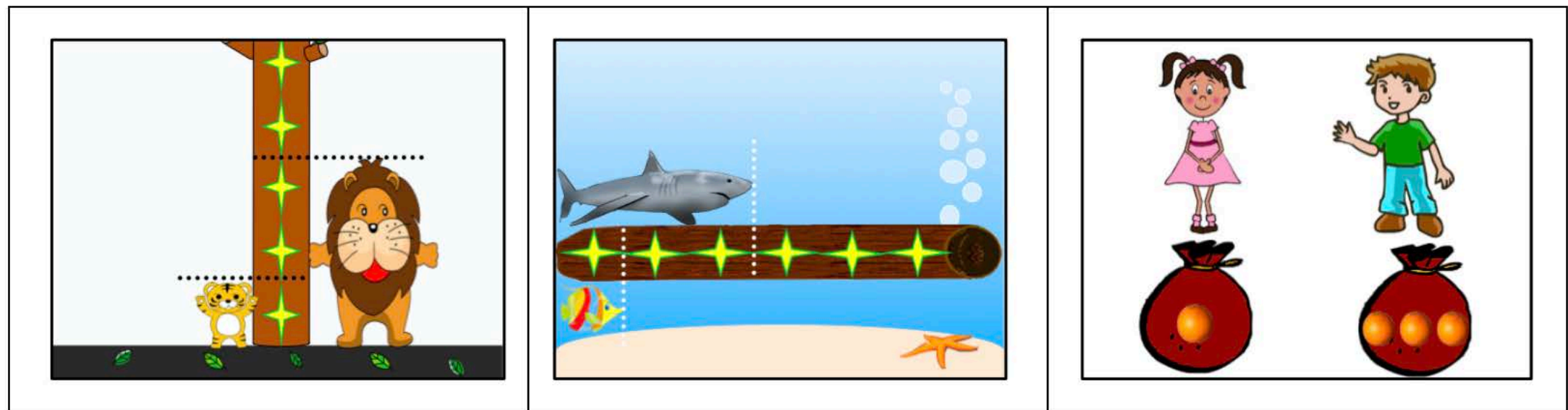


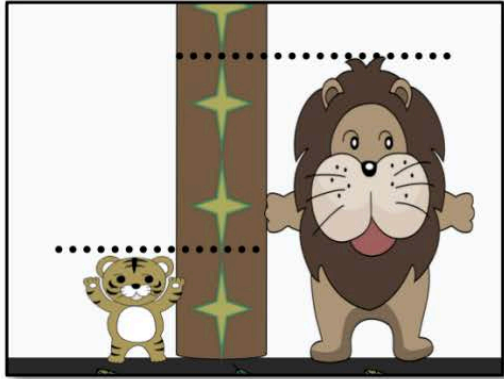
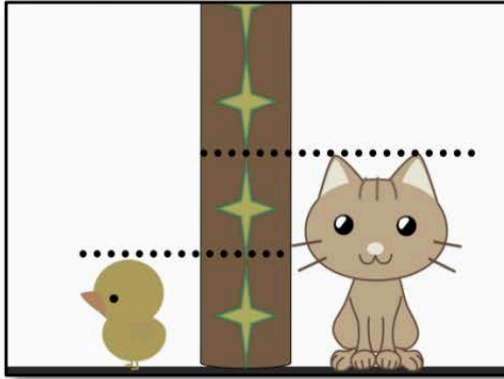
Fig. 1 Example of representative images appearing in tasks 1 and 2 of Experiment 1

*“The participant and puppet were first shown an animal (e.g., a tiger) against the tree with **chipanis**, and this animal’s height was indicated. They were then shown a second animal (e.g., a lion) on a separate screen, and the puppet made a prediction about the difference in height/length/quantity between the two animals (e.g., “I think the lion is 2 chipanis taller!”). The experimenter then said, “Let’s place the lion against the tree and the tiger to see if you’re right!”*

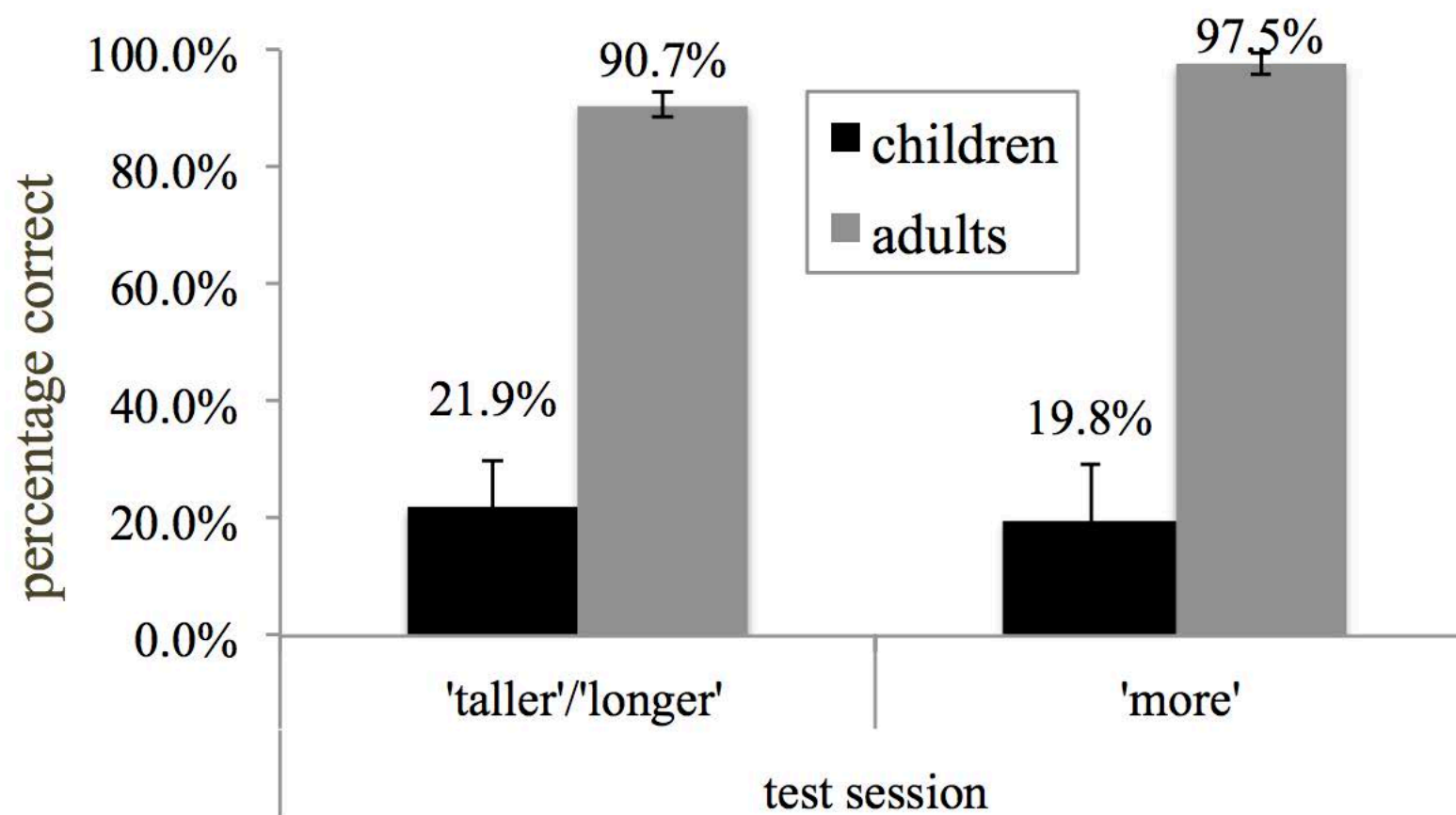
Arii et al.

- Trial types:

- ▶ In the **Differential** trials, the S was true under a differential comparative interpretation of the measurement expression (i.e., ‘2 chipanis taller’ / ‘2 more [nouns]’), and false under an absolute interpretation (i.e., ‘2 chipanis tall’ / ‘2 [nouns]’).
- ▶ In the **Absolute** trials, S was false under a differential comparative interpretation of the measurement expression, but true under an absolute interpretation.

Differential	Absolute
	
<i>The lion is 2 chipanis taller.</i>	<i>The cat is 2 chipanis taller.</i>
Differential True, Absolute False	Differential False, Absolute True

Arii et al.



The overall low percentage of correct responses indicates that they incorrectly rejected the utterances in the Differential trials, and incorrectly accepted the utterances in the Absolute trials.

Arii et al.

Replication w/ explicit standard (e.g. two chipanis taller than the lion)

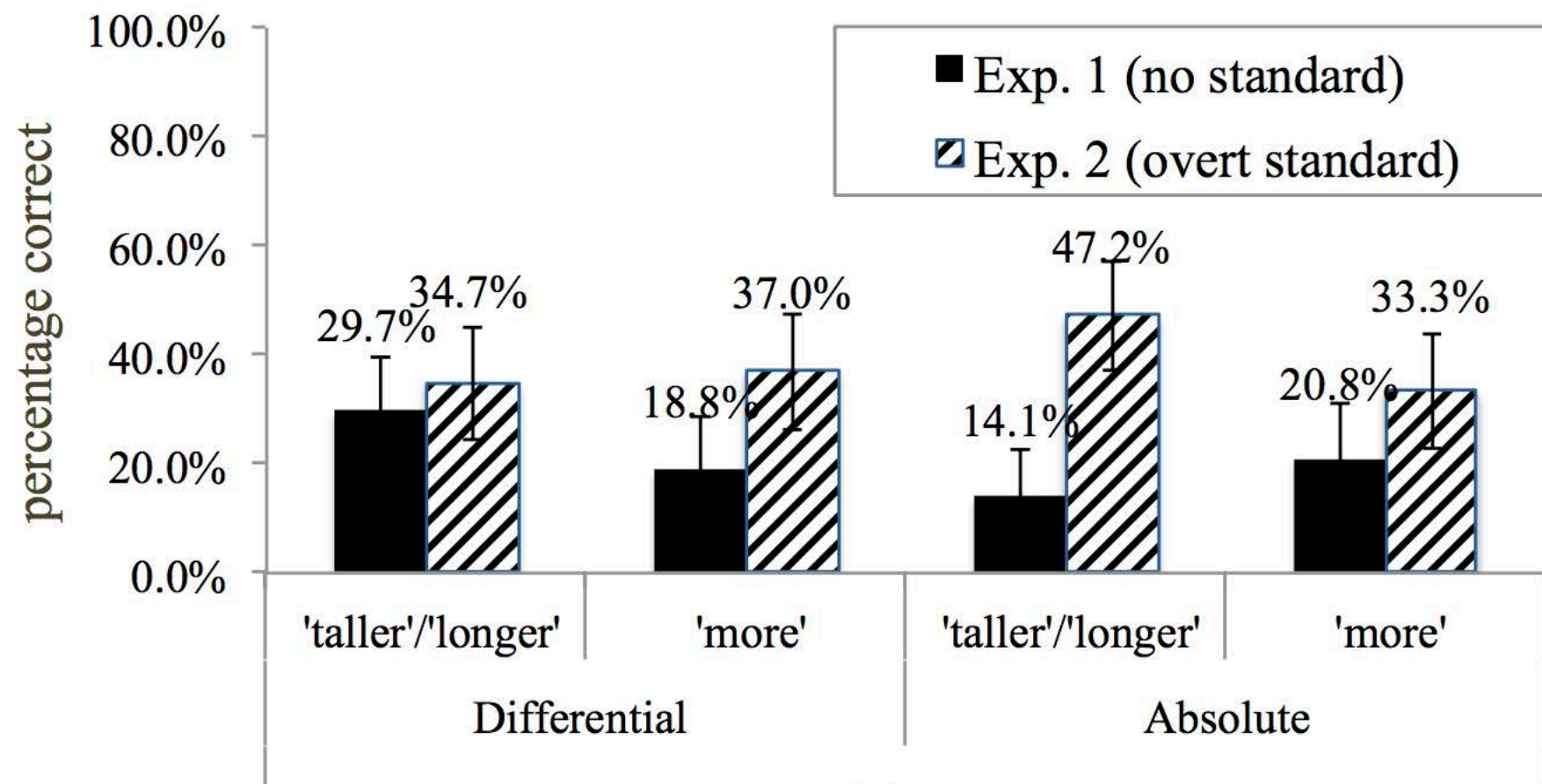
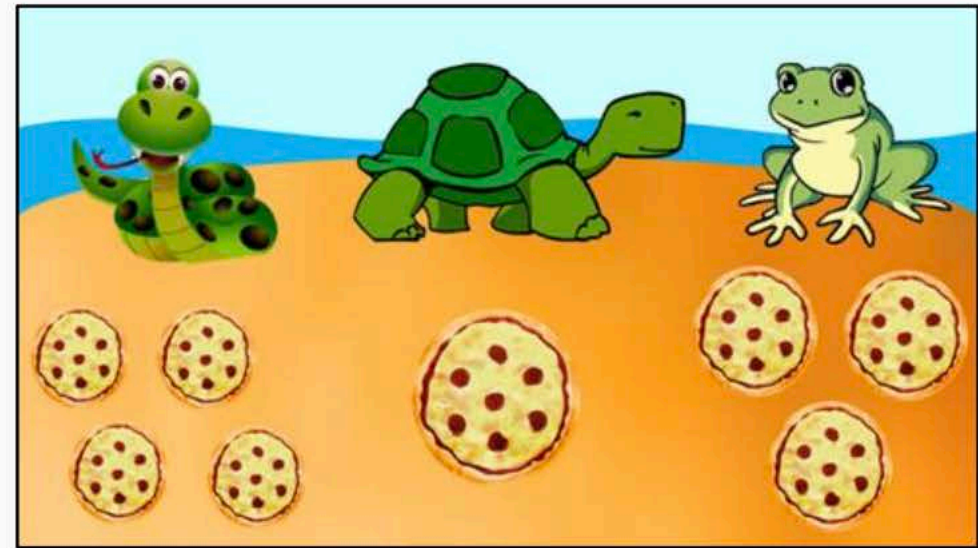


Fig. 5 Overall percentage correct for the English children in Experiments 1 and 2

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- Replication with amount comparatives
- 44 3-5yos, data collection ongoing



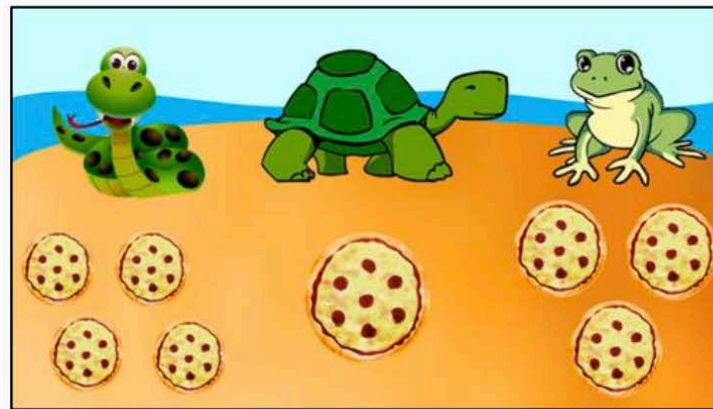
A: "The snake has more pizzas than the frog."

B: "The tortoise has more than two pizzas."

C: "The snake has one more pizza than the frog."

Hackl et al. 2020

- Replication with amount comparatives



- A: "The snake has more pizzas than the frog."
B: "The tortoise has more than two pizzas."
C: "The snake has one more pizza than the frog."

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	Adults	Children
A	Yes	Yes
B	Yes	Yes
C	Yes	No

Explanations

- Arii et al.: problem w/ syntax and semantics of differential comparatives
- Hackl et al.: conceptual problem w/ additivity; not yet true “arithmetic” knowers
- Some combination?

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