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IFIAS Workshop Report
ENERGY ANALYSIS AND ECONOMICS

*International Federation of Institutes for Advanced Studies**
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Twenty-seven economists and scientists from ten countries attended this Workshop, which considered the relationships between economic analysis and energy analysis. Energy analysis is a new field whose objective is the evaluation of resource flows in societal processes using physical units.

While we agreed that one of the principal roles of energy analysis is to furnish information that may be utilized in the allocation of the scarce resource energy, the Workshop unanimously concluded that this important function should not be interpreted as implying an energy theory of value. This conclusion rests on the simple observation, applicable across a wide range of institutional forms and degrees of technical development, that besides energy resources there are often indispensable primary inputs – labor, land, capital, non-energy minerals – with equal claim to having their scarcities (relative to the needs for them) expressed in the valuation system that guides allocation. However, many participants believe that the results of energy analyses would have their greatest impact if they were presented in a form suitable for incorporation into a valuation system.

Thus, the Workshop stressed the complementarity between economics and energy analysis. rather than elements of competition. The economist and the energy analyst alike believe that energy analysis can provide important physical data for the economic analysis of current productive and consumptive activity. Some Workshop members emphasized the use of energy analysis in assessment of activities in which there is clear evidence of market imperfections or failures. Additionally, energy analysis may be useful in technology assessment and in the evaluation of the impact of macroeconomic policies on energy demand. Technological assessments of this kind can also be used to define constraints within which a viable economic society must exist. It was clear that the interest of some energy analysts is not only in framing a response to scarcity of energy resources, but also in evaluating long-term destructive effects of intensive energy use.

Rapporteur's Note: This Report is drawn from several sources. Participants submitted both working papers and published articles for consideration at the Workshop. By common agreement, portions of these were assimilated into the Report without attribution. Session summaries were prepared daily by a rotating group of four or five. Recordings were made of several sessions, and extensive running notes were kept. Lastly, all participants had the opportunity to comment on an initial draft, and the majority of their suggestions have been incorporated in the final version.

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The Workshop discussions are not reported in chronological order, and there were only a few items about which there was a unanimity of views. Consequently, this Report is a synthesis that inevitably incorporates some of the prejudices of the rapporteur. Some of the energy analysts criticized the first draft of the Report for what they viewed as a strong emphasis on the incorporation of the data from energy analyses into economic decision-making rather than on its utilization in a more direct evaluative role. I have attempted to meet this criticism in the final draft in order to make this statement more fully representative of the opinions voiced at the Workshop. However, I am also willing to be counted as a strong supporter of the former view, and the reader should be forewarned.

Thomas Veach Long, II

1. Introduction

The Report of this Workshop will be interesting to a diverse audience: to the economist and the energy analyst primarily, but also to the politician, the governmental decision-maker, and the industrial manager. Others who will find the Workshop results intriguing are those who are concerned with intellectual interfaces. Through confrontation and resolution, criticism and reasoned reply, similarities and contrasts in goals and methods were delineated, and new insights were formed. By one participant's criterion, the purpose of such an endeavor is to change the way people think. On this basis, the Workshop was a success. There is little question that both the energy analyst and the economist left Sweden with remarkably modified views. Many of us feel that rumination over the issues that were raised will continue to produce substantive results.

This was the second Workshop on the subject of energy analysis that has been sponsored by IFIAS. The first Workshop was held in Guldsmedshyttan, Sweden, during August, 1974. At that meeting, the methodological problems of this young field were discussed, and a set of procedural recommendations were formulated. A summary of the recommendations is provided in appendix 1. Members of the first Workshop recognized that the interface between energy analysis and economics should be examined, and they recommended to IFIAS that a workshop on this subject to be held during the following summer.

There were twenty-seven participants in the second Workshop, assembled at the IBM Nordic Education Centre on the Lidingö peninsula outside of Stockholm during the last week of June, 1975. The members of the Workshop represented twelve countries. Included in this group were several of the earliest practitioners of energy analysis, a world-renowned econometrician, scientists and economists directly involved in the determination of national and international energy policies, academic economists and scientists, and an executive with a major industrial concern that has employed energy analysis in management for a number of years. We were particularly fortunate to count as a member of our group one of the 1975 Nobel Laureates in Economics. This diversity in backgrounds was evident throughout the proceedings.

2. What is energy analysis?

Broadly, energy analysis is a field devoted to studying societal use of a single aggregate resource, energy. Usually we think of energy as being provided only by fuels or by renewable sources such as solar, wind or hydro power generation. However, thermodynamics tells us that all materials have a potential for furnishing energy. This is even true of those that are not ordinarily considered to be fuels. The material flows in a process have associated with them flows of thermodynamic potential to do work.

Energy analyses quantitatively trace the changes in the thermodynamic potentials of materials as they pass through successive process stages. In productive processes, the thermodynamic potentials of materials often increase. This is because energy has been added to the materials through the application of heat energy from electricity or fuels or by doing work on the system. However, part of the heat and work energy is inevitably lost in the transfer process. Thermodynamic laws indicate that there are inviolable limits on the physical efficiencies of energy transfer processes.

There is an additional consideration. Heat and work that are added to the system in one part of a process may be lost quasi-simultaneously in another part of the process. Indeed, for a total process, the thermodynamic potential (stored energy) of the materials may actually decrease, even though there were additions of heat and work from external sources.

An example of this would be a driven exothermic chemical reaction process. An exothermic reaction is one that gives off heat, and the reactants have a higher thermodynamic potential than do the products. However, one can make the reaction go faster by raising the temperature of the system through the addition of heat. Industrial processes that use fuel and electrical energy but which result in a reduction in the thermodynamic potential of the materials are not uncommon.

One goal of energy analysis is to indicate where reductions in the energy requirements for total processes could be made – the pressure points for technological change. Possible reductions are assessed first by quantitatively evaluating the actual energy furnished to the process in the form of fuels and electricity. This is then compared with the actual change in the thermodynamic potential of the material. The difference in these two quantities is the energy that is lost in the process.

Thus, there are two quite different senses in which an energy analyst sees energy as being 'embodied' in a material. One sense is much the same as that of an economist when he speaks of 'embodied labor'. In this accounting sense, direct fuel and electrical energy that are utilized in the process form a portion of the 'embodied energy'. Additionally, the total fuel and electrical requirements for producing the inputs, tracing back to raw material extraction, are the indirect energy inputs.

The second sense in which energy is 'embodied' in the materials is as thermodynamic potential. A brief discussion of a few thermodynamic concepts is given in appendix 2. 'Embodied energy' in this sense can be used to do work and has a real physical meaning. Energy analysis focuses on the discrepancy between this real change in thermodynamic potential in a process and the 'embodied energy' requirements calculated by the accounting procedure. Note that a slippage loss of materials is captured by both evaluations. Thus, an energy analysis superimposed on a materials flow incorporates information about both energy and materials flows.

3. Energy analysis and economics

It may be helpful to put forward a few observations and to do away with some misconceptions in order that subsequent meetings do not have to pay the heavy search costs associated with finding a basis for interaction between these two fields of analysis. First, the common ground between energy analysis and economics is found in their parallel claims to be (in part) sciences of description. The motivating force behind the initial energy analyses was an attempt to construct accurate and all-encompassing descriptions of production and consumption processes. For example, there was the desire to account for externalities. These are not fully captured by operation of the market by definition. Thus, it is no accident that a majority of the early workers in energy analysis are physicists, physical chemists, and ecologists. These disciplines concentrate on precise specification as a basis for scientific progress.

Unfortunately, this primary goal has been obscured by the emphasis on the use of energy analysis in framing responses to perceived energy supply constraints. The concentration on the role of energy analysis in policy decisions has led economists to think of energy analysis as though it were a competing method for determining the efficient allocation of scarce resources over space and time, which is the focus of economics. Indeed, a few energy analysts apparently see this as its role, and suggestions of an 'energy theory of value' have peppered discussions of the subject. *It was the unanimous view of the participants that a value system based on the single factor energy is not satisfactory for analyzing modern market, mixed, or planned economies* (*vide infra*). This conclusion rests on a simple observation applicable across a wide range of institutional forms and degrees of technical development. Besides energy resources there are other indispensable primary inputs – labor, land, capital, non-energy minerals – with equal claim to having their scarcities expressed in the valuation system that guides allocation. For this reason, it is necessary that the precise description of technological processes not be limited to the energy inputs, but include all important inputs and outputs. *The principal goal of energy analysis is the development of a portion of the precise physical description of the operation of real-world processes. This* *description does not supplant that of economic analysis, but supports and complements it and may provide new perspectives.*

This should not be taken to diminish the role of energy analysis in developing policy alternatives. To the contrary, energy analysts can concentrate on determining those situations in which the physical description may provide a useful addition to market information. A variety of questions of this type can only be answered empirically. For example, are there circumstances under which energy analysis furnishes faster (and equally accurate) signals of impending critical situations than does the market? Operationally, under what conditions can one accept the reduced information content of energy analysis as compared to economic analysis, because the costs of carrying out the energy analyses are also smaller? Economic analyses may also require more time than energy analyses. Consequently, one may be prepared to sacrifice the more complete information incorporated in the economic data. A plant manager may employ energy analysis as a materials control technique because it furnishes him with a near-instantaneous picture of his operation that is not subject to market fluctuations. Economic analysis, which should be used in parallel, requires a time lag for formulation and is sensitive to variation in prices. These are questions of comparative sensitivity, accuracy, time responses and ease of informational organization that can be answered only by careful empirical evaluations.

Second, the descriptive framework of the energy analyst and the economist have different bases, and this can lead to thorny misconceptions. Traditionally, the economist has focused on inputs and outputs, regarding the transformation process from the former to the latter as a black box. However, there is substantial interest in incorporating more accurate process description, particularly among econometricians and other modelers. As one economist pointed out, energy and materials are washed out in macroeconometric models in order to avoid double counting. Only capital and labor are included as original productive factors, although land may also appear for the case of a frontier economy. Energy and materials may be incorporated in models of smaller scope. Conversely, the energy analyst trained in thermodynamics (see appendix 2) concerns himself with the transformation process and the factors that effect the change. From this viewpoint, energy is of prime importance because every process requires a change in energy (free energy, more precisely).

These are radically different frameworks and should be so recognized. The central question is whether energy is an *intermediate* good. This is answered affirmatively by the econometrician, acting from a methodological posture, and negatively by the energy analyst, whose different philosophical framework requires him to consider it a primary descriptive element. The economist is interested in fuels and feedstocks, and the energy analyst is interested in energy. As will be discussed below, a bridge between these two conceptual foundations may possibly be found in some economic literature dealing with engineering production functions.

A third point that came across strongly was that neither economics nor energy analysis are monoliths. Within each of these disciplines there is a broad range of points of view. In economics, one can identify the econometrician, whose outlook is partially determined by what is possibly measurable and what is required to measure it; the analytical economist, who concerns himself with the construction of the conceptual framework; and at least three forms of political economists, one sort emphasizing the capacity of a freely-operating market to compute efficient solutions and to implement them, another more interested in questions of distribution, and the third concentrating on the operation of planned economies. Of course, a number of subdivisions could be added to this abbreviated list.

Despite the short history of energy analysis, there are sharp contrasts between at least two schools of thought. The first argues that energy analyses stand alone, totally independent of economic analyses, and that choices can be made on the basis of either or both. This position emphasizes the possibility of carrying out any desired transformation of a material resource if sufficient energy is available. Energy should therefore be treated as a unique, *essential* resource. The second school basically contends that information from energy analyses can and should be incorporated into economics. This school tends to see energy as but one of the primary natural resources that dominate technological description. Water would be another. They suggest that economic decision-making can be improved by capturing all the physical information that it can. Of course, most energy analysts find themselves somewhere in the middle between these two cases.

Another important view of the relationship between energy analysis and economics transcends the other two positions. This is the idea that economic processes must operate within the constraints imposed by the physical and biological world. Technological descriptions like energy analyses are useful in that they permit quantitative assessments of the constraints. Thus proposals should be subjected to two assessment stages. First, viability should be determined on purely physical grounds. Then, value and possible tradeoffs can be analyzed using economic criteria.

A fourth observation is that the interface being explored is between two fields at vastly different stages of maturity. Economics has a long and largely successful history of being a useful tool in marshalling the potentials of society into productive activity. Energy analysis is a much newer endeavor and has little to point to in the way of historical contributions. There are many questions that energy analysis has not had time to answer, but this does not mean that they will not be or cannot be answered in quite a robust way. It is a field for which the data base is not yet sufficient to allow a response to questions of its utility, such as 'What can energy analysis do that economics doesn't do better?' Some felt that there is a possibility that energy analysis will not survive as a well-defined discipline. In view of this, it would have been all too easy for the economists present to criticize energy analysis soundly and then to dismiss it as a rudimentary exercise in tracing the flow of a single resource through economic society. To their credit, they did not, but instead, through gentle persuasion, contributed measurably to the growth of this young field.

4. General objectives and opening forays

The Workshop was organized about three objectives, to examine:

(1) energy analysis as a complement to financial accounting for evaluating new and alternative technologies;
(2) energy efficiency versus economic efficiency as a criterion for resource allocation; and
(3) the integration of physical information into economic behavioral relationships.

These objectives were quickly broadened when energy analysts suggested that the proper role of energy analysis is in evaluating the energy implications of policies, but not as the sole decision basis for determining policies. Further, they suggested that potential evaluative functions of energy analysis could be broken up into those appropriate for short-term, medium-term, and medium-long-term decisions.

Short-term:
(a) calculation of fuel price elasticities;
(b) evaluation of 'energy conservation' measures;
(c) testing production function specification and some price system assumptions.

Medium-term:
(d) disaggregated demand forecasting;
(e) evaluation of alternative energy sources.

Medium-long-term:
(f) documentation of *one* effect of resource depletion;
(g) prediction of when the costs of production will rise based on technical factors; the inclusion of time scales and non-linearities;
(h) description of 'points of futility' *associated with* 'technological (physical and biological) limits', and placing limits on allocation over finite times;
(i) setting limits on *ex ante* production functions;
(j) analyzing the stability of societal trajectories based on physical resource use.

A participant then outlined energy analysis from an economist's viewpoint. Several features of the outline formed a basis for subsequent discussions. It is worthwhile reconstructing the presentation in some detail because it is representative of an economist's perspective.

The economist asserted that the existence of energy analysis as a separate method is based on four propositions and that the burden of their proof rests with the energy analyst:

1. all energy resources are scarce;
2. this scarcity increases over time;
3. scarcity imperils the quality of life; and
4. society must focus on this scarcity by employing criteria of physical efficiency.

In order to respond to this perceived scarcity situation, the economist's impression is that energy analysis has developed two analytical objectives. One analytical objective is the determination of physical efficiencies at a micro level, often in a single process, in order to assess the state of technology. The second is the establishment of thermodynamic boundary conditions for these processes with the intent of defining the limits for energy husbandry efforts. Also, some energy analysts have championed a 'methodological tour de force', which economists clearly reject, in introducing an 'energy theory of value'. A good deal of discussion was devoted to this in subsequent sessions, and it will be dissected below.

Insofar as potential relationships with economic analysis are concerned, it appears that energy analysis could aid in identifying alternatives in responding to price changes, thereby helping to minimize search and information costs. More importantly, in the absence of well-formulated future plans or futures markets, energy analysis could aid by providing accurate imputed prices faster. The economist, drawing on analogues in his own field, quickly identifies a list of methodological problems. These include those of defining the system boundary, of the need for some assumption regarding the specification and aggregation of heterogeneous inputs such as the fuel mix in the system, and of incorporating values and prices into the analytical system. The economist approaches energy analysis with the attitude that the energy analyst must show when joules can be more useful than constant dollars in analyzing the decisions of economic society.

Some of the energy analysts sharply disagree with the assertion that the only use for their discipline is in response to the scarcity of energy resources. Their view is that the economist's 'scarcity' implies only those conditions reflected in the responses of a market. By contrast, energy analysis, in furnishing a scientific basis for energy husbandry, is equally well a diagnostic tool for indicating overly intensive energy use relative to physical constraints, even when these constraints are not yet associated with costs. An energy analyst concerned with intensive energy use presented the following example.

To a first approximation, the only way the earth can increase the amount of heat it radiates into space is through an increase in its surface temperature. According to the Stefan–Boltzmann radiation law, the annual solar energy input, E, is related to the surface temperature, T, by

$$E=\sigma A T^4,$$

where σ = Stefan–Boltzmann constant and A = radiating surface area. In thermal equilibrium this is also equal to the energy output into space.

Using an annual solar input of 3.6×10^{18} MJ/annum this formula gives a surface temperature of 280°K, which is reasonably close to that which is observed. Working from this equation, we can also derive a relationship between the change in energy input and change in surface temperature, namely

$$\frac{\mathrm{d}E}{E}=4\frac{\mathrm{d}T}{T}.$$

Thus if the energy input were increased by 1%, a net addition of 3.6×10^{16} MJ/annum, then the rise in surface temperature would be $\frac{1}{4}$% or 0.7°C. This could be serious, because a change of this magnitude could lead to a melting of the polar ice-caps. This level of fuel consumption is within our time-horizon. Two extrapolations of world fuel consumption are shown in fig. 1. That marked (b) assumes a constant growth of 5% p.a., the present growth rate. That marked (a) assumes a continued increase in the rate of growth. Both put the 1% solar level within our own time horizons. This example indicates why some energy analysts are concerned with abundance rather than scarcity of energy resources.

Moreover, scientists generally agree that local climatic effects will sound the alarm bells before the global problems arise. For example, according to some reports, London annually dissipates 20% of its annual solar input. The temperature in the middle of London is about 5°C higher in summer and 10°C higher in winter than its surroundings, which could lead to secondary local climatic disturbances.

This discussion is intended to be illustrative, rather than definitive. We do not intend for the values of parameters cited to be taken as precisely true, although the magnitudes are at least crudely correct. This example indicates that the data from energy analysis on waste heat production might be used to establish the physical constraints within which economic systems must operate. Here is one case where these arise from intensive energy use rather than from the constraint of energy scarcity.

There is another concern of energy analysis that is somewhat separable from considerations of energy resource scarcity in the first order. Energy analysis can be used as one indicator of what was termed by a Workshop member ‘the saturation of technical progress’. Particularly in the mineral extraction industries and in agriculture, increased fuel inputs may be required simply to maintain present production levels. Decreasing soil quality under intensive farming and the need to use even lower grades of mineral resources could lead to increasing requirements for all factors of production – capital and labor as well as energy.

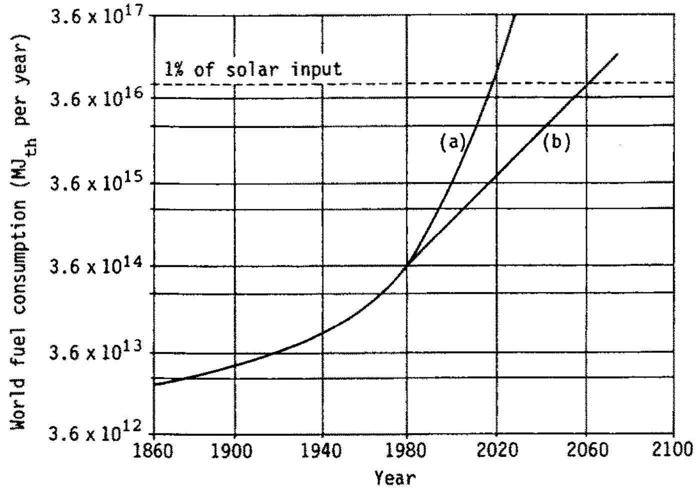

Fig. 1. Two extrapolations of past trends in world fuel consumption.

The historical experience in many of the extractive processes is that more intensive use of items of physical capital has resulted in decreasing requirements for both labor and energy inputs per unit output. However, there are suggestions that this trend has reversed in the case of energy use, and energy and capital may now be complementary factors in some industries.

The argument is that resource grades are decreasing at a rate greater than the rate of factor-saving technological innovation in these processes. Furthermore, the argument implies that energy analysis can be a sensitive indicator of this effect.

Of course, increasing efficiency in the use of capital and labor could generate greater productivity even in the face of the larger fuel requirements if ‘technical saturation’ did not lead to increased requirements for all factors. Technical saturation in the use of energy alone is important only to the degree that energy is scarce.

This reflects the long-range concern of many energy analysts with the more general question of how society uses all of its natural resources, including but not solely restricted to scarce energy resources. The degree to which energy analysis can furnish information about technological change is a question that must be investigated empirically.

Although the energy analysts did not agree with several portions of the economist's comments, it was clear that in order to establish an initial basis for communication, the problem of valuation had to be considered. The 'energy theory of value' had to be dealt with, if only to dismiss it.

5. Valuation and an energy theory of value

There are two senses in which the term 'valuation' is generally employed. The first is its use in a normative connotation as a synonym for a purposeful optimization that is imputed to society, such as the maximization of a social welfare function or the definition of efficient growth paths over time. Another sense of the term is as a description of the preferences of consumers as revealed in consumer demand in the market. The economists attending the Workshop felt that in order for energy analysis to become something, to realize its full-potential, an element of valuation must be introduced into its structure. In order to do this, it is necessary to adopt one of three postures. One choice would be to minimize energy use as the valuation procedure. A second is to recognize the existence of other scarce factors but to base their evaluations on some measure of 'embodied energy' only, and again minimize energy use. Finally, factors of production such as capital and labor could be entered and the optimization could be based on their scarcities as well, as is ordinarily done in economic analysis.

A few energy analysts have clearly put themselves on record as favoring a method of valuation based on energy minimization alone. Hannon (1973, p. 139) has stated:

> 'The adoption of a national – and consequently a personal – energy budget appears to be necessary . . . Individual allocation could be similar to that of our present economies, which reflect personal value, except that we would have to strive for the right to consume energy; the accrued currency would be regulated by the amount of energy budgeted for a given period . . . Recognition of the value of energy is equivalent to setting energy as the basis or standard of value. In doing so, society readmits itself into the natural system in which acknowledgment of energy's importance has never been lost.'

In a like vein, Odum suggests that a large number of economic concepts (*e.g.*, goods, wants, income, interest, inflation) are best analyzed in an energy language in which these terms are redefined. He uses these redefinitions to support a 'technocratic' value system with an 'energy certificate' as a 'money standard' [Odum (1971; 1973, p. 220)]. He postulates that money flows and energy flows circulate in opposite directions, and this leads him to assume that a functional relationship exists between these flows. 'Money is a counter current with the ratio of money to energy flow being price' [Odum (1974)]. The logic establishing this relationship is unclear, and he has unfortunately eschewed the use of conventional thermodynamics in energy analysis in favor of a system that seemingly directly incorporates an efficiency concept.

There are several superficial attractions to a value system based on a single factor. The system is holistic, and its predictive value is substantial. As Samuelson (1959, p. 1) has pointed out regarding the one-factor hypothesis: 'A spy can memorize (the technological coefficients) and know most of what there is to know about the economy.' But the shortcomings are also clarified. Energy analysts will already have realized that their methodology leaps right across production to consumption without mentioning a market connecting the two. Indeed, for a long-run one-factor Ricardian economy, the 'substitutability theorem' can be shown to hold, which states that even though possibilities for substitutions exist, no substitutability need be experienced [see Koopmans (1951, chs. VII–X)]. In such a world, barring joint production, relative prices are determined by technological coefficients and are independent of the mix of consumer demand. Thus, complete economic control is implied, whatever level of disaggregated decision-making is adopted.

The market economist has confronted single factor theories of value such as those based on land or on labor before. He has rejected them because they are weak in treating processes that involve more than one factor – in particular, processes that take time and therefore involve the participation of capital. The economists who were members of the Workshop felt that an energy theory of value was a poorer approximation to reality than the analytically similar labor theory of value. This is because, in addition to neglecting to treat time properly, there is no scarcity value ascribed to human labor. The energy analysts accepted this position. Thus, *the Workshop rejected the concept of an energy standard of value* – not because it is impossible to design such an allocation mechanism, but because such a system does not adequately describe the full texture within which human economic choices are made.

Before departing from the topic of valuation, perhaps we should take note of a distinction that may be significant in understanding this interface between energy analysis and economics. The distinction is between the use of the words 'choice' and 'decision'. Economics is concerned with the allocation of scarce productive resources among alternative uses either now or in the future. Allocation is a social phenomenon, involving the actions of groups of individuals. As viewed by neoclassical economic theory, the individual is required to choose but not to decide. The differentiation lies in allowing choice to admit possible noncognitive elements. Choice views alternatives through a preference function. A decision is the result of the resolution of a set of alternative actions by purely cognitive selection processes. The information from energy analysis can be used to add to the rational component of a choice or alone as a basis for decision.

The way society values scarce resources, such as energy, capital, labor and land, results from social preferences and associated social choice. Results from energy analyses can serve as the basis for some kinds of decisions and can aid in the formulation of cognitive selection criteria. But their use in social choice is limited to furnishing information for that portion of the choice that proceeds by a cognitive mechanism.

The economists maintain that energy analysis can have maximum impact if the analyses are carried out so that the data can be directly incorporated into a valuation system, whether in terms of competitive market prices or of imputed and computed shadow prices. Their position is that the embodiment idea fades and is not pertinent in describing price formation *if* possibilities of substitution of different primary factors of production are present. In opposition, many of the energy analysts argue that the most effective use of their analyses is not through incorporation into a valuation system, but as a direct input in policy assessment of societal choices. This assessment can then be compared and contrasted with the economic analysis that results from a valuation procedure.

The energy analysts maintain that their assessment can be used without valuation to reject technically non-feasible options. Also, should a mode of action seriously threaten the survival of mankind, technical assessments need not be subjected to further valuation procedures. For example, many would argue that clear and present danger to human health from ozone depletion by fluorocarbon spray propellants exists. They would contend that a ban on the manufacture and sale of spray products should be implemented on the basis of this assessment alone, without recourse to a further (economic) valuation procedure. In a sense, of course, this argues that a very high price should be imputed to use of these items.

Again, if a *positive* choice is to be made, we concluded that this can only be done through a valuation procedure that incorporates all indispensable primary inputs, including labor, land, capital, non-energy minerals, *and* precisely-analyzed energy requirements.

6. Economic and physical efficiency criteria

A second step in constructing a working relationship between the two fields is a clear-cut comparison of the economist's concept of efficient allocation of resources and the energy analyst's definition of an efficiency measure. There are a number of possible physical efficiency criteria that could be adopted. In appendix 2, definitions of both first and second law efficiencies are provided. Another possible choice is based on the waste factor described in 'Guidelines for energy analysis' (appendix 1).

More important than the particular choice of parameter is the rationale behind defining a physical criterion. A point that has not been sufficiently emphasized is that the first step in carrying out an energy analysis is the construction of a detailed picture of the mass flows in the real-world system chosen for study. This is, in principle, a complete materials balance. The material output of the process will require amounts of material inputs that are greater than would be required in the ideal case because of loss of material as 'waste' in the course of production. One could use the analysis to compare the actual material input requirements for a given output with those computed for the limiting case in which there is no slippage.

For a chemical process, the ideal input requirements of materials and energy can be evaluated from the stoichiometry of the reaction provided only that the reaction goes to completion. Those who are not trained in thermodynamics may find it surprising that energy has entered the picture even at this early stage. There has been no mention of fuels, nor need there be to this point. As discussed in an introductory section, each chemical species has a thermodynamic potential relative to other species, an energetic level that is physically determined. When two species of differing potential are brought into contact, they may react spontaneously. If heat is generated in this reaction, it may be used to do work, although one would not ordinarily consider either reaction species to be a fuel. For example, condensation of monomeric units into a polymer is one such heat-producing (exothermic) process. In this process, there is a reduction in the thermodynamic potential of the system associated with the generation and loss of heat.

Alternatively, another class of reactions may proceed only with the application of heat or electricity, raising the thermodynamic potential of the system. Purely mechanical processes are also describable using thermodynamics, but ideally they often involve little change in the thermodynamic potential of the materials. In summary, energy in the form of thermodynamic potential is already embodied in materials prior to their involvement in any production step, and it can be used to do work.

After the materials flows have been traced, the energy required in every process step is evaluated. It is important to distinguish between two different evaluations. One assesses the actual energy requirements for the system, with material slippage, and the other calculates the requirements for a hypothetical system operating at maximum thermodynamic efficiency with no material slippage. The analysis of the actual energy requirement for the process generally is carried out by determining from process data the material, fuel and electricity requirements for every step, following each material input back to its natural state.

We can assess the ideal energy requirements only if we first designate the constraints associated with the process. In traditional thermodynamics, there is no *time* constraint, so the ideal energy requirements are always evaluated for a reversible process (see appendix 2). The reversible process requires an infinite time, and the actual system will use extra thermodynamic potential in order to proceed at a finite rate. The thermodynamic requirements for this ideal process are calculated for the system in which there is no material slippage. Finally one compares the actual and the ideal by computing some sort of efficiency parameter.

We note that an energy analysis fits like a veil over the materials flow diagram. Because each material has its own gross energy requirement (which would be the thermodynamic potential of the material if the subprocess producing that material operated ideally), slippage of any component will result in a larger energy requirement as compared to the ideal system. Thus, the increase in energy requirements due to materials losses will be proportional to the energy requirements of the components that are used inefficiently.

Therefore, the energy analyst's definition encompasses somewhat more than simply an evaluation of the efficiency with which the single resource energy is used. *Achieving maximum physical efficiency requires both the efficient application of energy to the system and the careful husbandry of materials throughout, so that none are wasted.* Energy analyses are sensitive to both sources of inefficiency and incorporate two different forms of information.

The energy analyst maintains that his efficiency criteria are appropriate for assessing a trade-off in the physical world, that there is a natural valuation system operating in that world, but that it is only part of the larger realm of human activity. What it does not permit – and this is where economics enters – is an examination of how systems that require human labor and an investment in capital can efficiently combine these resources with those of the physical system.

Economic efficiency is attained if given resources (including capital, labor and natural resources) are combined in such a manner that a higher output of any desired good could be obtained only at the cost of a lesser output of some other desired good.

Let us investigate this concept in greater detail by analyzing fig. 2.[1] The economy represented in this figure consists of two industries, one engaged in the production of electrical energy and the other in the production of equipment. Each industry makes only one homogeneous product, and quantities of production are represented by coordinates y_1 and y_2, respectively. Both industries utilize labor. For simplicity, but without loss of generality, we can assume that one unit of labor input is required for one

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[1] For a precise discussion, see Koopmans (1953).

unit of output of electricity or of equipment. Thus, we will omit the third coordinate corresponding to labor and utilize a two-dimensional diagram.

Each point in the plot corresponds to a production technology available to the industry. A positive value for a coordinate indicates that the commodity is an output of the industry, while a negative value signifies that the industry utilizes the commodity as an input. The electrical power generation industry requires equipment and labor inputs and furnishes a net output of electrical energy, while the equipment industry requires electricity and labor to produce a net output of machinery. The electrical power generation industry can adopt methods A_1, B_1, C_1, and others indicated by points in the upper left-hand quadrant. The equipment industry has techniques represented by points A_2, B_2, $C_2, \ldots$ available to it. There is no joint production. The possibility that some labor will not be utilized can be handled by admitting a point at the origin O of the coordinate system, for which one unit of labor is expended without production of outputs.

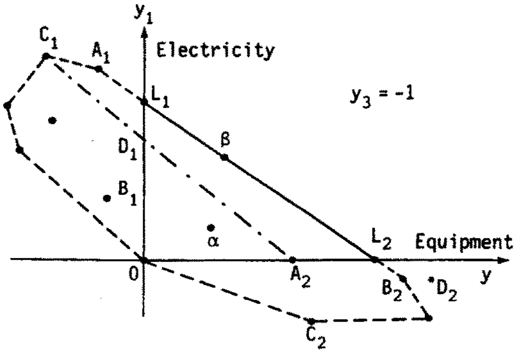

Fig. 2. The convex hull for the production of electrical energy and equipment (arbitrary units) with labor as a primary input ($y_3 = -1$).

The polygon created by connecting the points A_1 C_1, O, C_2, B_2 bounds the production possibilities, and every point on the boundary or inside can be achieved by the proper combination of techniques. If there are no exogenous sources of supply of either commodity, the only *attainable points* utilizing one unit of labor will lie within the triangle L_1OL_2. A point in the set of attainable points is efficient if there exists no attainable point that is superior in providing greater output of one commodity without diminishing the output of the other. The line segment L_1L_2 is the *efficient set*. Point α is not efficient because the output of both electricity and equipment can be increased within the attainable set, but point β is clearly on the efficient boundary. Consequently, for a two-industry input–output model, we have arrived at the set of possible combinations of techniques that represent efficient use of labor in production. These are combinations of only two methods, A_1 and B_2.

Observe that it would have been feasible to produce equipment using technique A_2 with a decrease in electrical energy requirements (per worker) and that electricity production could employ method C_1, with a higher net energy output (per worker). But the set of attainable points along A_2D_1 corresponds to a set of technique combinations that is everywhere inferior to those of L_1L_2. One function energy analysis can serve is to aid in the development of a technique for equipment production that utilizes one unit of labor but less energy per unit produced, as would be represented by point D_2. The efficient set would then fall along a line connecting A_1 and D_2.

One source of difficulty that the economist and energy analyst encounter in seeking a level of discussion is that, in loose terms, the economist concerns himself with fuels as intermediate goods and does not recognize energy in the abstract as a good. The energy analyst treats energy as an aggregate quantity that is a primary factor of production. More precisely, the economist deals only with specific forms of energy considered at points in the chain of extraction or interception and processing at which an option in extraction, conversion or utilization exists and may be exercised. This concern therefore encompasses a number of scarce *primary* energy sources such as uranium, oil, coal in the crust of the earth or elevated water. This disagreement is meaningful to the degree that sources of thermodynamic potential other than those ordinarily regarded as fuels are utilized by economic society for their ability to deliver this potential. For example, materials whose marketplace values are primarily determined by their structural properties or by their ability to provide other services desired by society are also sources of thermodynamic potential. The energy analyst is arguing that he is providing the information that society requires to make a knowledgable choice between use of a material for the energy it naturally embodies and its use based on some other characteristic. Through careful empirical evaluations, he is showing society the full range of options that it confronts.

Consequently, let us consider a two-industry economy in which there are two primary factors, energy (thermodynamic potential) and labor, each available in a given amount. The industries will be taken to be metal mining and equipment production. Each requires both primary factors. This economy can be analyzed with the aid of fig. 3. First, we will ignore any restriction on labor and make the assumptions utilized in discussing fig. 2. The attainable point set is defined by the two processes A_1 and A_2 and is L_1OL_2. Similarly, the attainable point set resulting from ignoring the restriction on energy, M_1OM_2, is defined by the pair of techniques B_1 and B_2. Taking both restrictions into account, the attainable point set consists of the quadrilateral OL_1DM_2, and the efficient production set lies along the two line segments L_1D and DM_2. Thus either of the two pairs of methods can be utilized in efficient production, and the efficient choice between the pairs depends on whether labor or energy is the limiting primary factor. Energy analysis can be employed in determining the points A_1 and A_2.

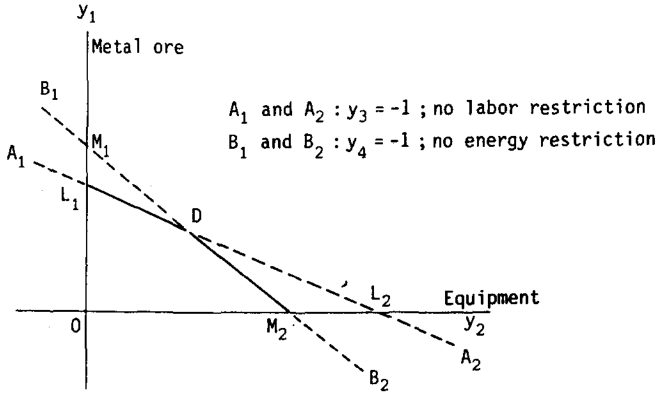

Fig. 3. Production with two primary factors, energy (y_3) and labor (y_4).

7. Engineering and economic production functions

There are a few serious attempts to explore the manner in which more comprehensive physical information can be introduced into economic behavioral relationships by utilizing a production function structure.[2] A few words about production functions are in order. The production function expresses the technological possibilities relating outputs and inputs that are faced by a productive unit. The productive unit considered may be an aggregate entity, such as a nation, and in that case the macro production function relationship is

$$Q = f(A, B, \ldots),$$

in which Q is the output quantity expressed as a flow and A, B,... are input quantities, again expressed as flows. For the macro function, the inputs are taken to be aggregates, such as flows of capital, K, and labor, L.

If the productive unit is an industry, a firm, a division or a process unit, the production function can be thought of as a microeconomic relationship, again expressing the technological possibilities confronting the micro unit.

[2] Chenery (1953), Hildreth–Reiter (1951), Koopmans–Reiter (1951), Borts (1952), Manne (1956, 1967), Manne–Markowitz (1961), Smith (1966), Danø (1966), Johansen (1972), Marsden et al. (1974), and some pre-World War II studies by Ragnar Frisch, Ivar Jantzen and others in the *Skandinavisk Tidskrift för Teknisk Ökonomi*, a journal that is no longer published.

The output is now thought of as a particular commodity flow, and the inputs are also flows of particular goods and services. One can also differentiate between long-run functions, in which all inputs can be modified in amount, and short-run functions, in which items of physical capital are taken as fixed. A production function formulation presumes that the output is the maximum possible from the group of input factors or, equivalently, that a given output is produced by a minimum quantity of inputs – a 'premaximization' presumption [Koopmans (1957)]. Economic relationships, such as marginal pricing, can be developed from the production functions.

In passing, it is amusing to note that perhaps the earliest paper in which marginal pricing is suggested is by one of the first energy analysts, Sir William Thomson (Lord Kelvin) (1881, p. 527).[3] His solution to the problem of the most economical size of a copper conductor for electrical transmission is still in use, and, as Smith (1966) points out, this preceded the precise formulation of marginal productivity theory by Walras (in the fourth edition of his *Elements*) by nineteen years and the formulation by Wicksteed (in *An Essay on the Coordination of the Laws of Distribution*) by thirteen years.

One of the first serious attempts to analyze the connection between the economic production function and a process description more firmly grounded in engineering practice was that of Chenery (1949, 1953). However, there are several earlier and contemporaneous papers in which technological information is specifically incorporated into economic treatments of production.

Briefly summarized, Chenery conceives of production as the result of a series of industrial processes. Each of these processes involves a change, which is most usually a change in form, of input materials effected through the application of energy. The industrial plant is a map of material flows stimulated by applications of energy. In a parallel way, the first step of an assessment of the energy requirements of a complex process involves setting up a complete materials balance and then evaluating the energy flows that overlay this. Chenery's *engineering* production function is a long-run, *ex ante*, microeconomic production function. Thus, all input quantities are variable and functions of engineering design parameters. This means that trade-offs – substitutions – that could only be made by complete system redesign are considered. One of the most useful aspects of energy analysis is that it provides a means of evaluating just that sort of substitution process.

Chenery sets up the following stripped-down definitional structure. Let $X = f(u_i, \ldots, u_m)$ be the *economic* production function, in which X is the (optimized) output quantity and the u_i's are input quantities. This set spans all the goods of the economy. The amount of each input is determined by

$$u_i = u_i(v_i, \ldots, v_n),$$

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[3] Another great thermodynamicist, G.N. Lewis, published a paper on political economics in a journal edited by F.Y. Edgeworth and J.M. Keynes. See Lewis (1925, p. 40).

where the v_i's are the engineering variables, the physical parameters that ultimately specify the process. The economic production function combines this set of equations and the engineering production function, which has the form:

$$X = \phi(v_i, \ldots, v_n).$$

Chenery discusses determination of this function for the compression and pipeline flow processes of natural gas transmission.

Following this work, Smith (1966), Danø (1966), Manne (1956, 1967), Manne–Markowitz (1961), and Johansen (1972) have published more extensive treatments of industrial production models. Smith's work has evoked substantial interest because he employs the long-run engineering production function as the foundation for a theory of investment. Smith is also apparently the first to consider productive systems that are best described by kinetic engineering production functions.[4] These functions describe the dependence of flow quantities of output (rates of production) on input stocks, and this mixture of stocks and flows in one equation is a strange bag for one trained in traditional economics. Recently, Marsden et al. (1974) investigated the forms of production functions arising from rate processes, and applied this apparatus to river water quality problems.

Currently, the only empirical energy-analysis model for the total U.S. economy is that of Herendeen (1973). This input–output model effectively incorporates a production function with fixed coefficients and no substitution, but only the energy portion of the production relationships are given. He has recently updated this matrix to reflect the technological coefficients from the 1967 data [Herendeen and Shiu (1975)], and this permitted a test of the projection procedures that had been employed. It is fair to say that the projection procedure that had been used, which consisted of deflating 1967 dollars to 1963 in the sector rows and renormalizing the energy rows by the energy/GNP ratio, did not appear successful. In this test, projections of the 1967 coefficients from 1963 data were compared to empirical 1967 energy coefficients obtained by sectoral energy-to-dollar conversions. Apparently even over this four-year period of relatively smooth economic growth substitution is important, particularly in certain sectors. Another difficulty associated with use of the energy input–output matrix arises from the datedness of the raw input data, which takes a number of years to surface from the inner sanctums of the government. Also, its high degree of aggregation, even when the economy is broken down into 362

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[4] Smith's analysis of chemical kinetic processes is somewhat in error. In general, it is not possible to determine the functional dependence of the rate of a chemical reaction on the reactant concentrations from a knowledge of the stoichiometric information contained in the chemical equation, as claimed by him (1966, pp. 50–51).

industrial sectors, makes it primarily useful for macro policy. Hopefully, we will soon find it possible to develop an input–output matrix in which the coefficients are obtained from process energy analyses, perhaps for both marginal and average technologies.

In this and the prior section, we have attempted to sketch the manner in which physical information enters traditional economic production theory, with primary attention to energy. In the usual theory, the firm is considered to have knowledge of its short-run economic production function or the marginal products of each resource. Acting as an instantaneous profit-maximizer and price-taker, it adjusts its use of each resource to the level at which the resource cost is equal to the value of its marginal product. The data from energy analysis consists of the inverses of the marginal productivities of energy. These are incorporated into the economic analysis of the firm when the entrepreneur sets output levels at the point at which the resource unit cost divided by the output price is equal to the marginal productivity. The marginal productivity theory of short-run operating behavior must also be compatible with a description of the economic agent who, confronted with a long-run production function, acts to maximize long-run profits. This is the area in which physical information also has a natural entry into economic behavioral relationships – through the entrepreneur's consideration of the broad spectrum of production options implied in an engineering production function.

Also, energy analysts have noted that one of the advantages of their method is that it permits an examination of tradeoffs between processes that form portions of a vertically integrated chain, but which are usually subjected to step-wise optimization. An energy analyst provided the following illustration. Admittedly, the example is oversimplified because it permits no adjustment to prices, and it is based on only a single resource. However, overall suboptimization resulting from the optimization of subsystems in real-world situations is well-recognized. Consider a theoretical example involving a steel works and a car manufacturer. The operation of steel furnaces can be represented by material inputs of pig-iron and steel scrap, a fuel input and an output of steel, as shown in fig. 4. The pig-iron is produced in a blast-furnace which, let us assume, consumes E_p units of energy per tonne of pig-iron. The steel scrap is not assigned an embodied energy, but the operation of the furnace requires E_f units of fuel per tonne of steel throughput. Thus from the steel-maker's point of view the total energy requirement per tonne of steel (E_0) is given by

$$E_0 = E_f + E_p(1-\beta).$$

This suggests that the larger the scrap input, the larger β, then the smaller is the energy requirement of steel. In this hypothetical example, the steel manufacturer decides to install more scrap-handling furnaces and increases the price he is prepared to pay for steel scrap.

At about the same time a car manufacturer is faced with a choice between two steel presses. Press A consumes 10 MJ per sheet pressed and rejects 10% of the plates as scrap. Press B requires 12 MJ per sheet, but doesn't reject any scrap. In the cause of energy conservation, and with the price of electricity rising (and the price of scrap rising) the price- and energy-conscious car manufacturer installs Press A.

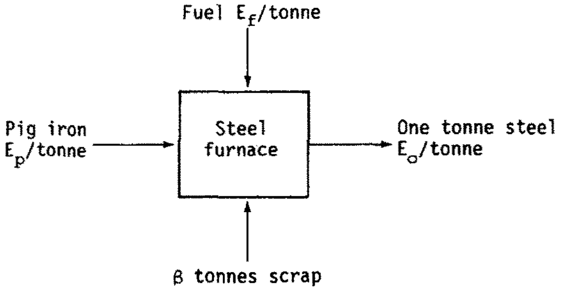

Fig. 4. The material and fuel inputs to a steel furnace. Note that to produce one tonne of output, the inputs are β tonnes of scrap and $(1-\beta)$ tonnes of pig-iron.

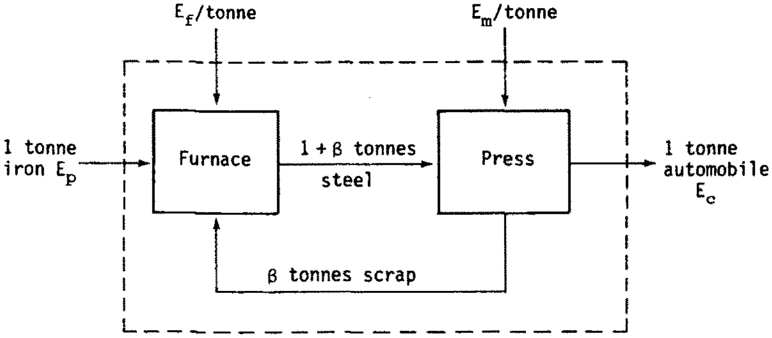

Fig. 5. The enlarged system including a car manufacturer who generates the steel scrap consumed by the furnace.

The net result of these two investment decisions is to increase the energy required to produce automobiles. This is illustrated in fig. 5, which shows the fuel input to the press as E_m per tonne throughout and all the car manufacturer's scrap being used by the steel plant. If the output car requires 1 tonne of steel then, by conservation of mass and without slippage, the pig-iron input must be 1 tonne. However the fuel consumed in both the steel furnace and car press is proportional to the total mass throughput. This includes β tonnes of scrap in addition to the 1 tonne flowing from input to output. Thus the total energy requirement of the car, E_c, is

$$E_c = E_p + (1+\beta)(E_f + E_m).$$

This shows that increasing the quantity of scrap generated and used, increasing β, increases the energy requirement. If significant externalities of any kind exist, then it is well-known that step-wise optimization does not yield the most efficient solution for the total system. Again, information coming from energy analysis that could be summarized in an engineering production function could be utilized by a firm in making investment decisions that involve more than one process step.

8. Optimization over time

The subject of the optimal rate of use of resources has prompted intense and sophisticated interest in the economic community.[5] The history of this topic can be traced back to an early paper by Hotelling (1931, p. 137), through a number of published papers.[6] The formulation that is generally employed is one drawn from optimal control theory (*vide infra*), in which an exogenously specified social welfare functional is maximized over time. A number of models assume that this functional takes the form

$$V = \int_0^T e^{-\rho t} u(c_t)\,dt,$$

in which ρ is the discount rate on utility and u is the utility of consuming c_t at time t. The treatment of Dasgupta and Heal (1973) clearly points out that the more general problem is one of joint optimization in a macroeconomic environment. How can one construct a program that provides efficient paths for *both* resource depletion and investment–investment that utilizes these resources? Of course, concern with the best societal use of natural resources moved a number of energy analysts to enter the field. Addressing this problem may be where energy analysis can most effectively enter economic description. Analysis of resource depletion must, at bottom, rest on the empirical analysis of physical production relationships, which can then be translated into value terms. In a sense this is a normative application of physical analysis.

Faced with the possibility of total depletion of specific mineral resources,

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[5] For an introduction to optimal resource use over time, see Koopmans (1973).

[6] Anderson (1972), Vousden (1973, p. 126), Schulze (1974, p. 53), Long (1975, p. 42), Koopmans (1964, p. 355), Ingham–Simmons (1975, p. 191), Smith (1973), Heal (1974), Dasgupta–Heal (1973).

physical analyses are needed to define what substitutabilities between resources may be feasible, and to evaluate upper and lower bounds on the magnitudes of these effects. This information can be used by economists in determining not only efficient but also fair allocation of these resources over time. Economists are devoting increasing attention to the normative question of intergenerational equity [see Rawls (1973), Arrow (1973), Solow (forthcoming), Brock and Scheinkman (1975)].

The traditional approach to intergenerational equity issues has been to discount future utilities at some positive rate of interest, as indicated above. In the early stages of the Workshop, an energy analyst posed the following question for the economists. Consider nuclear generation of electricity and the problem of storing wastes from a nuclear reactor. This process must continue over tens of thousands of years, and the storage process has energy requirements that may extend over the whole storage period. Also, take into account decreasing resource grades of energy resources, which result in a greater energy expenditure to produce an equivalent amount of fuel over time. In other words, let us examine the possibility that technological knowledge does not progress fast enough to overcome the rate of decrease in resource grade. In such a case, how should we discount the utility of consuming electricity generated by nuclear facilities over time?

The economists' reply was that there are certain hypothetical cases, involving an essential but exhaustible resource and no technical progress in its utilization, in which discounting has objectionable consequences. In a model for the optimal depletion of an essential exhaustible resource, discounting future utilities favors an earlier generation over any surviving later generation. This effect increases monotonically with the discount rate.[7] This statement might apply to energy resources if renewable energy sources, such as solar or wind power, were not feasible. Another situation in which one might want to suspend discounting is when there is irreversible damage to an ecological system, such as the depletion of a species to a population below survival level or when harm is done to the health of future generations of *mankind*. In these situations, we are putting the responsibility on later generations for our detrimental actions, and this takes us beyond economics *per se* into the realm of human rights. In most other cases, so long as capital is scarce, efficient allocation of resources can be furthered by the use of a uniform interest rate.

Energy analyses have recently been used in assessing the technological consequences of nuclear power construction scenarios. These were discussed informally as well as the energy analyses of the nuclear fuel cycle on which they are based. The values reported for the energy requirements for this process by independent investigators vary widely. The consensus was that

[7] For an introduction to optimal resource use over time, see Koopmans (1973).

conclusions should not be drawn from the evaluations available at the time of the Workshop because of this uncertainty.

It was suggested that an alternative to carrying out such evaluations for *ad hoc* scenarios would be to formulate the assessment problem in the language of optimal control theory [Hadley and Kemp (1971), Pontryagin et al. (1962)]. This, like any mathematical apparatus, forces us to state our question precisely, which often goes a long way toward determining the answer we get.

In energy analysis, economics and thermodynamics, the first step is the determination of what constitutes the system boundary. The relevant time-dependent behavior of the system is assumed to be completely described by specifying a finite number of variables called *state* variables. Each state variable may be a function of time. Let us abbreviate the set of state variables $y_1(t), \ldots, y_n(t)$ in vector notation as $\boldsymbol{y}(t)$. One assumes that the time behavior of the state variables is controlled by a set of control variables $(v_1, \ldots, v_m) = \boldsymbol{v}$ through a set of first-order differential equations:

$$\frac{\mathrm{d}\boldsymbol{y}}{\mathrm{d}t} = \boldsymbol{f}(\boldsymbol{v}, \boldsymbol{y}, t).$$

The system's trajectory over time is then viewed as specified by the state variables, $\boldsymbol{y}$. These are in turn determined by the control variables, $\boldsymbol{v}$, which are not arbitrary but take on values over time that are subject to deterministic control.

Specifying the optimal control problem involves choosing some measure of the effectiveness of the control process, a function of $\boldsymbol{v}$, $\boldsymbol{y}$ and t whose behavior over time is to be optimized. Calling this function $F(\boldsymbol{v}(t), \boldsymbol{y}(t), t)$, and

$$J[\boldsymbol{v}, \boldsymbol{y}] = \int_{t_0}^{t_1} F(\boldsymbol{v}(t), \boldsymbol{y}(t), t)\,\mathrm{d}t,$$

we require that the integral J, called the objective functional, be optimized over the set of control variables, with $\boldsymbol{y}(t)$ evolving according to the set of differential equations above. The integral in the first equation of this section is a special case of J. The specification is completed by giving an initial condition $\boldsymbol{y}(t_0) = \boldsymbol{y}_0$. Constraints may be placed on the state variables independent of the control variables.

This procedure has two advantages. The first is that precise specification aids in assessing the total implications of a particular option. Second, when the information is available, we should choose among alternatives that represent efficient rather than non-efficient programs. Each efficient program is the optimized trajectory associated with a particular specification of objective functional and constraints.

Optimal control problems can be formulated using only physical or economic variables. We can also use this apparatus to show one way in which the information from energy analysis can be introduced into economic optimization through defining constraints. For example, heat released by an electrical generation facility could have detrimental effects on a local climate. Because of this, the rate of heat released per unit area from the facility may be required to be less than or equal to a maximum acceptable amount. Physical analysis is required to define this constraint. A utility will choose to maximize its return (the objective functional) as a function of the rate at which it burns fuel in the face of this constraint.

Some participants felt that another potentially fruitful area of interaction between technological analysis and economics lies in the theory of investment. The principal problem facing a producer in a market economy is the formulation of a plan for investment in capital goods that will maximize his long-run profit. The economic literature on investment is rich, this topic is one of a good deal of current interest, but it was not discussed in detail at the Workshop.

Smith (1966) grounds his monograph on the problem of investment in a technological description that utilizes an engineering production function motif, and this can be recommended as a source that gives serious attention to the question of the incorporation of physical information into economic theory. We are interested in properly describing the real-life behavior of entrepreneurs making investment decisions in a state of knowledge characterized by manifest uncertainty, particularly with regard to the introduction of technological change, the non-existence of markets for future delivery and market imperfections. Theoretical formulations of the investor's decision process that *directly* incorporate information derived from technological analyses should be of importance.

9. Setting the limits

There was a strong consensus among both economists and energy analysts that one of the most promising applications of energy analysis and other technological assessments is in establishing the physical and ecological limits on economic processes. Such an effort would be useful in constructing intermediate- and long-range predictive models of large-scale systems. This information can be incorporated into econometric, input–output or systems dynamics models. In this application of energy analysis it may turn out to be unnecessary to augment the physical assessment with a valuation procedure. Energy analysis and other technological assessment procedures can be used without parallel economic analysis in testing the technological feasibility of societal options, and in rejecting non-feasible or unstable options. In a second stage, after the technological feasibility has been ascertained, other inputs must be measured and included in any affirmative test of stability on an economic basis. Finally, the desirability or efficiency of the options can be determined using economic analysis.

There are two different boundaries that can be established using energy analysis and technological assessment procedures, more generally. The first type of limit, familiar to most individuals who are concerned with predicting societal trajectories, is one which sets a maximum on a certain activity. Examples are easy to come by – the maximum pollution that can be absorbed by a lake before the limits on biological oxygen demand are reached, or the maximum amount of energy use that can be permitted before significant heating of the earth's atmosphere is observed. Maintaining the stability of the biophysical system implies certain limits on the maximum amounts (or rates) of use of resources.

More surprising to the economist was the statement that a kind of lower limit can also be physically defined. All transformations – chemical, electrical or purely mechanical – will require at a minimum the energy needed for the reversible process, which can be calculated. Again, for a short discussion of the concept of reversibility, see appendix 2. For example, the minimum energy that would be required to synthesize a petroleum hydrocarbon from carbon dioxide and water in atmospheric abundance in the year 2025 can be exactly assessed (and will be equal to the free energy of combustion of the hydrocarbon). This sort of assessment has strong predictive implications, and robust economic predictions must incorporate this kind of information. The delineation of upper and lower bounds is a tough problem, but it can be done in principle. The actual energy required for the process will be greater than the minimum because energy is needed to drive the process at a finite rate.

To illustrate the possibility of comparing a real system to a theoretical limit, let us refer to fig. 6. This illustrates the gains in energy efficiency over time in the production of ammonia from methane and air. As can be observed, this process is now operating moderately close to its thermodynamic limit, which is quite exceptional for industrial processes. Note also that it is possible to leap beyond any particular thermodynamic limit by going to a completely new technology for producing ammonia that does not employ reaction of air with methane. The thermodynamic limit for the new process can then also be calculated. However, it should be apparent that there are only a limited number of processes that utilize materials as abundant as air and natural gas (formerly), so that a feasible set of technology modifications (*e.g.*, production of NH_3 from the direct, catalyzed reaction of nitrogen and hydrogen) could be projected, and the minimum energy for each transformation could be evaluated. But no matter how we refine our technology, there is always some absolute thermodynamic minimum energy requirement for the production of any good, even if we allow ourselves free choice among all available inputs.

Let us explore a stability analysis in terms of physical variables. 'Stability' here means that the flows and stocks represented by the variables remain bounded during the entire period of interest, which can be extended at will.

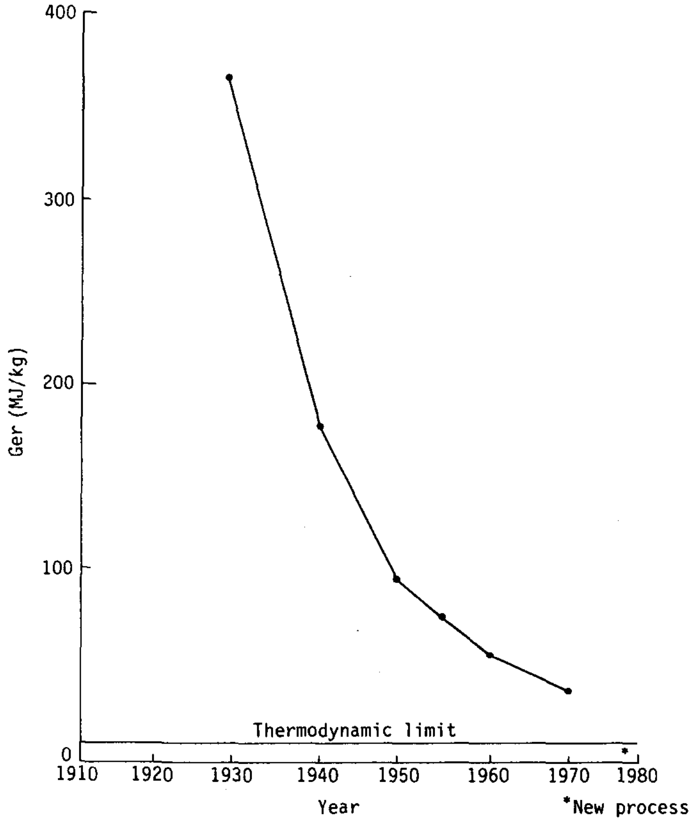

Fig. 6. Gross energy requirements (*GER*, see appendix 1) for the production of ammonia. Base reaction: $CH_4 + AIR = NH_3 + CO_2$. Thermodynamic limiting $GER = 17.5$ MJ/kg.

With judicious caution, we can temper rigor with realism to use the theory of idealized nonlinear behavior to study real problems, provided we do not inadvertently push our models to times when our hypotheses are no longer valid. We cannot push our models to *infinite* time, for example. Like all stability analyses, these analyses give us negative information only, in the sense of defining ranges of parameters, such as intensitivities of use, within which our life styles must remain. The physical analyses will only be useful if they sometimes set tighter bounds on the regions of stability than do the price-based bounds. Whether this will be the case is not yet known.

One particularly simple example is that of a physical resource that is slowly being consumed – vacant, primitive land, for example. The shadow (or real) price of this resource is initially zero or even negative, but its price will be positive when some threshold is passed. This is also the case for air and water as waste receptacles. These are the classic situations giving rise to externalities. Historically, it appears to be very difficult to stimulate or justify the scientific and technological work necessary to meet forthcoming non-zero prices of presently unpriced resources with enough lead time to meet the new scarcity in a technologically effective way. This is a statement in the language of classical economics of the simple, cynical truism that people won't start thinking about a problem until it already hurts to live with it.

Stability analysis is a recognized tool for economic study. We are not aware of the application of this approach in the context of resource use with empirical production functions and resources constraints, although there have been model treatments of the allocation of a single resource, both exhaustible and inexhaustible. With the data from physical analyses, it will be possible to use technical production functions that reflect current or alternative engineering practices to decide whether a real or potential pattern of resource use is stable within the selected set of constraints.

By way of illustration only, let us use a model for the system of production of an energy source (presumably coal) whose *stock* we denote by E, and one other resource that we may take to be steel, whose *stock* is S. These stocks represent inventories, not ultimate reserves. For example, E may represent already mined coal, or known reserves in mines, but not the total projected resource. For present purposes, take as the model a system invented and studied in detail by the Brussels school – Prigogine and Lefever (1968, p. 1695), Nicolis (1971, p. 209) – (and hence called the 'Brusselator') and others – Tyson (1973, p. 3949), Tyson and Light (1973, p. 4164), and extended to include a broader range of possibilities than were encompassed in the original work – Tyson (1975, p. 1010) and many references therein. Our simple illustration, based on the Brusselator, is the production of steel, waste heat and waste matter from ore and energy reserves,

$$\text{Reserves} \rightarrow \text{Energy Inventory}, \tag{1}$$

$$\text{Ore} + \text{Energy} \rightarrow \text{Steel} + \text{Waste Matter}, \tag{2}$$

$$2\ \text{Energy} + \text{Steel} \rightarrow 3\ \text{Energy}, \tag{3}$$

$$\text{Energy} \rightarrow \text{Waste Heat}. \tag{4}$$

The coefficients of energy in Step (3) need not be 2 and 3. These are simply the values for which the system has been most thoroughly studied. It is necessary that the system be nonlinear because of the real requirements for energy in the acquisition of energy. This nonlinearity lends both mathematical richness and portentous significance for policy-making to this problem. The equations relating *stocks* to *flows* (not flows to flows or stocks to stocks) take the form (when scaled to reduce the number of parameters to a minimum)

$$\frac{\mathrm{d}E}{\mathrm{d}t}=r-mE+\delta E^2S-\gamma E, \tag{5}$$

$$\frac{\mathrm{d}S}{\mathrm{d}t}=mE-\delta E^2S, \tag{6}$$

in which r = flow of energy reserves, and m = flow of ore reserves; δ and γ are parameters of the system, equal to the rate coefficients for steps (3) and (4), respectively. Step (4) involves the final evolution of waste heat from energy. Eq. (5) states that the rate at which the coal inventory is used is equal to the rate of step (1), r, plus the rate δE^2S at which more energy resources are developed, less the rates of steelmaking, mE, and heating, γE, shown by steps (2) and (4). The convenient but not necessarily realistic assumption is made in eq. (3) that we develop three units of new coal inventory for every two consumed in development. Note that both δE^2S and γE are rates of flow and that the units of the rate coefficients δ and γ put all these quantities into common units. Note also that the stocks E and S in (5) and (6) are inventories of intermediate factors, whereas r and m are flows. The units of the rate coefficients are such that the steady-state condition on the inventories, $\mathrm{d}E/\mathrm{d}t=\mathrm{d}S/\mathrm{d}t=0$ is a necessary condition for economic equilibrium. The analysis of stability consists of asking what happens to the system if it is displaced slightly from the steady state.

The analysis of the Brusselator [Lefever and Nicolis (1971, p. 267)] shows that the steady state may be unstable in the Lyapounov sense [Andronov et al. (1966)] if m/δ, the flow of the reserve of ore relative to the energy production rate coefficient δ, exceeds a critical rate of the form $[(r/m)^2+\gamma/\delta]$. However, the situation for energy use need not be altogether bleak even if the Brusselator model were realistic and ore consumption exceeded its critical value, because at least one limit cycle always exists in the unstable region. This means that the energy-using technical production system may have a long grace period, during which the rates of energy consumption and steel production oscillate but remain bounded. During this time, the system can readjust its technology to move to a stable range or extend the life of its limit cycle. The price of operating with the rate of extraction of ore above its critical value is having to live with a cyclic, rather than a steady-state condition. Empirical energy analysis would be the means to evaluate the parameters m, r, δ and γ, and, at one level deeper, the coefficients in relation (3). Let us once more emphasize that this particular simple system is meant to illustrate a direction for examining stability from the technical data of resource analyses, and has not been derived from observed data.

The aim of energy analysis in this area must be the evaluation of *realistic* estimates within which our economic system must operate. We cannot be satisfied with the theoretical knowledge that limit cycles are a formal possibility. We must have a quantitative idea of what rates of resource use are stable or consistent with limit cycles, what rates are clearly inconsistent with either, and, within the limit cycle scenario, how long we could expect the cycle to endure. The 'empirical economics' we call energy analysis (or resource analysis) is precisely the tool for this job.

10. The economists' critique of energy analysis

On the final day of the Workshop, there was a period in which the economists voiced their perceptions of the strengths and weaknesses of energy analysis. They believe that the field could be strengthened in three major respects.

(A) *The viewpoint of the discipline should be broadened to include other resources, and technological analysis in a larger sense.* One group of energy analysts responded that a program concerned with water resources employing similar evaluative methodology had already been initiated in their group as a step toward a general multi-resources program. Water was chosen, they said, because they sense that it will be the next resource to suffer severe supply constraints.

(B) In a similar vein, *the energy analyst should not be so concerned with a particular methodology.* Rather than adopting a definite set of questions to which the discipline is pledged to seek answers, it should reorient itself to ask what questions are important in making societal decisions? The straightjacket of formalism should be shed. The energy analyst's reply was that he does not see himself as operating under such strictures and that the definitional structure suggested in appendix 1 was adopted primarily to ease communication in the field. Furthermore, the methodology parallels that of thermodynamics, which permits evaluation of efficiencies and limits as discussed above.

(C) *To be of maximum value, some valuation procedure should be adopted. It was suggested that the data be organized and spelled out so that an economist who works with an input–output or process model can incorporate it directly.* The economists held that energy analysis paled compared to economics as an allocator of scarce resources, and that this is not a proper function for energy analysis. The importance of consumer reaction and modeling consumer behavior – about which energy analysis has little say – was emphasized as well as the efficacy of introducing a price system, even if one doesn't believe in its optimality, in order to introduce behaviour, particularly in making decisions over time periods. The rejoinder by the energy analyst is that pragmatically there are certain cases in which energy analysis may furnish faster or earlier signals than does economics, and that these situations should be empirically evaluated. He admits that in order to use energy analysis for any allocative function, some explicit or implicit valuation must be introduced. He also admits that any allocation procedure based on a single resource is inferior to that of efficient economic allocation. However, under conditions of energy supply constraint, governments may wish to evaluate the implications of policies on energy use. Furthermore, based on this evaluation, considerations of national defense or similar goals may dominate the desirable goal of efficient allocation of resources. Several energy analysts are actively developing input–output tables from process data to use in analyzing national energy policies.

11. The energy analysts' critique of economics

One senses that the existence of the field of energy analysis is felt by some to be in itself a criticism of economics. Energy analysis is directed toward providing information for planning decisions, both those made outside the operation of the price system and those where the market is not operating efficiently or with sufficient promptness. In this sense, the two systems of analysis are not antithetical, and energy analysis provides important inputs to economic analysis. For instance, energy analysis can provide a check on market operation by verifying the marginal pricing assumptions for this one resource. Also there are some economic occurrences that may be signaled more rapidly and with equal accuracy by energy analyses.

Thus, the principal comment directed at economics from the energy analyst's corner is that there should be a greater attention to the gathering of physical information appropriate to economic analyses and its incorporation. This is not to say that economists do not utilize such data or are unwilling to do so. As noted above, a number of economic studies have incorporated technological data directly, and the economists present emphasized the need to do so. Technological information is vital in developing accurate medium- and long-term models of economic systems, particularly in projecting technological change.

As is the case with any healthy discipline, the sharpest attacks come from within. There have been recent broad criticisms of the general equilibrium framework [Kaldor (1972, p. 1237), Kornai (1971)], by which is meant the rigorously-derived mathematical structure that is proposed as a description of market operation under static or stationary-state conditions, as well as a response to these [Koopmans (1974, p. 325)]. The main thrust of this critique is behavioral, emphasizing the discrepancies between observable market processes and the body of assumptions and proposed behavioral mechanisms that are required in order to develop a closed equilibrium system. The energy analysts emphasize only the need to modify or go beyond the equilibrium model, rather than the rejection called for in the economic criticism of Kaldor (1972) and Kornai (1971). Some of the questions for economists from energy analysts are within the province of this behavioral challenge, while others probe only the necessity to extend the elegant equilibrium theory in describing the real dynamic system that may be often characterized by disequilibrium.

(1) Is the general equilibrium assumption of market agents who operate almost exclusively on the basis of price information accurate or should other direct information bases, such as quantity, be incorporated?

(2) Does economic theory handle the problem of the dynamic evolution of social systems in an adequate manner? To what extent should exclusive control by market forces be allowed in the face of physical constraints and the irreversibility of some decisions?

(3) Can we expect a description of the economic arena that goes no farther than an equilibrium analysis when we observe that it is usually in a disequilibrium state? As noted above, the emphasis in thermodynamics is on the transitions due to disequilibrium rather than on equilibrium states.

At least one economist indicated that he was willing to support economics' current position with respect to each of these issues.

12. Economics–energy analysis interfaces

On the positive side, there were a number of suggestions from both economists and energy analysts of areas in which greater interaction between energy analysis and economics is possible. Some of these have been implied in the foregoing text, and some are suggestions that are promising but were not considered at length.

(1) The use of technological analysis in designing predictive economic models, especially for long-range planning.
(2) The use of energy analysis in testing for viability of proposed systems of production.
(3) The development of sophisticated descriptive process models that will help an economist frame more realistic descriptions of technological change than simply as an exponential function of time, a proxy for all time-dependent residuals.
(4) The determination of marginal input–output coefficients, for both marginal increases and decreases in output, in order to determine possible responses to sudden exogenous changes.
(5) The use of energy analysis in analyzing the operation of those economic sectors where there is government intervention and planning and also for better behavioral understanding of those sectors in which there is no direct intervention.
(6) The use of energy analysis in determining the relationship between the rate of a production process and its utilization of energy. By rate of production is meant the ratio of stocks of goods-in-process to the flow of output. One point that struck the economists as potentially important was the comment from energy analysts that less energy may be required per unit output as the rate of the productive process decreases or the duration of the process step increases. This indicates that under conditions of capital saturation, there is the possibility of decreasing the consumption of a resource by increasing the duration of the process step.
(7) The use of energy analysis in properly definong the characteristics utilized in a consumer production function [see Lancaster (1971)]. The utility of employing a characteristics space rather than a commodity space lies in the reduction in dimensionality, which is important in computer modeling. For example, in the heating of buildings the two primary characteristics would be heating convenience and heat energy.

In summary, both the economists and the energy analysts felt that the data being generated by energy analysis and the detailed process description can be of substantial value in sharpening the economic description of the system.

13. Issues for further thought

Near the close of the Workshop a subcommittee drafted a list of issues that they felt had either been raised by the discussion or remained from the original agenda presentations. Some of them represent modifications and refinements in the statements of the initial foci, and it may be useful to compare them with the list found at the beginning of section 4. Many could usefully be employed as bases for further interdisciplinary contact. The issues are not necessarily in order of priorities as seen by the Workshop, and the formulation and language is generally that of the subcommittee.

1. At what point in the continuum of production activities, from primary extraction to final demand, must a valuation system be selected or inferred?
2. Is process methodology the most valuable single contribution of energy analysis to economic analysis?
3. Are there consistent methods for allocating the energy inputs to the outputs of joint production which satisfy both energy-analytic and economic criteria?
4. What are the relative advantages and disadvantages of energy analysis done by a process model in comparison to that utilizing a macroeconomic energy input–output matrix?
5. Is the 'steady-state energy economy' compatible with finite resource bases? Are the assumptions about relative scarcities that underlie energy analysis relevant? What are viable trajectories for economies?
6. How can thermodynamic and other technological constraints be utilized in economic analysis? Where do energy-technology data fit into economic systems, and how can they be integrated into the economic calculus?
7. How relevant is economic discounting to longer term analyses of the future provided by energy analysis? Are questions of 'unacceptably' high and enduring risk or of supply security beyond discounting?
8. Is it possible for physical process models to provide better – *i.e.*, more reliable and faster – forecasts of the future? If so, how? What is the role of energy analysis in technology assessment?
9. Is there an efficient method for communication between economists and energy analysts?

14. Reports of empirical studies

During the course of the Workshop, several presentations were made of work-in-progress that is primarily empirical, and these studies are reported briefly below. Those desiring fuller descriptions should contact the individual contributors; see the list of participants at the end of the article.

(1) Asger Hansen, 'The Relation Between Energy and the Economy in Denmark Analysed in a Multi-Sector Growth Model'.

The impetus for this investigation was the knowledge that Denmark is bereft of domestic energy resources, and that the energy use choices faced are primarily on the consumption side of the ledger. This work forms a portion of a program analyzing 'Energy Choices in Denmark' that was initiated by IFIAS and the Niels Bohr Institute in April, 1974. A model of the Danish economy that incorporates an energy demand sub-model, an energy supply sub-model, and an economic activity multi-sectoral growth model was described. The general development of the economy is simulated by the *Multi-Sector-Growth (MSG) model*, which is based on the static input–output table for Denmark for the year 1966. The model describes development in accordance with perspective plan II (PPII) of the Danish Ministry of Finance (1973).

In its basic version the MSG model assumes unaltered technology throughout the entire forecasting period in the sense that raw materials requirements, including energy, are proportional to output. This assumption was singled out for criticism at the Workshop, and it was noted that variable coefficients will be incorporated. It was suggested that these coefficients be both dynamic and price responsive. Technical advances are reflected in increased capital and labor productivity with time, but not in modified raw materials requirements.

In order to relax the assumption of constant input coefficients the basic model can be supplemented with an *energy matrix* that gives a detailed description of the consumption of different kinds of energy in the production sectors in physical units. Process analysis information from the other sub-models can be used to change the energy coefficients. Thus the model facilitates simulation of various growth paths of the demand for energy. In the *net energy demand sub-model* a detailed technical treatment of ways of meeting a given net demand is undertaken by indicating the allocation pattern of energy resources to different sectors. For a given demand, alternative sources of supply that are economically efficient within the constraints of technology, environment, and reliability of supply are investigated in an *energy supply sub-model*. The supply model may generate heavy capital requirements for some solutions that may be inconsistent with the assumed or planned development of the economy. On the other hand, these solutions may have a positive effect on the economy through stimulating more efficient energy use. The MSG model thus serves to check the overall consistency of the assumptions of the various technical scenarios. Another important point that was raised at the Workshop was that the energy embodied in Danish imports and exports should be incorporated into this model if certain questions are to be answered.

(2) Bent Elbek, 'Energy Analysis of a National Economy'.

Closely related to the investigation reported above is Elbek's attempt to use energy analysis methodology to evaluate all primary processes in the Danish economy, process by process, in order to formulate a total picture of energy use in that country. Sectors investigated in varying degrees of detail are agriculture (farming, gardening, forestry and fishing), industry (iron, metals, paper, chemicals, stone and clay, food, transport equipment, machinery, textiles, and wood), transportation (foreign shipping, domestic transport, air transport, private transport, communications), services (*e.g.*, schools, hospitals, libraries, supermarkets), construction, and residential. Graphical displays of variations in energy, capital, and labor requirements of the aggregated sectors over the period 1950–1972 were also presented. This study is unfinished but impressive. In particular, it is one of the first to treat service and residential sectors in a direct manner rather than employing a 'money–energy' conversion.

(3) Ingemar Ståhl, 'An Input–Output Evaluation of the Energy Requirements for 1000 MW_e Forsmark I Light Water Reactor'.

This presentation outlined the merit-order structure of Swedish baseload electrical generation capacity, pointing out that closer matching of the price structure to marginal costs of generation could lead to a clearer perception of the cost of oil-fired generation facilities. A detailed (42-sector) input–output study of the energy requirements for constructing the Forsmark I reactor was described. This analysis, which utilizes 1971 prices, assesses reactor, construction, and turbine energy requirements, as well as estimating energy use at site. Fuel and electrical energy requirements are maintained as separate entries. The values reported in this careful evaluation were of reasonable magnitudes, but there was criticism of the exclusion of energy embodied in imported material components, which Ståhl estimates would add 40–60% to the total. No initial core fuel requirements were included. Also, it was noted that the use of conversions based on aggregate industrial sector energy/krona ratios can lead to substantial error because of the specialized nature of many of the reactor components.

In a separate session, there was a general discussion of the energy requirements for the construction of nuclear facilities and the generation of nuclear power. It appears that these requirements are significant, but the energy analyses that have been published to date exhibit substantial variations. The disagreements seem to stem from many of them being back-of-the-envelope exercises, and from the use of only money–energy conversions in several studies. Many participants felt that a careful disaggregated energy analysis based on physical inputs should be carried out before policy suggestions regarding nuclear facilities based on net-energy arguments are put forward.

(4) Lawrence Klein, 'A Summary of Methods of Introducing Variable Coefficients in Input–Output Models'.

A brief but detailed presentation of various methods of introducing dynamic and price-sensitive coefficients into input–output models to produce richer descriptive possibilities was given. The technical input–output module of an econometric system can be used flexibly. In particular, observed coefficients can be modified by engineering considerations of new processes. These can then be combined with statistical coefficients for the rest of the system. The three econometric approaches that were discussed, in order of increasing generality, involved the use of Cobb–Douglas, constant elasticity of substitution (CES) and translog functional descriptions. The CES function, employed in the Wharton model, was picked for closer examination, and the use of the model in assessing the effects of economic constraints or policy instruments was developed.

(5) J.M. Leathers, 'A Description of the Use of Energy Analysis in Dow Chemical USA'.

The history of the use of energy analysis as a materials management technique at Dow dates from the submission of a design for a balanced energy plant for expansion of chlorine production facilities in 1965. The design was accepted, and energy analysis has expanded so that the Dow Accounting Department now maintains financial and energy accounts side-by-side for all of its 600 productive units. This appears to be a particularly effective management method for a company whose inputs and outputs are energy-intensive, and its use at Dow has made an impression in the crucial area of profitability. The Dow system was described at length, including their treatment of wastes and by-products. Principal advantages appear to be the sensitivity of the energy requirements to modifications in the production system and the ease with which surveillance can be maintained on changes in these requirements. Also, in times of volatile price fluctuations companies may find it to their advantage to analyze their operations in physical rather than financial terms. A striking aspect of the Dow analytic system is that it is nearly identical to the one agreed upon in August, 1974 at the First IFIAS Workshop on Energy Analysis, although neither group was aware of the other's method until March, 1975.

(6) J.P. Charpentier, 'Distribution of Energy Consumption in the World (1971; 178 countries)'.

A plot of the distribution of the number of countries having a given energy use per capita as a function of that energy use is shown in fig. 7. A similar distribution is found for population, with the following differences: 72% instead of 75% (class III), 6% instead of 3% (class I), and 22% (identical) for class II. It again was pointed out that although this plot is interesting, it would be valuable to have an accompanying graph in which the energy embodied in imports and exports is included in the energy per capita figures. The reference used for the data in this plot is the *U.N. Statistics Handbook.*

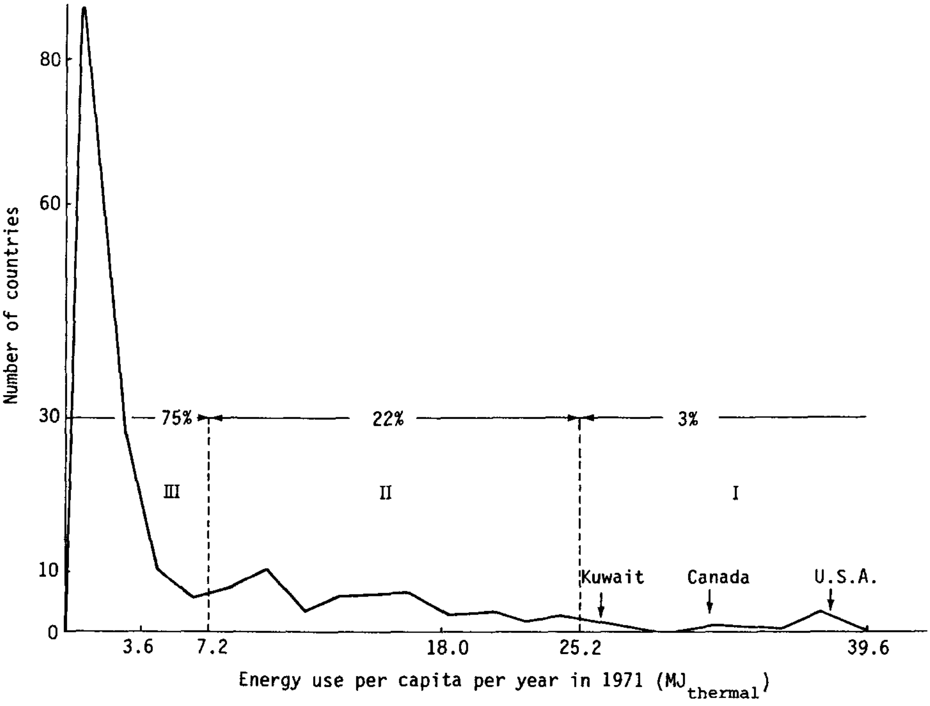

Fig. 7. Distribution of energy use per capita per year (1971) for 178 countries.

(7) Willem van Gool, 'An Informal Survey of the Use of Energy Analysis in The Netherlands'.

The energy supply situation in The Netherlands, and energy research and development activities were succinctly described. Two major problems must be faced: short-term dependency upon imported oil fo transport and the leveling off of indigenous natural gas production in *ca.* 1978. Investigations of future supply possibilities included thorough assessments of wind and solar energy. Energy analysis has been useful in examining the consumption options facing the Dutch over the long-term, and there are those who feel that it should be used in societal planning decisions in order to avoid severe constraints. For example, the energy supply options may be seen as facing the paired constraints.

Energy supply sources	Physical constraints
Fossil fuels, geothermal, nuclear energy	Thermal load
Fossil fuels	CO_2 load
Nuclear energy	Radioactive waste
Wind, solar, biochemical methods	Land and sea surface
Any source on a large scale	Local influence on climate

(8) Peter Roberts, 'Some Interesting Energy-Use Correlations for the British Economy'.

Several intriguing empirical results were presented, including a plot showing the energy-use-per-household as a function of income in the UK for the following sectors: services, fuel, transport, goods, food, housing, and alcohol and tobacco. A plot of the log of value-added-per-unit-mass produced versus the log of energy-requirements-per-unit mass over a wide spectrum of British industry was also introduced. This plot was linear and striking in its lack of scatter.

Appendix 1: Guidelines for energy analysis

This memorandum is a summary of recommendations adopted at the First Workshop on Energy Analysis held in Guldsmedshyttan, Sweden, 26–30 August, 1974, under the sponsorship of The International Federation of Institutes of Advanced Study (IFIAS). Twenty participants from ten countries took part; they were all engaged in studying some aspect of energy, and almost all have been active in analyzing how energy and related resources are used. A full report of the Workshop, with examples, has been published by IFIAS in Stockholm. The goal was the production of a set of definitions, conventions and standards to be recommended for general use by those working with the analysis of energy. The motivation was the need felt by the organizers and the participants to facilitate accurate communication in this fast-growing field, and to do this in a way that would make the information useful to people outside the subject.

The following summary presents only the final recommendations; the logic of choice and the considerations of alternatives are discussed in the full report.

Title and subject

The title 'Energy Analysis' is recommended for the endeavor consisting of the study of the energy, free energy, availability or any other thermodynamic quantity sequestered in the provision of goods or services. 'Sequestered' is employed in the sense of 'set apart', to indicate that energy may be tied up in the finished good or in the process materials, in addition to the energy used to do the work of the process. The title is intended to cover both the evaluation of energy and other thermodynamic quantities and the study of the implications of the results of the calculations.

Quantities and units

The quantities recommended for use in the presentation of energy analysis data are:

(a) the internal energy E;
(b) the enthalpy H, equal to E plus the product of pressure P and volume V;
(c) the Gibbs free energy G, which is defined as the enthalpy less the product of temperature T and entropy S, whenever it is feasible to evaluate this quantity.

Conventionally, the heating value that is recorded for a fuel is its heat of combustion at constant atmospheric pressure, which is an enthalpy, and energy analyses will customarily utilize enthalpies in process evaluations. Thus energy analyses will most often be enthalpy analyses. It is recommended that the *gross* heat of combustion be used; *i.e.*, the enthalpy of combustion of a fuel consisting of carbon, hydrogen and oxygen should be based on products that are gaseous CO_2 and *liquid* H_2O, with reactants and products at 273.15°K.

The Gibbs free energy change should be evaluated when feasible. There will doubtless be situations in which the availability A, which is equal to $H - (T_{\text{external}}S) + (P_{\text{external}}V)$, and the Helmholtz free energy F (equal to $E - TS$) will be useful also, but they are not recommended as normal forms of reporting data. Evaluations utilizing net heats of combustion, based on a combustion process that has as a final product water vapor (rather than liquid), may be valuable at times, but a statement explicitly noting their use rather than gross values must be included. It may be desirable to evaluate the internal energy E when it differs significantly from the enthalpy.

The unit of choice is the *joule* (J) and powers of ten thereof (megajoules, $MJ = 10^6$ J, for example) in accord with the Système International conventions. Metric units of mass are recommended. Thus, energy per weight of product is conveniently expressed in megajoules per kilogram (10^6 joules per 1000 grams), which is equivalent to gigajoules per metric tonne (10^9 joules per 10^6 grams).

Use of the following units is strongly discouraged by the Workshop in reporting energy analyses in technical media: British thermal units (Btu), kilowatt hours (kWh), and all units based on material consumption and therefore of variable value, such as metric tonnes of coal equivalent (tce), short tons of oil equivalent (toe) or barrels of oil (bbl). However their place in popularized presentations was recognized; it was strongly recommended that if these units are used, the data should also be given in standard metric units.

The system and levels of analysis

The system is that portion of the universe chosen for study. The system must be carefully defined by specifying its boundary. The system boundary separates those activities that are part of the question under analysis and are contained within the boundary from those activities that lie outside because they presumably have negligible impact on the question. Energy analysis often begins with a focus on a product and the process stage by which it is fabricated from material inputs. The system boundary can be defined so as to contain this process stage and none other, and energy analysis would then calculate how much energy is required to carry out this single step. But the system could be defined to account for the energy used to prepare material inputs that are themselves fabricated in prior stages. Another choice of system boundary would include the final process stage and the processes that generate the inputs to the final stage. A further regression would have the boundary enclose all of these activities plus those that produce the inputs to the stages that yield the fabricated first-stage inputs. This regression can be continued upstream until the system boundary encloses stages that employ only raw materials. Downstream, the boundary may or may not include discard or recycling.

Defining the system boundary also requires answering the question: what are the inputs to the system? Should one, for example, include the energy requirements for producing the capital equipment used in the stages of production? This problem and the Workshop's recommendations concerning systems and their boundaries can be described conveniently using fig. 8, which designates levels of regression. Level 1 is the level of direct energy input to the final process stage. An evaluation at this level would include fuels and electric energy supplied to the process but none of the energy requirements for prior steps, such as the generation of electricity. The consensus of the Workshop is that data from Level 1, which is sometimes useful for engineering purposes, is generally not sufficiently informative for decisions in those areas in which energy analysis is particularly cogent.

At Level 2, one includes the inputs to produce the materials used in the process and to provide the energy used at Level 1. For those inputs that are themselves manufactured commodities, there is a further regression. Level 3 takes into account the energy requirements of producing the capital equipment, as well as the first regression of the requirements for input materials considered at Level 2. Level 4 and higher levels continue the regression in the same way.

Where, in practical terms, does one stop in this regression? Sometimes one is only interested in a particular level, and sometimes it is impossible to carry the analysis beyond a partial evaluation at Level 2. The Workshop recommends that whenever possible, analyses be carried back to the level at which the contributions are comparable with the uncertainties in the contributions from preceding levels. Currently, this will often mean carrying out the evaluation through Level 2, or, sometimes, through Level 3. Frequently an analysis through Level 2 will include 90 to 95% of the energy requirements calculated through Level 4, so that analyses terminating at Level 2 will be useful representations.

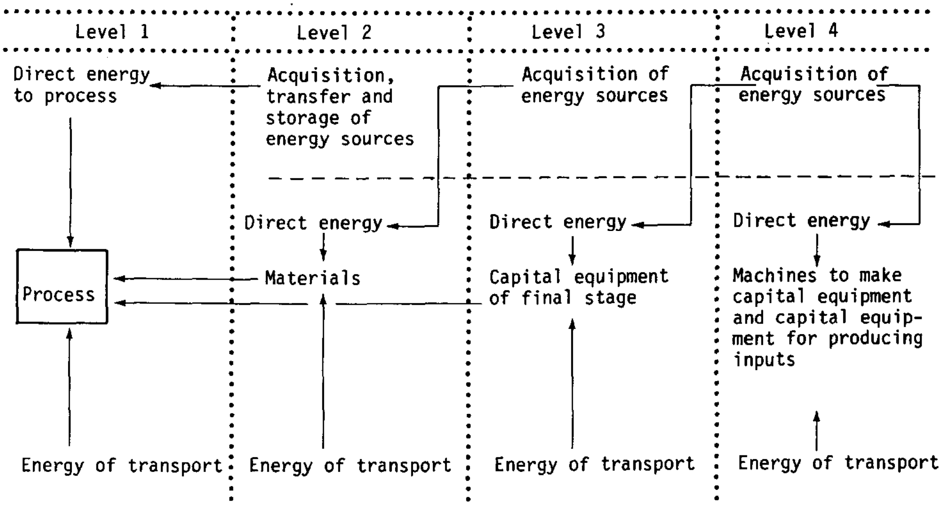

Fig. 8. Levels in the definition of the system boundary.

The increments to the total energy requirements, per unit of product, *tend* to decrease in magnitude as one goes to successively higher levels even though the number of inputs increases with level. Hence it is frequently appropriate to use more approximate and aggregated methods as one takes higher levels into account. (Note that the notion that the contributions diminish as the levels get higher is an approximation; there can easily be small contributions to the total coming from low levels, and occasionally, a large contribution from Level 3 or even Level 4.)

The Workshop emphasized the importance of specifying the level and the system boundary, in order to permit comparisons among different calculations.

Definitions of measures

The energy (or free energy, etc.) calculated for the system of interest is called the *energy requirement* (or free energy requirement, etc.). The term 'energy requirement' is recommended even when the figures refer to gross heats of combustion, *i.e.*, enthalpies, so long as the meaning is clear. Use of the term 'energy requirement' avoids the possible confusion that could arise from terms such as 'energy cost', which could be taken to mean the money costs of the fuels for the system.

Four measures of the energy (or free energy, enthalpy, etc.) requirements are defined:

(a) The conventional thermochemical changes in enthalpy (ΔH) and Gibbs free energy (ΔG) [also, but probably less frequently, the change in the internal energy, (ΔE) and Helmholtz free energy (ΔF)] of all the chemical and physical processes that occur within the system boundary. These are evaluated by enumerating all the chemical and physical transformations, such as combustion, chemical reduction of oxide ores, or evaporation, determining the enthalpy changes or Gibbs free energy change for each, and summing the contributions from each reaction and transformation.

(b) The Process Energy Requirement (*PER*) is the sum of the fuel energy supplied to drive all the process stages within the system boundary, which may include the production of inputs beyond Level 1.

(c) The Gross Energy Requirement (*GER*) is the Process Energy Requirement plus the gross heat of combustion of inputs that have alternative use as fuels. Whenever the *GER* includes any energy sources other than fossil fuels, care must be taken to specify how the energy embodied in fuelstocks is defined. For example, the variety of available technologies for energy production from fissionable materials allows a large range in the embodied energy one attributes to unit mass of material, so the value given to the *GER* will depend on the technology to which the definition refers.

(d) The Net Energy Requirement (*NER*) is the Gross Energy Requirement, less the gross heats of combustion of the products of the process.[8] This quantity reflects the net amount of energy required by a process if the products are finally used as fuel. If the *NER* is being evaluated for a real fuel, it is important to define whether the values taken into account are those of the actual heat derived from combustion (in which some energy may remain in uncombusted material), or the ideal (thermodynamic) heat of combustion.

The Workshop recommends that whenever depletion of resource bases is the concern, the *GER*'s, *NER*'s and *PER*'s be evaluated as free energies of combustion and so identified. To obtain free energy requirements, corresponding energy (enthalpy) requirements must be evaluated, and it would be helpful to have the energy (enthalpy) requirement figures available as well.

[8] Note that the heat of combustion of a hydrocarbon is defined to be a *positive* quantity.

An explicit statement should be made, as to whether the energy requirements are computed as free energies or enthalpies of combustion.

Evaluations using the conventional thermodynamic functions are well-approximated in many systems by the *PER*, which employs only heats of combustion. However, in a number of industrial processes, a significant fraction of the total energy employed is energy given off in chemical or physical transformations of the material being processed (exothermicity of reactions), which is not included in the *PER*. The *PER*, *GER* and *NER* are measures of how much we draw on our stocks of fuels, while the thermodynamic energy and free energy requirements encompass total changes in usable energy.

Comparisons between a hypothetical ideal process and a real process are made by defining the ideal process, and by evaluating absolute and relative measures of their differences:

(e) Free Energy Waste is the actual free energy requirement minus the ideal free energy requirement ($\Delta G_{\text{actual}} - \Delta G_{\text{ideal}}$). Similarly, the energy waste is the energy difference between real and ideal processes. These quantities can be calculated using total thermodynamic changes, *GER*'s, *NER*'s or *PER*'s, and must be so identified. The process upon which the ideal requirement is based must be defined explicitly.

(f) The *Waste Factor* (w) is the ratio of the Free Energy Waste to the actual free energy requirement ($w = (\Delta G_{\text{actual}} - \Delta G_{\text{ideal}})/\Delta G_{\text{actual}}$), and is thus a relative measure of the difference between real and ideal requirements. Again, the Waste Factor can be defined using energy changes or free energy changes, and may be calculated using total thermodynamic changes, *GER*'s, *NER*'s or *PER*'s. The efficiency measures defined in (e) and (f) are arbitrary, and other workers may wish to develop additional parameters that are particularly suited to their problem.

At least two meaningful per-unit-product values can be calculated for each of the energy requirement parameters of ΔG, ΔH, *GER*, *NER*, *PER*, Free Energy Waste (or Energy Waste) and Waste Factor. One is the *average* obtained by dividing total requirement by total output, and the other is the *marginal* value. The marginal value of an energy requirement is equal to the derivative of the energy requirement with respect to the amount of product, evaluated at the level of the last unit of product – the requirement for the last unit of output. Most data reported thus far have been averages, but both average and marginal requirements are being evaluated, and one must specify which values are presented.

Partitioning

If the product of interest is in joint production with others, there is an ambiguity as to how to allocate input requirements among the outputs. The allocation of energy inputs when a process generates more than one good or service is called partitioning. The Workshop recommends that, whenever possible, energy requirements be partitioned according to a physical parameter. With several fuel products, for example, it would be natural to partition energy inputs according to the energy embodied in the various outputs. It is also helpful to report the total unpartitioned requirements, so that people using the data can devise their own partitioning schemes. Obviously, for some policy applications, one might wish to partition according to product money values.

Further definitions

Direct energy is the gross enthalpy of combustion of fuels plus direct electrical energy used in a process or process stage, equivalent to the energy requirements of Level 1.

Delivered energy (or *Delivered free energy*) is the output of an energy-analysis system delivered to a consumer.

Energy intensity (or *Free energy intensity*) is the energy requirement per unit money value or product, such as megajoules per dollar value.

All these quantities can be calculated using ΔG's or ΔH's, *GER*'s, *NER*'s or *PER*'s.

Process analysis is analysis based on the vertical flow of materials to yield a specific product or small set of products, such as a house, an automobile or a bushel of wheat.

Input–output analysis, as in economics, treats the evaluation problem as one in which multiple inputs yield multiple outputs. The term has been applied largely to linear (matrix) relationships, but it need not be so restricted.

Graphic presentation of process analysis

Data from process analyses can be presented in flow charts. The Workshop recommends that this be done and a set of conventions for the chart representation was adopted. The symbols are shown in fig. 9, and should be displayed in the sequence indicated in the figure.

Rectangle: name of process stage;
Triangle with vertex in flow direction: the *PER* for the process stage just named;
Cart or bogey: the energy requirement for transport in this stage;
Diamond: the energy requirement for capital;
Oval: the name and amount of product from this stage.

These representations may be used for diagrams displaying ΔG's or ΔH's, *GER*'s, *NER*'s or *PER*'s. To include the energy embodied in input materials that could be used as fuels, in order to represent the *GER* in a flow chart, use an *upper-half semicircle* for the enthalpy of combustion of any input that has value as a combustible fuel. To represent the *NER*, by including the fuel energy embodied in the products, use a *lower-half semicircle* for the enthalpy of combustion of any output that has value as a combustible fuel.

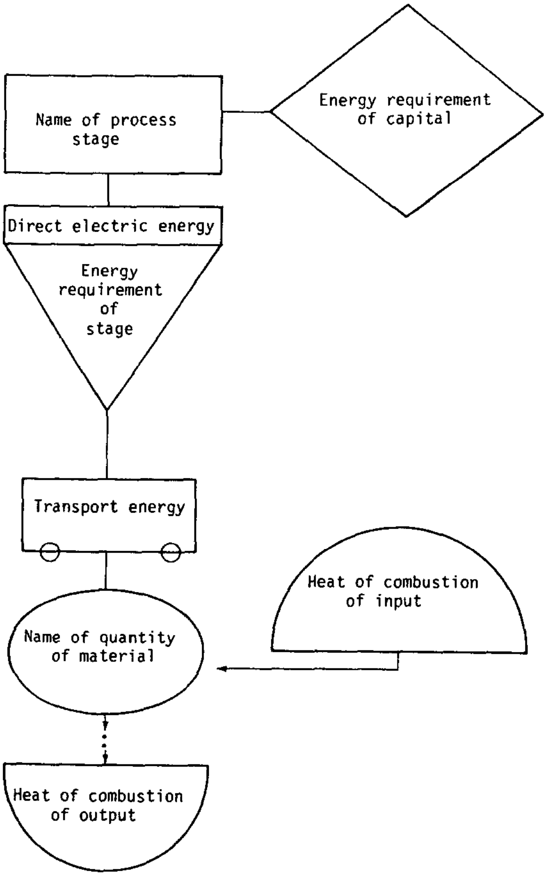

Fig. 9. Symbols for energy analysis flow diagrams.

If one wishes to show direct electric energy explicitly, the quantity of electric energy should be written into a box atop the triangle containing the total energy requirement for the process stage or for the particular input stream in the process stage.

To compile the data into a flow chart, one fixes the unit of *product* and works upstream, filling in the amounts of materials and energy that ultimately go into the supplying of that unit of product. Fig. 10 is an example of a flow chart for the production of one tonne of aluminum.

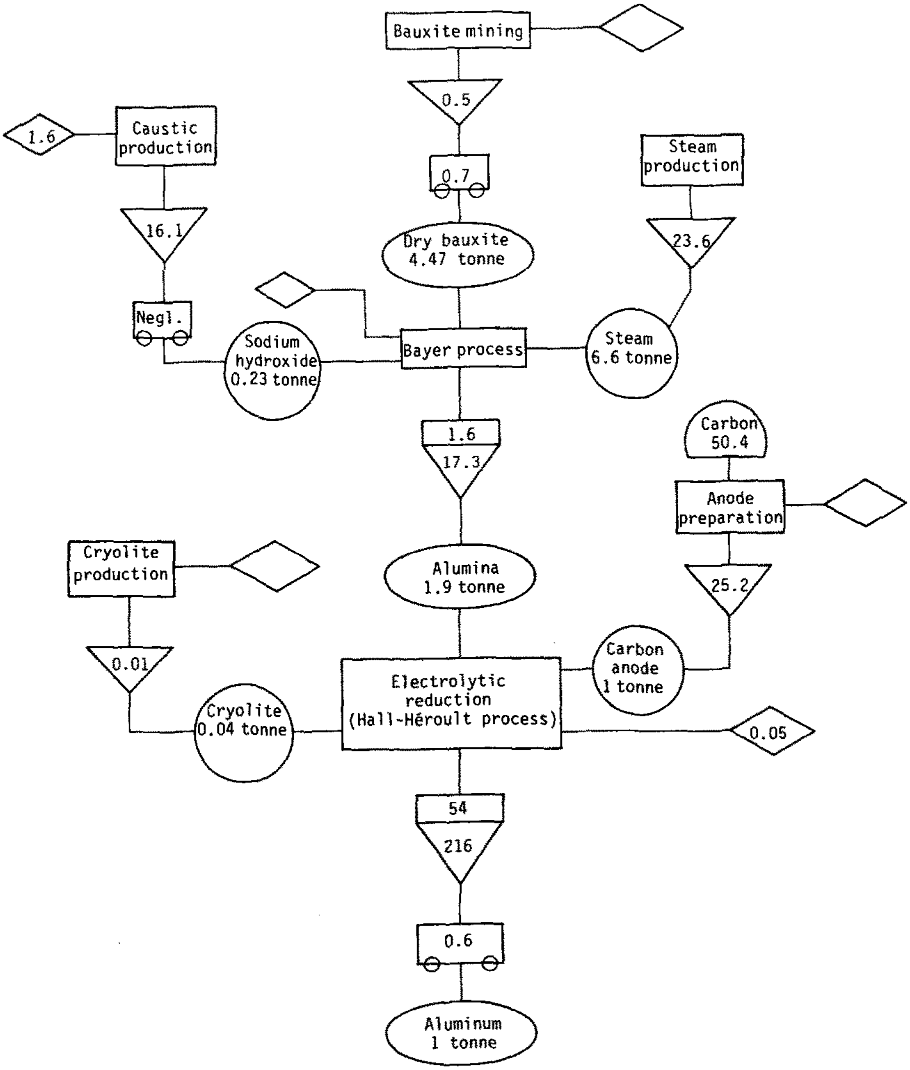

Fig. 10. Typical flow diagram for production of 1 metric tonne of aluminum. Energy units are gigajoules (10^9 joules). Empty diamonds signify capital requirements that have not been evaluated. Figures are based largely on British production. For this analysis, $PER = 1.6 + 16.1 + 0.5 + 0.7 + 23.6 + 17.3 + 25.2 + 0.01 + 0.05 + 216 + 0.6 = 302$ GJ/tonne; $GER = PER + 50.4 = 352$ GJ/tonne.

As with all the generalized energy requirements, the units that go into the triangles, diamonds and carts of the flow diagram may be energies, enthalpies or free energies.

Simple addition of all the numbers in the triangles, carts and diamonds (but *not* the amounts of electricity in boxes on the triangles) gives the *PER*. Adding the values in upper-half semicircles to the *PER* gives the *GER*, and finally, subtracting the lower-half semicircles to the *GER* gives the *NER*.

Appendix 2: A thermodynamics primer

Thermodynamics is that branch of physical sciences that describes *changes* involving the transformation of heat and work into each other. It focuses on the driving forces responsible for the changes and the circumstances under which change will be predicted to occur, *i.e.*, with disequilibrium.

The language used by the thermodynamicist is much the same as that adopted by the economist in analyzing a Lyapounov growth model. First, a physical *system* is defined by a precise description of its boundaries and by specifying the interaction of the system with the rest of the universe outside these boundaries. The macroscopic *state* of the system is determined by a set of measurable properties, the *state variables*. For a gas, these would be: pressure, P; temperature, T; volume, V; composition, C; and energy, E.

For a change in which the system passes from one state to another, the difference in any of these *properties* between the two states is independent of the transition path and determined solely by the states. An *equation of state* expresses the mathematical relationship between all independent state variables. For example, the equation of state for a dilute gas containing non-interacting particles is $PV = nkT$, in which n is the total number of particles and k is a proportionality constant.

A system is in an *equilibrium state* if it returns to this (original) state after a slight and temporary modification in external conditions. The properties of a system in an equilibrium state undergo no observable changes, even over an indefinite time, unless perturbed by external changes.

Let us examine one example of a system in an equilibrium state, that formed by the chemical reaction of methane and air that is utilized in ammonia production:

$$3CH_4 + 2N_2 + 3O_2 \leftrightarrow 4NH_3 + 3CO_2.$$

When separated from each other, methane (CH_4), nitrogen (N_2) and oxygen (O_2) are in equilibrium states. Upon bringing them together under the proper conditions, they will react to form ammonia (NH_3) and carbon dioxide (CO_2). The system's equilibrium state will contain a mixture of all five molecules.

The description of how a system gets from its initial state to the final state is called the *transition path.* A particular transition mode that is very important in thermodynamic conceptualizations is the *reversible path,* which is a succession of equilibrium states. Anything that can happen in accordance with physical law does, and this path defines the most efficient mode of transition available to the system. Because the change is thought of as occurring over a succession of infinitesimal states, the duration of the changes is infinite – the transition is carried out infinitely slowly. By reversible is meant that the path can be followed exactly backward to the initial state without any change of state variables.

Thermodynamics relates the set of state variables to the process variables heat, Q, and work, W, which are the instruments of change, through two basic laws. Both of these laws are empirical, but they are well-tested.

First law of thermodynamics

The change in the energy of the system is equal to the heat absorbed by the system less the work done by the system, $\Delta E = Q - W$. The change in energy ΔE is a perfect differential and is independent of path, depending only on the initial and final states of the system. This law is an equivalent formulation of the law of conservation of energy.

Second law of thermodynamics

This law grapples with the idea of non-conservation of some physical variable. In order to formulate it, we define a new state variable, the entropy, S, by specifying the change in the entropy of the system between initial and final states

$$\Delta S \equiv \int \frac{\mathrm{d}Q_{\text{reversible}}}{T},$$

in which $\mathrm{d}Q_{\text{reversible}}$ is the differential of heat along a reversible path for the system, and the absolute temperature, T, can be physically interpreted to be the average energy per particle. The second law states that the magnitude of the right-hand side integral is always greater than or equal to its magnitude over the actual path followed by the system:

$$\Delta S \equiv \int \frac{\mathrm{d}Q_{\text{reversible}}}{T} \geqq \int \frac{\mathrm{d}Q_{\text{actual}}}{T}.$$

An alternate statement of the second law is that for any change in a system, there must be an increase in the entropy of the universe, which is the sum of the changes in the entropies of the system and everything outside it, its surroundings.

Beginning with these simple definitions, we can investigate what it means to set thermodynamic limits. For example, let us consider a change occurring in a closed system, with no heat exchanged with the surroundings so that $dQ = 0$ everywhere along the path. Then $\Delta S > 0$ for any real change or $\Delta S = 0$ for a change conceived of as being carried out along a reversible path. In examining thermodynamic limits, it is convenient to define a few additional quantities that are also state properties because they are composed of state variables. The enthalpy of the system, H, is defined to be equal to the energy of the system plus the system pressure times its volume, $H = E + P_{\text{system}} V$, and the enthalpy change is $\Delta H = \Delta E + \Delta(P_{sy} V)$. This is a useful quantity for processes occurring at constant pressure, say open to the atmosphere, so that $\Delta H = \Delta E + P_{sy} \Delta V$. Now, let us ask what is the enthalpy of a process carried out reversibly at constant pressure:

$$\begin{tikzcd}
\Delta H = \Delta E + P_{sy}\Delta V \arrow[d] \\
\Delta H = Q - W + P_{sy}\Delta V \arrow[d] \\
\Delta H = Q - P_{su}\Delta V + P_{sy}\Delta V \arrow[d] \\
\Delta H = Q_{\text{reversible}}
\end{tikzcd}$$

First law.

W = work done by system against surrounding
= (force) (distance)
= ($P_{\text{surroundings}}$)(volume).
Only pressure–volume work is considered.

$P_{su} = P_{sy}$ for a change carried out reversibly (and only for a reversible change).

Thus, for a system in which constant pressure is maintained, the enthalpy change is equal to the heat absorbed by the system when the process is carried out in the thermodynamic limit of complete reversibility. The enthalpy change for combustion under a constant pressure of one atmosphere is the value utilized by the energy analyst in assigning an energy content to fuels.

There are three other thermodynamic state quantities that arise in setting thermodynamic limits, each defined for a frequently observed situation: the Helmholtz free energy, $F \equiv E - TS$; the Gibbs free energy, $G \equiv E + PV - TS$; and availability, $A \equiv E + P_{\text{surroundings}} V_{\text{system}} - T_{\text{surroundings}} S_{\text{system}}$. Under conditions of constant temperature and process reversibility, the change in the Helmholtz free energy is equal to the negative of the maximum amount of work that the system does on the surroundings, $\Delta F = -W_{\text{maximum}}$:

$$\begin{array}{ll}
\Delta F = \Delta E - \Delta(TS) & \\
\downarrow & T = \text{constant.} \\
\Delta F = \Delta E - T\Delta S & \\
\downarrow & \text{First law.} \\
\Delta F = Q - W - T\Delta S & \\
\downarrow & \text{Definition of } S. \\
\Delta F = Q - W - T \displaystyle\int \frac{\mathrm{d}Q_{\text{reversible}}}{T} & \\
\downarrow & \text{Reversibility.} \\
\Delta F = -W_{\max} &
\end{array}$$

The work term, $W_{\max}$, may be composed of electrical and chemical as well as mechanical work. If one examines a reversible process under conditions of constant temperature and pressure, the PV term in the Gibbs free energy cancels the mechanical work component of $W_{\max}$ to yield

$$\Delta G = W_{\text{maximum, non-mechanical}}.$$

For a spontaneous transition of a system that begins and ends in equilibrium with its (constant temperature and pressure) environment, the free energy change for the *system* must be negative, $\Delta G_{\text{system}} < 0$. Availability is an especially interesting measure in that it allows one to assess the maximum heat and work that can be exchanged between a system and the surroundings as a result of the previously constrained system's returning to equilibrium with its environment.

Building on this foundation, it is possible to define several efficiency criteria. 'First-law efficiency' is defined to be the net work done by a working cycle of a system, divided by the heat absorbed from a high temperature source by a low temperature sink. By mental construction of an ideal heat engine, one can show that the ideal first law efficiency is equal to $(T_{\text{high}} - T_{\text{low}})/T_{\text{high}}$, an identification that actually requires use of the second law. 'Second law efficiency' is defined to be equal to the utilized amount of availability, divided by the availability in the state of highest availability. For the purpose of energy analysis, it has been suggested that a related parameter, the 'energy waste factor' be defined, $w = (\Delta G_{\text{actual}} - \Delta G_{\text{reversible}})/\Delta G_{\text{actual}}$. For most real processes, this factor will be significantly greater than zero because energy is expended in effecting the change at a finite rate.

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